STACKING SEQUENCE OPTIMIZATION OF THE ANTI-BUCKLED GRAPHITE/EPOXY LAMINATED COMPOSITES FOR MINIMUM WEIGHT USING GENERALIZED PATTERN SEARCH ALGORITHM

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ABSTRACT

STACKING SEQUENCE OPTIMIZATION OF THE ANTI-BUCKLED GRAPHITE/EPOXY LAMINATED COMPOSITES FOR MINIMUM WEIGHT USING GENERALIZED PATTERN SEARCH ALGORITHM

Composite materials have been increasingly used during the last decades due to their properties such as low weight, high stiffness, superior fatigue and corrosion resistance. They have been used in aerospace, automobile, marine applications and etc. Composite materials being an expensive but efficient technology to get minimum weight structures, it is logical to make an attempt to find out how to design properly optimum laminated composite plates with no reduction in their strength. The aim of the thesis is to find the optimum stacking sequence to obtain the minimum thickness (weight) of laminated composite plates in different loadings and plate dimensions under buckling constraint. Moreover, a comparison study of conventional and continuous designs are performed to determine the effect of stacking sequence on weight. The objective function is the critical buckling load factor. Fiber angles of the composite plates are taken as continuous design variables and the plate is assumed to be balance and symmetric. Composite plates made of graphite/epoxy have been considered in this thesis. A combination of Generalized Pattern Search Algorithm (GPSA) and Genetic Algorithm (GA) has been considered as an optimization method. All the results show that the loading conditions and dimensions of composite plates are significant in stacking sequences optimization of laminated composite materials in terms of maximum critical buckling load factor and minimum thickness.

ÖZET

GENELLEŞTİRİLMİŞ MODEL ARAMA ALGORİTMASI KULLANILARAK BURKULMAYAN TABAKALI GRAFİT/EPOKSİ KOMPOZİTLERİN MİNİMUM AĞIRLIK İÇİN TABAKA DİZİLİMLERİNİN OPTİMİZASYONU

Son yıllarda, fiber katkılı tabakalı kompozit malzemeler, yüksek dayanıklılığa sahip ve hafif olmalarından dolayı, otomobil ve havacılık gibi birçok mühendislik uygulamalarında kullanılmaktadır. Kompozit malzemeler pahalı olmasına karşın hafif yapılar elde edebilmek için elverişli bir teknolojidir. Bu yüzden, dayanımlarında azalma olmaksızın optimum tabakalı kompozit plakaların tasarımlarını bulmaya çalışmak mantıklı bir girişim olacaktır. Bu çalışmada, minimum kalınlıkta ve burkulmaya dayanıklı çok katmanlı kompozit malzemelerinin optimum tabaka dizilimi tasarımları farklı en-boy oranlarında ve yükleme koşullarına göre incelenmiştir. Ayrıca yaygın olarak kullanılan açılar (geleneksel) ve sürekli açılarla yapılan tabaka dizilimleri karşılaştırılıp, ağırlık üzerindeki etkiside belirlenmiştir. Kritik burkulma yükü faktörü amaç fonksiyonu olarak alınmıştır. Grafit /epoksi kompozit plakalar, balans ve simetrik bir yapıya sahip plakalar olarak değerlendirilmiştir. Sürekli fiber yönlenme açıları da tasarım parametresi olarak düşünülmüştür. Genelleştirilmiş model arama algoritması (GPSA) ve genetik algoritmasının (GA) kombinasyonu optimizasyon methodu olarak kullanılmıştır. Bütün sonuçlar incelendiğinde, yükleme koşullarının ve plaka ölçülerinin tabakalı kompozitlerin maksimum burkulma yük kapasitesi ve minimum ağırlık açısından kritik önem taşıdığı ve fiber yönlenme açılarınında kırılmanın burkulmadan veya kırılma kriterlerinden kaynaklandığını belirlemede etkili olduğunu göstermektedir.

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CHAPTER 1

INTRODUCTION

Fiber-reinforced composite material is material system composed of two or more macro constituents that differ in shape and chemical composition and which are insoluble in each other. Fiber-reinforced composites are used extensively in the form of relatively plate, and consequently the load carrying capability of composite plate against buckling has been intensively considered by researchers under various loading and boundary conditions. In many engineering structures such as columns, beams, or plates, their failure develops not only from excessive stresses but also from buckling. They have been used in aerospace, automobile, marine applications and etc. due to their low weight, high stiffness, superior fatigue and corrosion properties.

Optimization of composite structures is a recent issue, because both optimization techniques and composite structures have been developed during the last few decades and therefore, the conjunction of both of them is even more recent. Composite materials being an expensive but efficient technology to get minimum weight structures, it is logical to make an attempt to find out how to design properly optimum laminated composite plates with no reduction in their strength.

Many researchers have investigated studies using various optimization techniques considering several optimization objectives in the literature for the optimization of laminated composite materials. Generalized pattern search, genetic algorithm, simulated annealing algorithm, tabu search and ant colony optimization are most commonly used method of the stochastic optimization methods in the literature. One of the most commonly used the design objective is weight or thickness minimization of the laminated composite plates.

The earliest attempt in composite optimization seems to be due to Foye (1968)**,** who studied the minimum weight optimum design of laminated composite for strength and membrane stiffness. Multiple in-plane loading conditions were considered, and a random search method was used to find ply orientation angles, such that the strength and stiffness requirement would be satisfied with the smallest number of plies. Another procedure for the optimum design of laminates was reported by Waddoups (1969).

Minimum weight designs are obtained by considering strength constraints under multiple distinct loading conditions. Either the Tsai-Hill or the maximum strain criterion may be used, and all laminae are assumed to behave linearly to failure.

Many researchers tried to make a better laminated composite material either by increasing static strength of composite laminates or reducing the weight for a given thickness. Especially, genetic algorithm, which is stochastic optimization method, was firstly studied to obtain the minimum thickness design of composite laminated plate by Riche and Haftka (1995). The thinnest symmetric and balanced laminates satisfying the 4-ply contiguity constraint which do not fail because of the fact that buckling or excessive strains are obtained (Le Riche and Haftka 1995). Minimum weight design of composite laminates is presented using genetic algorithm considering the failure mechanism based, maximum stress and Tsai-Wu failure criteria as design constraints for different in-plane loading conditions and different ply orientations, which are defined as the design variable (Naik, et al. 2008).

The problem of weight minimization of composite plates subjected to critical buckling load and maximum displacement constraints was studied by Adali, Richter and Verijenko (1997) where the fibre orientations are the design variables. The results for both single and multiple load conditions show that a minimum plate total thickness is reached. In stacking sequence optimization studies, the usage of fiber orientations angles as design variables can be divided into two groups. The first one contains research that modeled the orientation angle as a continuous variable by Adali (2003). This approach may lead to a non-optimal or out of feasible region stacking sequence during the manufacturing. The second one contains studies that modeled the orientation angle as a discrete variable by Irisarri (2009) and Erdal and Sonmez (2005) . In this study adopts the continuos modeling of the orientation angles.

Some researchers have included the minimum weight design to minimum deflection or minimum cost design objectives. The minimum deflection and weight designs of laminated composite plates with four layers considering various boundary conditions, varying aspect ratios and different loading types are given separately using the finite element method based on Mindlin plate theory in conjunction with optimization routines (Walker, et al. 1997).

The buckling load capacity of a composite plate under in-plane compressive loads is crucial for the design of the composite structures. The buckling could cause a premature failure of the structure. Therefore, buckling load maximization is a critical issue that many researchers deal with. The stacking sequence design of a composite laminate for buckling load maximization considering strain failure has been studied using genetic algorithm by Haftka and Le Riche (1993). Their study involved the application of genetic algorithm technique to stacking sequence and minimum number of laminae optimization problems, to a simple rectangular composite laminate, subjected to buckling, strain and contiguity constraints.

The optimization of the critical buckling load of composite plates is an issue of great interest. Therefore, researchers have studied the optimum design of composite laminates for buckling load. In Chao and Koh (1975) and Hirano (1979), the ply thickness and number of plies are constant while the fibre orientations vary. The closed form solution of the buckling problem of simply supported composite plates is used and an optimization search is conducted for plates with different aspect ratios and load conditions. Since the total thickness is constant, the structural weight is also constant and the critical budding load is maximized.

Erdal and Sonmez (2005) presented optimization of laminated composite plate for maximum buckling load using simulated annealing that is one of the most popular stochastic optimization techniques.

Soykasap and Karakaya (2007, 2009) have investigated the critical buckling load for various load cases such as biaxial load and uniaxial load, and different plate aspect ratios. They showed that, not only uniaxial and biaxial loadings but also pure shear loading and the combination of shear and biaxial loadings change the optimal solutions for maximum buckling load.

Generalized pattern search algorithm (GPSA) represent a subclass of direct search algorithms, in which the minimizer of a continuous function is sought without the use of derivatives. GPSA is a mostly local search method and the use of the algorithm in composite optimization is very few. GPSA has been used for optimal stacking sequence of a 64-layer composite plate made of graphite epoxy by Karakaya and Soykasap (2009). Generalized pattern search algorithm has not been used for the stacking sequence optimization of the laminate composites before Karakaya and Soykasap because this algorithm is mostly a local algorithm. They have compared the method with GA and GPSA, and concluded that the Genetic Algorithm is expensive but more effective in finding distinct global optima than generalized pattern search algorithm.

Another study about GPSA is minimization of energy consumption in the workplace and house was studied by Wetter and Wright (2003). They have compared GPSA with GA. Genetic Algorithm has given better solution because they have thougt that the problem has many local minima. But they have added GPSA is more effecient.

Hough, Kolda and Torczon (2000) have studied on GPSA in their earlier work "while the theory for pattern search assumes that an objective function is continuously differentiable, pattern search methods can be effective on nondifferentiable (and even discontinuous) problems precisely because they do not explicitly rely on derivative information to drive the search."

In this study, optimal stacking sequence designs of laminated composite plates for maximum buckling load and minimum weight are determined using generalized pattern search algorithm (GPSA). Symmetric and balanced composite plates which are simply supported on four sides, are analyzed under different load conditions and aspect ratios (length to width). Fiber orientation angle in each layer is taken as a design variable and the orientation angle as a continuous variable. Design constraint is based on critical buckling load factor.

CHAPTER 2

COMPOSITE MATERIALS

2.1. Definition and Basic Characteristics

A composite material consists of two or more separable materials which are combined in a macroscopic structural unit (Gibson, 1994). The most important advantage of using a composite structural material compared to other structural materials such as metals, ceramics or polymers, is that the specific strength (the ratio of tensile strength to density) of composites is much higher. In addition in applications such as aircraft or spacecraft where weight reduction is important, composites can play a significant role.

The main idea of composite structures is to utilise the materials in the best possible way by tailoring the material to the application. A composite material is thus not just used in an immediate form but designed to meet the specified requirements. From mechanics of materials it is well known that the maximum stresses happen in a certain direction. Thus, having uniform strength of the material in all directions leads to a natural "oversizing" in the non-maximum directions. This oversizing is strongly reduced in laminated composite structures as the material is designed to have directional strength where needed.

The properties of a composite material depend on the properties of the constituents, geometry and distribution of the materials. One of the most important parameters for composite materials is volume fraction and it represents fiber volume ratio of composite structure. Volume fraction can be defined by burn-out test. The distribution of reinforcing fibers defines homogeneity or heterogeneity. The more heterogeneity areas of composite structure, the higher are the possibility of failure in the weakest areas.

The fiber orientation and geometry cause the isotropy or orthotropy. If the composite material properties such as stiffness, strength, thermal expansion and thermal conductivity are the same in all directions, it is called that isotropic composite. Otherwise an anisotropic material properties are vary with direction or fiber orientation

(Daniel and Ishai 1994). Common metals, such as steel and aluminum alloys, are isotropic, due to showing the same property values for all directions. However, a fiber-reinforced composite is not isotropic materials because of that the properties depend on the direction of measurement.

Directional strength is not the only possibility of tailoring a composite material to the application. A wide variety of properties can be improved by the use of composite materials. Some of these properties are strength, stiffness, fatigue life, weight and etc. (Jones, 1999). In addition, composites show more superior properties than single phase material. For example, some of carbon reinforced composites are ten times stronger than steel and lighter. On the other hand, fabricating techniques used in composite production increase the cost of composites, for this reason, the main challenge of the composite world is to reduce cost of the laminated materials (Staab, 1999). The most commonly used advanced composites are polymer matrix composites which have a polymer (e.g., epoxy, polyester, Vinyl Ester, urethane) reinforced by fibers (e.g., carbon, graphite, aramids, kevlar).

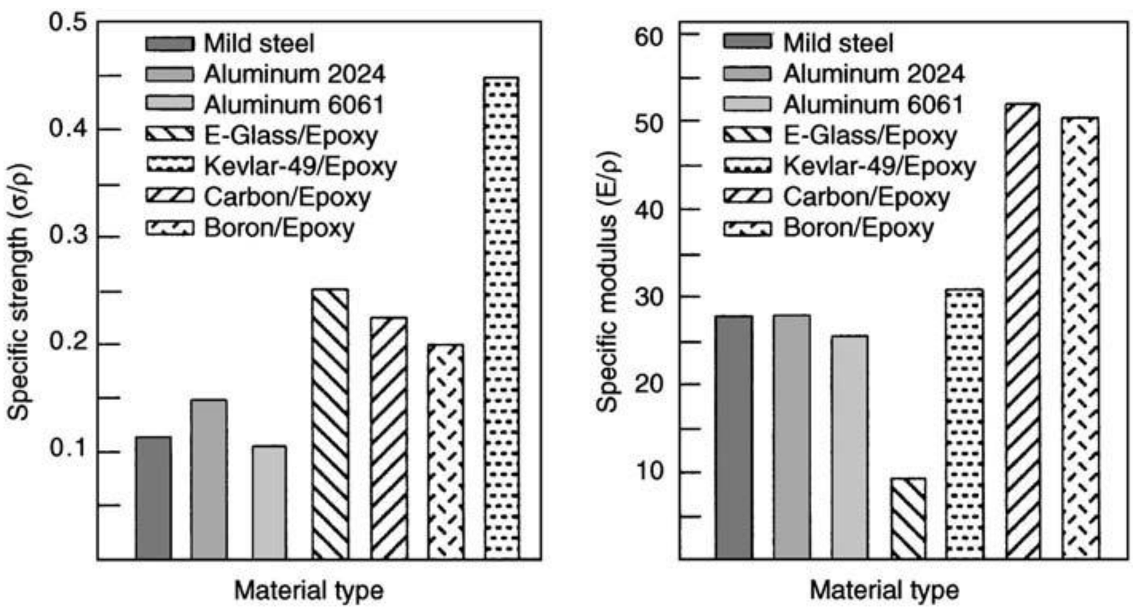

Figure 2.1. Specific strength and stiffness comparison for selected composites and conventional bulk materials (Source: Staab,1999)

A composite offers strength-to-weight and stiffness-to-weight ratios superior to those of conventional materials. Figure 2.1 represent the comporasion of strength and stiffness for several composite material As seen in these figures, a wide range of specific strength and specific modulus are available. In some instances strength may be a primary consideration, while in others the stiffness is more important. In all cases shown the specific strength for the composite material systems is better than that for the conventional materials, whereas the specific modulus is not always superior (Staab, 1999).

Although composite materials have certain advantages over conventional materials, composites also have some disadvantages (Jan Gou);

- Fabrication of composites' costs are higher than metals .
- Mechanical definition of a composite structure is more complicated than a metal structure.
- Repair of composites is a complex process compared to that for metals
- Composites don't have a high combination of strength and fracture toughness compared to metals.
- Composites don't necessarily give higher performance in all the properties used for material selection: strength, toughness, formability, joinability, corrosion resistance, and affordability.

2.2. Types and Classification of Composite Materials

Composite materials are usually categorized according to the type of reinforcement used. Most made composite materials are produced by two materials. A reinforcement material called fiber and a base material, called matrix material.

Composite materials are commonly formed in three different types. Fibrous composites, which consist of fibers of one material in a matrix material of another. Particulate; which are composed of macro size particles of one material in a matrix of another. Laminated composites, which are made of layers of different materials, including composites of the first two types. Each has unique properties and application potential, and can be subdivided into specific categories (J.N. Reddy,2004).

A fibrous composite consists of either continuous (long) or chopped (whiskers, short) fibers suspended in a matrix material (Staab,1999). A continuous fiber is geometrically defined as having a very high length-to-diameter ratio. They are generally stronger and stiffer than bulk material. The geometry and physical makeup of a fiber are somehow crucial to the evaluation of its strength and must be considered in structural applications.

A whisker (discontinuous) is generally considered to be a short, stubby fiber. Discontinuous fibers have actually the same near crystal sized diameter as a fiber, though the length to diameter ratio can be in the hundreds (Jones, 1999).

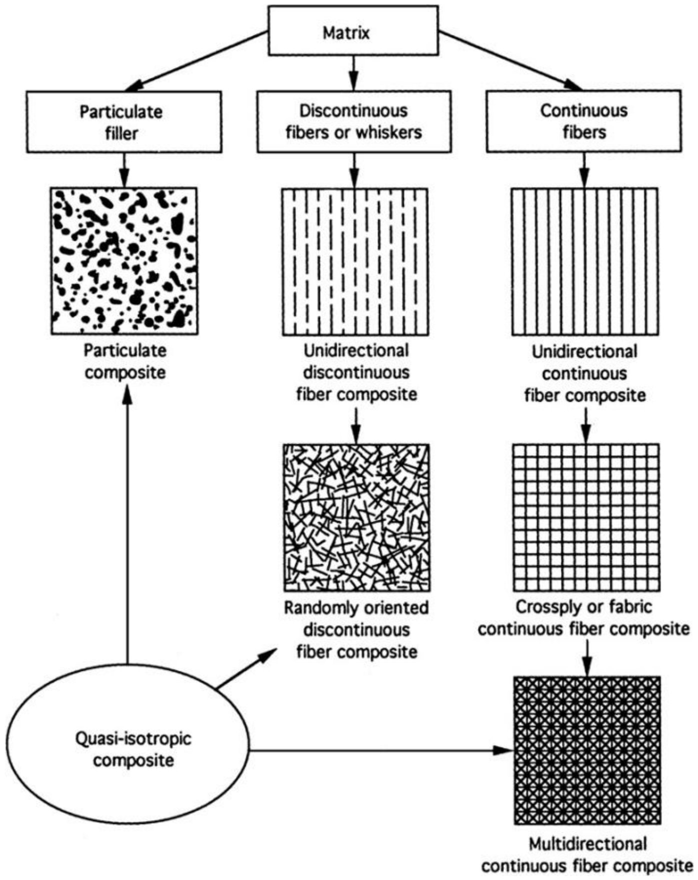

Figure 2.2. Classification of composite material systems
(Source: Daniel and Ishai 1994)

The reinforcements in the composites can be manufactured as a whiskers or fibers. hence, they have either random or biased orientation. Material systems composed of discontinuous reinforcements are considered single layer composites. The discontinuities can manufacture a material response that is anisotropic, but in many instances the random reinforcements manufacture nearly isotropic composites. Continuous fiber composites can be either single layer or multilayered. The single layer continuous fiber composites can be either unidirectional or woven, and multilayered composites are generally referred to as laminates. Fiber composites consist of matrices as the continuous phase, reinforced fibers (short (discontinuous) or long (continuous)) and an interface. Carbon, graphite, aramids, boron, and kevlar can be selected as fibers for composites and they are generally anisotropic. Resins such as epoxy, vinylester, polyester; metals such as aluminum, magnesium or titanium, and ceramics such as calcium–alumina silicate are examples of matrices. Continuous fiber matrix composite materials include unidirectional or woven fiber laminas. Laminas are stacked on top of each other at various angles to form a multidirectional laminate.

The table below presents properties of various kinds of fibres.

Table 2.1. Properties of glass, aramid and carbon fibres (Source: Zobel H,2004)

Typical properties	Fibres					
	glass		aramid		carbon	
	E-Glass	S-Glass	Kevlar 29	Kevlar 49	HS (High Strength)	HM (High Modulus)
Density ρ [g/cm³]	2,60	2,50	1,44	1,44	1,80	,190
Young´s Modulus E [GPa]	72	87	100	124	230	370
Tensile strength R_m [MPa]	1,72	2,53	2,27	2,27	2,48	1,79
Extension [%]	2,40	2,90	2,80	1,80	11,00	0,50

The material response of a continuous fiber composite is generally orthotropic. Figure 2.3. is schematics of both types of fibrous composites.

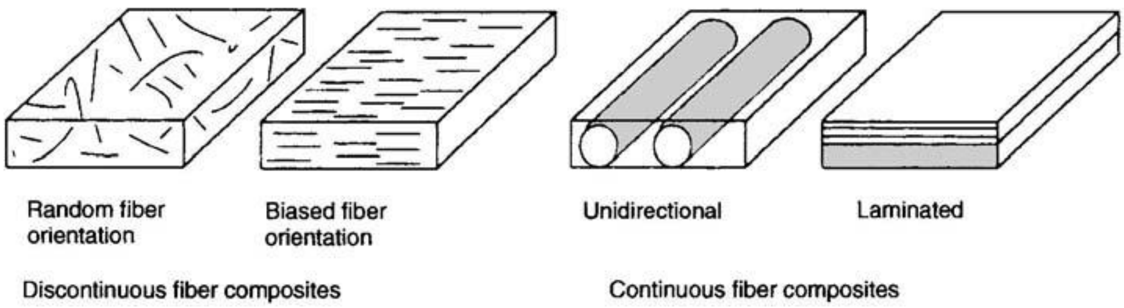

Figure 2.3. Schematic representation of fibrous composites (Source: Staab,1999)

A particulate composite is defined as being composed of particles suspended in a matrix. Particles can have virtually any shape, size or configuration. Examples of well-known particulate composites are concrete and particle board. They are isotropic because of the randomness of particle distribution. Particular composites can be categorized into four groups. These are nonmetallic particles in a nonmetalic matrix (glass feinforced with mica flakes), metallic particles in nonmetallic matrices (aluminium particles in polyurethane), metallic particles in metallic matrices (lead particles in copper alloys) and nonmetallic particles in metallic matrices (silicon carbide particles in aluminium). A flake composite is generally composed of flakes with large ratios of platform area to thickness, suspended in a matrix material (particle board, for example). A filledskeletal composite is composed of a continuous skeletal matrix filled by a second material: for example, a honeycomb core filled with an insulating material. Particulates cause improved strength, increased operating temperature, oxidation resistance of composite materials. The response of a particulate composite can be either anisotropic or orthotropic. Such composites are used for many applications in which strength is not a significant component of the design. Figure 2.4 is schematic of several types of particulate composites (Staab,1999).

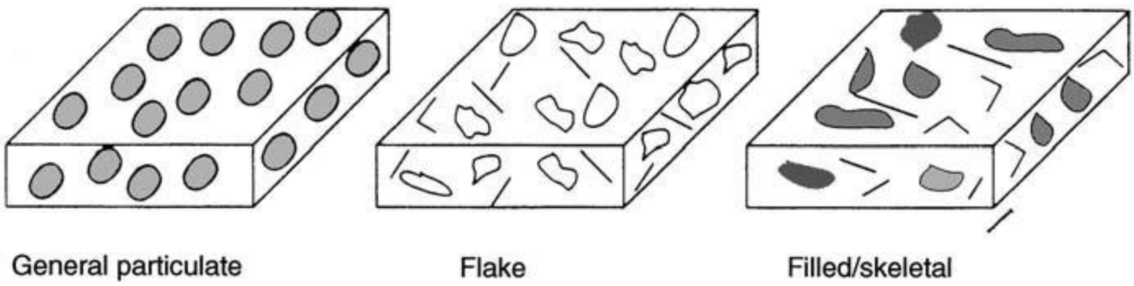

Figure 2.4. Schematic representation of particulate composites (Source: Staab,1999)

Laminate which is made by stacking a number of very thin layers of fibers can be defined as assemblages of layers of fiber-reinforced composite materials. In order to control the stacking sequence of various layers in a composite laminate properties including in-plane stiffness, bending stiffness, strength, and coefficients of thermal expansion, it can be generated Laminate composite is made of two dimensional sheets or panels that have a preferred high strength direction such as in wood and continuous and aligned fibre reinforced plastics. Layers are stacked and subsequently cemented together such that the orientation of high strength direction varies with each successive layer laminations may also be constructed using fabric material such as cotton, paper etc. Thus a laminate composite has relatively high strength in a number of directions. However the strength in any given direction is of course lower than it would be if all fibres were oriented in that direction..

2.3. Fundamental Composite Material Terminology

Some of the more prominent terms used with composite materials are defined below.

A lamina has been described as a thin single layer of composite material. An individual layer of the laminate. Also known as a ply. A lamina or ply is a typical sheet of composite materials, which is generally of a thickness of the order 1 mm. Both unidirectional and woven lamina are schematically shown in Figure 2.5.

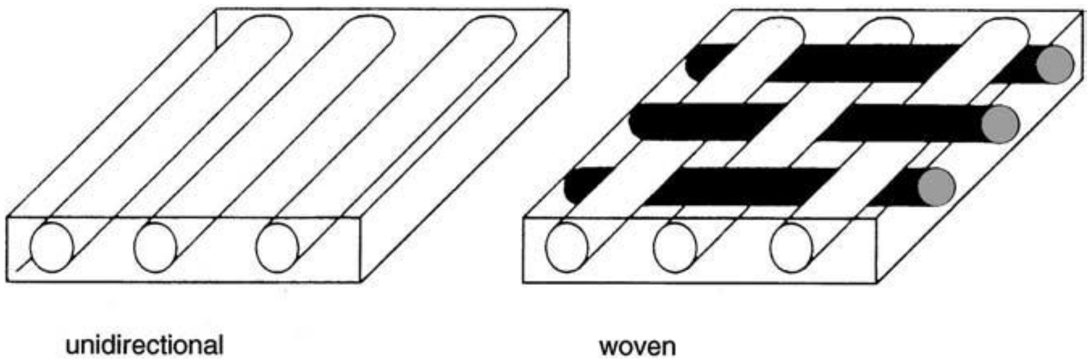

Figure 2.5. Schematic representation of unidirectional and woven composite lamina (Source: Staab,1999)

A laminate is constructed by stacking a number of lamina in the direction of the lamina thickness. A laminate is a stack of lamina, as illustrated in Figure 2.6, oriented in a specific manner to achieve a desired result. The layers are usually bonded together with the same matrix material as in the single lamina (Altenbach,2004).

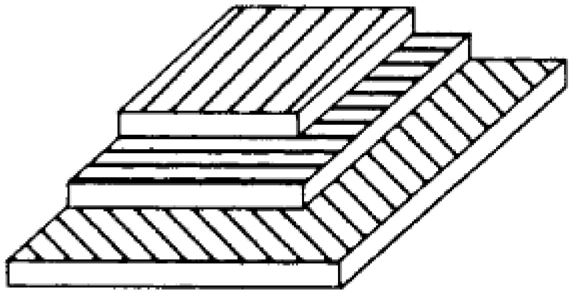

Figure 2.6. Schematic of a laminated composite (Source: Staab,1999).

Designers use reinforcements to make the composite structure or component stronger. The most extensively used reinforcements are boron, glass, graphite (often referred to as simply carbon), and Kevlar, but there are other types of reinforcements such as alumina, aluminum, silicon carbide, silicon nitride, and titanium.

The matrix is the binder material that supports, allocates, and protects the fibers. It provides a path by which load is both transferred to the fibers and redistributed among the fibers in the event of fiber breakage. The fibers typically has a higher density, stiffness, and strength than the matrix. Matrices can be brittle, ductile, elastic, or plastic. They can have either linear or nonlinear stress-strain behavior. In addition, the matrix material must be capable of being forced around the reinforcement during some stage in the manufacture of the composite. Fibers must often be chemically treated to ensure proper adhesion to the matrix. The most commonly used matrices are carbon, ceramic, glass, metal, and polymeric. Each has special appeal and usefulness, as well as limitations.

The most common advanced composites are polymer matrix composites including a thermoset (e.g., epoxy, polyimide, polyester) or thermoplastic (poly-ether-ether-ketone, polysulfone) reinforced by thin diameter fibers (e.g., graphite, aramids, boron). These composites have high strength, simple manufacturing technique and low cost. Metal matrix composites consist of metals or alloys (aluminum, magnesium, titanium, copper) reinforced with boron, carbon (graphite) or ceramic fibers. The materials are widely used to provide advantages over metals such as steel and aluminum. Some of these advantages contain higher specific strength and modulus by low density metals such as aluminum and titanium, lower coefficients of thermal expansion, such as graphite. Ceramic matrix composites have ceramic matrices (silicon carbide, aluminum oxide, glass-ceramic, silicon nitride) reinforced with ceramic fibers. Ceramic matrix composites have many advantages, some of these advantages are high strength, hardness, high service temperature limits for ceramics, chemical inertness and low density. However fracture toughness is low for ceramic matrix composites. Carbon-carbon composites use carbon fibers in the carbon or graphite matrix. They have excellent properties of high strength at high temperature, low thermal expansion and low density.

2.4. Applications of Composites

Composite structures are extensively used nowadays in range of components for automotive, aircraft, marine, satellite and even in consumer products such as golf, ski, and tennis. Military aircraft designers were among the first to recognize the importance of composites for weight reduction since performance of these vehicle heavily depend on weight. Examples of aircrafts with composite components are Boeing 757 and 756 where most of the body, wing and empennage are made out of long fiber and woven composite. Although composite have been used for a variety of aircraft parts, yet they have not been used for engine components due to their temperature and fuel exposure limitation (Gibson, 1994).

The main structural applications for fiber-reinforced composites are in the field of military and commercial aircrafts, for which weight reduction is critical for higher speeds and increased loads. Figure 2.7 shows the use of composite material in Boeing 787. With the use of carbon fibers in the 1970s, carbon fiber-reinforced composites have become the primary material in many wing, fusel age, and empennage components. The structural integrity and durability of components have increased confidence in their performance and developments of other structural aircraft components, so increasing amount of composite materials are used in military aircrafts. For example, the F-22 fighter aircraft also contains 25% by weight of carbon fiber reinforced polymers; the other major materials are titanium (39%) and aluminum (16%). The stealth aircrafts are almost all made of carbon fiber-reinforced polymers because of design features that have special coatings, reduce radar reflection and heat radiation. Furthermore, many fiber-reinforced polymers are used in military and commercial helicopters for making baggage doors, fairings, vertical fins, tail rotor spars and so on (Mallick 2007).

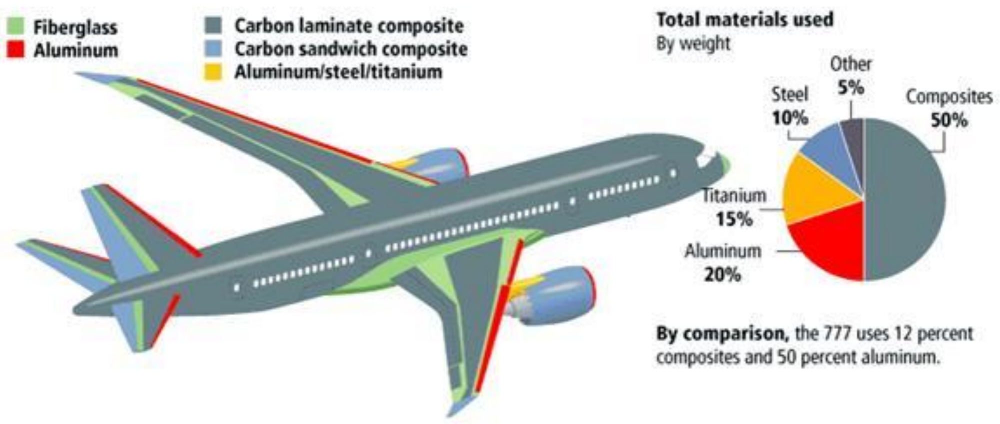

Figure 2.7. Composite Materials used in Boeing 787 body
(Source: Bintang, 2011)

Everyone is familiar, to some degree, with space activities. However, few are conversant with the role that various composite materials play in these activities. Weight savings are a crucial arena for space structures because of the enormous cost of boosting every structure from earth into space. Thus, composite materials are playing a compelling role in virtually all space structures, but not as much as they will in the future as more applications are developed.

Some graphite epoxy structures can be tailored to have a zero coefficient of thermal expansion, a big advantage for large antennas that must pass in and out of the sun, yet maintain dimensional stability for accuracy of pointing the signal. For example, a graphite epoxy truss is used to stabilize and support the Hubble Space Telescope (Jones, 1999).

Composite materials have been a part of the automotive industry for several decades, with early application in the 1953 Corvette. The automotive industry faces many challenges, including increased global competition, the need for higher-performance vehicles, a reduction in costs and tighter environmental and safety requirements. The materials used in automotive engineering play key roles in overcoming these issues: ultimately lighter materials mean lighter vehicles and lower emissions. Composites have been the reduced weight and parts consolidation opportunities the material offers, as well as design flexibility, corrosion resistance, material anisotropy, and mechanical properties For this reasons, composites are being used increasingly in the automotive industry.

Figure 2.8. 1953 Chevrolet Corvette- the first production car to use structural composite materials.

CHAPTER 3

MECHANICS OF COMPOSITE MATERIALS

The mechanics of materials considers the concepts of stresses, strains, and deformations in structures exposed to mechanical and environmental effects for example temperature, moisture, and radiation. A typical composite structure consists of a system of layers bonded together. The layers can be made of different isotropic or anisotropic materials, and have different structures, thicknesses, and mechanical properties. In contrast to typical layers whose primary properties are determined experimentally, the laminate characteristics are usually calculated using the information concerning the number of layers, their stacking sequence, geometric and mechanical properties which should be known. The design steps from micromechanics (which takes into account the fiber and matrix properties and matrix properties) through macromechanics (which treats the properties of composite) to structural analysis(Mallick, 2007). A laminated composite is made by stacking a number of such orthotropic sheets at specific orientations to get composite materials with desired characteristics. When then use the existing theory of laminated plates to examine macromechanically such laminated composites (Chawla,1998). These steps are illustrated in Figure 3.1. fiber-reinforced composite materials are inhomogeneous and non-isotropic (orthotropic). For this reason, the analysis of mechanics of fiber-reinforced composite materials are much more complicated than that of traditional materials .

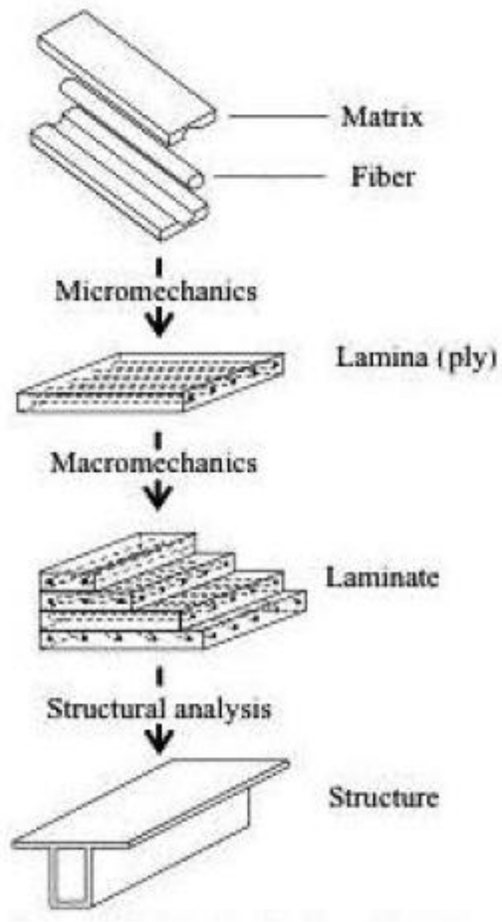

Figure 3.1. The levels of analysis for a structure made of laminated composite (Source: Kollar,2003)

The mechanical analysis of fiber-reinforced composites are performed in two levels: (i) macromechanical analysis, (ii) micromechanical analysis. These terms can be defined as follows:

Micromechanics: Mechanical analysis of the materials on the level of the individual constituents(the microscopic level). This study is generally performed with the aid of a mathematical model describing the response of each constituent material.

Macromechanics: The study of composite material behavior wherein the material is presumed homogeneous and the effects of the constituent materials are detected only as averaged apparent properties of the composite. micromechanics to determine the properties of individual layers and then use only these layer properties to describe the composite.

3.1. Macromechanics of Composite Laminates

Classical lamination theory based on classical plate theory is only valid for thin laminates and used to analyze the infinitesimal deformation of laminated structures. In this theory, it is assumed that laminate is thin and wide, perfect bounding exists between laminas, there exist a linear strain distribution through the thickness and all laminas are macroscopically homogeneous and behave in a linearly elastic manner (Kaw, 2006). Thin laminated composite structure subjected to mechanical in-plane loading (N_x, N_y) considered in this thesis is shown in Figure 3.1. Cartesian coordinate system x, y and z define global coordinates of the layered material. A layer-wise principal material coordinate system is denoted by 1, 2, 3 and fiber direction is oriented at angle θ to the x axis. Representation of laminate convention for the n-layered structure with total thickness h is given in Figure 3.2

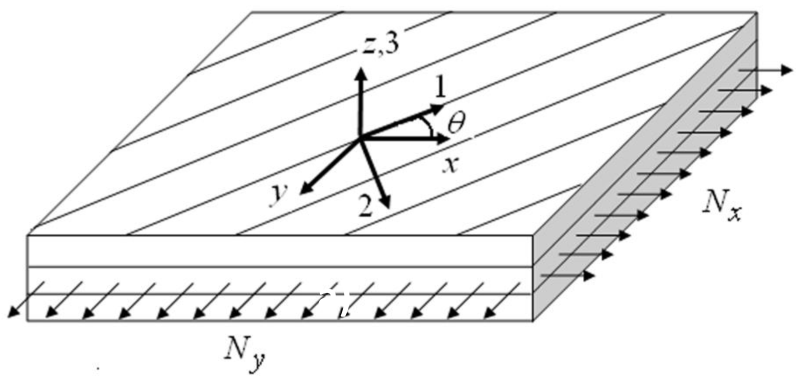

Figure 3.2. A thin fiber-reinforced laminated composite subjected to in plane loading

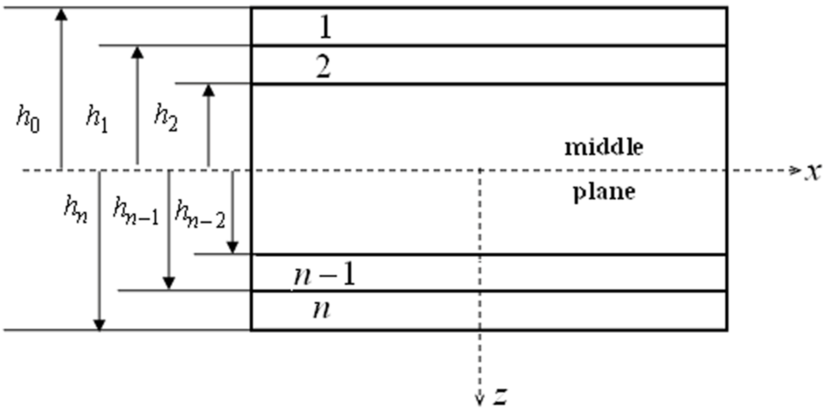

Figure 3.3. Coordinate locations of plies in a laminate

In most structural applications, composite materials are used in the form of thin laminates loaded in the plane of the laminate. Consequently, composite laminates can be considered to be under a condition of plane stress with all stress components in the out-of-plane direction (3-direction) being zero.

The strains at any point in the laminate to the reference plane can be written as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_s \end{bmatrix} = \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_s^o \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{bmatrix} \tag{3.1}$$

The stress-strain relation for the *k*-th layer of a composite plate based on the classical lamination theory can be written in the following form;

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \varepsilon_{xy}^o \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right) \tag{3.2}$$

where $[\overline{Q}_{ij}]_k$ are the components of the transformed reduced stiffness matrix, $[\varepsilon^o]$ is the mid-plane strains $[\kappa]$ is curvatures.

The elements of the transformed reduced stiffness matrix $[\overline{Q}_{ij}]$ expressed in Equation 3.2 can be defined as in the following form;

$$\overline{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \tag{3.3}$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4) \tag{3.4}$$

$$\overline{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \tag{3.5}$$

$$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})sc^3 - (Q_{22} - Q_{12} - 2Q_{66})s^3c \tag{3.6}$$

$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})sc^3 \tag{3.7}$$

$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(c^4 + s^4) \tag{3.8}$$

where stiffness matrix quantities $[Q_{ij}]$ are

$$Q_{11} = \frac{E_1}{1 - \nu_{21}\nu_{12}} \tag{3.9}$$

$$Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{21}\nu_{12}} \tag{3.10}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{21}\nu_{12}} \tag{3.11}$$

$$Q_{66} = G_{12} \tag{3.12}$$

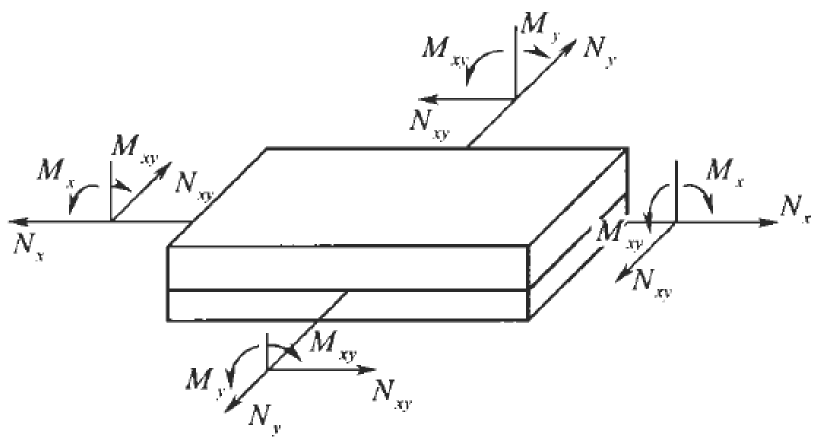

Figure 3.4. Stress resultants and couples applied to reference plane of layer.
(Source: Vasiliev)

The principal stiffness terms, Q_{ij}, depend on elastic properties of the material along the principal directions, E_1, E_2, G_{12}, ν_{12}, and ν_{21}. The in-plane loads (N_x, N_y and N_{xy}) and the moments (M_x, M_y, M_{xy}) in general have the following relations;

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{3.13}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{3.14}$$

The matrices [A], [B] and [D] given in Equation 3.13 and 3.14 are extensional stiffness, coupling stiffness and bending laminate stiffness, respectively. These matrices can be defined as;

$$A_{ij} = \sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_k - h_{k-1}) \tag{3.15}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \tag{3.16}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k \left(h_k^2 - h_{k-1}^2\right) \tag{3.17}$$

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. Combining Equation 3.13 and Equation 3.14 gives six simultaneous linear equations and six unknowns as;

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} \tag{3.18}$$

The extensional stiffness matrix [A] relates the resultant in-plane forces to the in-plane strains, and the bending stiffness matrix [D] relates the resultant bending moments to the plate curvatures. The coupling stiffness matrix [B] couples the force and moment terms to the mid-plane strains and mid-plane curvatures (Kaw 2006).

Now, stresses and strain expressions based on classical lamination theory can be expressed by local coordinate system (1, 2). The relation between the local and global stresses in an angled lamina can be written as in the following form;

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} \tag{3.19}$$

Similarly, the local and global strains are written as follows:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} = [R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \tag{3.20}$$

Where

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{3.21}$$

Transformation matrix [T] used in order to obtain the relation between principal axes (1, 2) and reference axes (x, y), is given by

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \qquad c = Cos\theta, \quad s = Sin\theta \tag{3.22}$$

3.2. Buckling Analysis of a Laminated Composite Plate

Determining the buckling load capacity of a composite plate under in-plane compressive loads is crucial for the design of the composite structures since the buckling of composite plates usually occurs at a lower applied stress and generates large deformations. The buckling could yield a premature failure of the structure. For the buckling analysis, we assume that the only applied loads are the in-plane compressive forces and other mechanical and thermal loads are zero.

When the stress resultants N_x, N_y and N_{xy} are uniformly loaded and w is the pre-buckling deformation, the equation of equilibrium in the direction normal to plate is defined as

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2\left(D_{12} + 2D_{66}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \qquad (3.23)$$

For simply supported plate with no shear load, N_{xy} is zero. In order to simplify the equation of equilibrium, the in-plane forces are defined as follows:

$$N_x = -N_0 \qquad N_y = -kN_0 \qquad k = \frac{N_y}{N_x} \qquad (3.24)$$

The simply supported boundary conditions on all four edges of the rectangular plate (Figure 3.5) can be defined as

$$w(x,0) = 0 \quad w(x,b) = 0 \quad w(0,y) = 0 \quad w(a,y) = 0 \qquad (3.25)$$

$$M_{xx}(0,y) = 0 \;\; M_{xx}(a,y) = 0 \quad M_{yy}(x,0) = 0 \quad M_{yy}(x,b) = 0 \qquad (3.26)$$

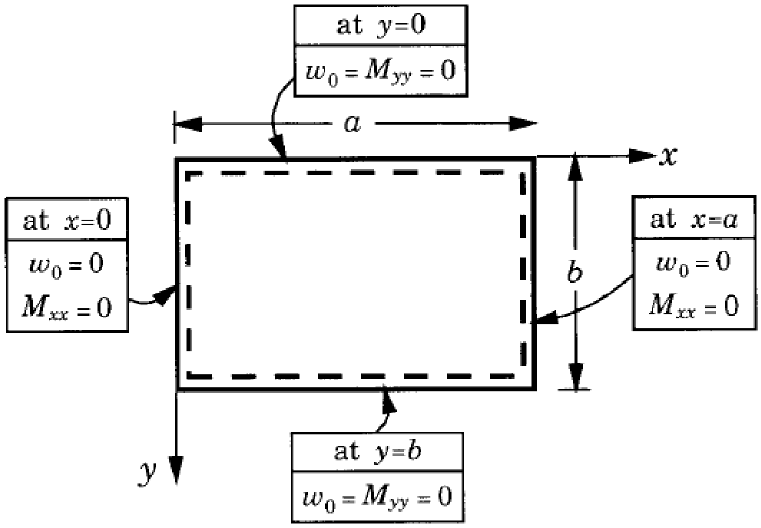

Figure 3.5. Geometry, coordinate system, and simply supported boundary conditions for a rectangular plate (Source: Reddy 2004)

As in the case of bending, Navier approach may be used for the solution considering simply supported boundary condition

$$w(x, y) = W_{mn}\sin(\alpha x)\sin(\beta y) \tag{3.27}$$

Substituting Equation 3.27 into Equation 3.23, we have obtained the following equation:

$$0 = \left\{-\left[D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right] + (\alpha^2 + k\beta^2)N_0\right\} \times W_{mn}\sin\alpha x \sin\beta y \tag{3.28}$$

For nontrivial solution $(W_{mn} \neq 0)$, the expression inside the curl brackets should be zero for every *m* and *n* half waves in x and y directions. Then we obtain

$$N_0(m,n) = \frac{d_{mn}}{(\alpha^2 + k\beta^2)} \tag{3.29}$$

where $$d_{mn} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 \tag{3.30}$$

$$\alpha = \frac{m\pi}{a} \tag{3.31}$$

$$\beta = \frac{n\pi}{b} \tag{3.32}$$

where a is the length of the plate, b is the width of the plate. Substituting Equation 3.24 into Equation 3.29, the buckling load factor λ_b is determined as

$$\lambda_b(m,n) = \pi^2\left[\frac{m^4 D_{11} + 2(D_{12} + 2D_{66})(rmn)^2 + (rn)^4 D_{22}}{(am)^2 N_x + (ran)^2 N_y}\right] \tag{3.33}$$

Where r is the plate aspect ratio (a/b). The buckling mode is sinusoidal and if the plate is loaded as $N_x^a = \lambda_b N_x$ and $N_y^a = \lambda_b N_y$, the laminate buckles into m and n half waves in x and y directions, respectively. The smallest value of λ_b over all possible combinations of m and n is the critical buckling load factor λ_{cb} that determines the critical buckling loads for a specified combination of N_x and N_y in Equation 3.34. If λ_{cb} is larger than 1, the laminate can sustain the applied loads N_x and N_y without buckling (Gurdal, et al. 1999).

$$\begin{bmatrix} N_{x_{cb}} \\ N_{y_{cb}} \end{bmatrix} = \lambda_{cb} \begin{bmatrix} N_x \\ N_y \end{bmatrix} \quad (3.34)$$

The combinations of m and n result in the lowest critical buckling load and, which is not easy to find. When composite plate is subjected to in-plane uniaxial loading and is simply supported for all edges, the minimum buckling load occurs at $n = 1$. The value of *m* depends on bending stiffness matrix (D_{ij}) and the plate aspect ratio(a/b). Therefore, it is not clear which value of *m* will provide the lowest buckling load (Vinson 2005). The critical buckling load factor λ_{cb} limits the maximum load which the laminate can withstand without buckling and it is the smallest value of λ_b under appropriate *m* and *n* values. Unless the plate has a very high aspect ratio or extreme ratios of D_{ij} s, the critical values of *m* and *n* are small (Gurdal, et al. 1999). The optimization problem which we have considered in the thesis study is to find the optimum configurations of composite plates which have the maximum critical buckling load factors λ_{cb}. The values of *m* and *n* are taken to be 1 or 2 in order to result in a good estimate of buckling load capacity for this reason smallest value of λ_b (1, 1), λ_b(1, 2), λ_b(2, 1) and λ_b(2, 2) are considered in order to make a good prediction with respect to critical buckling load factor (Erdal and Sonmez 2005).

CHAPTER 4

OPTIMIZATION

4.1. General Information

All of us are optimizers. We all make decisions that maximize our welfare in some way or another. Often the welfare we are maximizing may come later in life. By optimizing, it reflects our evaluation of future benefits versus current costs or benefits forgone. For instance, a small savings in a mass-produced part will cause substantial savings for the corporation. In many industries such as aircraft, marine, automotive, weight minimization of laminated composite material can impact fuel efficiency, increased payloads or performance.

Optimization, is a mathematical procedure for determining optimal allocation of scarce resources. Maximizing or minimizing some function relative to some set, often representing a range of choices available in a certain situation. The function allows comparison of the different choices for determining which might be "best". Engineers have to take many decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. the process of adjusting the inputs of a device, mathematical process, or experiment to find the minimum or maximum output. It can be seen from Figure 4.1 that if a point x^* corresponds to the minimum value of function $f(x)$, the same point also corresponds to the maximum value of the negative of the function, $-f(x)$. Consequently, without loss of generality, optimization can be taken to mean minimization because the maximum of a function can be found by searching for the minimum of the negative of the same function (Rao 2009).

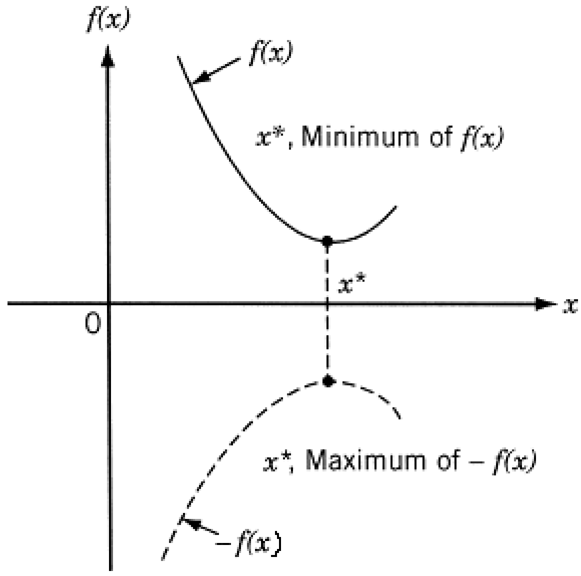

Figure 4.1. Minimum and maximum of objective function (*f(x)*)
(Source: Rao 2009)

An optimization algorithm is a procedure which is executed iteratively by comparing various solutions till an optimum or a satisfactory solution is found.

With the advent of computers, optimization has become a part of computer-aided design activities. There are two distinct types of optimization algorithms widely used today: (a) deterministic algorithms, (b) stochastic algorithms. These terms can be defined as follows:

(a) Deterministic Algorithms: They use specific rules for moving one solution to other. These algorithms are in use to suite some times and have been successfully applied for many engineering design problems.

(b) Stochastic Algorithms: The stochastic algorithms are in nature with probabilistic translation rules. These are gaining popularity due to certain properties which deterministic algorithms do not have.

It is impossible to apply single formulation procedure for all engineering design problems, since the objective in a design problem and associated therefore, design parameters vary product to product different techniques are used in different problems. Purpose of formulation is to create a mathematical model of the optimal design problem, which then can be solved using an optimization algorithm. Figure 4.2 shows an outline of the steps usually involved in an optimal design formulation.

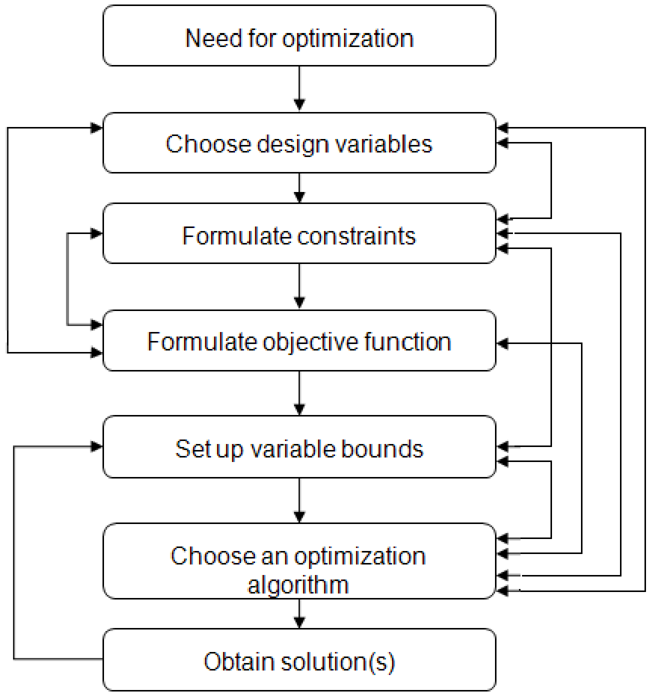

Figure 4.2 A flowchart of the optimal design procedure

Composite design problems generally are very complicated and it is imppossible to solve by the traditional optimization techniques. In these cases, the use of stochastic optimization methods such as Genetic Algorithms (GA), Generalized Pattern Search Algorithm (GPSA) and Simulated Annealing (SA) are appropriate. In composite laminate design problems, derivative calculations or their approximations are impossible to obtain or is often costly. Therefore, stochastic search methods have the advantage of requiring no gradient information of the objective functions and the constraints. In this thesis, GPSA has been considered and used with some modification for design of the laminated composites. In the following subsection, steps of the algorithm are shortly overviewed.

4.2. Definition of Optimization Problem

An optimization or a mathematical programming problem can be defined as follows

$$\text{Find} \quad X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \text{ which minimizes } f(x) \tag{4.1}$$

subject to the constraints

$$g_i(X) \le 0, \qquad i = 1, 2, ..., m$$

$$l_i(X) = 0, \qquad i = 1, 2, ..., p$$

where X is an n-dimensional vector called the design vector, $f(X)$ is termed the objective function, and $g_i(X)$ and $l_i(X)$ are known as inequality and equality constraints, respectively. The number of variables n and the number of constraints m and/or p are not necessary to be related in any way. The optimization problem stated in Equation 4.1 is called a constrained optimization problem. There are not any constraints in some optimization problems which are called unconstrained optimization problems (Rao 2009).

The formulation of an optimization problem begins with identifying the underlying design variables, which are primarily varied during the optimization process. A design problem usually involves many design parameters, of which some are highly sensitive to the proper working of the design. These parameters are called design variables in the parlance of optimization procedures. Other (not so important) design parameters usually remain fixed or vary in relation to the design variables. The first thumb rule of the formulation of an optimization problem is to choose as few design variables as possible. The outcome of that optimization procedure may indicate whether to include more design variables in a revised formulation or to replace some previously considered design variables with new design variables.

The constraints represent some functional relationships among the design variables and other design parameters satisfying certain physical phenomenon and certain resource limitations. The nature and number of constraints to be included in the formulation depend on the user. Constraints may have exact mathematical expressions or not.

4.3. Generalized Pattern Search Algorithm

Generalized pattern search (GPS) algorithms were decribed and examined by Torczon (1997) for derivative-free unconstrained optimization on continuously differentiable functions using positive spanning directions later extended to take nonlinear constrained optimization problems into account. Pattern search algorithms are a direct search method well capable of solving global optimization problems of irregular, multimodal objective functions, without the need of calculating any gradient or curvature information, especially for addressing problems for which the objective functions are not differentiable, stochastic, or even discontinuous (Torczon, 1997). As opposed to more traditional local optimization methods that use information about the gradient or partial derivatives to search for an optimal solution, pattern search algorithms compute a sequence of points that get closer and closer to the globally optimal point. At each iteration, the algorithms poll a set of points, called a mesh, around the current point — the point computed at the previous iteration of the algorithms, looking for a point whose objective function value is lower than the value at the current point. If this occurs, the poll is called successful and the point they find becomes the current point at the next iteration. If the algorithms fail to find a point that improves the objective function, the poll is called unsuccessful and the current point stays the same at the next iteration. The mesh is formed by adding the current point to a scalar multiple (called mesh size) of a set of vectors (called a pattern). In addition to polling the mesh points, pattern search algorithms can perform an optional step at every iteration, called search. At each iteration, the search step applies another optimization method to the current point. If this search does not improve the current point, the poll step is performed (Lewis and Torczon, 2002). GPSA has some collection of vectors that form the pattern and has two commonly used positive basis sets; the maximal basis with $2N$ vectors and the minimal basis with $N+1$ vectors.

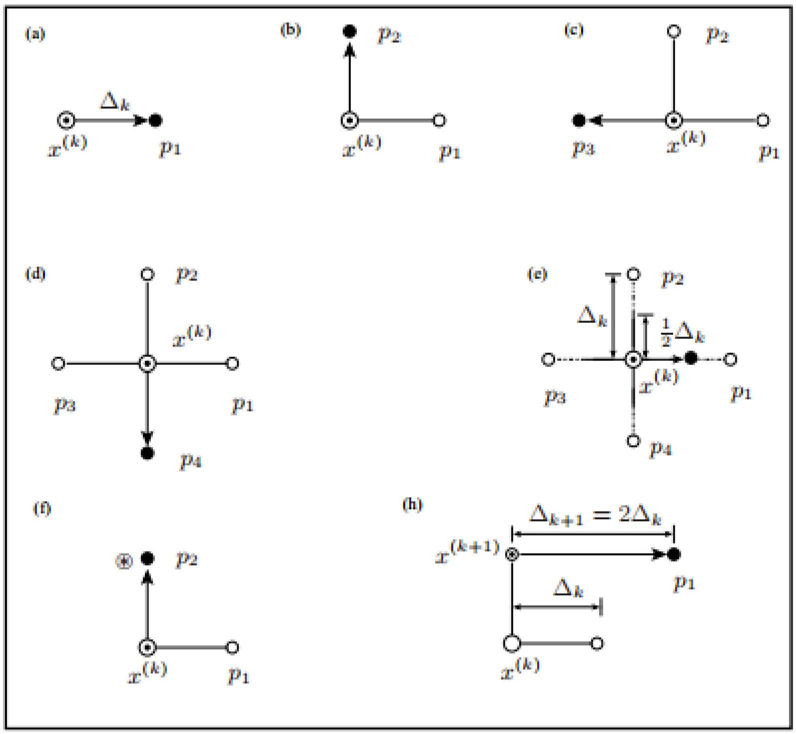

Figure 4.3 How the poll steps works in the GPS method

In order to clarify the algorithm, a laminated composite plate optimization problem including two independent variables θ_1 and θ_2 in the objective function has been considered. In this case, pattern consists of the vectors $v_1 = [1 \quad 0]$, $v_2 = [0 \quad 1]$ $v_3 = [-1 \quad 0], v_4 = [0 \quad -1]$ for the positive basis $2N$ or $v_1 = [1 \quad 0], v_2 = [0 \quad 1]$ $v_3 = [-1 \quad -1]$ for the positive basis N+1. These cases are shown in Figure 4.4.

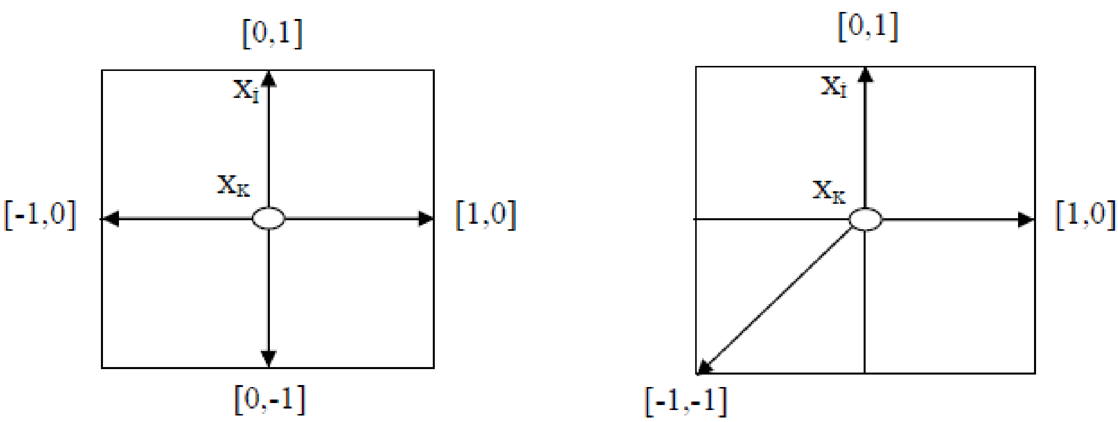

Figure 4.4 Positive basis set 2N and N+1

The mesh is defined by set of spanning directions pattern vectors by a scalar called mesh size. The mesh size is defined as follows:

$$\Delta^{m} = \frac{\left\| x_{k} - x_{k-1} \right\|}{\left\| d_{k} \right\|} \tag{4.2}$$

where x_{k-1} is previous point; x_k is new point; d_k is the length of the corresponding direction.

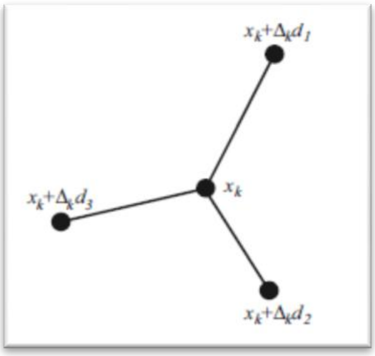

Figure 4.5 Search points and directions

The pattern search begins at a provided initial point vector θ_0. In this example problem, $\theta_0 = [20 \quad 60]$, the *mesh size* Δ^m=5 and positive basis 2*N* are taken into account. At the first iteration, the following *mesh points* can be calculated as

$$[1 \quad 0] \times 5 + [20 \quad 60] = [25 \quad 60]$$

$$[0 \quad 1] \times 5 + [20 \quad 60] = [20 \quad 65]$$

$$[-1 \quad 0] \times 5 + [20 \quad 60] = [15 \quad 60]$$

$$[0 \quad -1] \times 5 + [20 \quad 60] = [20 \quad 55]$$

and the algorithm computes the objective function at these *mesh points* before polls (Karakaya & Soykasap, 2009; Spall, 2003; Mathworks 2008b). If the algorithm finds an objective function value which is smaller than the value at $\theta_0 = [20 \quad 60]$, the poll at corresponding iteration is called as “successful”. Supposing the vector $[20 \quad 65]$ satisfies the condition, the algorithm sets the next point in the sequence equal to $\theta_1 = [20 \quad 65]$. After obtaining a successful poll, the algorithm multiplies the current mesh size by *expansion factor*. For example, if the *expansion factor* is taken as 2, the *mesh size* for the second iteration becomes 5x2=10 and the mesh at the second iteration is to be

$$[1 \quad 0] \times 10 + [20 \quad 65] = [30 \quad 65]$$

$$[0 \quad 1] \times 10 + [20 \quad 65] = [20 \quad 75]$$

$$[-1 \quad 0] \times 10 + [20 \quad 65] = [10 \quad 65]$$

$$[0 \quad -1] \times 10 + [20 \quad 65] = [20 \quad 55]$$

Now, suppose that $\theta_2 = [20 \quad 75]$ produce smaller objective function value than the value at $\theta_1 = [20 \quad 65]$. This procedure repeats until none of the *mesh points* has a smaller objective function value than the value at last (say *n*) successful poll iteration. This poll is called as “unsuccessful” in the corresponding iteration. In this case, the algorithm does not change the current point at the next iteration as $\theta_{n+1} = \theta_n$

In such a case, the algorithm multiplies the current mesh size by given *contraction factor* and the algorithm then polls with a smaller mesh size. The algorithm stops when any of the stopping criteria conditions satisfied.

4.4. Matlab Optimization Toolbox

MATLAB Global Optimization Toolbox provides methods that search for global solutions to problems that include multiple maxima or minima. It contains global search, multistart, pattern search, genetic algorithm, and simulated annealing solvers. These solvers to solve optimization problems where the objective or constraint function is continuous, discontinuous, stochastic, does not possess derivatives, or includes simulations or black-box functions with undefined values for some parameter settings.

4.4.1. Patternsearch Solver

Global Optimization Toolbox includes three direct search algorithms: generalized pattern search (GPS), generating set search (GSS), and mesh adaptive search (MADS). While more traditional optimization algorithms use exact or approximate information about the gradient or higher derivatives to search for an optimal point, these algorithms use a pattern search method that implements a minimal and maximal positive basis pattern. The pattern search method handles optimization problems with nonlinear, linear, and bound constraints, and does not require functions to be differentiable or continuous.

The *Patternsearch* solver interface has two separated parts: problem set up(objective functions, start point, linear inequalities, linear inequalities, lower and upper bounds, nonlinear constraint function and result screen) and options(*Poll, search, mesh, algorithm settings, cache, stopping criteria, plot functions, output function, display to command window, user function evaluation*). *Poll* option consists of the following sub-options: *poll method*, *complete poll* and *polling order*. These sub options are responsible the controlling of the pattern search poll of the mesh points at each iteration.

Poll method *(PollMethod)* specifies the pattern the algorithm uses to create the mesh. There are two patterns for each of the classes of direct search algorithms: the generalized pattern search (GPS) algorithm, the generating set search (GSS) algorithm, and the mesh adaptive direct search (MADS) algorithm. These patterns are the Positive basis 2*N* and the Positive basis *N*+1.

Complete poll *(CompletePoll)* specifies whether all the points in the current mesh must be polled at each iteration. Complete Poll can have the values On or Off. Complete poll to On means that the algorithm polls all the points in the mesh at each iteration and chooses the point with the smallest objective function value as the current point at the next iteration. Complete poll to Off means which is the default value that the algorithm stops the poll as soon as it finds a point whose objective function value is less than that of the current point. The algorithm then sets that point as the current point at the next iteration.

Search options specify an optional search that the algorithm can perform at each iteration prior to the polling. If the search returns a point that improves the objective function, the algorithm uses that point at the next iteration and omits the polling.

Complete search *(CompleteSearch)* applies when Search method to GPS Positive basis Np1, GPS Positive basis 2N,GSS Positive basis Np1, GSS Positive basis 2N, MADS Positive basis Np1, MADS Positive basis 2N, or Latin hypercube are setted. Complete search can have the values On or Off.

In patternsearch, a *search* is an algorithm that runs before a poll. The search attempts to locate a better point than the current point. (Better means one with lower objective function value.) If the search finds a better point, the better point becomes the current point, and no polling is done at that iteration. If the search does not find a better point, patternsearch performs a poll. By default, patternsearch does not use search. The Figure 4.6 illustrated patternsearch with a search method contains a flow chart of direct search including using a search method. The main reason to use a search method, to obtain a better a global solution.

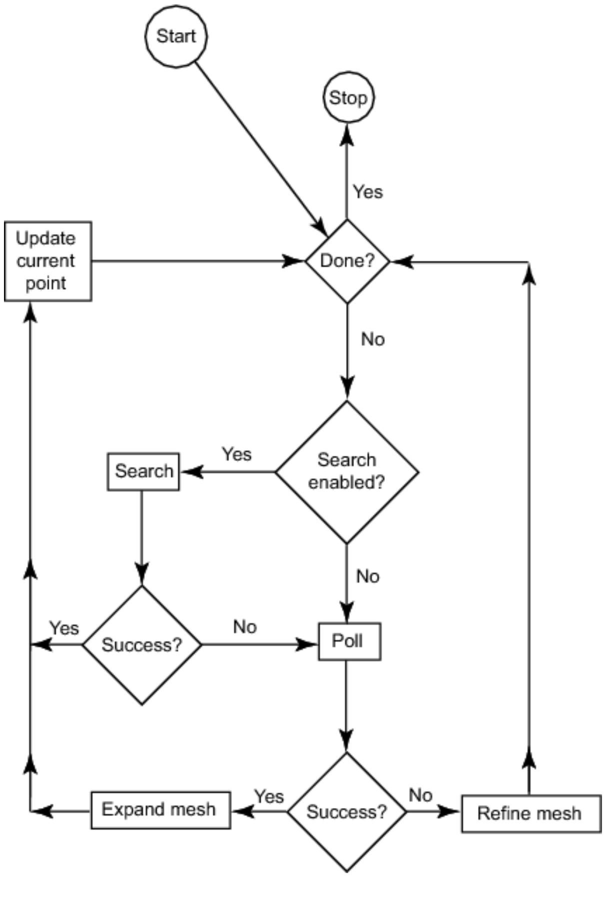

Figure 4.6 A flowchart of pattern search includes search method

Figure 4.7 represents the parameter selection steps for the GPSA analysis of patternsearch solver user interface. In Table 4.1 generalized pattern search algorithm parameters used in the model problems approach have been listed.

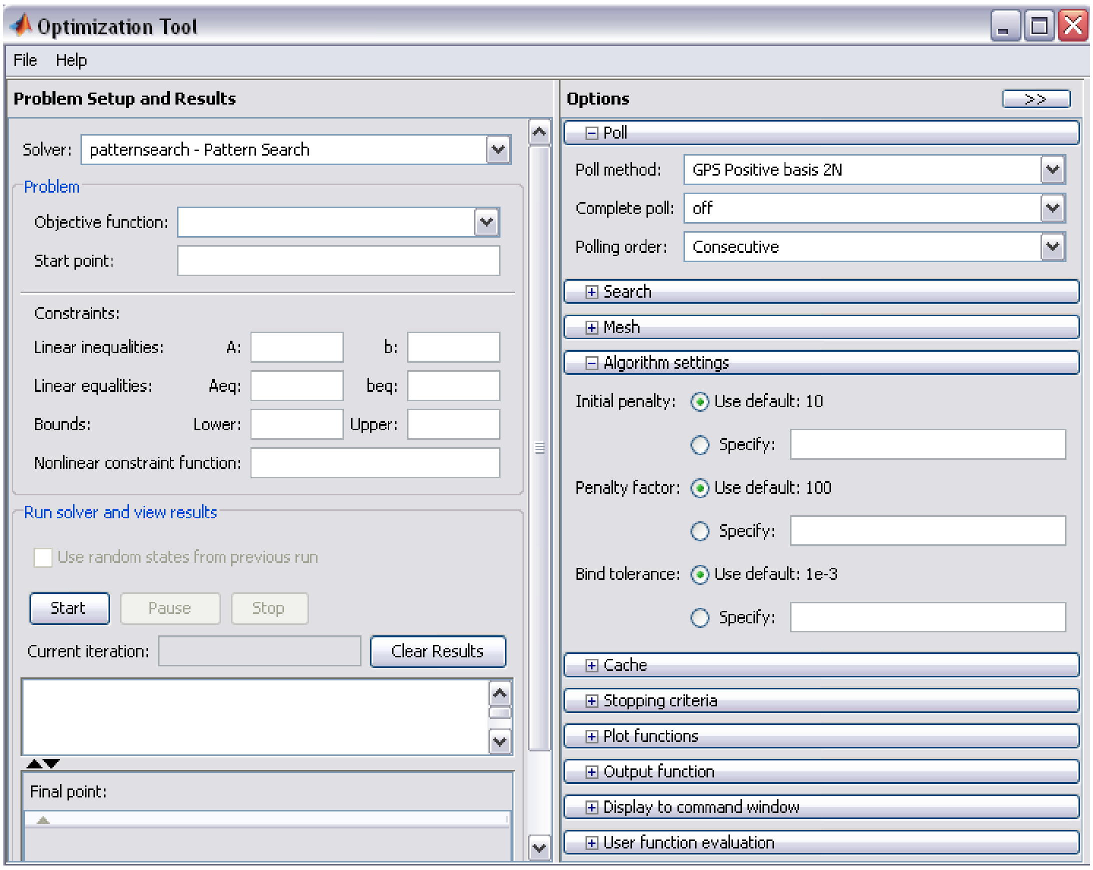

Figure 4.7 Matlab optimization toolbox patternsearch solver user interface

Table 4.1. GPSA solver parameters used in the problems

Poll Method	GPS Positive basis 2N
Complete poll	off
Polling Order	Consecutive
Complete search	on
Search method	GA
Mesh initial size	1.0
Mesh Max size	infinity
Mesh Accelerator	off
Mesh Rotate	on
Mesh Scale	on
Mesh Expansion factor	2.0
Contraction factor	0.5
Initial penalty	1.0
Penalty factor	100
Bind tolerance	10^-3
Stopping criteria	Mesh tolerance=10^-6 Max iterations= 200*number of variables Max function evaluations=2000*number ofvariables Time limit=infinity X tolerance=10^-6 Function tolerance=10^-6 Nonlinear constraint tolerance=10^-6

In this thesis, genetic algorithm is used as search method. Some specific information are given the following subheading.

4.4.1.1. Genetic Algorithm

The Genetic Algorithm (GA) is a stochastic optimization and search technique which allows to obtain alternative solutions for some of the complex engineering problems such as increasing composite strength and light weight structures, etc. GA method utilizes the principles of genetics and natural selection. This method is simple to understand and uses three simple operators: selection, crossover and mutation. Genetic Algorithm always considers a population of solutions instead of a single solution at each iteration. It has some advantages in parallelism and robustness of genetic algorithms. It also improves the chance of finding the global optimum point and helps to avoid local stationary point. However, GA is not guaranteed to find the global optimum solution to a problem. GA has been applied to the design of a variety of composite structures ranging from simple rectangular plates to complex geometries.

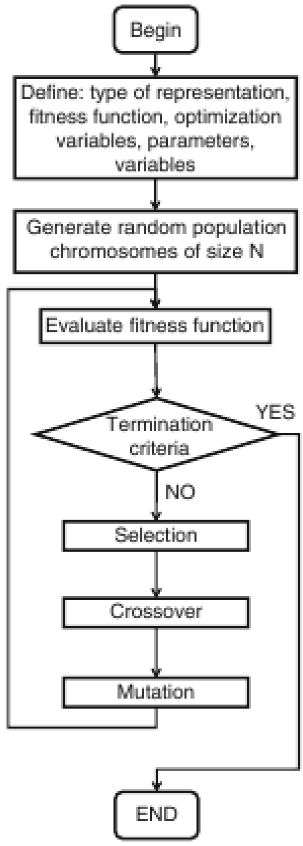

Figure 4.8. Flow chart of genetic algorithm
(Source: Cepin 2011)

CHAPTER 5

RESULTS AND DISCUSSIONS

5.1. Problem Statement

In this thesis study, the stacking sequence design of light-weighted laminated composite plates resisting to buckling have been considered.

The graphite/epoxy laminated composite plates under consideration are rectangle, simply supported on four sides with a length of a and width of b and subjected to in-plane compressive loads N_x and N_y, as shown in Figure 5.1. The length of plate $a = 0.508$ m and a ply thickness $t = 0.25$ mm.

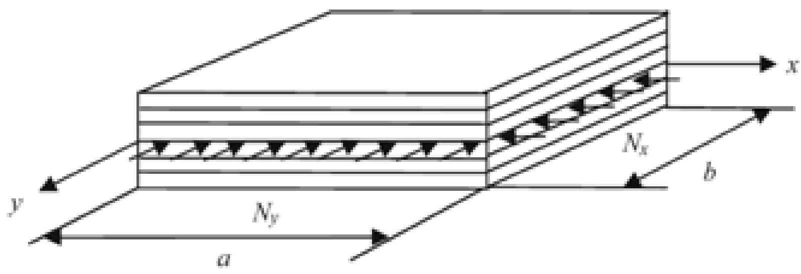

Figure 5.1. Composite plate subjected to in-plane compressive loadings
(Source: Soykasap and Karakaya 2007)

Different loading cases have been considered: N_x=1000 N/mm, 2000 N/mm, 3000 N/mm in design process. N_y and b have been calculated from the load ratio k (N_x / N_y) and the plate aspect ratio r (a/b) accordingly. The plate designs have been studied under loading ratios; k =1/2, 1, *2* and plate aspect raitos; r =1/2,1,2.

In design process, fiber orientation angles are taken as design variables and the allowable orientation angles are continuous $(-90 \le \theta \le 90)$. The number of design variables is reduced from *n* to *n/4* because the plate the plate has assumed to be balance and symmetric which n represented the number of layers. The representation of stacking sequences of n layered composite plate can be given as;

$$[\pm\theta_1 /\pm \theta_2 /\pm \theta_3 /\pm \theta_4 /\pm \theta_5 /\pm \theta_6 /\pm \theta_7 /\pm \theta_8 /\pm \theta_9 /\pm \theta_{10} / \dots /\pm \theta_{(n/4)-1} /\pm \theta_{n/4}]_s$$

The optimization problem can be represent as follows,

Find : $\{\theta_i\},\ \theta_i \in \{-90,90\} \quad i = 1,\dots, n$

Minimize : Weight

Subject to : Critical buckling load, $\lambda_{cb} \geq 1$

In order to obtain the plates with minimum weight which could resist to buckling, n should be 4 at least. Then, n is increased 4 by 4 untill the buckling criteria is reached, $\lambda_{cb} \geq 1$.

The elastic properties of the layers have been taken from a previous study (Akbulut and Sonmez 2008) and given in Table 5.1.

Table 5.1. Elastic properties of Graphite/Epoxy (T300/5208)
(Source: Akbulut and Sonmez 2008)

Property	Graphite/Epoxy (T300/5208)
Young's modulus, E_1 (GPa)	181
Young's modulus, E_2 (GPa)	10.3
Shear modulus, G_{12} (GPa)	7.17
Poisson rate, υ_{12}	0.28

The critical buckling load factor (λ_{cb}) has been used as an objective function in optimization. The objective function for each design has been obtained using the MATLAB *Symbolic Math Toolbox* and the algorithm is given in Appendix A. In order to obtain optimum stacking sequences of laminated composite material, λ_{cb}, the critical buckling load factor (Equation 3.33) is maximized by using generalized pattern search algorithm. Here, the smallest value of λ_b (1, 1), λ_b(1, 2), λ_b(2, 1) and λ_b(2, 2) is taken as the critical buckling load factor (λ_{cb}).

5.2. The Verification of Algorithms in Matlab

In this thesis, the optimum stacking sequence designs have been examined considering buckling and minimum weight. The algorithms for buckling analysis are written in MATLAB. Before optimization process, the verification of algorithms considering the critical buckling load factor (Equation 3.33) is satisfied by using the some literature studies.

In the stacking sequence optimization of the laminated composites, *Generalized Pattern Search Algorithm* (GPSA) from MATLAB Global Optimization Toolbox has been used. In order to increase the reliability of GPSA and also to find the optimum design, 30 searches are independently tested and the GPSA parameters used are shown in Table 4.1

Firstly, the critical buckling load factor algorithm is verified from the study of Karakaya and Soykasap (2009). They have used the genetic algorithm and generalized pattern search algorithm for optimum stacking sequences of a composite plate. Buckling load factor of the plate is maximized for different load cases (*k*=1/2, 1 and 2) and aspect ratios (*r*=1/2, 1 and 2). The obtained optimum fiber orientation angles have been converted to manufacturing values such as 90, 0, and ± 45 orientations. Using specifications of the model problems, the optimum critical buckling load factor values are achived and it is seen that the results are very closed when compared to the related study (Table 5.2). This means that algorithm could yield reliable results.

Table 5.2. Results of different algorithms for buckling analysis

Loading Cases		**λ_{cb} (Karakaya and Soykasap 2009)**	**λ_{cb} (Present Study)**
a/b	**N_x/N_y**		
2	1	695,781.30	695,781.3
1	1	242,823.10	242,823.1
1/2	1	173,945.30	173,945.3
2	2	1,057,948.30	1,057,949.2
1	2	323,764.00	323,764.3
1/2	2	206,492.9	206,492.9
2	1/2	412,985.80	412,986
1	1/2	161,882.10	161,882.1
1/2	1/2	132,243.50	132,243.6

5.3. Optimization Results and Discussion.

The results of this study are presented in Tables (5.3-5.10). Firstly, the optimum stacking sequences for minimum thickness under various in-plane loads and aspect ratios are obtained by using *Generalized Pattern Search Algorithm.* Critical buckling loads in x and y directions are calculated. Then, the designs obtained are checked by critical buckling load. The all fiber orientation angles vary continuously.

The laminated composite plate is subjected to N_x=1000 N/mm and optimum designs have been investigated depending on various loading ratios in Tables 5.3-5.5. Table 5.3 shows the optimum designs of laminated composite plates for the aspect ratio *r*=1/2. For these cases λ_{cb} and stacking sequences are calculated and these values are listed in the table. The optimum composite plate designs which resist to buckling and have minimum thickness are shown in grey color. As it can be understood from Table 5.3, when the same number of layers is taken into consideration, buckling load factor increases with increasing load ratio. This fact not only accounts for the values in Table 5.3 but also in Table 5.4 and Table 5.5.

Table 5.3. Optimum stacking sequence for N_x=1000 N/mm r=0.5

N_x/N_y	Layer number	λ_{cb}	Stacking Sequence
0.5	4	0.00036	$[\mp 28]_s$
	8	0.00285	$[\pm 28/\mp 28.2]_s$
	12	0.0096	$[\mp 28.5/\mp 25.8/\pm 34.1]_s$
	16	0.02278	$[\mp 28/\pm 27.4/\mp 29.5/\pm 29.5]_s$
	20	0.04443	$[\pm 28.9/\mp 25.1/\pm 29.2/\pm 30.1/\mp 42.8]_s$
	24	0.07677	$[\pm 27/\mp 29.2/\mp 29.5/\pm 22.7/\pm 35.6/\pm 38.1]_s$
	28	0.12191	$[\mp 28.9/\mp 29.5/\mp 26.7/\pm 24.6/\pm 22.4/\pm 35.3/\pm 44]_s$
	32	0.18179	$[\pm 28.1/\mp 27.2/\mp 29/\mp 34.4/\pm 23..8/\pm 17.3/\mp 20.3/\pm 40]_s$
	36	0.25911	$[\pm 29.6/\mp 26.8/\pm 30.6/\pm 24.1/\pm 27/\mp 29.7/\mp 27.3/\pm 9.3/\mp 23.8]_s$
	40	0.35478	$[\mp 29.1/\pm 29.3/\mp 22.7/\mp 23.3/\mp 33.8/\pm 27.7/\pm 36.1/\mp 34.4/\mp 29.2/ \pm 3.4]_s$
	44	0.47278	$[\mp 28.3/\mp 29.6/\pm 29.5/\pm 27.6/\pm 22.3/\mp 21/\mp 34.1/\pm 30.3/\pm 33.2/ \mp 29.7/\pm 0.6]_s$
	48	0.61164	$[\pm 31/\mp 20.4/\mp 25.5/\pm 29/\mp 31.4/\mp 32.2/\pm 21.8/\mp 38.8/\pm 37.1/ \mp 32.7/\pm 44.8/\pm 45]_s$
	52	0.77988	$[\mp 27.9/\mp 24.1/\mp 30.1/\pm 32.8/\pm 26.6/\pm 29/\mp 30.9/\pm 19.1/\mp 27.8/ \mp 24/\mp 37.5/\pm 44.7/\mp 43.8]_s$
	56	0.97389	$[\pm 28.5/\pm 31/\pm 26.1/\mp 24.2/\pm 28.8/\mp 22.9/\pm 32.3/\pm 26/\mp 31.6/ \mp 26.9/\mp 44/\mp 22/\pm 41.5/\pm 44.4]_s$
	60	1.19839	$[\pm 27/\mp 27.7/\pm 26.8/\mp 26.7/\mp 27.9/\mp 32.2/\pm 27.9/\mp 30.4/\pm 38.2/ \mp 27.9/\pm 13.2/\pm 30.1/\mp 18.1/\pm 44.3/\mp 39.1]_s$
1	4	0.00046	$[\mp 19.3]_s$
	8	0.00367	$[\pm 19.4/\pm 18.8]_s$
	12	0.01238	$[\pm 21.1/\pm 11.8/\mp 30]_s$

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Table 5.3 (Cont.)

1	16	0.02938	$[\mp17.9/\mp22.8/\mp17.7/\mp20.1]_s$
	20	0.05738	$[\mp20.2/\mp17.6/\mp21.6/\mp11.6/\mp35.5]_s$
	24	0.0992	$[\mp20.4/\pm17.1/\pm20.4/\pm20.3/\mp18.3/\pm1]_s$
	28	0.15752	$[\pm19.8/\pm20.6/\mp16.8/\pm19.4/\mp21/\pm9.2/\mp25.1]_s$
	32	0.23482	$[\pm16.5/\pm21.3/\mp14.3/\pm22.1/\pm26.6/\pm26.6/\pm22.4/\mp29.6]_s$
	36	0.33433	$[\mp20.8/\pm22.8/\pm21.6/\pm16.9/\pm7.9/\pm4/\mp15.9/\mp18.9/\pm6.6]_s$
	40	0.45914	$[\pm20.6/\pm20.7/\pm17.1/\mp20.3/\pm18.3/\pm16.5/\pm10.8/\pm25.1/\pm22.1/\pm1]_s$
	44	0.61031	$[\pm17.4/\mp19.5/\pm15.6/\pm23.9/\pm19.8/\pm19.4/\pm30.9/\pm14/\pm1/\mp29.7/\pm26.4]_s$
	48	0.79268	$[\pm15.4/\pm22.5/\pm20.5/\pm18/\pm18.8/\mp25.4/\mp9.9/\mp18.9/\mp26.2/\pm26.1/\mp18.7/\pm35.6]_s$
	52	1.00849	$[\mp20.5/\mp22.1/\pm17.6/\pm18.1/\pm17.8/\mp18/\mp21/\pm24.4/\mp7.5/\pm12/\mp13.2/\mp8.9/\pm28.3]_s$
2	4	0.00053	$[\mp11.3]_s$
	8	0.00425	$[\mp11.2/\pm11.8]_s$
	12	0.01434	$[\pm12.3/\pm9.1/\mp2]_s$
	16	0.03398	$[\mp9.5/\mp13.9/\pm13.2/\mp1]_s$
	20	0.06633	$[\pm13.3/0/\pm17/\pm0.4/\mp8.7]_s$
	24	0.11459	$[\pm7.3/\pm7.6/\mp17.3/\pm20/\pm14.2/\mp15.5]_s$
	28	0.18196	$[\pm17.1/\mp6.9/\pm7.3/\pm0.3/\mp11.3/\mp4]_s$
	32	0.27179	$[\mp13/\pm14.2/\mp7.7/\pm3.1/\mp12.8/\mp1/\mp1.1/\mp0.2]_s$
	36	0.38699	$[\pm11.3/\pm9.9/\mp16.1/\mp11.7/\mp0.1/\mp9.9/\mp0.2/\mp11.7/0]_s$
	40	0.53074	$[\pm13.6/\pm8.1/\mp16.3/\pm4/\pm13.2/0/\pm9.6/\pm1.3/\mp2/\pm1.1]_s$
	44	0.70614	$[\mp15.5/\mp16.2/\mp1/\mp2.3/\mp13.7/\pm3.4/\pm0.7/0/\pm1.5/\pm1/\pm1]_s$
	48	0.91633	$[\mp16.2/\mp18.7/\pm1/\pm1.6/\mp0.4/\pm7.7/\pm1.9/\mp4.7/\pm9.2/0/\mp0.9/\mp9]_s$
	52	1.16561	$[\mp3.6/\pm14/\mp17.1/\mp1/\mp13.7/\pm6.6/\pm14/\pm11.5/\pm0.1/\mp19.8/\pm17.8/\pm26.9/0]_s$

Table 5.4 shows the optimum designs of laminated composite plates for the plate aspect ratio r = 1 It can be observed that all possible fiber orientations consist of combinations of +45 or -45 angles which are discrete values.

Table 5.4. Optimum stacking sequence for N_x=1000 N/mm r=1

N_x/N_y	Layer number	λ_{cb}	Stacking Sequence
0.5	4	0.00041	$[\mp 45]_s$
	8	0.00327	$[\mp 45/\pm 45]_s$
	12	0.01102	$[\pm 45/\mp 45/\pm 45]_s$
	16	0.02613	$[\mp 45_2/\pm 45_2]_s$
	20	0.05103	$[\pm 45/\mp 45_2/\pm 45_2]_s$
	24	0.08819	$[\pm 45/\mp 45/\pm 45_4]_s$
	28	0.14004	$[\pm 45_5/\mp 45_2]_s$
	32	0.20904	$[\mp 45/\pm 45_3/\mp 45/\pm 45_3]_s$
	36	0.29764	$[\mp 45_3/\pm 45/\mp 45_2/\pm 45/\mp 45_2]_s$
	40	0.40828	$[\pm 45_2/\mp 45_4/\pm 45/\mp 45_3]_s$
	44	0.54342	$[\pm 45_2/\mp 45_5/\pm 45_3/\mp 45]_s$
	48	0.70551	$[\mp 45_4/\pm 45/\mp 45_4/\pm 45_3]_s$
	52	0.89699	$[\mp 45/\pm 45_5/\mp 45_6/\pm 45]_s$
	56	1.12032	$[\pm 45_4/\mp 45_2/\pm 45_7/\mp 45]_s$
1	4	0.00061	$[\mp 45]_s$
	8	0.0049	$[\pm 45_2]_s$

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Table 5.4 (Cont.)

1	12	0.01654	$\left[\mp 45_3\right]_s$
	16	0.03919	$\left[\pm 45_2/\mp 45/\pm 45\right]_s$
	20	0.07655	$\left[\mp 45/\pm 45_2/\mp 45_2\right]_s$
	24	0.13228	$\left[\mp 45_5/\pm 45\right]_s$
	28	0.21006	$\left[\mp 45_2/\pm 45/\mp 45_2/\pm 45/\mp 45\right]_s$
	32	0.31356	$\left[\pm 45/\mp 45/\pm 45/\mp 45_3/\pm 45/\mp 45\right]_s$
	36	0.44645	$\left[\mp 45_2/\pm 45_2/\mp 45/\pm 45_4\right]_s$
	40	0.61242	$\left[\mp 45/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45_2/\mp 45/\pm 45_2\right]_s$
	44	0.81513	$\left[\pm 45_5/\mp 45/\pm 45_4/\mp 45\right]_s$
	48	1.05826	$\left[\pm 45_5/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45_3\right]_s$
2	4	0.00082	$\left[\pm 45\right]_s$
	8	0.00653	$\left[\pm 45_2\right]_s$
	12	0.02205	$\left[\pm 45_2/\mp 45\right]_s$
	16	0.05226	$\left[\mp 45_4\right]_s$
	20	0.10207	$\left[\pm 45_2/\mp 45_3\right]_s$
	24	0.17638	$\left[\pm 45/\mp 45_2/\pm 45/\mp 45/\pm 45\right]_s$
	28	0.28008	$\left[\pm 45_2/\mp 45_2/\pm 45_2/\mp 45\right]_s$
	32	0.41808	$\left[\pm 45/\mp 45/\pm 45/\mp 45_3/\pm 45/\mp 45\right]_s$
	36	0.59527	$\left[\pm 45_2/\mp 45_3/\pm 45/\mp 45/\pm 45_2\right]_s$
	40	0.81656	$\left[\pm 45_2/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45_2/\mp 45\right]_s$
	44	1.08684	$\left[\mp 45_3/\pm 45/\mp 45_3/\pm 45/\mp 45_3\right]_s$

Table 5.5 shows the optimum designs of laminated composite plates for the plate aspect ratio r = 2.

Table 5.5. Optimum stacking sequence for N_x=1000 N/mm r=2

N_x/N_y	**Layer number**	**λ_{cb}**	**Stacking Sequence**
0.5	4	0.00106	$[\mp 78.7]_s$
	8	0.0085	$[\mp 78.1/\pm 81.2]_s$
	12	0.02867	$[\pm 79.8/\pm 77.2/\pm 72.2]_s$
	16	0.06797	$[\pm 77.8/\mp 79/\pm 82.8/\mp 85.3]_s$
	20	0.13269	$[\pm 76.1/\mp 89.3/\pm 77.3/\mp 77.1/\pm 77.9]_s$
	24	0.22927	$[\pm 74.7/\mp 80.2/\mp 88.9/\mp 85.2/\mp 81.6/\pm 87.6]_s$
	28	0.22927	$[\mp 77.3/\mp 80.5/\mp 83.4/\mp 72.7/\pm 83.8/\mp 76.5/\mp 79]_s$
	32	0.54354	$[\mp 85.1/\pm 77.1/\pm 77.4/\mp 75.8/\mp 74.6/\mp 73.7/\mp 82.1/\pm 74.4]_s$
	36	0.77377	$[\pm 80.3/\mp 78.9/\pm 71.9/\mp 80.4/\pm 89/\pm 81.1/\mp 86.2/\mp 82.8/\pm 88.4]_s$
	40	1.06164	$[\mp 78.9/\pm 74.2/\pm 87.3/\pm 80.8/\mp 78/\mp 77.8/\mp 78.5/\mp 79.1/\mp 77.2/\mp 79.9]_s$
1	4	0.00184	$[\pm 70.7]_s$
	8	0.0147	$[\pm 70.4/\pm 72.9]_s$
	12	0.0496	$[\pm 69.6/\pm 72.7/\pm 78]_s$
	16	0.11755	$[\pm 70.3/\pm 70.5/\pm 74.9/\pm 59.8]_s$
	20	0.22933	$[\pm 70/\pm 75.1/\pm 68.8/\mp 93.5/\mp 49.8]_s$
	24	0.39644	$[\mp 73.3/\pm 66.1/\mp 70.6/\pm 70.9/\pm 79/\mp 80.4]_s$
	28	0.62947	$[\pm 68.1/\pm 73.7/\mp 67.4/\pm 79.6/\mp 70.1/\mp 71.8/\pm 74.8]_s$
	32	0.94031	$[\mp 70.3/\pm 68.8/\mp 70.6/\mp 71.9/\pm 70.6/\pm 82.3/\mp 85.8/\pm 65.7]_s$

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Table 5.5 (Cont.)

1	36	1.33806	$[\mp 72.9/\pm 68.9/\pm 72.7/\mp 71.6/\pm 70.4/\pm 61/\mp 69.1/\mp 63.3/\pm 54.7]_s$
2	4	0.00285	$[\pm 62]_s$
	8	0.02278	$[\mp 62.1/\pm 61.5]_s$
	12	0.07681	$[\pm 60.8/\pm 65.1/\pm 64.5]_s$
	16	0.18208	$[\mp 62.8/\pm 59.3/\pm 64.1/\pm 67.7]_s$
	20	0.35563	$[\mp 63/\pm 62.1/\mp 59.8/\pm 56.9/\pm 71.8]_s$
	24	0.61361	$[\pm 63/\pm 65.1/\pm 56.7/\pm 59.9/\pm 56.2/\pm 51.6]_s$
	28	0.97553	$[\pm 64.2/\mp 60.8/\pm 59.5/\pm 63.4/\mp 57.4/\pm 65.8/\mp 61.4]_s$
	32	1.45621	$[\pm 61.3/\mp 63.5/\mp 62.7/\pm 64.3/\mp 58.3/\pm 57.2/\mp 52.3/\mp 64.8]_s$

As a result of the above three tables, the stacking sequences include both continuous and discrete fiber angles depending on the aspect ratios. As mentioned briefly in the previous comments, stacking sequences hold continuous fiber angles in plate aspect ratios of 2 and 1/2 and stacking sequences hold discrete fiber angles when the plate aspect ratio is 1. It is also observed that the maximum buckling load factors corresponding to aspect ratios 1/2, 1 and 2, have been obtained for combination of *(m,n)* values λ_{cb} (1,2), λ_{cb} (1,1) and λ_{cb} (2,1), respectively. It can be noted from the tables that when the aspect ratio is increased, number of layers decrease at same applied load and same loading ratios and consequently laminated composite plates become lighter.

The effect of applied load has also been investigated. The results are shown in Tables 5.6 and 5.7 compared with the results of Table 5.5. In all these tables loading ratios and aspect ratios are the same. In Table 5.6, the applied load equals to 2000 N/mm. In Table 5.7, the applied load equals to 3000 N/mm. It is seen that, as expected, when applied loads are increased, optimum number of layers increases. Contrary to expectations, the applied load is increased by 2 times yet optimum number of layers does not increase by 2 times.

Table 5.6. Optimum stacking sequence for N_x=2000 N/mm r=2

N_x/N_y	Layer number	λ_{cb}	Stacking Sequence
0.5	4	0.00053	$[\pm 78.7]_s$
	8	0.00425	$[\mp 78.7/\pm 78.6]_s$
	12	0.01434	$[\mp 78.4/\pm 78.6/\mp 87.5]_s$
	16	0.03398	$[\pm 78.5/\mp 78.7/\pm 83.2/\mp 65.8]_s$
	20	0.06637	$[\pm 77.3/\mp 79.1/\mp 82.4/\pm 80.4/\pm 82.9]_s$
	24	0.11469	$[\pm 79/\pm 77.1/\mp 77.5/\pm 89/\pm 81.6/\pm 75.3]_s$
	28	0.18208	$[\pm 78.5/\pm 75.6/\pm 85.1/\pm 82.9/\pm 76/\pm 76.5/\pm 65.1]_s$
	32	0.27182	$[\mp 78.1/\mp 79.6/\pm 74.9/\mp 85.2/\mp 77.6/\mp 88.2/\mp 81.7/\mp 81.6]_s$
	36	0.38697	$[\mp 82/\pm 77.6/\pm 82.4/\pm 74/\pm 78.3/\pm 73/\pm 77.8/\mp 75.8/\pm 66.9]_s$
	40	0.53078	$[\pm 76.9/\mp 76.4/\mp 84.7/\mp 78.6/\mp 81.9/\mp 82/\mp 86.9/\pm 72.4/\mp 62.9/\mp 65.3]_s$
	44	0.70625	$[\mp 87.4/\mp 73/\pm 78.6/\mp 77.1/\pm 86.2/\mp 74.3/\mp 76.8/\pm 83.4/\pm 85.2/\pm 76.9/\mp 72.7]_s$
	48	0.91708	$[\pm 80.4/\mp 85.3/\pm 72.3/\mp 76.6/\mp 81.8/\pm 78.1/\mp 77.7/\pm 79.2/\mp 83.3/\pm 72.9/\pm 89.5/\mp 72.9]_s$
	52	1.16605	$[\mp 76.8/\pm 77.2/\pm 84.7/\pm 72.4/\mp 83/\pm 87.9/\mp 79/\mp 83/\pm 82.8/\pm 86.1/\pm 65.8/\pm 75.2/\mp 59.8]_s$
1	4	0.00092	$[\mp 70.7]_s$
	8	0.00735	$[\pm 70.2/\mp 73.8]_s$
	12	0.0248	$[\mp 71.2/\mp 68.8/\mp 72.5]_s$
	16	0.05879	$[\mp 71.1/\pm 70.5/\pm 68.5/\pm 74.6]_s$
	20	0.11463	$[\mp 70.5/\mp 65.9/\mp 76.9/\pm 89.3/\pm 79.4]_s$
	24	0.19834	$[\mp 71.1/\mp 73/\mp 69/\mp 65/\pm 70.3/\mp 67.8]_s$

(cont. on next page)

Table 5.6 (Cont.)

1	28	0.31461	$[\mp72.4/\mp69.2/\mp75.6/\mp65.4/\pm68.7/\pm57.7/\mp50.7]_s$
	32	0.46941	$[\pm66/\mp75.8/\pm72.7/\pm72.2/\pm75.3/\mp62.7/\pm69/\pm48.9]_s$
	36	0.66925	$[\mp68.9/\pm67.9/\pm72.3/\pm76.6/\mp74/\pm70/\mp69.7/\pm74.8/\mp74.7]_s$
	40	0.91728	$[\pm68.2/\mp69.8/\pm68.7/\pm80.3/\pm66.9/\pm80.5/\mp75.1/\mp68.1/\pm63.9/\mp80.4]_s$
	44	1.21927	$[\mp68.7/\pm65.1/\mp79.6/\mp68.9/\mp74.2/\pm85.7/\mp64.2/\mp66.1/\pm83.4/\mp67.5/\mp70]_s$
2	4	0.00142	$[\pm 62]_s$
	8	0.01139	$[\pm62.1/\pm 61.3]_s$
	12	0.03841	$[\pm62.1/\mp60.4/\pm71.8]_s$
	16	0.09088	$[\mp61.5/\mp 65.9/\pm 55.4/\pm 51]_s$
	20	0.1776	$[\mp63.5/\pm 62.8/\pm 55.3/\pm 62.6/\pm 60.4]_s$
	24	0.30633	$[\pm65.3/\mp 62.8/\mp 58.8/\pm 53/\mp 53/\mp 60]_s$
	28	0.48716	$[\mp59.9/\mp61.1/\mp69/\pm61.3/\mp58.9/\pm65.6/\mp47.4]_s$
	32	0.728	$[\pm60/\mp63.4/\mp60/\mp62.9/\mp64.1/\mp71.1/\pm66.8/\mp64.3]_s$
	36	1.03422	$[\mp57.9/\mp64/\pm59.2/\pm62.7/\mp66.9/\pm71.5/\pm74.9/\mp62.6/\pm55]_s$

Table 5.7. Optimum stacking sequence for N_x=3000 N/mm r=2

N_x/N_y	Layer number	λ_{cb}	Stacking Sequence
0.5	4	0.00035	$[\mp78.7]_s$
	8	0.00283	$[\mp78.2/\pm83.5]_s$
	12	0.00956	$[\pm78.4/\pm77.8/\mp90]_s$

(cont. on next page)

Table 5.7 (Cont.)

N_x/N_y	Layer number	λ_{cb}	Stacking Sequence
0.5	16	0.02265	$[\pm77.2/\mp84.5/\mp74.8/\mp85.6]_s$
	20	0.04424	$[\mp79.4/\mp74.7/\pm84.5/\mp87.8/\pm84.2]_s$
	24	0.07644	$[\mp81.7/\pm74.3/\mp78.5/\mp82.2/\pm82.1/\pm74.8]_s$
	28	0.12135	$[\mp78.6/\pm83.3/\mp73.2/\pm78/\pm73.9/\pm81.8/\pm80.7]_s$
	32	0.18115	$[\mp77.5/\mp77.5/\mp87.8/\mp77.2/\mp88.1/\mp72.6/\pm68.5/\pm54.8]_s$
	36	0.25791	$[\mp78.1/\mp82.4/\mp72.6/\pm90/\pm79.2/\mp78.5/\pm83.1/\pm68.7/\mp69.1]_s$
	40	0.35374	$[\mp80.2/\pm76.7/\pm89.6/\mp77.8/\pm72.9/\pm81.7/\mp83.5/\pm63.9/\pm71.9/\pm73.4]_s$
	44	0.47096	$[\mp82.4/\mp75.7/\pm80.2/\pm82.3/\pm73.1/\pm76.9/\mp81.3/\pm84.6/\pm68.1/\mp81.3/\mp72.6]_s$
	48	0.61127	$[\mp79.6/\mp80/\mp80.3/\pm76/\mp82.5/\mp67.7/\pm85.7/\mp79.9/\pm82.5/\mp82.1/\pm85.3/\pm77.1]_s$
	52	0.7776	$[\pm77.6/\mp76.4/\mp80.5/\pm81.3/\pm81/\mp74.7/\pm82.6/\mp85/\pm75.9/\mp78.9/\pm76.2/\mp85.3/\mp58.4]_s$
	56	0.9708	$[\pm77/\pm88.8/\mp74.7/\mp75.3/\mp80.9/\pm89.6/\mp77.4/\mp73/\mp87.7/\pm86.9/\mp72.9/\mp84.3/\mp76.7/\pm62.4]_s$
	60	1.19416	$[\mp84.9/\pm77.2/\pm73.7/\pm82.3/\mp80.3/\mp76.8/\pm79.4/\mp72.3/\mp79.1/\pm81.7/\pm85.5/\mp88.6/\mp87.5/\mp65.2/\pm78.5]_s$
1	4	0.00061	$[\pm70.7]_s$
	8	0.0049	$[\mp70.3/\mp73.5]_s$
	12	0.01651	$[\mp68.3/\pm76.6/\pm81.7]_s$
	16	0.03918	$[\mp71.1/\mp68/\mp76.1/\mp71.9]_s$
	20	0.07646	$[\pm68.3/\pm75.4/\pm68.1/\pm78.8/\mp59]_s$
	24	0.13218	$[\mp68.1/\pm71.5/\pm74.2/\pm77/\pm64.9/\pm78.1]_s$
	28	0.20991	$[\pm68.8/\pm71.4/\mp74.4/\mp66.3/\mp73.7/\mp84.1/\mp68.3]_s$
	32	0.3129	$[\mp69.2/\mp65.3/\mp71.4/\pm80/\pm78.3/\mp79/\mp78.8/\pm60.7]_s$

(cont. on next page)

Table 5.7 (Cont.)

1	36	0.44576	$[\mp 67.5/\pm 75/\pm 72.2/\pm 64.6/\mp 75.2/\pm 70.3/\pm 83.1/\pm 70.4/\pm 89.3]_s$
	40	0.6113	$[\pm 66.5/\mp 69/\mp 74.3/\mp 71.2/\mp 72/\pm 79.7/\pm 78.5/\pm 89.9/\pm 51.8/\mp 56.6]_s$
	44	0,81422	$[\pm 74.8/\mp 69.1/\pm 68.4/\mp 67.1/\pm 72/\pm 67.3/\pm 72.7/\mp 88.8/\pm 62.8/\mp 81.8/\pm 85.3]_s$
	48	1.05549	$[\pm 73.5/\mp 74.8/\pm 66.6/\pm 65.6/\mp 64.2/\mp 68.3/\pm 84.3/\pm 83.7/\mp 77.1/\mp 63.4/\pm 56/\pm 61.5]_s$
2	4	0.00095	$[\pm 62]_s$
	8	0.00759	$[\mp 61.7/\mp 64]_s$
	12	0.02559	$[\mp 61.4/\mp 65/\mp 52.6]_s$
	16	0.06067	$[\mp 62.2/\pm 62.8/\pm 62.6/\mp 45.5]_s$
	20	0.11846	$[\pm 62.2/\mp 60.8/\mp 61.5/\pm 70.6/\mp 45.2]_s$
	24	0.20463	$[\pm 64.4/\pm 62.8/\pm 59.8/\pm 59.5/\pm 52.8/\pm 47]_s$
	28	0.32462	$[\mp 59.1/\mp 64.2/\mp 61.1/\pm 60.2/\pm 75.4/\pm 68.8/\mp 88.5]_s$
	32	0.48543	$[\pm 63.9/\pm 58.8/\pm 61.9/\pm 63.1/\pm 63.3/\mp 59.9/\pm 61.7/\pm 80.3]_s$
	36	0.69068	$[\mp 61.2/\pm 63.4/\pm 58.1/\mp 64.4/\pm 67.9/\pm 58.1/\pm 60.5/\mp 67.7/\pm 53]_s$
	40	0.94687	$[\mp 59.9/\pm 59.9/\pm 60.5/\mp 65.5/\mp 65.1/\mp 72.7/\pm 60/\pm 65.5/\pm 57.4/\pm 45.9]_s$
	44	1.26058	$[\mp 62.3/\pm 61.3/\pm 58.7/\mp 59.4/\pm 64.1/\mp 71.8/\mp 62.4/\pm 59.6/\pm 72.1/\pm 65.7/\pm 54.7]_s$

The next three tables (Tables 5.8, 5.9 and 5.10) show the laminated composite plate weights at the optimum number of layers (optimum thickness). Both aspect ratio and loading ratio are taken as 1/2, 1 and 2.

Table 5.8. Weight of the optimum composite plates for N_x=1000 N/mm

N_x	r	N_x/N_y	Optimum Number of Layers	Optimum weight (kg)
1000	0.5	0.5	60	12.39
		1	52	10.74
		2	52	10.74
	1	0.5	56	5.78
		1	48	4.96
		2	44	4.54
	2	0.5	40	2.07
		1	36	1.86
		2	32	1.65

Table 5.9. Weight of the optimum composite plates for N_x=2000 N/mm

N_x	r	N_x/N_y	Optimum Number of Layers	Optimum weight (kg)
2000	0.5	0.5	72	14.86
		1	68	14.04
		2	64	13.21
	1	0.5	68	7.02
		1	60	6.19
		2	56	5.78
	2	0.5	52	2.68
		1	44	2.27
		2	36	1.86

Table 5.10. Weight of the optimum composite plates for N_x=3000 N/mm

N_x	r	N_x/N_y	Optimum Number of Layers	Optimum weight (kg)
3000	0.5	0.5	84	17.34
		1	76	15.69
		2	72	14.86
	1	0.5	80	8.26
		1	68	7.02
		2	64	6.61
	2	0.5	60	3.1
		1	48	2.48
		2	44	2.27

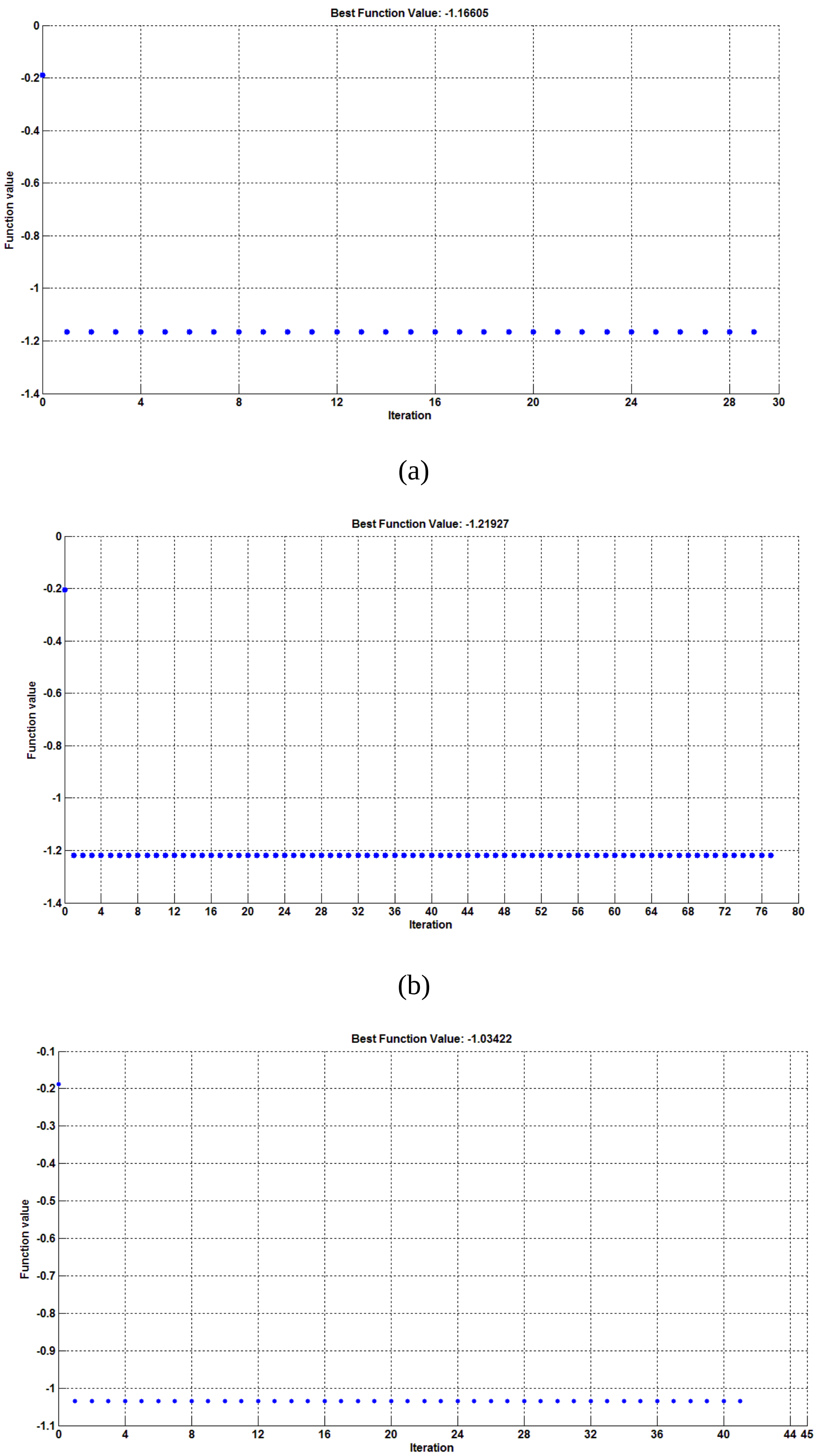
As seen in Table 5.8, Table 5.9 and Table 5.10, the lightest weight values have been obtained in the plates with aspect ratio of 2 and loading ratio of 2. It is possible to obtain more lighter laminated composite plates which resist to buckling at the same loading ratio and same applied load just minimizing the geometry (r=2). It can be observed that the designed plates having aspect ratio of 2 is more resistant than the others in terms of buckling. For instance, at the load ratio k=1/2 and applied load N_x =3000 N/mm two different cases r=1/2 (plate dimensions a=0.508 and b=1.016) and r=2 (plate dimensions a=0.508 and b=0.254) are examined. It is eventually found that plate weights are 17.34 kg and 3.1 kg, respectively. Therefore, it can be concluded from here that plate aspect ratio has a significant effect on the minimum weight.

The effect of the stacking sequences on weight has been investigated by means of making comparisons between conventional and continuous designs. Table 5.11 shows the optimum weights of the related design cases for N_x =1000 N/mm, a/b=2 and N_x/N_y=1/2, 1, 2.

Table 5.11. Weight and stacking sequences of the optimum composite plates for both conventional designs and continuous designs

N_x/N_y	Number of layer	λ_{cb}	Stacking Sequence	Optimum Weight (kg)
0.5	40	1.0616	$\left[\mp 78.9/\pm 74.2/\pm 87.3/\pm 80.8/\mp 78/\mp 77.8/\mp 78.5/\mp 79.1/\mp 77.2/\mp 79.9\right]_s$	2.07
	48	1.0347	$[0/90]_{12s}$	2.48
	48	1.2527	$[\pm 45]_{12s}$	2.48
1	36	1.3380	$[\mp 72.9/\pm 68.9/\pm 72.7/\mp 71.6/\pm 70.4/\pm 61/\mp 69.1/\mp 63.3/\pm 54.7]_s$	1.85
	40	1.0675	$[0/90]_{10s}$	2.07
	40	1.3049	$[\pm 45]_{10s}$	2.07
2	32	1.4562	$[\pm 61.3/\mp 63.5/\mp 62.7/\pm 64.3/\mp 58.3/\pm 57.2/\mp 52.3/\mp 64.8]_s$	1.65
	36	1.2887	$[0/90]_{9s}$	1.85
	32	1.1135	$[\pm 45]_{8s}$	1.65

As it can be understood from the table, when the weight is taken into consideration, continuous designs have more advantage among conventional ones. For continuous designs, weight reduction of the plates are obtained between the range of 10.6% and 16%. These results show that continuous designs have important role on weight reduction and higher buckling load capacity. Surprisingly, for N_x/N_y=2 both continuous design and conventional design (±45) have same weight but the continuous design has much higher load capacity than the conventional one. In conventional designs, stacking sequences for ±45 have better performance than stacking sequences for 0/90. The conventional designs are mostly used in industry due to the manufacturing ease but the results show that designers could prefer the continuous designs where the weight is an important parameter.



(a)

(b)

(c)

Figure 5.2. GPSA iteration steps for model problems N_x = 2000 N/mm, r=2 (a), k = 0.5, (b) k= 1, (c) k = 2

The performance of GPSA for various design cases (Nx = 2000 N/mm loading, r = 2 plate aspect ratio, k = 1/2, 1, 2 load ratios) depending on function values and generations is shown in Figure 5.2.

The best function value corresponds to critical buckling load factor (objective function) value at each iteration. The number of iteration determines in which the algorithm stops. It is observed from the figures that the best function values does not improve after the second iteration and converges to the optimal point for each case.

In this study, the optimization process was performed 30 times in order to see the effectiveness and reliability of the algorithm. The best function values for specific case (Nx = 1000 N/mm, r = 2, k = 2) have been represented for each run in Figure 5.3. It is observed that five global points have been achieved in the range of 1,43814 and 1,45621 and shown in red color.

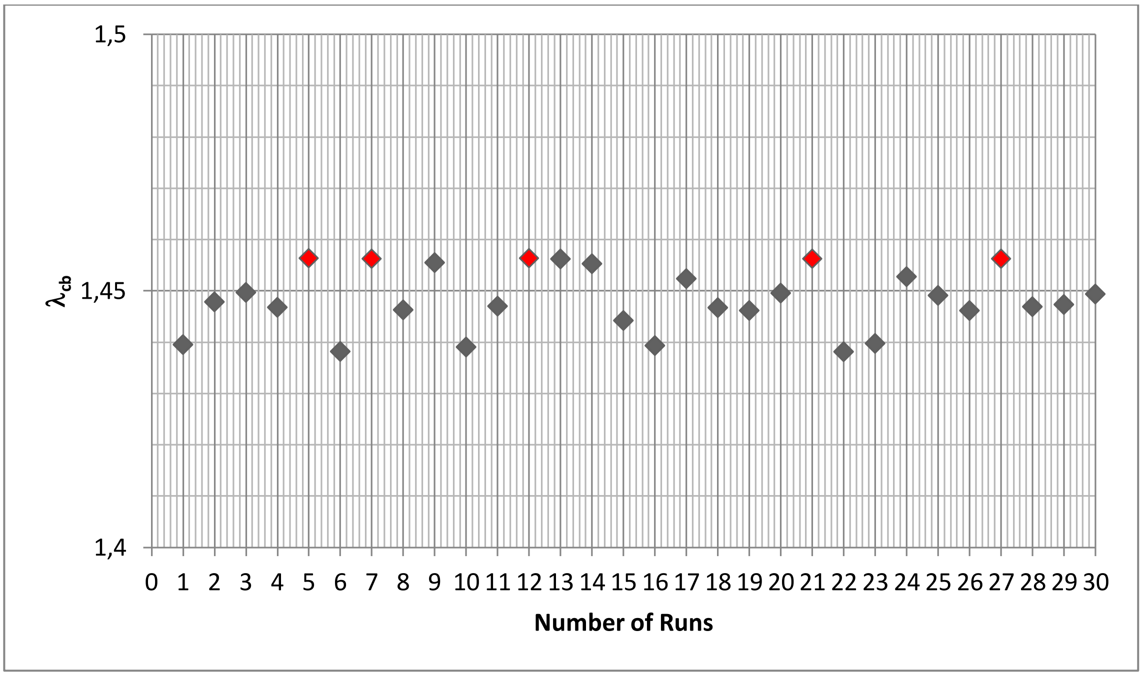


Figure 5.3. Probability of obtaining a global optimum versus number of runs

CHAPTER 6

CONCLUSIONS

The aim of this thesis is to find the optimum stacking sequence design of composite laminates for minimum thickness subject to a buckling constraint and the objective function is the critical buckling load factor. Fiber angles of the composite plates are taken as continuous design variables. The number of design variables is reduced from *n* to *n/4* because the plate has assumed to be balanced and symmetric which *n* represented the number of layers. The optimization has been performed for different loading ratios (*k* =1/2, *k* =1, *k* =2) and plate aspect ratios (*r* =1/2, *r* =1, *r* =2). Loading cases has been considered as N_x=1000 N/mm, 2000 N/mm, 3000 N/mm in design process. Composite plates made of graphite/epoxy have been considered in this thesis. The length of plate $a = 0.508$ m and a ply thickness $t = 0.25$ mm. N_y and *b* have been calculated from the load ratio *k* and the plate aspect ratio *r* accordingly. A stochastic search technique Generalized Pattern Search algorithm (GPSA) has been considered as an optimization method. MATLAB Global Optimization tool has been used in the optimization process. In order to increase the reliability of GPSA and find the best designs, GPSA is specialized by using GA as search method and setting GPS positive basis set 2N as poll method and 30 searches are independently carried out.

Before optimization process, the verification of algorithms is very important to ensure that optimum results are achieved. Therefore, the accuracies of the critical buckling load factor and optimization algorithm are checked by using the literature studies.

As it can be understood from the tables, buckling load capacity of laminated plates of the same N_x and *r* values increases with the increasing loading ratio for the same number of layers. The designed plates having aspect ratio of 2 is more resistant than the others in terms of buckling. The stacking sequences include both continuous and discrete fiber angles depending on the aspect ratios. Stacking sequences hold continuous fiber angles in plate aspect ratios of 2 and 1/2 and stacking sequences hold discrete fiber angles when the plate aspect ratio is 1. It can be noted from the tables that when the aspect ratio is increased, optimum number of layers decrease for same applied load and same loading ratios and consequently, laminated composite plate can become lighter. Considering all investigated cases, the lightest weight values have been obtained in the plates with aspect ratio of 2 and loading ratio of 2. As a result, it is found that aspect ratio has a significant effect on the minimum weight and buckling resistance.

A comparison study of conventional and continuous designs are performed to determine the effect of stacking sequence on weight and the result showed that the optimal continuous designs have better buckling capacity and lighter than conventional designs. Even if it seems that the use of cenventional designs in industry is adventageous in terms of manufacturing ease, continuous angles enable significant weight reduction.

It can also be concluded that all the results showed that critical buckling load factor is an important parameter to determine buckling resistance and weight minimization. The critical buckling load factor should be as high as possible in order to increase buckling resistance. Thus, the number of layers must be increased or geometry (length to width) of the composite plate should be changed. On the other hand, if the weight minimization of composite plate has been taken into consideration, the critical buckling load factor should be equal to one or slightly higher than one. Therefore the number of layers of composite plate can be decreased according to critical buckling load factor.

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APPENDIX A

MATLAB COMPUTER PROGRAMS

In this part, the computer program is shown. the computer program generating the objective functions and GPSA codes generated by Global Optimization Toolbox are given.

```
function f=discrete_bucklinghakan(x)
% x = round(x);
% 1 psi=6894.757 Pa
% 1 in=0.0254 m
E11=127.6e9; %[psi]
E22=13e9; %[psi]
G12=6.4e9; %[psi]
v12=0.3;
a=0.508; %[in]
Nx=1000000; %[lbf/in] load in the x-direction
k=1; % load ratio
Ny=Nx/k;
r=0.5; % plate aspect ratio
b=a/r;
N1=40; % number of plies
N4=N1/4;
N2=N1/2;
tp=(0.127e-3) *N1; % total plate thickness [in]
v21=v12*(E22/E11);

Q11 =E11/(1-v12*v21);
Q12=v21*E11/(1-v12*v21);
Q22=E22/(1-v12*v21);
Q66=G12;
Q = [ Q11 Q12 0; Q12 Q22 0; 0 0 Q66];
D=zeros(3,3);

M=N1+1;

for j=1:M
z(j)=-tp/2+(j-1)*tp/N1;
end
j=1;

for i=1:N4
x1(j)=x(i);
x1(j+1)=-x(i);
```

```
j=j+2;
end

for i=1:N2
x(i)=x1(i);
end

for i=1:N2
x(N2+i)=x(N2-i+1);
end

for k=1:N1
m=cos(x(k)*pi/180);
n=sin(x(k)*pi/180);

Qbar11=Q11*m^4+2*(Q12+2*Q66)*n^2*m^2+Q22*n^4;
Qbar12=(Q11+Q22-4*Q66)*n^2*m^2+Q12*(n^4+m^4);
Qbar22=Q11*n^4+2*(Q12+2*Q66)*n^2*m^2+Q22*m^4;
Qbar16=(Q11-Q12-2*Q66)*n*m^3+(Q12-Q22+2*Q66)*n^3*m;
Qbar26=(Q11-Q12-2*Q66)*n^3*m+(Q12-Q22+2*Q66)*n*m^3;
Qbar66=(Q11+Q22-2*Q12-2*Q66)*n^2*m^2+Q66*(n^4+m^4);
Qbar=[Qbar11 Qbar12 Qbar16;Qbar12 Qbar22 Qbar26;Qbar16 Qbar26 Qbar66];

D(1,1)=D(1,1)+Qbar11*(z(k+1)^3-z(k)^3)/3;
D(1,2)=D(1,2)+Qbar12*(z(k+1)^3-z(k)^3)/3;
D(1,3)=D(1,3)+Qbar16*(z(k+1)^3-z(k)^3)/3;
D(2,2)=D(2,2)+Qbar22*(z(k+1)^3-z(k)^3)/3;
D(3,3)=D(3,3)+Qbar66*(z(k+1)^3-z(k)^3)/3;
D(2,3)=D(2,3)+Qbar26*(z(k+1)^3-z(k)^3)/3;

End

D(2,1)=D(1,2);
D(3,2)=D(2,3);
D(3,1)=D(1,3);

m1=1;
n1=1;
LAMDA1=pi^2*(m1^4*D(1,1)+2*(D(1,2)+2*D(3,3))*m1^2*n1^2*r^2+n1^4*r^4*D(2,
2))/(m1^2*a^2*Nx+r^2*a^2*n1^2*Ny);

m1=1;
n1=2;
LAMDA2=pi^2*(m1^4*D(1,1)+2*(D(1,2)+2*D(3,3))*m1^2*n1^2*r^2+n1^4*r^4*D(2,
2))/(m1^2*a^2*Nx+r^2*a^2*n1^2*Ny);

m1=2;
n1=1;
LAMDA3=pi^2*(m1^4*D(1,1)+2*(D(1,2)+2*D(3,3))*m1^2*n1^2*r^2+n1^4*r^4*D(2,
2))/(m1^2*a^2*Nx+r^2*a^2*n1^2*Ny);
```

```
m1=2;
n1=2;
LAMDA4=pi^2*(m1^4*D(1,1)+2*(D(1,2)+2*D(3,3))*m1^2*n1^2*r^2+n1^4*r^4*D(2,2))/(m1^2*a^2*Nx+r^2*a^2*n1^2*Ny);

LAMDA=[LAMDA1 LAMDA2 LAMDA3 LAMDA4]
f=-min(LAMDA)
```