MINIMUM WEIGHT DESIGN OF CARBON/EPOXY LAMINATED COMPOSITES FOR MAXIMUM BUCKLING LOAD USING SIMULATED ANNEALING ALGORITHM

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ABSTRACT

MINIMUM WEIGHT DESIGN OF CARBON/EPOXY LAMINATED COMPOSITES FOR MAXIMUM BUCKLING LOAD USING SIMULATED ANNEALING ALGORITHM

Composite materials have been mostly used in engineering applications such as aerospace, automotive, sports equipment, marine because of their high specific strength-to-weight and stiffness-to-weight ratios. Weight reduction and buckling load capacity are critical issue for the engineering application. Accordingly, in this thesis, identification of optimum fiber orientations and laminate thicknesses of the composite plates resisting to buckling under given loading conditions and aspect ratios are investigated. Furthermore, a comparison study on continuous and conventional designs is performed to determine the effect of stacking sequence on weight. Symmetric and balanced *N*-layered carbon/epoxy composite plates are considered for optimization process. Critical buckling load factor is taken as objective function and fiber orientations which are considered continuous are taken as design variables. Simulated Annealing (SA) algorithm is specialized by using *fmincon* as hybrid function and this optimization method is used to obtain the optimum designs. Maximum critical buckling load factor and minimum thickness and hence minimum weight are achieved and shown in tables. As a result, it is observed that loading conditions and plate dimensions play an important role on stacking sequence optimization of lightweight composite laminates for maximum buckling load capacity.

ÖZET

BENZETİMLİ TAVLAMA ALGORİTMASI KULLANILARAK MAKSİMUM BURKULMA YÜKLEMESİ İÇİN TABAKALI KARBON/EPOKSİ KOMPOZİTLERİN MİNİMUM AĞIRLIK TASARIMI

Kompozit malzemeler yüksek dayanım ve sertlik özelliği sergilemeleri bakımından havacılık, otomotiv, spor ekipmanları ve denizcilik gibi mühendislik uygulamalarında sıklıkla kullanılmaktadır. Ağırlık azaltımı ve burkulma yükü kapasitesi mühendislik uygulamaları için önemli bir durum teşkil etmektedir. Buna bağlı olarak, bu tez çalışmasında, minimum kalınlıkta ve burkulmaya dayanıklı çok katmanlı kompozit malzemelerinin optimum tabaka dizilimi tasarımları değişken yükleme ve en boy oranlarına göre incelenmiştir. Buna ek olarak yaygın açı tasarımlarıyla sürekli açı tasarımları karşılaştırılıp, ağırlık üzerindeki etkisi belirlenmiştir. Optimizasyon sürecinde değişken sayıda karbon/epoksi plakalardan oluşmuş simetrik ve balans çok katmanlı kompozit yapılar değerlendirilmiştir. Kritik burkulma yükü faktörü amaç fonksiyonu olarak, sürekli fiber açı oryanatasyonu da tasarım değişkeni olarak alınmıştır. Hibrit fonksiyon olarak *fmincon* kullanılıp benzetimli tavlama algoritması özelleştirilmiştir ve bu optimizasyon yöntemi optimum tasarımların elde edilmesinde kullanılmıştır. Burkulma yükü faktörü, minimum kalınlık ve buna bağlı olarak minimum ağırlık tablolarda gösterilmektedir. Sonuç olarak yükleme koşulları ve plaka en-boy oranlarının çok katmanlı hafif kompozit malzemelerin tabaka dizilimi optimizasyonunda büyük rol oynadığı gözlemlenmiştir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1. Literature Survey

Composite materials have been increasingly used in a wide range of applications, particularly for lightweight structures that have strong stiffness and strength requirements such as aerospace, automotive and other engineering applications. However, because of their high cost, the composite structures should be optimized to obtain the best structure. Although the use of composite material instead of metallic material is attractive for many structural applications, the analysis and design of composite materials is more complex than those of metallic structures. The best composite structural design that meets all requirements of a specific application can be achieved by tailoring configuration of a laminate, i.e. fiber orientation, ply thickness, stacking sequence, reinforcement geometry ,volume fraction of reinforcement. For this purpose, many researchers conducted studies to get the best design by using optimization technique. One of the optimization techniques is analytical which utilize gradient information of the objective function and/or constraints to state the decreasing direction of the objective function. In this type of optimization problems could have many locally optimum configurations due to the design variables. Therefore, most of the analytical optimization methods are inefficient and may stick to one of the local minima. On the other hand, stochastic optimization methods are more suitable for such cases and can converge to the global optimum point regardless of the initial design point. They reside in the balance between intensification (exploitation of past search experience) and diversification (exploration of the entire solution space), and this feature gives the ability of the search process to escape from local optima and efficiently sample the most promising regions of the search space. Genetic algorithm (GA), simulated annealing algorithm (SA), tabu search (TS), pattern search and ant colony optimization (ACO) are the stochastic optimization methods which are used the most commonly in composite optimization.

Weight or thickness minimization of the laminated composite plates is so important to obtain light-weight due to high specific strength and stiffness. Various investigations have been conducted to find the best stacking sequence configuration of composite laminates under different loading and constraint conditions in order to get minimum weight. Akbulut and Sonmez (2008) have studied minimum thickness (or weight) optimization of laminated composite plates which are subjected to in-plane loading. In their study, fiber orientations angles and layer thickness are taken as design variables and the optimization problem is solved using direct simulated annealing which is a reliable global search algorithm. They also have extended their previous study and they take both in-plane and out-of-plane loading into account. A new variant of the simulated annealing algorithm is used to minimize the thickness (or weight) of laminated composite plates. As design variables, fiber orientation and number of plies; as critical failure mode, Tsai-Wu and maximum stress are taken (Akbulut and Sonmez 2011). A previously developed genetic algorithm for laminate design is thoroughly revised and improved by Le Riche and Haftka (1995) to determine the minimum thickness of composite laminated plates. Constraints were generated into objective function as a penalty function. The purpose of the optimization is to find the best design that will not fail because of buckling or excessive strains.

The minimum weight and the minimum material cost of laminated plates subjected to in-plane loads have been studied using genetic algorithm. Three different failure criteria -maximum stress, Tsai-Wu and the Puck failure criterion- are described as constraints and tested independently. Ply orientations, the number of layers and the layer material are taken as design variables (Lopez, et al. 2009). Narayana Naik et al. (2008) investigated the minimum weight design of composite laminates using the failure mechanism based, maximum stress and Tsai-Wu failure criteria. A genetic algorithm is utilized for the optimization study. They represented the effectiveness of the new failure mechanism based failure criterion. It includes fiber breaks, matrix cracks, fiber compressive failure and matrix crushing.

The buckling load capacity is very critical issue for thin and large composite plates. For this reason, buckling load maximization drew attention of researchers. Many researchers have been studied buckling load maximization using different optimization methods. Buckling load maximization for two-dimensional composite structures subject to given in-plane static loads has been examined to find globally optimum design by Erdal and Sonmez (2005). An improved version of simulated annealing algorithm is used to solve this problem. They make an improvement on a computer code and the results are calculated for different load cases. Kim and Lee (2005) analyzed the optimal stacking sequence of laminated composite plates for the maximum buckling load under several different loadings, such as uniaxial compression, shear, biaxial compression, and the combination of shear and biaxial loadings. Fiber orientations are taken design variables and critical buckling load is taken as an objective function and optimization problem is solved using a genetic algorithm. Similarly, Maximum buckling load capacity of laminated composite plate is presented using genetic algorithm by Soykasap and Karakaya (2007). Plate is considered simply supported and subject to in-plane compressive static loads. The critical buckling loads are obtained for several cases and different plate aspect ratios. In the latter case, the researchers have carried out the comparison of genetic algorithm and generalized pattern search algorithm for optimal stacking sequence of a composite to maximize the critical buckling loads in order to obtain performance of both algorithms (Karakaya and Soykasap 2009). In addition to these, genetic algorithm and simulated annealing have been utilized to maximize natural frequency and buckling loads of simply supported hybrid composite plates by Karakaya and Soykasap (2011). These techniques have been compared depending on number of function evaluations and the capability of finding best configurations. Another optimization method, ant colony optimization (ACO) which is a metaheuristic search technique has been used to find the lay-up design of laminated panels for maximization of buckling load with strength constraints by Aymerich and Serra (2008). A specific problem is selected as a test-case to compare the developed ACO algorithm with genetic algorithm (GA) and tabu search (TS) to state the computational efficiency and the quality of results. In another study, optimization of laminated composites that are subjected to uncertain buckling loads is carried out and the buckling load is maximized under worst case in-plane loading. The design variables are taken as ply angles and the optimization problems are solved both continuous and discrete using nested solution method (Adali, et al. 2003). Tabu search which is a heuristic search technique is used to find optimum stacking sequence of a laminate for buckling response, matrix cracking, and strength requirements. The specific studies previously investigated using genetic algorithm are utilized to compare relative performance of tabu search (Pai, et al. 2003).

Additionally, various optimization objectives have been solved using simulated annealing algorithm in the literature. For instance, The optimal design of laminated composite plates with integrated piezoelectric actuators are investigated using refined finite element models based on equivalent single layer high-order shear deformation theories. These models are generated with simulated annealing, a stochastic global optimization technique, in order to find the optimal location of piezoelectric actuators and also to find the optimal fiber reinforcement angles. Main objective of this study is maximizing the buckling load of the composite adaptive plate structure in both cases (Correia, et al. 2003). Deng et al. (2005) have studied a constant thickness optimization of laminated composite using simulated annealing (SA). In this paper, the edging stress of a composite plate with constant thickness is taken as an objective function and the optimum stacking sequence has been investigated. The design of laminates with required stiffness properties have been studied by Javidrad and Nouri (2011). Optimization problem which is finding the optimum lamination stacking sequence is calculated by minimizing a cost function composed of the relative difference between the calculated effective stiffness properties and weight of trial laminate and the desired properties. Number of layers and orientation of fibers in each layer group are taken as design variables. A modified simulated annealing method is used to solve the problem and the main features of this algorithm and results are represented.

In this thesis, optimal stacking sequence designs of laminated composite plates for maximum buckling load capacity and minimum weight are studied using simulated annealing algorithm (SA). Composite plates are assumed symmetric and balanced and simply supported on four edges. They are investigated for under various load conditions and aspect ratios (length to width). Design variables are fiber orientation angle in each layer and they are taken as a continuous variable. The optimum designs are calculated considering the critical buckling load factor.

CHAPTER 2

COMPOSITE MATERIALS

2.1. Introduction

Composite material is a structural material that is composed of two or more constituents which are combined at a macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the other one is the matrix. The material which is the reinforcement might be in the form of fibers, particles, or flakes. In general matrix phase materials are continuous (Kaw 2006). The above explanation which is more general and can include metals alloys, plastic co-polymers, minerals, and wood. Fiber-reinforced composite materials differ from the materials given above in that the components are different at the molecular level and are mechanically separable. Fiber-reinforced composite materials consist of fibers and matrix. Both fiber and matrix maintain their physical and chemical identities in this form, yet they generate a new material which has a combination of properties which cannot be achieved with either of the constituents acting alone (Mazumdar 2002).

Historically, the use of fiber-reinforcement is quite old. For instance; in ancient Egypt, bricks were made with chopped straw to be stronger. Moreover; mud huts were reinforced with grasses and thin sticks by Africans. Studies show that bonded with a mixture of cow dung and mud, woven sticks were used to build house walls in England 1500 B.C. (Armstrong 2005). Composites were generated to optimise material properties, mechanical (mainly strength), and chemical and/or physical properties. Afterwards, the importance of thermal and electrical optimization as well as optical and acoustical properties can be realized. Since the early 1960s demands for materials that are stiffer and stronger yet lighter in aeronautic, energy, civil engineering and in various structural applications have increased. Unfortunately, monolithic engineering material could not afford the expectations. This need and demand certainly has stimulated the concept of combining different materials in an integral composite structure (Akovali 2001).

Figure 2.1. shows the comparison of composites and fibers to the other traditional materials in terms of specific strength on yearly basis.

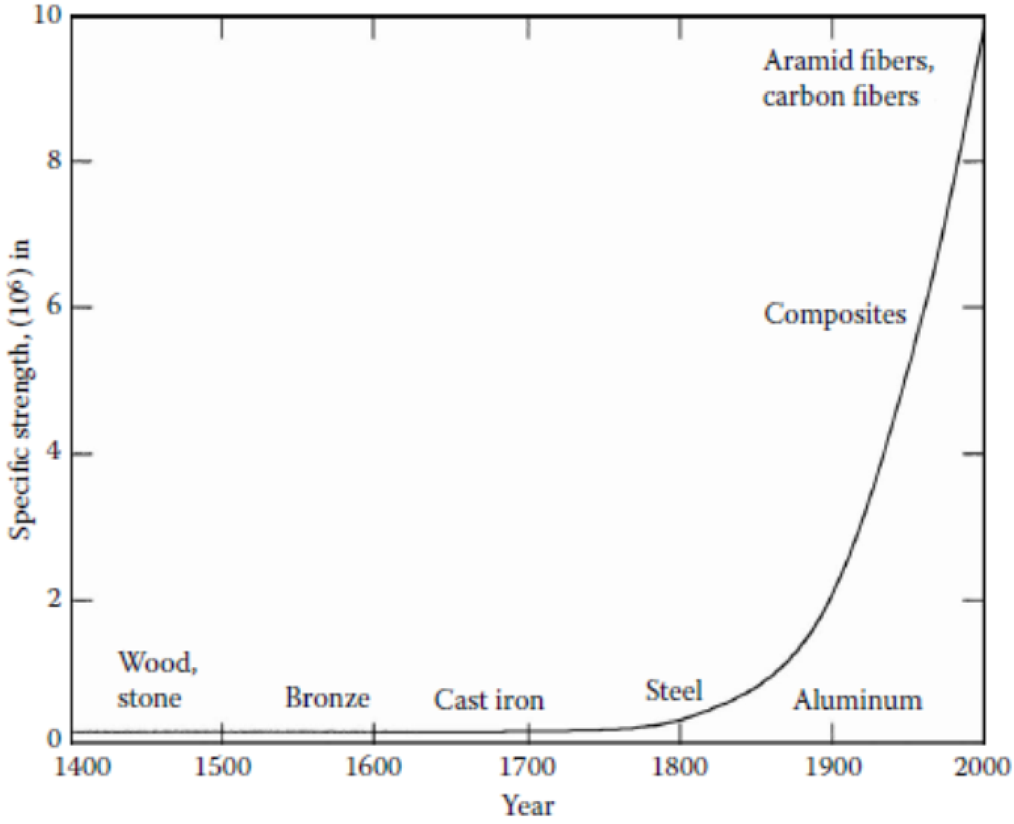

Figure 2.1. Specific strength as a function of time of use of materials (Source: Kaw 2006)

High tensile and high modulus fibres (such as carbon, silicon carbide and alumina) generated in the 1970s, and were used to reinforce high-performance polymer, metal and ceramic matrices. A new group of advanced composite material (ACM) were developed to produce extremely strong and stiff composite material. Matrix is one of points to reach to proper ACM structures. The main object for the ACM is to make densities of matrices as small as possible with the highest temperatures. A relationship between density and service temperature for different materials are presented in Figure 2.2. The arrow in the figure shows the trend for advanced materials which has its peak at high application temperatures and low densities (two important criteria for the next generation spacecraft's being lighter and faster) (Akovali 2001).

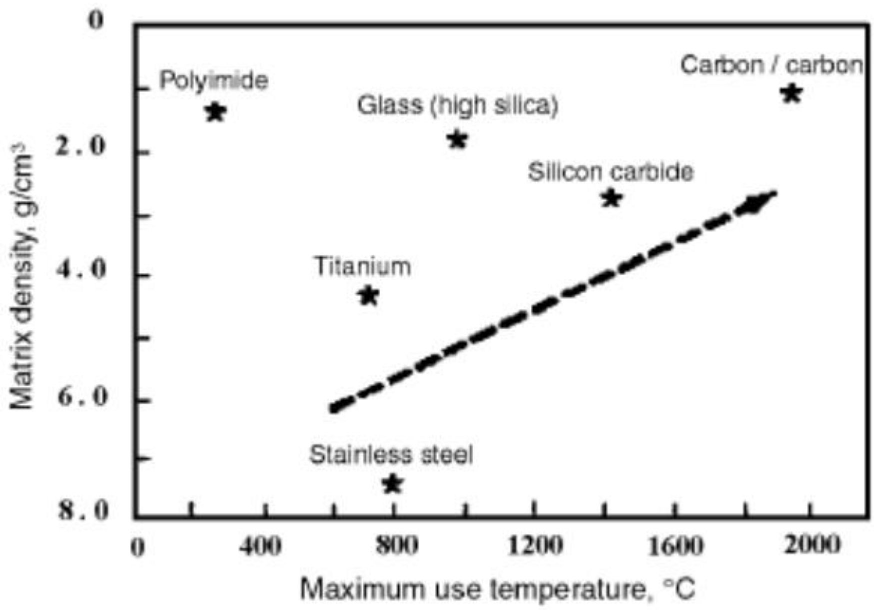

Figure 2.2. Density versus maximum use temperature for some materials (Source: Akovali 2001)

Composite materials have an important role in modern industry since they are capable of providing higher stiffness/density and strength/density ratio in the design and manufacture of advanced materials. Due to these ratios, composite materials can be used in various applications where the weight and strength of the structure are highly significant design parameters. Composite materials not only improve the performance but also increase the efficiency of such structures. Therefore; understanding of the behaviour of these materials under arising loads must be established to ensure structural and safe performance (Taqieddn 2005).

Unidirectional composites, cross-ply and quasi-isotropic laminated composites and monolithic metals specific modulus and specific strength properties of the typical composite fibers are given in Table 2.1.

Table 2.1. Specific Modulus and Specific Strength of Typical Fibers, Composites and Bulk Metals (Source: Kaw 2006)

Material Units	Specific gravity	Young's modulus (GPa)	Ultimate strength (MPa)	Specific modulus (GPa-m^3/kg)	Specific strength (MPa-m^3/kg)
System of Units: SI					
Graphite fiber	1.8	230.00	2067	0.1278	1.148
Aramid fiber	1.4	124.00	1379	0.08857	0.9850
Glass fiber	2.5	85.00	1550	0.0340	0.6200
Unidirectional graphite/epoxy	1.6	181.00	1500	0.1131	0.9377
Unidirectional glass/epoxy	1.8	38.60	1062	0.02144	0.5900
Cross-ply graphite/epoxy	1.6	95.98	373.0	0.06000	0.2331
Cross-ply glass/epoxy	1.8	23.58	88.25	0.01310	0.0490
Quasi-isotropic graphite/epoxy	1.6	69.64	276.48	0.04353	0.1728
Quasi-isotropic glass/epoxy	1.8	18.96	73.08	0.01053	0.0406
Steel	7.8	206.84	648.1	0.02652	0.08309
Aluminum	2.6	68.95	275.8	0.02652	0.1061

The exclusive properties make ACM convenient for different applications. For example, advanced polymers composites are not only lightweight but they also offer excellent strength, stiffness and design versatility. Furthermore some polymers have good chemical resistance and dielectric strength. There are three main factors that may affect the competition between composites and traditional engineering materials which are the cost, reliability and the degree of complexities involved (Akovali 2001).

The most common form of fiber-reinforced composites in structural application is a laminate, which is made by stacking a number of thin layers of fibers and matrix and combining them into the desired thickness. Fiber orientation in each layer as well as the stacking sequence of various layers in a composite laminate can be controlled to create a wide range of physical and mechanical properties for the composite laminate (Mallick 2007). If the plies are stacked at various angles, the lay-up is called a laminate. When there is a single ply or a lay-up in which all of the layers or plies are stacked in the same orientation, it is called a lamina and they are shown in Figure 2.3. (Campbell 2010).

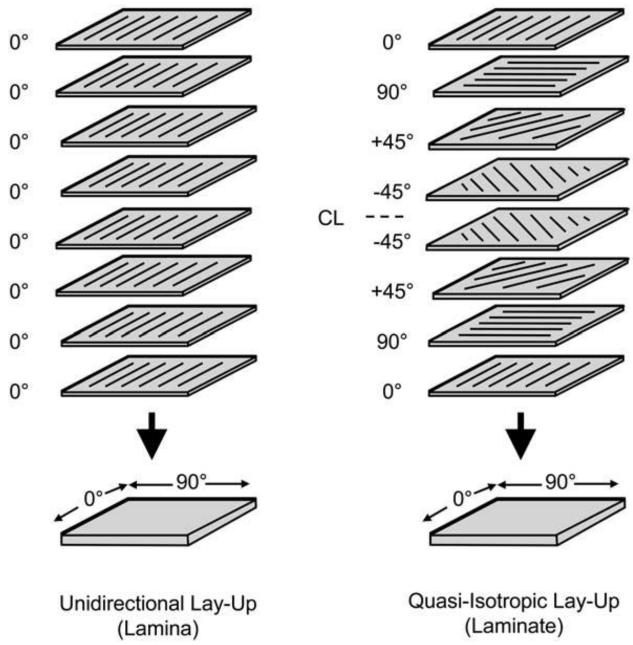

Figure 2.3. Lamina and laminate lay-ups (Source: Campbell 2010).

2.2. Classification of Composite Materials

Composites usually comprise of a reinforcing material embedded in a matrix. The effective method to improve overall properties is to combine dispersed phases in to the matrix, which can be an engineering material such as ceramic, metal or polymer. Hence, ceramic matrix composites, metal matrix composites (MMC) or polymer matrix composites (PMC) –or ceramic/metal/polymer/ composites-, carbon matrix composites (CMC) or even hybrid composites can be obtained. While composite matrices have low modulus, reinforcing elements are typically 50 times stronger and 20-150 times stiffer. MMC and CMC structures are enhanced to provide higher temperature applications (>316 ^{o}C), where PMC are usually inadequate.

Composites are usually preferred for their structural properties where the most commonly using reinforcement component is in particulate or fibrous form and that is why it’s no longer above given before can be restricted to such systems that have a continuous/discontinuous fibre or particle reinforcement, all in a continuous supporting phase, the matrix. Volume fraction of reinforcement phase is usually %10 or more. Thus; three common types of composites can be described as: particle-strengthened, discontinuous fibre reinforced and/or continuous fibre reinforced composites depending on the size and/or aspect ratio and volume fraction of reinforcing phase. In addition to these types of composites, another group of composite system is laminar composites (or simply laminates) where reinforcing agents are in the form of sheets stacked together and are often impregnated with more than one continuous phase in the system (Akovali 2001).

Some of the fiber-matrix combinations of composite materials are shown in Table 2.2.

Table 2.2. Types of Composite Materials (Source: Daniel and Ishai 1994)

Matrix type	**Fiber**	**Matrix**
Polymer	E-glass	Epoxy
	S-glass	Polyimide
	Carbon (graphite)	Polyester
	Aramid (Kevlar)	Thermoplastics
	Boron	(PEEK, polysulfone, etc.)
Metal	Boron	Aluminium
	Borsic	Magnesium
	Carbon (graphite)	Titanium
	Silicon carbide	Copper
	Alumina	
Ceramic	Silicon carbide	Silicon carbide
	Alumina	Alumina
	Silicon nitride	Glass-ceramic
		Silicon nitride
Carbon	Carbon	Carbon

2.3. Types of Matrix

%30-%40 of composite structure is usually consisted of the matrix. It has a number of functions (Akovali 2001).

- It binds the constituents together and specifies the thermo-mechanical stability of the composite
- It preserves the reinforcements from abrasion and environment,
- It helps to scatter the applied load by acting as a stress-transfer medium,
- It provides durability, interlaminar toughness and shear/ compressive/ transverse strengths to the system in general , and ,
- It keeps the desired fibre orientations and spacing in specific structures

2.3.1. Metal Matrix Composites

Below given the important characteristics of metals which make them preferable matrix materials in MMC structures (Akovali 2001).

- Higher application temperature ranges,
- Higher transverse stiffness and strengths,
- High toughness values,
- Exemption from the moisture effects and the danger of flammability and have high radiation resistances,
- High electric and thermal conductivities,
- Fabrication via the use of conventional metal working equipment. However, MMC have also some disadvantages.
- Most metals are heavy
- Metal are sensitive to interfacial degradation at the reinforcement and matrix interface and are sensitive to corrosion
- Producing the MMC has usually high material and fabrication costs and related composite technology is not well matured yet.

Aluminium, copper and magnesium are the most common metals which employed in MMC. Within these, an application temperature of aluminium is at and above 300^{o}C and its alloys are normally used with boron or borsic filaments. 6061 aluminium is used more often than either 2024 or 1100. 6061 aluminium has a good combination of strength, toughness and corrosion resistance on the other hand 2024 provides the highest strength. Titanium can be used at 800^{o}C and generally borsic fibers are used as a matrix. Magnesium alloys are usually used with graphite reinforcement and also magnesium matrices with boron fibres present excellent interfacial bond and outstanding load redistribution characteristics.

Where the materials with high conductivities are needed, usually copper MMC are used. The high density and limited upper-application-temperature values of monolithic copper can be compensated by adding of graphite fibres, which generate MMC with reduced density, increased stiffness, increased application temperatures and improved thermal conductivities.

Aluminide matrices are usually used for advanced gas turbine engine component and are also called intermetallic. In general, they have excellent oxidation resistance, low density and high melting temperature. To produce excellent intermetallics, the reinforcing fibres are expected to provide both toughening and strengthening to the system and for this, they must be chemically compatible with the matrix and should have similar cte with the matrix.

In principle, all metals present degradation of properties at very high temperatures, hence there is a thermal limitation in use even for the MMC as well (Akovali 2001).

2.3.2. Ceramic Matrix Composites

Ceramics are consisting of metallic and non-metallic elements. These are the important characteristics of ceramic matrix:

- Having a very high application temperature range (>2000^{o}C), hence they provide advanced heat engine application,
- Having low densities,
- Usually having very high elastic modulus values.

Brittleness is the major disadvantage of the ceramic matrix materials, because it makes them easily susceptible to flaws. On the other side of being brittle, they usually lack of uniformity in properties and have low thermal and mechanical shock resistances, as well as low tensile strengths. Due to the brittleness, existence of even minor surface flaws, scratches or internal defects can cause a disaster. In fact, the main target is the production of tough ceramic materials (Akovali 2001).

Common ceramic matrix materials can be categorized in the four main groups:

- glass ceramics (such as lithium aluminosilicate),
- oxides (such as alumina),
- nitrides (such as silicone nitride), and
- carbides (such as silicone carbide).

Silicone nitride matrices are specially employed for the production of ceramic matrix composite systems where strong, tough, oxidation resistant and very high temperature/ high heat flux resistant materials are needed. In addition to this, silicone carbide provides high strength, high toughness, and high oxidation resistance and high thermal conductivities for structural applications at temperatures above 1400^{o}C. Also, the use of reaction-formed silicone carbide reinforcement is asserted to have certain advantages. Properties of some ceramics are given in Table 2.3.

Table 2.3. Typical examples of some ceramic matrices (Source: Akovali 2001)

	Density ρ (g cm^{-3})	Young's Modulus (GPa)	Yield Strength (MPa)	CTE (10^{-6} K^{-1})
Borosilicate glass	2.2	60	100	3.5
Soda glass	2.5	60	100	8.9
Mullite		143	83	5.3
MgO	3.6	210–300	97–130	13.8
Si_3N_4	3.2	310	410	2.25–2.87
Al_2O_3	3.9–4.0	360–400	250–300	8.5
SiC	3.2	400–440	310	4.8
Glass – ceramics:				
Lithium aluminosilicate	2.0	100	100–150	1.5
Magnesium aluminosilicate	2.6–2.8	120	100–170	2.5–5.5

2.3.3. Carbon Matrix Composites

Carbon, as the matrix material, is usually employed with carbon fibres in composite systems. The resistance to high temperature is the main advantage of the carbon matrix, and the fact that it gains extra strength at elevated temperatures. The other advantages of the C/C composites are having high strength-to-weight and high stiffness-to-weight values, high dimensional stabilities and high resistance to fatigue. And also these composites are used in the medical field, since carbon is chemically and biologically inert, it can be kept sterile inside the human body and can be used as prosthetic devices. Additionally, this composite has been used for some time in replacement hips or joint, due to the similarity to bone. Many internal or surgical implant devices are already being produced, preferably from carbon composites, because they also present greater corrosion and chemical resistance than stainless steel or selective metal alloys, in addition to their inherent satisfactory fatigue and toughness characteristics. On the contrary, the cost of the materials used and the cost of the fabrication is so high (Akovali 2001).

2.3.4. Polymer Matrix Composites

Polymers are mostly organic compounds and consist of carbon, hydrogen and other non-metallic elements. The most developed composite materials group is PMC and they have found widespread applications. One of the important advantages is can be easily fabricated into any large complex shape.

Generally PMC are a synergistic combination of high performance fibres and matrix and also called reinforced plastics. In these systems, the fibre provides the high strengths and moduli and the other form is the matrix distribute the load and helps resistance to weathering and to corrosion. Thus, in PMC, strength is almost directly related to the basic fibre strength and it can be further improved at the expense of stiffness. One of the unresolved main objectives is to optimize the stiffness and fibre strength, and it is under serious consideration.

Thermosetting or thermoplastic polymers can be utilized as the matrix component. Thermoplastic PMC soften upon heating at the characteristic glass transition temperature (T_g) of the polymer which, usually, are not too high (upwards of 220 ºC). Hence, for thermoplastic PMC:

- They have a limit in the application temperatures,
- They can be easily produced by use of the conventional plastic processing techniques,
- Their shape can be easily remade with heat and pressure and also they offer the potential for the higher toughness and the low cost-high volume processing of composite structures,
- One of the main disadvantages of thermoplastics is their rather large CTE values, which may lead to a mismatch in their composites and their sensitivities towards environmental-mostly hygrothermal-effect (i.e., absorption of moisture causes swelling in the composite structure).

Polyolefinics (polyethylene, polypropylene), vinylic polymers (polyvinyl chloride (PVC)), polyamides (PA), polyacetals, polyphenylenes (polyphenylene sulphide (PPS)), polysulphone and polyetheretherketone (PEEK) are the most commonly used materials as a thermoplastic matrix. Some of their characteristic properties are shown in Table 2.4.

Conversely, thermosetting PMC are crosslinked and shaped during the final fabrication step, after which they do not soften by heating. They bonded each other in a covalent-bond, and have insoluble and infusible three-dimensional network structure. To provide the process ability, thermosetting resins are typically available in the special B-stage. A B-stage epoxy is a system wherein the reaction between the resin and the curing agent/hardener is not complete. Due to this, the system is in a partially cured stage. When this system is then reheated at elevated temperatures, the cross-linking is complete and the system fully cures (masterbond). Most of the potential disadvantages ot he thermoset and the thermoplastics are nearly the same, although heat resistances are much higher and there is no softening point involved in the case of thermosets. In brief, they have both application temperature limit, both are susceptible to environmental degradation due to radiation/moisture and even atomic oxygen. They have rather low transverse strengths and because of the mismatch in CTE between reinforcement and the matrix there may be very high residual stresses (Akovali 2001).

Table 2.4. Typical properties of some thermoplastic and thermosetting matrices (Source: Akovali 2001)

Property	Epoxy	Thermosetting polyimides	PEEK	Polyamide-imide	Polyether imide	Poly-sulphone	PPS
Density (g cm^{-3})	1.15–1.4	1.43–1.46	1.30	1.38		1.25	1.32
Elastic Modulus (GPa)	2.8–4.2	3.2					
Flexural Modulus (MPa)	15–35	35	40	50	35	28	40
Tensile Strength (MPa)	35–130	55–120	92	95	105	75	70
Compressive Strength (MPa)	140	187					
CTE (10^{-5} °C)	4.5–11	5–9		6.3	5.6	9.4–10	9.9
Thermal Conductivity (W m^{-1} K^{-1})	0.17–0.2	0.36					
Water Absorption (24 h, %)	0.1	0.3	0.1	0.3	0.25	0.2	0.2
T_g (°C)	130–250	370					
Continuous Service Temperature (°C)	25–85	260–300	310		170	175–190	260

Polyesters (unsaturated), epoxies and polyimides are the most common thermoset polymer matrix materials. Generally with the polyster glass fibres are used, their advantages are being inexpensive and somewhat resistant to environmental exposure and lightweight with useful temperatures up to 100°C. Polyesters are used in various constructions, automotive and in general for most of the non-aerospace applications. They have poor impact, hot/wet mechanical properties, limited shelf file and high curing shrinkages, because of these reasons they could not be used high performance applications. Epoxies are more expensive than polyesters and have lower shrinkage on curing. They have good hot/wet strength, excellent mechanical properties, dimensional stability, good adhesion to a variety of reinforcements and a better moisture resistance. There are lots of different types and different formulations exist for epoxies and they have slightly higher maximum application temperature. Polyimides have a much higher application temperature but their fabrication is difficult so that most of the high performance PMC have epoxies as matrices (Akovali 2001).

2.3.5. Hybrid Composite Materials

Hybrid composite material is the newest group of various composites. The general definition of the hybrid material is a material that includes two moieties blended on the molecular scale. Commonly one of these constituents is inorganic and the other one organic in nature (Kickelbick 2007). When more than one type of fibre is used to increase cost-performance effectiveness, it is called hybrid composite material. In a composite system using carbon fibre as reinforcement, the cost can be minimised by reducing its content while maximising the performance by optimal placement and orientation of the fibre. Another example of such a composite is aramid reinforced aluminium laminate (ARALL). It is composed of high strength aluminium alloy sheets interleafed with layers of aramid fibre reinforced adhesive.

Other HCM include: nanocomposites, functionally gradient materials, hymats (hybrid materials), interpenetrating polymer networks and liquid crystal polymers (Akovali 2001).

2.4. Reinforcement Forms

The general sense, composites are the result of a combination of substantial volume fractions of high strength, high stiffness reinforcing components with lower modulus matrix. The properties of composites are related to form of components such as their relative amount and geometry.

Composites can be classified by the geometry of the reinforcement into three groups as particle reinforced, fibre reinforced and structural composites.

Composites which have the dispersed phase for particle (or particulate) are observed as equiaxed and consist components in spherical, rod, flake-like and such shapes with approximately equal axes. It should be recognized that in the case of additional characteristics such as porosity, these characteristics may not be intact. Fillers in filled systems which might not show any reinforcement or whose particles are utilized to extend the material so as to reduce the cost, cannot be taken as particulate reinforced.

Fibre reinforced composites, where the dispersed phase is fibrous with a larger lengthto-diameter ratio;

In situations where structural composites are seen as combinations of composites and as homogenous materials (i.e., laminates and sandwich panels), a change in the shape or even size of particulates and the sizes of fibres in the first two of these groups depending on the type of processing employed during the processing stage is possible. Materials in fibre form are generally much stronger and stiffer than any other form. As a result of this, fibre reinforcements are remarkably preferred (Akovali 2001).

The reinforcing constituents in a composite structure can be discontinuous (either in the form of dispersions/particles, flakes, whiskers, discontinuous short fibres with different aspect ratios) or continuous (long fibres and sheets); although particulate and fibrous form are the most commonly used as a reinforcing component (Akovali 2001). Proper selection of the fiber type, fiber volume fraction, fiber length, and fiber orientation is very important, since it directly affects the following characteristics of a composite laminate (Mallick 2007).

- Density
- Tensile strength and modulus
- Compressive strength and modulus
- Fatigue strength as well as fatigue failure mechanisms
- Electrical and thermal conductivities
- Cost

The type and quantity of the reinforcement determine the composite material properties. Figure 2.4 shows that the highest strength and modulus are obtained with continuous-fiber composites (Campbell 2010).

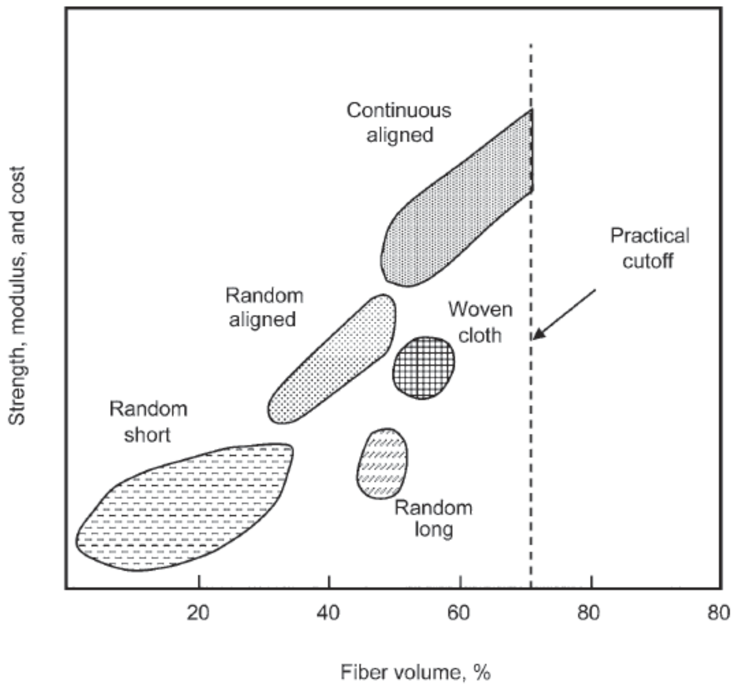

Figure 2.4. Influence of reinforcement type and quantity on composite performance (Source: Campbell 2010)

2.5. Application of Composite Materials

As a matter of fact that there is a continuing explosion of new applications (Herakovich 1997). Today, it is difficult to find any industry that does not take advantages of the composite materials. Many reason can be counted for the growth in composite applications, but the main object is about composites are strength and lightness. There are lots of composite applications but it would be impossible to list them all. Some of the significant applications are aerospace, transportation, automotive, sporting goods, marine and infrastructure.

The first industry is the aerospace to notice the benefits of composite materials. Composite materials such as glass, carbon and Kevlar have been used in producing of aircraft, spacecraft, satellites, space telescopes, the space shuttle, missiles, rockets and helicopters. Specific stiffness and specific strength, design tailorability, and fatigue resistance are the primary reasons for using composite in aerospace. Lowering the weight in the military aircrafts is another important point to increase the payload capacity as well as the missile range. After implementation of composite in aircraft, in the range of 20 to 35% mass reductions were achieved. Figure 2.5. shows the typical composite structures used in military aircraft (Mazumdar 2002).

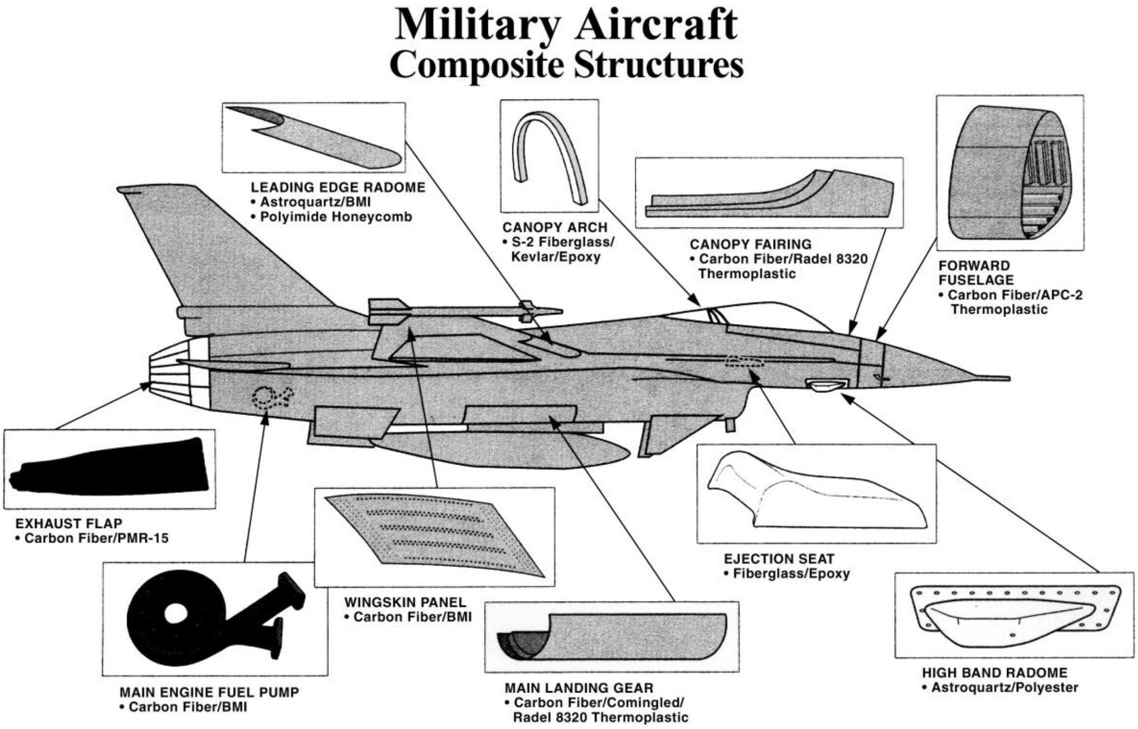

Figure 2.5. Typical composite structures used in military aircraft (Source: Mazumdar 2002)

Lower weight and greater durability (improved corrosion resistance, fatigue life, wear and impact resistance) are the reasons for choosing composites in automotive applications. Drive shafts, fan blades, springs, bumpers, interior panels, tires, belts and engine parts are all examples of automotive applications. Carbon and glass fiber composites are used in manufacturing the hybrid composite drive shaft by pultrusion. These shafts are significantly lighter. They are fabricated as a single component and this method makes them less expensive (Herakovich 1997).

Examples of athletic and recreational equipment such as tennis rackets, baseball bats, helmets, skis, hockey sticks, fishing rods, windsurfing boards can be made of composite materials (Fig.2.6.). The advantages of these products are easy handling, comfort and light in weight so that they provide higher performance.

Figure 2.6. Composite improve the performance of sports equipment (Source: Campbell 2010)

The reasons for choosing composites in marine applications include corrosion resistance and light weight. These benefits are translated into fuel efficiency and higher cruising speed. Composite materials are used in a variety of construction and civil structures such as bridges. Light weight is not the primary reason for construction. However, reduced maintenance, erection costs, handling and life cycle-costs are the some advantages of composite materials in construction. They also gains considerable amount of time for repair and installation and thus minimizes the blockage of traffic.

One of the world's fastest-growing energy sources is wind power. Composite materials are used in blades for large wind turbines to improve electrical energy generation efficiency. Besides these common application fields, composites are used in medical such as wheelchair and implant devices, in electronic such as chips, in military such as helmets, bulletproof vests, portable bridges and lighter weapons (Herakovich 1997; Mazumdar 2002).

CHAPTER 3

MECHANICS OF COMPOSITE MATERIALS

3.1. Introduction

Structural designers aim is to find the best possible design using the least amount of resources. The success of design measurability is related to strength or stiffness and weight or cost. For this reason the best design sometimes signifies either the lowest weight (or cost) with limitations on the stiffness properties or the maximum stiffness design with prescribed resources and strength limits. With the introduction of composite materials, designers achieved a new flexibility through the use of variables that directly affect the properties of the material, and therefore optimal design of structures has acquired a new meaning. It is now possible to tailor the properties of the structural material in order to enhance structural performance and provide requirements of a specific design situation.

There are so many issues that make the design task complex while producing the composite part of design. The mechanics of laminated composite materials can be analyzed in two distinct levels, commonly referred to as macromechanics and micromechanics (Gurdal, et al. 1999).

Mechanical analysis in the micromechanics level is made examining the interactions of the constituents on the microscopic level. This study investigates the state of the stress and deformation in the constituents such as matrix failure (tensile, compressive, shear), fiber failure (tensile, buckling, splitting) and interface failure (debonding).

An analysis in the macromechanics level considers the material homogeneous and investigates the interaction of the individual layers of a laminate with one another on a macroscopic scale and their effects on the overall response quantities of the laminate. At the laminate level the macromechanical analysis is used in the form of lamination theory to analyse overall behaviour as a function of lamina properties and stacking sequence (Daniel and Ishai 1994).

3.2. Classical Lamination Theory

Laminate theory can be defined as analytical stress-strain analysis of the arbitrary laminated structures (with plane laminae) subjected to mechanical or thermal load. Arbitrary number of layers, layer thicknesses and material type (isotropic, anisotropic) can be considered. Stresses and strains within layers, apparent laminate properties or total deformation of the laminate (bending, twisting) could be calculated using classical laminate theory (Montan University). In this theory, it is assumed that laminate is thin and wide, perfect bounding exists between laminas, there exist a linear strain distribution through the thickness and all laminas are macroscopically homogeneous and behave in a linearly elastic manner (Kaw 2006)

Thin laminated composite structure subjected to mechanical in-plane loading (N_x, N_y) and coordinates are shown in Figure 3.1. Cartesian coordinate system *x, y* and *z* defines global coordinates of the layered composite material. A layer-wise principal material coordinate system is represented by 1, 2, and 3 and and fiber direction is oriented at angle θ to the *x* axis.

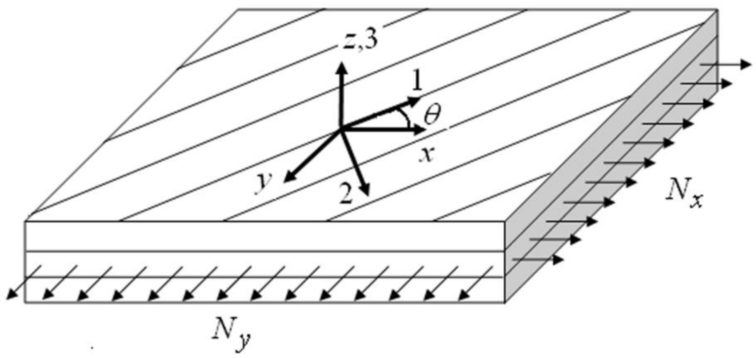

Figure 3.1. A thin fiber-reinforced laminated composite subjected to in plane loading

Representation of laminate convention for the *n*-layered structure with total thickness *h* is given in Figure 3.2.

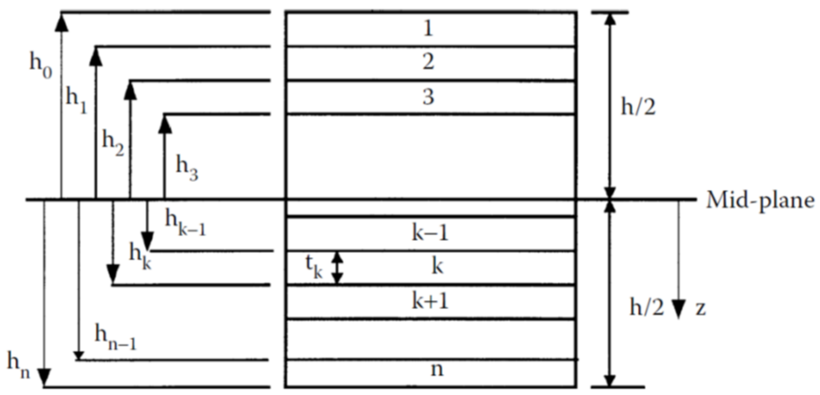

Figure 3.2. Coordinate locations of plies in a laminate (Source: Kaw 2006)

The strains at any point in the laminate to the reference plane can be written as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_s \end{bmatrix} = \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_s^o \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{bmatrix} \tag{3.1}$$

The in-plane stress components are related to the strain components for the *k*-th layer of a composite plate will be as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \varepsilon_{xy}^o \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right) \tag{3.2}$$

where $[\overline{Q}_{ij}]_k$ are the components of the transformed reduced stiffness matrix, $[\varepsilon^o]$ is the mid-plane strains $[\kappa]$ is curvatures. Transformation matrix [T] used in order to obtain the relation between principal axes (1, 2) and reference axes (x, y), is given by

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \qquad c = Cos\theta, \quad s = Sin\theta \tag{3.3}$$

The elements of the transformed reduced stiffness matrix [$\overline{Q}_{ij}$] expressed in Equation 3.2 can be defined as in the following form:

$$\overline{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \tag{3.4}$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4) \tag{3.5}$$

$$\overline{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \tag{3.6}$$

$$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})sc^3 - (Q_{22} - Q_{12} - 2Q_{66})s^3c \tag{3.7}$$

$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})sc^3 \tag{3.8}$$

$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(c^4 + s^4) \tag{3.9}$$

where

$$Q_{11} = \frac{E_1}{1 - \nu_{21}\nu_{12}} \tag{3.10}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{21}\nu_{12}} \tag{3.11}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{21}\nu_{12}} \tag{3.12}$$

$$Q_{66} = G_{12} \tag{3.13}$$

The principal stiffness terms, Q_{ij}, depend on elastic properties of the material along the principal directions, E_1, E_2, G_{12}, υ_{12} and υ_{21}. The in-plane loads (N_x, N_y and N_{xy}) and the moments (M_x, M_y and M_{xy}) in general have the following relations:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{3.14}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{3.15}$$

The matrices [A], [B] and [D] given in Equation 3.14 and 3.15 are extensional stiffness, coupling stiffness and bending laminate stiffness, respectively. These matrices can be defined as

$$A_{ij} = \sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_k - h_{k-1}) \tag{3.16}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \tag{3.17}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k \left(h_k^2 - h_{k-1}^2\right) \tag{3.18}$$

For the layers of a symmetric laminate with orthotropic layers, there is no coupling between in-plane loading and out of plane deformation, so the coupling stiffness matrix (B_{ij}) is equal to zero. The load-deformations relations are therefore reduced to

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} \tag{3.19}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{3.20}$$

It is seen that the $[A]$ matrix is the extensional stiffness matrix relating the in-plane stress resultants $(N's)$ to the midsurface strains $(\varepsilon_0's)$ and the $[D]$ matrix is the flexural stiffness matrix relating the stress couples $(M's)$ to the curvatures $(\kappa's)$. Since the $[B]$ matrix relates $M's$ to $\varepsilon_0's$ and $N's$ to $\kappa's$, it is called bending-stretching coupling matrix (Vinson 1999).

The relation between the local and global stresses in each lamina can be expressed by the following transformation matrix.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} \tag{3.21}$$

Similarly, the local and global strains are written as follows:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} = [R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \tag{3.22}$$

where $$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

3.3. Buckling Analysis of a Laminated Composite Plate

Buckling is the most critical issue for thin and large laminated composite plates subject to in-plane compressive loads. For the buckling analysis, we assume that the plates is subjected to only the in-plane compressive forces and other mechanical and thermal loads are zero (Reddy 2004). When the stress resultants N_x, N_y and N_{xy} are uniformly loaded and w is the pre-buckling deformation, the equation of equilibrium in the direction normal to plate is described as

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}=N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x \partial y}+N_y\frac{\partial^2 w}{\partial y^2} \quad (3.23)$$

For simply supported plate with no shear load, N_{xy} is zero. In order to simplify the equation of equilibrium, the in-plane forces are defined as follows:

$$N_x=-N_0 \qquad N_y=-kN_0 \qquad k=\frac{N_y}{N_x} \quad (3.24)$$

The simply supported boundary conditions on all four edges of the rectangular plate (Fig. 3.3) can be defined as

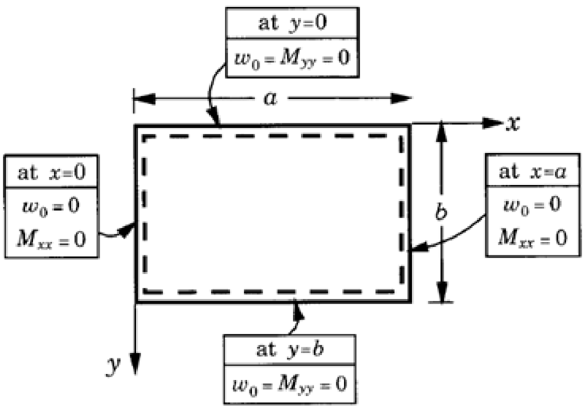

Figure 3.3. Geometry, coordinate system and simply supported boundary conditions for a rectangular plate (Source: Reddy 2004)

$$w(x,0)=0 \quad w(x,b)=0 \quad w(0,y)=0 \quad w(a,y)=0 \tag{3.25}$$

$$M_{xx}(0,y)=0 \;\; M_{xx}(a,y)=0 \quad M_{yy}(x,0)=0 \quad M_{yy}(x,b)=0 \tag{3.26}$$

As in the case of bending, Navier approach may be used for the solution considering simply supported boundary condition

$$w(x,y)=W_{mn}\sin(\alpha x)\sin(\beta y) \tag{3.27}$$

Substituting Equation 3.27 into Equation 3.23, we have obtained the following equation:

$$0=\left\{-\left[D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4\right]+(\alpha^2+k\beta^2)N_0\right\}\times W_{mn}\sin\alpha x\sin\beta y \tag{3.28}$$

For nontrivial solution $\left(W_{mn}\neq 0\right)$, the expression inside the curl brackets should be zero for every *m* and *n* half waves in x and y directions Then we obtain

$$N_0(m,n)=\frac{d_{mn}}{(\alpha^2+k\beta^2)} \tag{3.29}$$

where $$d_{mn}=D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4 \tag{3.30}$$

$$\alpha=\frac{m\pi}{a} \tag{3.31}$$

$$\beta=\frac{n\pi}{b} \tag{3.32}$$

where *a* is the length of the plate, *b* is the width of the plate. Substituting Equation 3.24 into Equation 3.29, the buckling load factor λ_b is determined as

$$\lambda_b(m,n)=\pi^2\left[\frac{m^4D_{11}+2(D_{12}+2D_{66})(rmn)^2+(rn)^4D_{22}}{(am)^2N_x+(ran)^2N_y}\right] \tag{3.33}$$

where r is the plate aspect ratio (a/b). The buckling mode is sinusoidal and if the plate is loaded as $N_x^a = \lambda_b N_x$ and $N_y^a = \lambda_b N_y$, the laminate buckles into *m* and *n* half waves in x and y directions, respectively. The smallest value of λ_b over all possible combinations of *m* and *n* is the critical buckling load factor λ_{cb} that determines the critical buckling loads for a specified combination of N_x and N_y in Equation 3.34. If λ_{cb} is larger than 1, the laminate can sustain the applied loads N_x and N_y without buckling (Gurdal, et al. 1999).

$$\begin{bmatrix} N_{x_{cb}} \\ N_{y_{cb}} \end{bmatrix} = \lambda_{cb} \begin{bmatrix} N_x \\ N_y \end{bmatrix} \tag{3.34}$$

The combinations of *m* and *n* result in the lowest critical buckling load and, which is not easy to find. When composite plate is subjected to in-plane uniaxial loading and is simply supported for all edges, the minimum buckling load occurs at *n=1*. The value of m depends on bending stiffness matrix (D_{ij}) and the plate aspect ratio (a/b). Therefore, it is not clear which value of *m* will provide the lowest buckling load (Vinson 2005). In case of biaxial loading, as composite plate has low aspect ratio, or low ratios of the D_{ij}, the critical values of *m* and *n* should be small (Gurdal, et al. 1999). For this reason, the smallest value of λ_b (1, 1), λ_b(1, 2), λ_b(2, 1) and λ_b(2, 2) are considered in order to make a good prediction with respect to critical buckling load factor (Erdal and Sonmez 2005).

CHAPTER 4

OPTIMIZATION

4.1. General Information

People as well as nature incline to optimize. On the one hand, airline companies tend to optimize in terms of scheduling crews and organizing the aircraft production for minimizing cost. While investors create portfolios that avoid excessive risks to achieve a high rate of return, manufacturers seek for maximum efficiency in the design and operation of their production processes. On the other hand, physical systems aim for a state of minimum energy. For instance, the molecules in an isolated chemical system react with each other until the total potential energy of their electrons is minimized. Or, rays of light follow paths that minimize their travel time (Nocedal and Wright 1999).

Optimization is a crucial issue in science along with the analysis of physical systems. For optimization, at first we should specify some *objective*, a quantitative measure of the performance of the system under study. This objective could be profit, time, potential energy, or any quantity or combination of quantities that can be represented by a single number. Certain characteristics of the system, called *variables* or *unknowns* can be decisive on the objective. At first, the values of the variables that optimize the objective should be found. The variables are often seen as restricted, or *constrained*, in some way (Nocedal and Wright 1999). The increased number of design variables might be both as ease or difficulty for the designer. There are many ways of controls to fine-tune the structure to meet design requirements, but the high number of variables causes the additional responsibility of choosing those design variables which are essential for determining their values for the best solution of the design problem. The potentiality of attaining an efficient design that is resistant against numerous failure mechanisms, coupled with the difficulty in selecting the values of a large set of a design variables renders mathematical optimization a natural tool for the design of laminated composite structures (Gurdal, et al. 1999).

Generally, an optimization problem has an objective function (fitness function) that determines efficiency of the design. Objective function can be classified into two groups: single objective and multi-objective. An optimization process is usually performed within some limits that determine the solution space. These limits are defined as constraint. Lastly, an optimization problem has design variables, which are parameters that are changed during the design process. Design variables can be dispersed (continuous) or discrete (limited continuous). A special case of discrete variables are integer variables. The standard formulation of optimization problem can be stated as follow:

$$\begin{aligned} &\textbf{minimize} \quad && f(x) && x \in \boldsymbol{X} \\ &\textbf{such that} \quad && h_i(x) = 0, && i = 1,\ldots\ldots., n_e \\ & && g_j(x) \le 0, && j = 1,\ldots\ldots., n_g \end{aligned} \tag{4.1}$$

$$x^L \le x \le x^U \tag{4.2}$$

where $f(x)$ is an objective function, $g(x)$ and $h(x)$ are inequality and equality constrains, respectively. Here, x^L and x^U define lower and upper bounds. Generally, although the objective function is minimized, for the cases of the engineering problems, it is maximized. For instance, stiffness and buckling load factor are maximized for laminated composite material. In order to convert a minimization problem into maximization problem, the sign of the objective function is changed (Fig. 4.1). In other words, so as to maximize $f(x)$, we can minimize - $f(x)$ (Gurdal, et al. 1999).

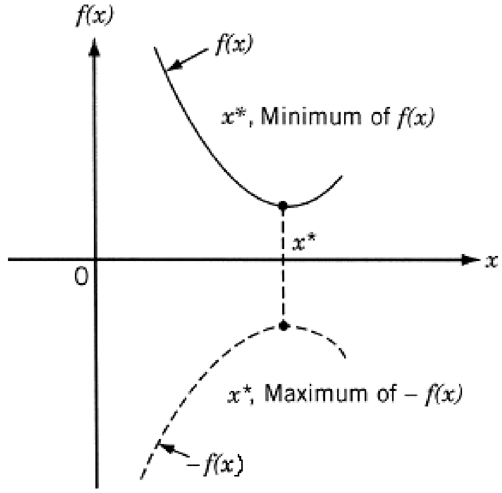

Figure 4.1. Minimum and maximum of objective function (*f(x)*) (Source: Rao 2009)

Optimization sustains the engineers a tool that is necessary in finding the best design among a great number of designs. Design optimization of composite structures was analysed to be a global optimization problem, with multiple local optima and complex design space. A deterministic algorithm, where a monotonically decreasing value of an objective function is iteratively created, may stuck into any local optimum point rather than globally optimum one in relation to the starting point. Thus, the choice of initial design is an important factor in its success. In this respect, the common approach might be to employ the algorithm several times starting from different configurations with the aim that one of the first positions be sufficiently close to the globally optimum configuration, and then to find out the lowest value as the globally optimum solution. If the starting point is outside the feasible region, the algorithm may converge to a local optimum within the infeasible domain which may be regarded as another disadvantage.

A global optimization method has to be employed in structural optimization problems so that absolute optimum of an objective function without being sensitive to the starting position can be found. At this point, stochastic optimization techniques can be evaluated as quite suitable. Moreover, one can propound several advantages of stochastic optimization. Firstly, they are not sensitive to starting point. Not only can they search a large solution space, but they can also escape local optimum points as they allow occasional uphill moves. Due to these aforementioned advantages, the simulated annealing algorithm (SA) is one of the most popular stochastic optimization technique (Erdal and Sonmez 2005).

4.2. Simulated Annealing Algorithm

4.2.1. Introduction

Simulated annealing (SA) is a stochastic approach and it is based on simulation of the statistical process of growing crystals using the annealing process to reach its global minimum internal energy configuration (Arora 2004). In this approach, a solid (metal) is heated to a high temperature and brought into a molten state. The atoms could move freely respect to each other in this state. However, the movements of atoms get restricted by lowering the temperature. As the temperature reduces, the atoms tend to get settled and the crystals forms have the minimum possible internal energy. The formation of crystals is basically related with cooling rate. When molten metals' cooling rate is very fast, it means that temperature is reduced at a very fast, it may not be able to achieve the crystalline state; instead, it may reach a polycrystalline state having a higher energy state compared to that of the crystalline state. In engineering applications, there will be defects inside the material at the end of the rapid cooling. Thus the temperature of the heated solid (molten metal) needs to be reduced at a slow and controlled rate in order to attain the lowest energy state (internal energy). This cooling at a slow rate is named as annealing (Rao 2009).

Table 4.1 shows how physical annealing can be mapped to simulated annealing.

Table 4.1. Relationship between Physical Annealing and Simulated Annealing (Source: Kendall 2014)

Thermodynamic Simulation	Combinatorial Optimization
System States	Feasible Solutions
Energy	Objective
Change of State	Neighbouring Solutions
Temperature	Control Parameter
Frozen State	Heuristic Solution

Using these mappings any combinatorial optimization problem can be converted into an annealing algorithm (Kendall 2014).

4.2.2. Procedure

The simulated annealing method is used to achieve the minimum function value in a minimization problem by simulating the process of slow cooling of molten metal. The cooling phenomenon is simulated by introducing a temperature-like parameter and Boltzmann's probability distribution is used to control the system. The distribution implies the energy of a system in thermal equilibrium at temperature T and it can be written in the following form (Rao 2009).

$$P(E) = e^{-E/kT} \tag{4.1}$$

where *P(E)* represents the probability of achieving the energy level *E*, *k* denotes the Boltzmann's constant and *T* is temperature. Equation (4.1) demonstrates that at low temperatures, the system has a small probability of being at a high-energy state; however, at high temperatures the system could be at any energy state. This shows that the convergence of the simulated annealing algorithm can be controlled by controlling the temperature *T*, when the Boltzmann's probability distribution is used as a search process.

4.2.3. Algorithm

The SA algorithm can be explained as follows. Start with an initial design vector x_1 and a high value of temperature *T*. Generate a new design point randomly in the neighbourhood of the currents design point and evaluate function values and find the difference.

$$\Delta E = \Delta f = f_{i+1} - f_i \equiv f(x_{i+1}) - f(x_i) \tag{4.2}$$

If f_{i+1} is smaller than f_i (with a negative value of Δf), accept the point x_{i+1} as the next design point. Otherwise, when Δf is positive, accept the point x_{i+1} as the next design point only with a probability $e^{-E/kT}$. This means that if the value of a randomly generated number is larger than $e^{-E/kT}$, accept the point x_{i+1}; otherwise, reject the point x_{i+1}. This completes one iteration of the SA algorithm. If the point x_{i+1} is rejected, then the process of generating a new design point x_{i+1} randomly in the vicinity of the current design point, evaluating the corresponding objective function value f_{i+1}, and deciding to accept x_{i+1} as the new design point. A predetermined number of new points x_{i+1} are tested at any specific value of the temperature *T* to simulate the obtainment of thermal equilibrium at every temperature. The initial temperature *T* plays an important role in the successful convergence of the SA algorithm. For example, if the initial temperature *T* is too large, it needs a larger number of temperature reductions for convergence. On the contrary, if the initial temperature is chosen to be too small, the searching may be incomplete and it might be stuck in the local minima.

Simulated Annealing algorithm could be written step by step to simplify the method:

1. Start with an initial vector x_1 and assign a high temperature value to the function

2. Generate a new design point randomly and find the difference between the previous and current function values

3. Specify whether the new point is better than the current point.

4. If the value of a randomly generated number is larger than $e^{-\Delta E/kT}$, accept the point x_{i+1}

5. If the point x_{i+1} is rejected, then the algorithm produces a new design point x_{i+1} randomly. However, it should be noted that the algorithm accepts a worse point based on an acceptance probability (Rao 2009).

Figure 4.2 presents a flowchart of this process.

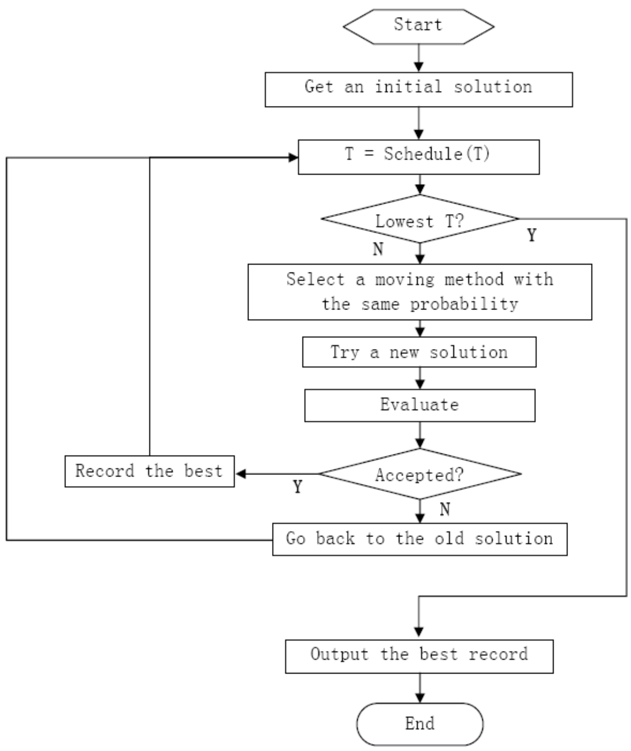

Figure 4.2. A typical flowchart of simulated annealing (Source: Sheng and Takahashi 2012)

4.2.4. Features of the Method

Some of the features of simulated annealing are as follows:

- The algorithm is heuristic. It means that it is not guaranteed to get the optimum solution; they are designed to give an acceptable solution for typical problems in a reasonable time.
- The method can be used to solve mixed-integer, discrete, or continuous problems.
- The quality of the final solution is not affected by the initial guesses, except that the computational effort may increase with worse starting designs.
- The design variables need not be positive (Rao 2009).
- SA is able to avoid getting trapped in local minima.
- SA often suffers from slow convergence.
- Its main advantages over other local search methods are its flexibility and its ability to approach global optimality.
- SA methods are easily tuned.
- There is a clear tradeoff between the quality of the solutions and the time required to compute them (Busetti 2003).

4.3. Matlab Optimization Toolbox

MATLAB Optimization Toolbox provides algorithms to solve optimization problems using Simulated Annealing (SA), Genetic Algorithm (GA) and Direct Search (DS). All of these methods could be used in design of composite materials. Many researchers have been used all these methods in their optimization problem (Erdal & Sonmez, 2005; Akbulut & Sonmez, 2008; Karakaya & Soykasap, 2009). *Ga, gamultiobj, simulannealbnd, patternsearch* are the some solvers of the toolbox and simulannealbnd is used in this thesis to solve the optimization problem.

4.3.1. Simulannealbnd Solver

This solver comprises of two main parts in the MATLAB Optimization Toolbox solvers: (i) Problem set up and results, (ii) Options. In problem set up and results part, *Objective function* defines the function we want to optimize in other words the fitness of the optimization problem of the user. *Start point* represents the initial point for the Simulated Annealing search. Additionally, lower and upper bounds can be given for the design parameters in the *bounds* sub-options. Options part comprises of five main sub-options: *stopping criteria, annealing parameters*, *acceptance criteria*, *data type* and *hybrid function*. *Stopping criterias* define the conditions to determine when to stop the solver. In short, *TolFun*; termination tolerance on function value, *MaxIter*; maximum number of iterations allowed, *MaxFunEvals*; maximum number of objective function evaluations allowed, *TimeLimit*; the algorithm stops after running for TimeLimit seconds, *ObjectiveLimit*; minimum objective function value desired. *Annealing parameters* are related with the process of the algorithm. *Annealing function* specify function that is utilized to generate new points for the next iteration. The *fast annealings'* step length equals the current temperature and direction is uniformly random whereas *Boltzmann annealings'* step length equals the square root of the temperature and direction is uniformly random, too. *Annealing* is the technique of closely controlling the temperature when cooling a material to ensure that it reaches an optimal state. *Reannealing* raises the temperature after the algorithm accepts a certain number of new points, and starts the search again at the higher temperature. Reannealing avoids the algorithm getting caught at local minima. Specify the reannealing schedule with the *ReannealInterval* option. *Reannealing interval* determine the number of points to accept before re-annealing process. The algorithm systematically lowers the temperature, storing the best point found so far. The *TemperatureFcn* option specifies the function the algorithm uses to update the temperature. Let *k* denote the annealing parameter. (The annealing parameter is the same as the iteration number until reannealing). *Temperature update function* options are: *Exponential temperature* updates the temperature by lowering as InitialTemperature*0.95^k. *Logarithmic temperature* updates that temperature by lowering as InitialTemperature/log (k). *Linear temperature* updates that temperature by lowering as InitialTemperature /k. Initial temperature define the temperature at the beginning of the run. The algorithm determines whether the new point is better or worse than the current point. If the new point is better than the current point, it becomes the next point. If the new point is worse than the current point, the algorithm can still make it the next point. The algorithm accepts a worse point based on an acceptance function. This acceptance functions is $1/(1+\exp(\Delta/\max(T)))$. Δ denotes the difference between new and old objectives. Smaller temperature leads to smaller acceptance probability. Also, larger Δ leads to smaller acceptance probability. *Data type* can assign: *Double* for double-precision numbers.

Figure 4.3 represents the parameter selection steps for the SA analysis of simulannealbnd solver user interface.

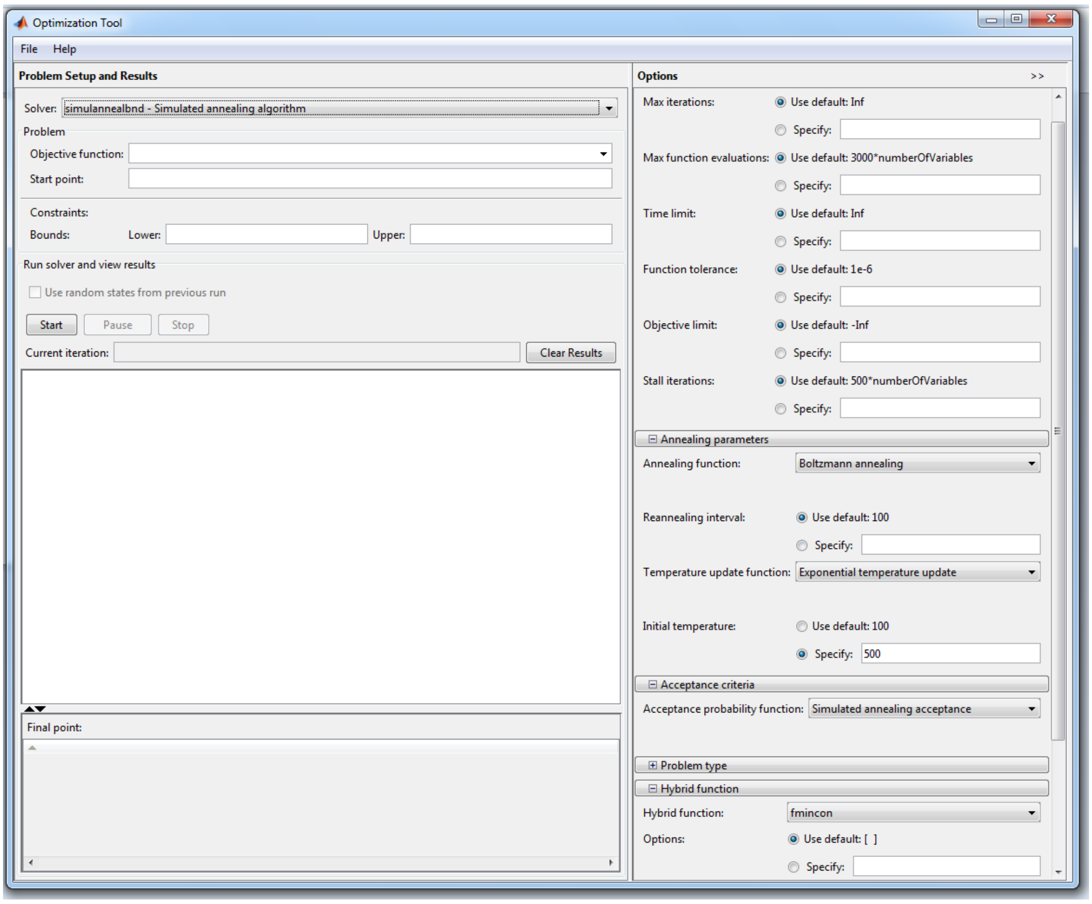

Figure 4.3. Matlab optimization toolbox simulannealbnd solver user interface.

Parallel processing is an attractive way to speed optimization algorithms. Simulannealbnd does not run in parallel automatically. However, it can call hybrid functions that take advantage of parallel computing *Hybrid function* determine an alternate solver that runs at specified times. The selections consist of *fminsearch* that can be used only for unconstrained problems, *patternsearch* that is used if you specify bounds, *fminunc* that is utilized only for unconstrained problem, *fmincon* can be used only for constrained problems.

In this study, *fmincon* is used as a hybrid function. It attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. This is generally referred to as constrained nonlinear optimization or nonlinear programming. *fmincon* uses a Hessian as an optional input. This Hessian is the second derivatives of the Lagrangian. *Fmincon* has four algorithms which are related with Hessians forms. *The active-set* and *sqp* algorithms do not accept a user-supplied Hessian. They compute a quasi-Newton approximation to the Hessian of the Lagrangian. *The trust-region-reflective* can accept a user-supplied Hessian as the final output of the objective function. Since this algorithm has only bounds or linear constraints, the Hessian of the Lagrangian is same as the Hessian of the objective function. *The interior-point algorithm* which is used in this optimization process can accept a user-supplied Hessian as a separately defined function—it is not computed in the objective function.

In Table 4.2 Simulated Annealing algorithm parameters used in the problems have been listed. The previous studies in the literature have been examined and it is decided to set initial temperature to 500.

Table 4.2. Simulated Annealing solver parameters used in the problems

Annealing function	Boltzmann
Reannealing interval	100
Temperature update function	Exponential temperature update
Initial temperature	500
Acceptance probability function	Simulated annealing acceptance
HybridFcn	fmincon
Stopping criteria	Max iterations= infinity Max function evaluations = 3000*number of variables Function tolerance=10^-6

CHAPTER 5

RESULTS AND DISCUSSION

5.1. Problem Definition

The composite panel under consideration is simply supported on four sides with a length of *a* and width of *b*. The panel is subject to given in-plane compressive loads N_x and N_y in the x and y directions, respectively (Fig.5.1). The laminate is symmetric, balanced about the midplane and has the ply thickness of *t*. The aim of this study is to find the optimum stacking sequence designs of laminated composite plates having the minimum thickness which resist to buckling under the given loading conditions and different aspect ratios. These conditions are provided with critical buckling load factor.

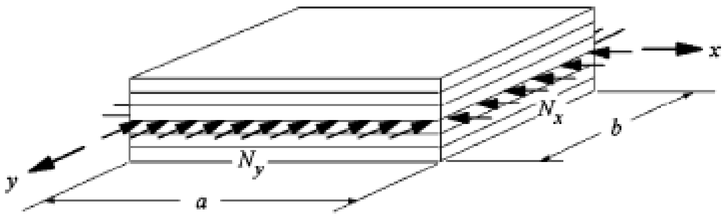

Figure 5.1. A symmetric laminate under compressive biaxial loads.
(Source: Soykasap and Karakaya 2007)

Composite plates are made of carbon/epoxy and each layer is 0.25 mm thick. The length of plate *a* equals to 0.508 m and the width of plate *b* changes with respect to aspect ratios *a/b*=1, 2, 1/2 . The optimization process is performed by increasing the number of layers four by four since the laminate is symmetric and balanced, and also minimum layers could be at least four. This optimization process is continued till optimum configuration satisfying the critical buckling load factor constraint ($\lambda_{cb} \geq 1$). In this study, the plate optimization is examined under various loading conditions such as N_x=1000 N/mm, 2000 N/mm, 3000 N/mm and N_y changes with respect to loading ratios N_x/N_y =1, 2, 1/2. The elastic properties of layers have been taken from a previous study (Lopez, et al. 2009) and presented in Table 5.1. Fiber orientation angles are treated as continuous design variables ($-90 \le \theta \le 90$) during the optimization process. The representation of stacking sequence of *N*-layered composite plate can be given as

$$[\pm\theta_1 / \pm\theta_2 / \pm\theta_3 / \pm\theta_4 / \pm\theta_5 / \pm\theta_6 / \pm\theta_7 / \pm\theta_8 / .... / \pm\theta_{n-1} / \pm\theta_n]_s$$

Here *n* corresponds to *N/4* since the plate is considered symmetric and balanced. The optimization problem can be described as follow;

Find : $\{\theta_k\},\ \theta_k \in \{-90,90\}\ \ k = 1,...,n$

Minimize : Weight

Subject to : Critical buckling load factor ($\lambda_{cb} \ge 1$)

Table 5.1. The elastic properties of Carbon/Epoxy layers (Source: Lopez, et al. 2009)

Longitudinal modulus	E_1=116600 MPa
Transverse modulus	E_2=7673 MPa
In-plane shear modulus	G_{12}=4173MPa
Poisson's ratio	υ_{12}=0.27
Density	ρ=1605 kg/m^3

In order to obtain optimum stacking sequences of laminated composite material, the critical buckling load factor (λ_{cb}) (Equation 3.33) is maximized by using simulated annealing algorithm. Here, the smallest value of λ_b (1, 1), λ_b(1, 2), λ_b(2, 1) and λ_b(2, 2) is taken as the critical buckling load factor.

5.2. Optimization Results and Evaluation

In this study, the laminated composite plates subjected to in-plane loads have been investigated. For this reason, Simulated Annealing Algorithm optimization method in MATLAB *Global Optimization Toolbox* is used for the given load ratios and plate aspect ratios. The optimum stacking sequence designs have been analysed considering buckling and minimum weight. In order to provide efficiency and reliability of SA,

- 30 independent searches have been performed for each case,
- SA Toolbox options have been adjusted as in Table 4.2,
- Matlab nonlinear optimization solver *fmincon* is used in order to apply a hybrid algorithm.

Firstly, in order to show that the optimization algorithms are reliable, the algorithms related to objective function (critical buckling load factor) have been verified using some convenient results from previous studies in the literature. The results of buckling load factor algorithm verification for loading cases specified in the study of Karakaya and Soykasap (2009) are given in Table 5.2.

Table 5.2. Verification of objective function algorithm

Loading ratio (N_x/N_y)	Aspect ratio (a/b)	λ_{cb} (Karakaya and Soykasap 2009)	λ_{cb} (Present Study)
1	2	695781.3	695781.3
1	1	242823.1	242823.1
1	1/2	173945.3	173945.3
2	2	1057948.3	1057948.3
2	1	323764	323764
2	1/2	206492.9	206492.9
1/2	2	412985.8	412985.8
1/2	1	161882.1	161882.1
1/2	1/2	132243.5	132243.5

In Table 5.2, it has been show that the optimum critical buckling load factor values are very close to compared study. This means that algorithm could yield reliable results.

The optimum stacking sequence of composite plates which have the minimum thickness and could resist to buckling for various plate aspect ratios in different loading conditions are calculated by using simulated annealing algorithm. The results are given in Tables 5.3-5.11.

As a first design problem, N_x and *a/b* are taken as a constant and they are equal to 1000 N/mm and 0.5, respectively. Different loading conditions are examined to obtain the optimum designs of laminated composites. And then the critical buckling loads factors (λ_{cb}) and optimum stacking sequences are shown in Table 5.3. The optimum design solutions which have minimum thickness and sufficient ($\lambda_{cb} \geq 1$) buckling load factor are displayed in grey scale. It is observed that buckling load capacity increases with respect to increasing loading ratios for the same number of layers.

Table 5.3. Optimum stacking sequence designs for N_x=1000 N/mm and *a/b* =1/2

N_x/N_y	**Number of Layers**	λ_{cb}	**Stacking Sequence**
1/2	4	0.0002	$[\pm 27.8]_s$
	8	0.0018	$[\pm 27.8/\mp 27.7]_s$
	12	0.0062	$[\pm 27.9/\mp 27.8/\pm 26.9]_s$
	16	0.0147	$[\pm 27.8/\mp 28.1/\mp 27/\mp 29.2]_s$
	20	0.0287	$[\pm 27.9/\mp 27.9/\mp 27.8/\mp 27.7/\pm 15.1]_s$
	24	0.0496	$[\mp 27.8/\pm 27.8/\pm 27.8/\pm 27.8/\pm 27.7/\mp 27]_s$
	28	0.0789	$[\mp 27.8/\mp 27.8 \pm 27.8/\pm 27.8/\pm 27.7/\pm 27.8/\mp 28.2]_s$

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Table 5.3. (cont.)

1/2	32	0.1177	[±27.8/±27.8/±27.8/±27.8/∓27.9/∓27.8/±28.3/∓36.1]$_s$
	36	0.1677	[∓27.9/±28/±27.9/±27.4/±27.7/∓27.2/±28/∓27.4/±28.4]$_s$
	40	0.2300	[±27.8/∓27.8/∓27.8/±27.8/±27.8/±27.8/∓27.9/±28/±26.4/±32.8]$_s$
	44	0.3061	[∓28/∓27.8/∓27.9/∓28.3/±26.6/±27.9/∓28.4/∓27.7/±30.8/ ∓15.7/∓2.4]$_s$
	48	0.3975	[∓27.9/±27.9/±27.9/∓27.8/±27.8/∓27.8/±27.8/∓27.7/±27.5/ ∓27.1/±25.4/∓8.1]$_s$
	52	0.5054	[∓27.8/∓27.9/±27.8/±27.6/±28.1/∓28.1/±27.3/∓27.9/∓28.6/ ∓26.9/∓25.9/±28.6/±17.3]$_s$
	56	0.6312	[±27.9/∓27.9/∓28/±27.7/∓27.6/∓27.6/∓27.7/±27.8/∓27.1/ ±28/±27.3/∓27/±25.5/∓31.3]$_s$
	60	0.7764	[∓27.9/±27.9/±27.9/±27.8/±27.9/±27.9/±27.3/±27.5/±27.9/ ±26.4/∓27.7/±25.7/∓25.9/∓27.9/∓18.7]$_s$
	64	0.9422	[±27.9/±27.9/∓27.9/±27.8/±27.8/∓27.8/±28/∓27.7/±28/∓28.2/ ±26.8/±28.2/±26.3/±22/∓22.3/∓20.4]$_s$
	68	1.1302	[±27.7/±27.8/±27.8/∓27.8/±27.9/±27.7/∓27.8/∓28.2/∓28.1/∓27.7/ ∓27.7/±27.6/±27.9/∓26.2/±27.0/∓25.6/∓30.2]$_s$
1	4	0.0003	[∓19.2]$_s$
	8	0.0023	[∓19.2/±19.2]$_s$
	12	0.0080	[±19.3/∓18.9/∓19.2]$_s$
	16	0.0189	[∓19.4/∓19.2/∓18.3/∓17.1]$_s$
	20	0.0370	[±19.2/∓19.2/±19.1/∓18.6/±22.2]$_s$
	24	0.0639	[∓19.2/±19.2/∓19.2/∓19.2/±19.1/∓12.9]$_s$
	28	0.1015	[±19.2/±19.2/∓19.2/±19.2/±19.3/±19.3/±15,3]$_s$
	32	0.1516	[∓19.3/±19.3/∓19.4/±19.2/∓19.2/∓18.1/∓14.7/±20.9]$_s$
	36	0.2158	[±19.3/±19.2/±19.2/±19.2/±19.2/±18.5/±19.4/∓18.7/∓5.4]$_s$

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Table 5.3. (cont.)

	40	0.2961	[±19.2/±19.2/∓19.1/∓19.2/∓18.9/∓19.7/∓19/∓18.5/±22.4/±17]$_s$
	44	0.3914	[∓19.3/∓19.3/∓19.3/±19.3/∓19.1/±18.6/±19.2/±18.6/±19.1/∓19.1/∓19]$_s$
	48	0.5117	[±19.2/∓19.2/∓19.2/∓19.2/∓19.2/±19.2/±19.2/±19.2/∓19.2/∓19.1/±19/±18.2]$_s$
1	52	0.6506	[∓19.2/±19.2/±19.2/∓19.2/±19.3/±19.3/±19.2/∓19/∓19.4/±18.6/∓18.5/±15.2/±16.5]$_s$
	56	0.8126	[∓19.3/±19.2/∓19.2/∓19.2/∓19.2±19.2/∓19.1/∓19.2/∓19.1/±19.1/∓18.7/∓18.5/±14.0/∓3.7]$_s$
	60	0.9994	[±19.3/∓19.1/±19.3/∓18.8/∓18.4/±20.4/±19.5/±19.5/∓18.6/∓20.2/∓19.7/∓18.6/±16.7/±11.8/∓8.7]$_s$
	64	1.2129	[±19.4/∓19.4/∓19.4/±19.3/∓19.3/±19.3/∓19.1/∓18.8/∓19/∓19/±18.9/±17.3/±17.6/∓16/±3/∓4.5]$_s$
	4	0.0003	[∓11.2]$_s$
	8	0.0027	[±11.2/±11]$_s$
	12	0.0092	[∓11.1/±11.5/∓11.8]$_s$
	16	0.0218	[±11.1/∓11.5/∓11.5/±6.4]$_s$
	20	0.0427	[±10.3/±11.6/∓12.2/±13.7/∓8.6]$_s$
	24	0.0738	[±11.1/∓11.2/∓11.7/±11.9/∓7.5/±12.4]$_s$
	28	0.1172	[∓11.3/∓13.2/∓10.4/∓8.3/±8.4/∓8.2/∓2.1]$_s$
2	32	0.1749	[±11.2/±11/±11.1/±10.9/±11.5/∓11.6/∓15.2/∓5.4]$_s$
	36	0.2491	[∓11.2/∓11.3/∓11.3/∓11.2/∓11/±11.3/∓11.2/±11/∓9.2]$_s$
	40	0.3417	[±11.6/±11.7/∓11/∓11.9/±11.8/∓5.9/∓8.8/±12.7/∓9.6/±11.7]$_s$
	44	0.4548	[±11/∓11.2/±10.1/∓10.7/∓11.4/∓12.6/∓12.6/±12.9/∓14.2/∓9.6/∓3.9]$_s$
	48	0.5905	[±9.1/∓11.6/±12.2/∓10.8/±11.6/±14.2/±11/∓7.2/±12.7/±13.2/±14.9/±7.3/∓14.7]$_s$
	52	0.7507	[±11.7/∓9.8/±11.6/±12.0/∓11.4/∓11.4/∓12.0/∓13.5/±6.7/∓10.7/±8.4/∓6.7/∓4.0/±15.2]$_s$
	56	0,9377	[±11.7/∓9.8/±11.6/±12.0/∓11.4/∓11.4/∓12.0/∓13.5/±6.7/∓10.7/±8.4/∓6.7/∓4.0/±15.2]$_s$
	60	1.1533	[∓10.5/±12/∓11/∓12.4/∓11.9/±12.2/±12.5/±10.3/±5.9/∓9/∓2.8/±2.5/±14/±17.8/∓33.7]$_s$

The optimum designs of laminated composite plates for the plate aspect ratio *a/b* =1 and N_x= 1000 N/mm are calculated and the results are shown in Table 5.4. It is seen that all possible fiber orientations consist of ±45 angles which are discrete values.

Table 5.4. Optimum stacking sequence designs for N_x=1000 N/mm and *a/b* =1

N_x/N_y	Number of Layers	λ_{cb}	Stacking Sequence
1/2	4	0.0002	[±45]$_s$
	8	0.0021	[±45/±45]$_s$
	12	0.0071	[±45/±45/±45]$_s$
	16	0.0169	[∓45/∓45/∓45/±45]$_s$
	20	0.0331	[±45/∓45/∓45/±45/∓45]$_s$
	24	0.0573	[∓45/±45/±45/±45/∓45/±45]$_s$
	28	0.0910	[∓45/±45/∓45/±45/∓45/∓45/±45]$_s$
	32	0.1358	[∓45/∓45/∓45/∓45/∓45/∓45/±45/±45]$_s$
	36	0.1934	[∓45/±45/∓45/±45/±45/±45/∓45/±45/±45]$_s$
	40	0.2653	[±45/∓45/±45/±45/±45/∓45/±45/∓45/±45/±45]$_s$
	44	0.3531	[∓45/∓45/∓45/∓45/∓45/±45/∓45/∓45/±45/∓45/∓45]$_s$
	48	0.4584	[∓45/±45/∓45/±45/±45/∓45/±45/∓45/±45/±45/∓45/∓45]$_s$
	52	0.5829	[∓45/±45/∓45/±45/±45/∓45/±45/∓45/∓45/±45/∓45/±45/±45]$_s$
	56	0.7280	[∓45/∓45/±45/±45/∓45/±45/∓45/±45/±45/∓45/±45/∓45/45/45]$_s$
	60	0.8954	[±45/∓45/±45/∓45/±45/∓45/∓45/±45/∓45/±45/±45∓45/±45/ ±45/∓45]$_s$
	64	1.0867	[±45/∓45/∓45/∓45/∓45±45/±45/∓45/±45/±45/∓45/±45/±45/±45/ ∓45/±45]$_s$

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Table 5.4. (cont.)

1	4	0.0004	$[\pm 45]_s$
	8	0.0031	$[\mp 45/\pm 45]_s$
	12	0.0107	$[\mp 45/\mp 45/\pm 45]_s$
	16	0.0254	$[\pm 45/\mp 45/\mp 45/\pm 45]_s$
	20	0.0497	$[\mp 45/\pm 45/\pm 45/\mp 45/\pm 45]_s$
	24	0.0859	$[\mp 45/\mp 45/\mp 45/\pm 45/\pm 45/\mp 45]_s$
	28	0.1365	$[\pm 45/\mp 45/\pm 45/\pm 45/\pm 45/\mp 45/\pm 45]_s$
	32	0.2037	$[\mp 45/\pm 45/\pm 45/\mp 45/\pm 45/\pm 45/\mp 45/\pm 45]_s$
	36	0.2901	$[\mp 45/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45/\mp 45/\pm 45/\pm 45]_s$
	40	0.3979	$[\mp 45/\mp 45/\pm 45/\mp 45/\pm 45/\pm 45/\mp 45/\pm 45/\pm 45/\pm 45]_s$
	44	0.5297	$[\mp 45/\mp 45/\pm 45/\mp 45/\pm 45/\pm 45/\pm 45/\pm 45/\pm 45/\pm 45/\pm 45]_s$
	48	0.6877	$[\mp 45/\pm 45/\mp 45/\mp 45/\pm 45/\pm 45/\mp 45/\mp 45/\mp 45/\pm 45/\mp 45/\mp 45]_s$
	52	0.8743	$[\pm 45/\mp 45/\mp 45/\mp 45/\mp 45/\mp 45/\mp 45/\pm 45/\mp 45/\mp 45/\mp 45/\mp 45/\mp 45]_s$
	56	1.092	$[\mp 45/\pm 45/\mp 45/\mp 45/\pm 45/\pm 45/\pm 45/\pm 45/\pm 45/\pm 45/\pm 45/\pm 45/\pm 45/\mp 45]_s$
2	4	0.0005	$[\pm 45]_s$
	8	0.0042	$[\pm 45/\mp 45]_s$
	12	0.0143	$[\mp 45/\pm 45/\pm 45]_s$
	16	0.0339	$[\pm 45/\mp 45/\pm 45/\mp 45]_s$
	20	0.0663	$[\mp 45/\pm 45/\pm 45/\mp 45/\mp 45]_s$
	24	0.1146	$[\pm 45/\mp 45/\pm 45/\mp 45/\mp 45/\pm 45]_s$
	28	0.1820	$[\pm 45/\mp 45/\pm 45/\mp 45/\pm 45/\pm 45/\pm 45]_s$

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Table 5.4. (cont.)

2	32	0.2716	[±45/∓45/±45/∓45/±45/±45/∓45/±45]$_s$
	36	0.3868	[±45/±45/∓45/±45/±45/±45/±45/∓45/±45]$_s$
	40	0.5306	[±45/∓45/±45/∓45/±45/∓45/∓45/±45/∓45/±45]$_s$
	44	0.7062	[∓45/±45/±45/±45/±45/∓45/∓45/∓45/±45/±45/∓45]$_s$
	48	0.9169	[∓45/∓45/∓45/∓45/∓45/∓45/∓45/±45/∓45/∓45/∓45/±45]$_s$
	52	1.1658	[∓45/∓45/±45/∓45/±45/∓45/±45/∓45/±45/±45/∓45/±45/∓45]$_s$

Optimum fiber orientations for the increasing number of layers are calculated and results have been presented in Table 5.5. Aspect ratio is taken as 2 and N_x is equal to 1000 N/mm for the laminated composite plates

Table 5.5. Optimum stacking sequence designs for N_x=1000 N/mm and *a/b* =2

N_x/N_y	Number of Layers	λ_{cb}	Stacking Sequence
1/2	4	0.0007	[±78.8]$_s$
	8	0.0055	[±78.8/∓78.7]$_s$
	12	0.0185	[±78.9/±78.7/∓77.6]$_s$
	16	0.0437	[∓78.9/±78.9/±78.1/∓76.4]$_s$
	20	0.0854	[∓79/±78.7/±78.5/±78.4/±76.6]$_s$
	24	0.1476	[∓79.1/∓79/±78.6/∓77.9/±76.5/±82.7]$_s$
	28	0.2344	[±78.6/∓79/±79.7/±78.4/±78.1/∓76.8/±72.7]$_s$
	32	0.3500	[∓78.7/±78.1/∓79.1/∓78.8/∓81/∓79.2/±77.6/∓81.5]$_s$
	36	0.4983	[±78.4/±78.9/∓79.5/±79.5/∓77.8/±79.4/±78.3/±76.7/±78.1]$_s$

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Table 5.5. (cont.)

1/2	40	0.6835	[∓79/∓78.6/∓78.3/±80/±78.9/∓78.1/∓79.3±76.9/±81.4/∓74.6]$_s$
	44	0.9097	[∓78.5/±80/±79.1/∓77.9/±79.1/∓77.8/∓78.5/∓78.1/±77.6/∓81.4/ ±80.2]$_s$
	48	1.1810	[±81.1/∓78.3/∓78.2/±78.2/∓78/±78.1/±77.8/∓79.1/±79/±77/ ±79.1/∓63.2]$_s$
1	4	0.0012	[±70.8]$_s$
	8	0.0095	[∓70.8/∓70.7]$_s$
	12	0.0320	[±70.9/∓70.8/±70.2]$_s$
	16	0.0758	[±70.9/±70.8/±70.6/∓69.3]$_s$
	20	0.1481	[∓70.9/∓70.8/∓70.8/±70.6/∓69.3]$_s$
	24	0.2559	[∓70.8/∓70.8/∓70.8/±70.8/±70.6/∓69.3]$_s$
	28	0.4063	[±70.9/∓70.9/∓70.8/±71/±70/∓69.3/∓69.6]$_s$
	32	0.6065	[∓70.8/∓70.8/∓70.8/∓70.8/±70.8/∓70.7/±70.6/±69.3]$_s$
	36	0.8635	[±71/±71/±70.9/∓71/±70.8/∓70/±69./±67.5/∓59.6]$_s$
	40	1.1843	[±71.5/±71.4/±71.3/∓70.5/±70.5/±69.8/±68/±66.1/±62.3/∓56.1]$_s$
2	4	0.0018	[∓62.2]$_s$
	8	0.0147	[∓62.2/±62.2]$_s$
	12	0.0497	[∓62.2/∓62.2/∓61.9]$_s$
	16	0.1178	[∓62.2/±62.2/∓62.2/±62.2]$_s$
	20	0.2301	[±62/∓62.3/∓62.8/∓62.2/∓59.3]$_s$
	24	0.3975	[∓62.3/±62.3/∓62.2/∓62/∓61.4/∓57.1]$_s$
	28	0.6313	[∓62.1/∓62/∓62.5/∓62.7/±62.5/±62.5/±58.5]$_s$
	32	0.9423	[±62.4/±62.4/∓62.1/±62.1/±62.4/∓60.8/∓58.9/∓58.3]$_s$
	36	1.3416	[∓62.3/∓62.2/∓62.5/±62.2/±62.2/±60.6/±62.9/∓61.7/∓61.8]$_s$

It can be understood from Tables 5.3-5.5 that the stacking sequences have both continuous and discrete fiber angles which depend on the aspect ratios. When the plate aspect ratios are 1, stacking sequences have discrete fiber angles. On the other hand, fiber angles are continuous when the plate aspect ratios are 1/2 and 2. It can be also seen that the maximized critical buckling load factor values depends aspect ratios. When the plate aspect ratio decreases, optimum number of layers increases. Therefore, it can be said that the thickness of the laminate is getting higher. It is also observed that the maximum critical buckling load factors corresponding to aspect ratios, 1/2, 1 and 2, have been obtained for the combination of (*m*,*n*) values λ_b (1, 2), λ_b (1, 1) and λ_b (2, 1), respectively.

The effect of applied loads on plate having the same geometry has been investigated and the results are presented in Tables 5.6 and 5.7. The loadings are taken as 2000 N/mm and 3000 N/mm, respectively. When it is compared with the results of Table 5.5 (N_x=1000 N/mm). It can be concluded, as expected, the optimum number of layers are increased with increasing applied loads. Contrary to expectations, the applied load is increased by 2 times yet optimum number of layers does not increased by 2 times.

Table 5.6. Optimum stacking sequence designs for N_x=2000 N/mm and a/b =2

N_x/N_y	Number of Layers	λ_{cb}	Stacking Sequence
1/2	4	0.0003	$[\pm 78.8]_s$
	8	0.0027	$[\mp 78.8/\mp 78.8]_s$
	12	0.0092	$[\mp 78.9/\pm 78.5/\mp 79.4]_s$
	16	0.0219	$[\mp 78.8/\pm 78.8/\pm 78.3/\pm 79.9]_s$
	20	0.0427	$[\mp 79.4/\pm 78.7/\mp 78/\mp 76.7/\pm 75.9]_s$
	24	0.0738	$[\pm 78.7/\mp 79.3/\mp 80.2/\pm 78.4/\mp 78.9/\mp 76.1]_s$

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Table 5.6. (cont.)

1/2	28	0.1172	[±79.2/∓78.2/±80.3/∓78.7/±76.8/±74.9/∓75]$_s$
	32	0.1750	[±79.3/±78.5/∓78.6/∓78.9/∓79.7/±76.6/∓76.3/±71.4]$_s$
	36	0.2491	[∓78.8/±78.6/±78.2/∓78.7/±81.4/∓77.4/∓80.4/±78.1/∓75.8]$_s$
	40	0.3417	[∓79.2/±78.9/∓78.3/±77.7/∓80.5/∓79.8/∓77.1/∓78.2/∓77/∓72]$_s$
	44	0.4549	[∓78.9/∓79.4/∓78.9/±78.4/±78.5/∓79.9/±78.1/±78.5/∓74.1/ ±77.7/∓69.4]$_s$
	48	0.5906	[±78.8/∓78.8/∓79/∓78.8/±79.47∓78.6/±78.3/∓78.2/∓78.7/±77 / ±76/±72.6]$_s$
	52	0.7508	[∓79.1/±78.8/∓78.7/±78.9/±79/∓78.7/±78.8/∓78.6/∓78.6/∓77.5/ ±77.4/±76.1/∓74.7]$_s$
	56	0.9378	[∓80.1/∓78.6/±78.5/∓78.7/±78.3/±78.8/∓79.1/±78.7/∓78.9/±77.7/ ∓75.5/∓75.37±73.2/±73.3]$_s$
	60	1.1534	[∓78.8/±78.8/∓78.8/∓78.8/±79/∓78.8/∓79.1/±78.6/±78.6/∓78.8/±78.4/ ±78.4/±77.2/∓76.1/±70.2]$_s$
1	4	0.0006	[±70.8]$_s$
	8	0.0047	[∓70.8/±70.8]$_s$
	12	0.0160	[±70.7/±71.1/±70.5]$_s$
	16	0.0379	[−70.9/∓70.8/±70.4/±69.2]$_s$
	20	0.0740	[∓70.9/∓70.6/±71.2/∓70.5/∓71.6]$_s$
	24	0.1279	[∓70.9/±70.8/±70.7/±70.6/±70.6/∓70.7]$_s$
	28	0.2032	[∓70.9/∓70.8/∓70.9/∓70.8/∓70.7/±70.5/±68.5]$_s$
	32	0.3033	[∓71/∓70.7/∓70.8/±70.6/±70.2/±71.4/∓70.6/∓68.9]$_s$
	36	0.4318	[±70.8/∓70.9/±70.7/±70.9/∓70.8/±70.7/∓70.6/∓70.7/±67.6]$_s$
	40	0.5923	[∓70.8/∓70.6/∓70.9/±70.6/±70.7/∓70.8/∓70.6/∓73.5/∓72.5/±60.8]$_s$
	44	0.7883	[∓71.2/±71.1/±70.9/∓70.9/±70.8/∓70.3/∓70/∓69.7/∓68.3/∓65.2/ ±52.7]$_s$
	48	1.0234	[∓70.7/±70.8/±71/∓71.1/∓70.7/±70.8/±70.8/±71.2/∓70.8/±68.9/ ±64.4/∓53.1]$_s$

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Table 5.6. (cont.)

2	4	0.0009	$[\mp 62.2]_s$
	8	0.0074	$[\mp 62.2/\mp 62]_s$
	12	0.0248	$[\pm 62.2/\pm 62.2/\pm 62.1]_s$
	16	0.0589	$[\mp 62.2/\mp 62.2/\pm 62.1/\pm 61.6]_s$
	20	0.1150	$[\mp 62.2/\mp 62.2/\pm 62.2/\pm 62.1/\pm 61.6]_s$
	24	0.1988	$[\mp 62.2/\pm 62.2/\mp 62.2/\pm 62.2/\pm 62.1/\pm 61.5]_s$
	28	0.3156	$[\mp 62.4/\pm 62.1/\pm 62.2/\mp 62.2/\mp 62.1/\mp 61.5/\pm 53.1]_s$
	32	0.4711	$[\pm 62.3/\mp 62.3/\mp 62.3/\pm 62.2/\pm 62/\pm 61.7/\mp 60.6/\mp 53.7]_s$
	36	0.6708	$[\pm 62.3/\pm 62.3/\mp 62.3/\mp 62.2/\pm 62.2/\pm 62/\pm 61.6/\pm 60.5/\pm 53.5]_s$
	40	0.9202	$[\mp 62.3/\pm 62.1/\pm 62.1/\pm 62.3/\pm 62.8/\pm 62.6/\mp 62/\pm 61.4/\mp 55.8/\pm 57]_s$
	44	1.2248	$[\mp 62.2/\mp 62.1/\pm 62.1/\pm 62.8/\mp 62.5/\mp 62.3/\mp 62/\mp 62.1/\pm 60.4/\mp 58.7/\pm 52.2]_s$

Table 5.7. Optimum stacking sequence designs for N_x=3000 N/mm and *a/b* =2

N_x/N_y	**Number of Layers**	λ_{cb}	**Stacking Sequence**
1/2	4	0.0002	$[\pm 78.8]_s$
	8	0.0018	$[\mp 78.8/\mp 78.9]_s$
	12	0.0061	$[\pm 78.7/\mp 79/\pm 80.1]_s$
	16	0.0146	$[\pm 79.8/\mp 78/\pm 76.6/\pm 75.8]_s$
	20	0.0285	$[\mp 79/\mp 78.6/\mp 78.9/\mp 77.3/\mp 79.2]_s$
	24	0.0492	$[\mp 78.9/\mp 79.6/\pm 78.1/\pm 78/\mp 77.6/\pm 70.6]_s$

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Table 5.7. (cont.)

1/2	28	0.0781	[±79.5/±79.4/∓78.5/∓76.7/∓77.3/∓77.8/∓77.2]$_s$
	32	0.1166	[±78.9/∓78.7/∓78.7/∓78.8/∓78.6/±78.4/∓82.4/±75.1]$_s$
	36	0.1661	[±78.6/∓79.3/∓77.9/∓78.8/∓79.8/±79.4/±78.7/∓77.7/±70.9]$_s$
	40	0.2278	[±79.2/∓78.9/∓78.8/∓78.6/∓78.8/±78.4/∓78.6/±78.1/±73.1/±68]$_s$
	44	0.3032	[∓78.6/±79/±79.3/±78.4/±79.2/∓79.6/∓78.5/±76.8/±79/∓75.1/ ±69.2]$_s$
	48	0.3937	[±79/∓79/∓78.1/∓78.7/∓79.1/∓79.6/±77.3/±83.2/±76.9/∓76.3/ ∓74/∓74.6]$_s$
	52	0.5006	[±78.7/±79.1/±79.2/∓78.7/±78.6/±78.7/∓78.8/±78.5/±78.7/±77.3/ ∓78.2/∓74.2/±75.8]$_s$
	56	0.6251	[±78.7/∓79.3/∓79.2/±78.6/±80.1/±78.7/±77.9/±78/±78.1/∓79.7/ ±77.9/±72.8/∓74.2/∓37.6]$_s$
	60	0.7689	[±78.8/±78.6/±79.3/∓78.8/∓78.9/∓79.3/∓79/∓78.3/∓78.4/±78.8/ ±79/∓77.7/∓74.8/±68.7/±64.1]$_s$
	64	0.9331	[±78.5/∓77.4/±82.8/∓78.6/∓79.4/±78.2/±78.7/±77.9/∓77.5/±78.7/ ∓78.4/∓76.2/±80.8/±76./6±76.5/±70.2]$_s$
	68	1.1192	[±77.7/±82.4/±77.5/±77.8/±79.9/∓79.1/∓77.3/±77.4/±80.5/∓78/ ±81.9/∓78.3/∓78.4/∓77.1/±83.9/∓72.6/±66.5]$_s$
1	4	0.0004	[±70.8]$_s$
	8	0.0032	[±70.8/±70.9]$_s$
	12	0.0107	[±70.9/±70.6/±69.9]$_s$
	16	0.0253	[±71.1/∓70.5/∓70.1/±70.8]$_s$
	20	0.0494	[∓71/∓70.7/±70.6/±70.3/±67.9]$_s$
	24	0.0853	[∓70.9/∓70.8/±70.8/∓70.7/±70.2/±67.1]$_s$
	28	0.1354	[∓70.9/∓70.9/∓70.8/∓70.6/∓70.6/∓69.9/∓68.1]$_s$

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Table 5.7. (cont.)

1	32	0.2022	$[\mp 70.8/\mp 70.8/\pm 70.8/\mp 70.7/\pm 71.7/\pm 70.5/\pm 70/\mp 67.1]_s$
	36	0.2879	$[\pm 70.8/\pm 70.9/\pm 70.8/\pm 70.8/\mp 70.8/\pm 70.8/\mp 70.7/\pm 70/\pm 67.2]_s$
	40	0.3949	$[\mp 70.8/\pm 70.8/\mp 70.8/\pm 70.8/\mp 70.8/\mp 70.8/\pm 71.1/\mp 69.5/\mp 70/\pm 69.4]_s$
	44	0.5256	$[\pm 70.8/\pm 70.8/\mp 70.8/\mp 70.8/\pm 70.8/\mp 70.8/\pm 70.8/\pm 70.8/\pm 70.5/\pm 70.1/\mp 67.1]_s$
	48	0.6823	$[\pm 70.8/\mp 70.8/\mp 70.8/\pm 70.7/\mp 70.9/\mp 70.8/\mp 70.7/\pm 71.1/\pm 70.9/\pm 70.4/\mp 71.1/\pm 55.9]_s$
	52	0.8674	$[\mp 71.1/\pm 71/\pm 71/\pm 71/\pm 70.9/\mp 71.2/\pm 70.5/\mp 70.2/\mp 69.8/\pm 68.5/\mp 66.8/\mp 62.7/\pm 50.9]_s$
	56	1.0834	$[\pm 71/\mp 71/\pm 71/\mp 71/\mp 71/\pm 71/\mp 70.7/\mp 70.9/\mp 70.1/\mp 69.1/\pm 69.1/\mp 66.7/\mp 63.6/\pm 54.4]_s$
2	20	0.0767	$[\pm 62.2/\mp 62.2/\mp 62.2/\mp 62.1/\pm 61.2]_s$
	24	0.1325	$[\mp 62.2/\mp 62.2/\pm 62.2/\mp 62.2/\pm 62.1/\pm 61.2]_s$
	28	0.2104	$[\mp 62.2/\mp 62.2/\pm 62.2/\pm 62.2/\mp 62.2/\pm 62/\pm 61.2]_s$
	32	0.3141	$[\pm 62.2/\mp 62.2/\mp 62.8/\pm 61.7/\pm 62.1/\pm 62.4/\pm 59.3/\pm 55.6]_s$
	36	0.4472	$[\mp 62.3/\pm 62.3/\pm 62.3/\pm 62.2/\mp 62.1/\pm 61.9/\pm 61.3/\pm 59.7/\pm 51.3]_s$
	40	0.6135	$[\pm 62.6/\mp 62.4/\mp 62.1/\mp 62/\pm 62.2/\pm 61.1/\pm 61.7/\mp 61.1/\pm 60.4/\mp 59.5]_s$
	44	0.8165	$[\pm 62.2/\mp 62.3/\pm 62.3/\pm 62.3/\mp 62.2/\mp 62.1/\mp 62/\pm 62.1/\pm 61.3/\mp 59/\pm 46.6]_s$
	48	1.0601	$[\mp 62.1/\pm 62.2\mp 62.2/\mp 62.3/\mp 62.4/\mp 62.1/\mp 62.5/\pm 62.2/\pm 61.8/\mp 61.4/\mp 60.2/\mp 53.2]_s$

Tables 5.8-5.10 show the optimum designs for N_x=1000, 2000, 3000 and *a/b*=1/2, 1, 2. Plates with optimum number of layers (optimum thickness) and hence corresponding weights are tabulated in these tables.

Table 5.8. Weight of the optimum composite plates for N_x=1000 N/mm

N_x	N_x/N_y	a/b	Optimum Number of Layers	Optimum Weight (kg)
1000	1/2	1/2	68	14.08
		1	64	6.62
		2	48	2.48
	1	1/2	64	13.25
		1	56	5.79
		2	40	2.07
	2	1/2	60	12.42
		1	52	5.38
		2	36	1.86

Table 5.9. Weight of the optimum composite plates for N_x=2000 N/mm

N_x	N_x/N_y	a/b	Optimum Number of Layers	Optimum Weight (kg)
2000	1/2	1/2	84	17.39
		1	80	8.28
		2	60	3.10
	1	1/2	76	15.73
		1	72	7.45
		2	48	2.48
	2	1/2	76	15.73
		1	64	6.62
		2	44	2.27

Table 5.10. Weight of the optimum composite plates for N_x=3000 N/mm

N_x	N_x/N_y	*a/b*	**Optimum Number of Layers**	**Optimum Weight (kg)**
3000	1/2	1/2	96	19.88
		1	92	9.52
		2	68	3.52
	1	1/2	88	18.22
		1	80	8.28
		2	56	2.89
	2	1/2	84	17.39
		1	72	7.45
		2	48	2.48

All considered cases have been calculated and optimum laminated composites plate weights have been shown in the above tables. It is observed that the plate with *a/b=2*, it is possible to acquire lighter composite plates which could not be buckled in the same design conditions. To make it clear, the first condition which is N_x=1000 N/mm and N_x/N_y = 1/2 could be examined. It is seen from Table 5.8 that when the aspect ratio of laminated composite is 1/2, its weight is found 14.08 kg. On the other hand, when the aspect ratio of laminated composite is 2, its weight is obtained as 2.48 kg. Therefore, It can be understood that taking the laminated composite plate with smaller geometry (minimizing thickness) has important effect on buckling.

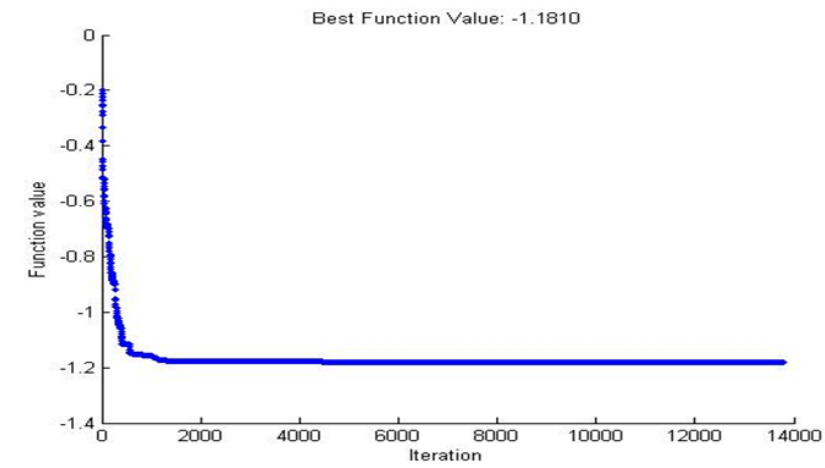
In order to find out the effect of the stacking sequence on weight, conventional designs are firstly examined and subsequently compared to the continuous designs which are obtained as a result of optimization. These comparisons are shown in Table 5.11. N_x and *a/b* are taken 1000 N/mm and 1/2, respectively.

Table 5.11. Weight of the optimum composite plates for both conventional and continuous design

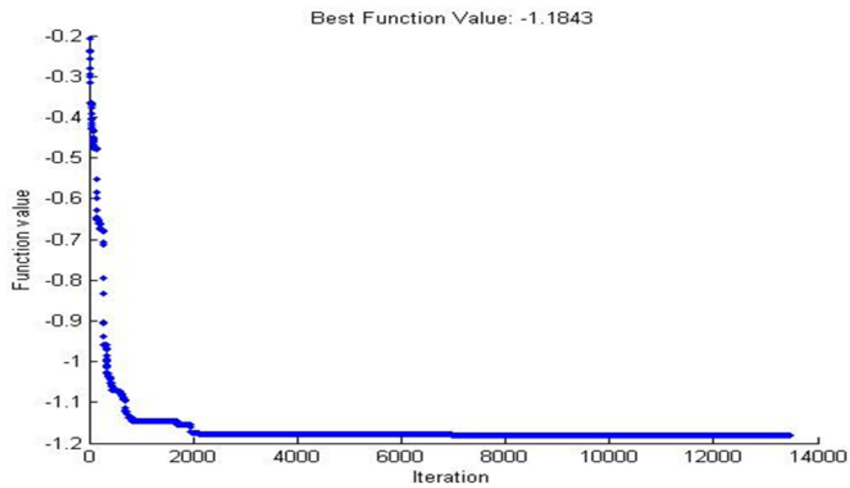
N_x/N_y	Number of Layers	λ_{cb}	Stacking Sequence	Weight (kg)
1/2	72	1.0276	$[\pm 45]_{18s}$	14.91
	76	1.0619	$[0/90]_{19s}$	15.73
	68	1.1302	$[\pm27.7/\pm27.8/\pm27.8/\mp27.8/\pm27.9/\pm27.7/\mp27.8/\mp28.2/\mp28.1/\mp27.7/\mp27.7/\pm27.6/\pm27.9/\mp26.2/\pm27.0/\mp25.6/\mp30.2]_s$	14.08
1	68	1.0388	$[\pm 45]_{17s}$	14.08
	72	1.0957	$[0/90]_{18s}$	14.91
	64	1.2129	$[\pm19.4/\mp19.4/\mp19.4/\pm19.3/\mp19.3/\pm19.3/\mp19.1/\mp18.8/\mp19/\mp19/\pm18.9/\pm17.3/\pm17.6/\mp16/\pm3/\mp4.5]_s$	13.25
2	68	1.1542	$[\pm 45]_{17s}$	14.08
	68	1.0274	$[0/90]_{17s}$	14.08
	60	1.1533	$[\mp10.5/\pm12/\mp11/\mp12.4/\mp11.9/\pm12.2/\pm12.5/\pm10.3/\pm5.9/\mp9/\mp2.8/\pm2.5/\pm14/\pm17.8/\mp33.7]_s$	12.42

In Table 5.11, it is observed that it is possible to achieve lighter composites using continuous design. These designs provide significant weight reduction. It can be seen that approximately 5%-12% weight reductions can be obtained. It is known that conventional designs have many advantages in industry, however weight reduction and higher buckling load capacity are attractive for the manufacturer where the lightweight is important. Because of these reasons, manufacturers are inclined to produce continuous designs.

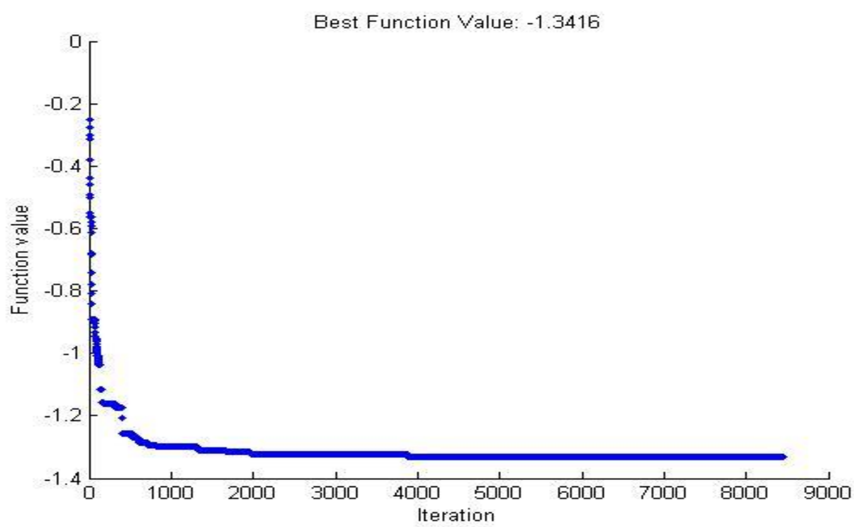
Simulated Annealing algorithm for different cases have been studied and some specific cases (N_x =1000 N/mm loading, *a/b* = 2 plate ratio, N_x/N_y = 1/2, 1, 2 load ratios) depending on the best function values and iterations are given in Figure 5.2. The best function value corresponds to critical buckling load factor (objective function) value. It is seen from the figures that firstly the best function values start to converge quickly and then improve more slowly. Finally, it reaches the optimum point after satisfying the stopping criteria (Func. tolerance=10^-6).



(a)



(b)



(c)

Figure 5.2. The best function values of the objective functions at each iteration in SA for (a) Nx/Ny = 1/2, (b) Nx/Ny = 1, (c) Nx/Ny = 2, N_x =1000 N/mm is taken as constant

The effectiveness and reliability of the algorithm can be shown in details by considering the specific case (N_x =1000 N/mm, a/b = 2, N_x/N_y = 2). The best function values for this case have been represented for each run in Figure 5.3. It is observed that six global optima have been achieved in the range of 1.3402 and 1.3416 and shown in red.

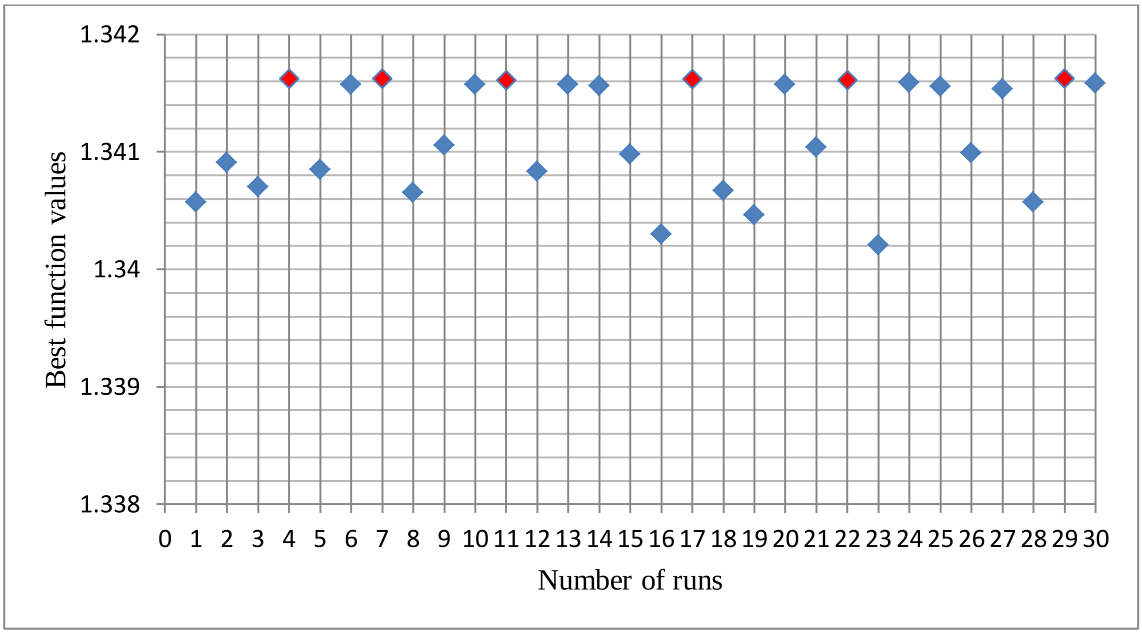


Figure 5.3. Best function values versus number of runs

CHAPTER 6

CONCLUSIONS

In this thesis, the stacking sequences of N–layered carbon/epoxy laminated composite plates subjected to in-plane compressive loading have been investigated and the plates are assumed symmetric and balanced. The aim of this study is to find the optimum stacking sequence designs of laminated composite plates having the minimum thickness which resist to buckling under the given loading conditions (N_x/N_y =1, 2, 1/2) and different aspect ratios (*a/b*=1, 2, 1/2) . N_x has been taken as 1000 N/mm, 2000 N/mm and 3000 N/mm; the length of the plate *a* is taken constant and equals to 0.508 m. The critical buckling load factor is taken as objective function and fiber angles of the composite plates are taken as continuous design variables. A stochastic search technique Simulated Annealing algorithm (SA) has been considered as an optimization method. MATLAB Global Optimization tool has been utilized in optimization process. In order to increase the reliability of SA and obtain the best design, nonlinear optimization solver *fmincon* is used as hybrid function and 30 independent searches have been performed.

The specific studies previously investigated in the literature are utilized to verify the optimization algorithm of critical buckling load factor.

Optimization of the composite plates which has specified design conditions has been studied and the optimum designs in terms of buckling have been investigated to obtain the lightweight composite. The optimization is performed by increasing the number of layers four by four and each of the configurations is checked to obtain the sufficient critical buckling load factor ($\lambda_{cb} \geq 1$).

As the result of the optimization, buckling load capacity increases with respect to increasing loading ratio for the same number of layers. In addition to this, it is observed that plate aspect ratios have significant effect on critical buckling load factor. When the aspect ratio is equal to 1/2 (a/b), the smallest critical buckling load factor is obtained and hence the resistance to buckling becomes low. On the other hand, the most resistant design is achieved when the plate aspect ratio is equal to 2. The stacking sequences are found to have both continuous and discrete fiber angles depending on the aspect ratios. The optimum fiber orientation angles are formed as the combination of ± 45 and ∓ 45 for all load ratios and $a/b = 1$. Continuous fiber orientations angles are obtained when the aspect ratio is equal to 1/2 and 2. As it can be understood from the tables, when the aspect ratio is increased, for each applied load case the optimum number of layers decreases at each loading ratio. It means that it is possible to acquire lighter composite plates which could not be buckled with the same applied loads and loading ratios using smaller geometry. Minimum thickness and hence minimum weight have been achieved by using the aspect ratio of 2. In addition to this, conventional and continuous designs have been compared to determine the effect of the stacking sequence on weight. As a result of this, it can be said that the optimum continuous designs have higher buckling load capacity and they are lighter than conventional designs.

It is understood that critical buckling load factor plays an important role to specify the optimum design of laminated composites. In order to obtain composite plates having a sufficient buckling load capacity ($\lambda_{cb} \geq 1$) and minimum thickness, the number of layers or geometry of the composite plate should be changed. It is also concluded that use of the geometrically smaller plates has an important effect on buckling and weight optimization.

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APPENDIX A

MATLAB COMPUTER PROGRAM

In this section, the computer program calculating the buckling load factor of the laminated composites is given.

```
function f=discrete_buckling(x)
% x = round(x);
% 1 psi=6894.757 Pa
% 1 in=0.0254 m
E11=116.6e9; %[psi]
E22=7.673e9; %[psi]
G12=4.173e9; %[psi]
v12=0.27;
a=0.508; %[in]
Nx=5000000; %[lbf/in] load in the x-direction
k=2; % load ratio
Ny=Nx/k;
r=2; % plate aspect ratio
b=a/r;
N=64; % number of plies
N4=N/4;
N2=N/2;
tp=0.25e-3*N; % total plate thickness [in]
v21=v12*(E22/E11);
q11=E11/(1-v12*v21);
q12=v21*E11/(1-v12*v21);
q22=E22/(1-v12*v21);
q66=G12;


D=zeros(3,3);
M=N+1;
for j=1:M
z(j)=-tp/2+(j-1)*tp/N;
end
j=1;
for i=1:N4
x1(j)=x(i);
x1(j+1)=-x(i);
j=j+2;
end
for i=1:N2
x(i)=x1(i);
end
```

```
for i=1:N2
x(N2+i)=x(N2-i+1);
end


for k=1:N
m=cos(x(k)*pi/180);
n=sin(x(k)*pi/180);
Q11=q11*m^4+2*(q12+2*q66)*n^2*m^2+q22*n^4;
Q12=(q11+q22-4*q66)*n^2*m^2+q12*(n^4+m^4);
Q22=q11*n^4+2*(q12+2*q66)*n^2*m^2+q22*m^4;
Q16=(q11-q12-2*q66)*n*m^3+(q12-q22+2*q66)*n^3*m;
Q26=(q11-q12-2*q66)*n^3*m+(q12-q22+2*q66)*n*m^3;
Q66=(q11+q22-2*q12-2*q66)*n^2*m^2+q66*(n^4+m^4);
Q=[Q11 Q12 Q16;Q12 Q22 Q26;Q16 Q26 Q66];
D(1,1)=D(1,1)+Q11*(z(k+1)^3-z(k)^3)/3;
D(1,2)=D(1,2)+Q12*(z(k+1)^3-z(k)^3)/3;
D(1,3)=D(1,3)+Q16*(z(k+1)^3-z(k)^3)/3;
D(2,2)=D(2,2)+Q22*(z(k+1)^3-z(k)^3)/3;
D(3,3)=D(3,3)+Q66*(z(k+1)^3-z(k)^3)/3;
D(2,3)=D(2,3)+Q26*(z(k+1)^3-z(k)^3)/3;
end

D(2,1)=D(1,2);
D(3,2)=D(2,3);
D(3,1)=D(1,3);


m1=1;
n1=1;
LAMDA1=pi^2*(m1^4*D(1,1)+2*(D(1,2)+2*D(3,3))*m1^2*n1^2*r^2+n1^4*
r^4*D(2,2))/(m1^2*a^2*Nx+r^2*a^2*n1^2*Ny);
m1=1;
n1=2;
LAMDA2=pi^2*(m1^4*D(1,1)+2*(D(1,2)+2*D(3,3))*m1^2*n1^2*r^2+n1^4*
r^4*D(2,2))/(m1^2*a^2*Nx+r^2*a^2*n1^2*Ny);
m1=2;
n1=1;
LAMDA3=pi^2*(m1^4*D(1,1)+2*(D(1,2)+2*D(3,3))*m1^2*n1^2*r^2+n1^4*
r^4*D(2,2))/(m1^2*a^2*Nx+r^2*a^2*n1^2*Ny);
m1=2;
n1=2;
LAMDA4=pi^2*(m1^4*D(1,1)+2*(D(1,2)+2*D(3,3))*m1^2*n1^2*r^2+n1^4*
r^4*D(2,2))/(m1^2*a^2*Nx+r^2*a^2*n1^2*Ny);
LAMDA=[LAMDA1 LAMDA2 LAMDA3 LAMDA4];
f=-min(LAMDA);
```