RENORMALIZATION GROUP INVARIANTS IN MINIMAL SUPERSYMMETRIC STANDARD MODEL

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ABSTRACT

RENORMALIZATION GROUP INVARIANTS IN MINIMAL SUPERSYMMETRIC STANDARD MODEL

This thesis work is devoted to a detailed study of the renormalization group invariants (RG invariants) in minimal supersymmetric standard model (MSSM). The RG invariants are those Lagrangian parameters or combinations of the parameters, which exhibit no dependence on the energy scale up to the loop order with which the renormalization group equations (RGEs) are constructed.

In this work, following an introductory chapter on standard model of electroweak and strong interactions as well as supersymmetry and supersymmetric field theories, we discuss construction of renormalization group equations in supersymmetric models, in particular, the minimal supersymmetric standard model with holomorphic and non-holomorphic soft terms. We finally concentrate on construction and phenomenological implications of the RG invariants in the minimal supersymmetric standard model with and without non-holomorphic supersymmetry breaking terms.

ÖZET

MİNİMAL SÜPERSİMETRİK STANDART MODELDE RENORMALİZASYON GRUP ENVARYANTLARI

Bu tez çalışması minimal süpersimetrik standart modelde (MSSM) renormalizasyon grup envaryantların (RG envaryantları) ayrıntlı bir çalışmasıdır. Renormalizasyon grup envaryantları, renormalizasyon grup denklemlerinin oluşturduğu halka mertebesine kadar enerji skalasına bağımlılık göstermeyen Lagrangian parametreleri yada bu parametrelerin kombinasyonudur.

Bu çalışmada, elektrozayıf ve kuvetli etlileşimlerin standart modelin yanı sıra süpersimetri ve süpersimetrik alan teorilerine giriş bölümünü takiben, süpersimetrik modelde renormalizasyon grup denklemlerini özellikle minimal süresimetrik standart modele özellikle holomorfik ve non-holomorfiğin yumşak terimleri içeren minimal süpersimetrik standart modelde, renormalizasyon grup denklemlerinin oluşturulmasına çalıştık. Son olarakta non-holomorfik süpersimetrik kırılmış terimleri içeren ve içermeyen minimal süpersimetrik standart modelde renormalizasyon grup envaryantlarının oluşturulması ve fenomenolojik emplikasyonları üzerinde yoğunlaştık.

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CHAPTER 1

INTRODUCTION

This thesis work is devoted to analysis and discussion of the renormalization group (RG) invariants in the minimal supersymmetric standard model (MSSM). Basically, Renormalization group equations (RGEs) determine how a given parameter in a Lagrangian field theory varies with the energy scale (or distance scale probed). Certain parameters or combinations of the parameters may turn out to be RG invariant (or, equivalently, scale invariant), that is, they do not vary with energy scale at all. Such parameters turn out to be viable probes of the underlying model since they express correlations among the model parameters in a way independent of the energy scale. This implies that measurements at different colliders (which run at different center of mass energies) of RG invariants must return the same answer. This requirement implies that such invariants can be used to test both experimental measurements and consistency of the underlying model up to the accuracy with which RGEs are obtained. In this work, we will make use of one-loop RGEs of the MSSM parameters to construct invariants out of them.

In Chapter 2 below, we will give a brief introduction to supersymmetry by first reviewing the SM and then pointing out the problems it has in its scalar sector (Higgs sector). Then we give reasons for and basic structure of supersymmetry as a further symmetry principle to account for ultraviolet catastrophe that the SM Higgs sector faces. Basic concepts of a generic supersymmetric field theory *i.e.* superspace, superfields, construction of supersymmetric Lagrangians and superpotential all will be discussed in Chapter 2.

In Chapter 3 we will introduce the MSSM by giving its particle spectrum, gauge structure, and superpotential. We will therein give also why and how supersymmetry is broken in a safe way so that problems encountered in the SM Higgs sector are not regenerated. We will, in particular, introduce soft supersymmetry breaking terms in Chapter 3, and discuss possibility of holomorphic and non-holomorphic soft terms separately. By holomorphic soft terms we mean supersymmetry breaking, gauge invariant, mass-dimension three polynomials of scalars, which consist of no conjugated fields (such supersymmetry breaking terms are usually a replica of the superpotential with superfields being replaced by their scalar components). The non-holomorphic soft terms are of similar structure; however, they contain hermitian-conjugates of scalars (except for fermion bilinears that they can contain) with no tension with gauge invariance. The MSSM with non-holomorphic soft terms is a more general model than the one with holomorphic soft terms and thus deserves of a separate analysis.

In Chapter 3, we will give a detailed discussion of RGEs for a general softly broken supersymmetric theory. Their applications to MSSM with holomorphic and non-holomorphic soft terms are given in Appendices.

In Chapter 4, we will discuss derivations and possible applications of the RG invariants within MSSM with and without non-holomorphic soft supersymmetry breaking terms. We will discuss the two cases separately and discuss their phenomenological implications by examining certain RG invariants. It is worthy of noting that the two cases, holomorphic and non-holomorphic soft supersymmetry breaking terms, possess various RG invariants, which demonstrate their underlying structural differences. In a collider environment, these structures will give distinct structures.

In Chapter 5 we conclude the work.

CHAPTER 2

SUPERSYMMETRY

In this chapter, we will give introduction to supersymmetric theories. We will first give a brief overview of the standard model of electroweak and strong interactions (SM) and then motivate and describe supersymmetric models, in particular, the minimal supersymmetric model which is nothing but a direct supersymmetrization of the SM.

2.1. The Standard Model

All known particle physics phenomena are well-described within the Standard Model (SM) of elementary particles and force carriers. The SM (Salam 1967, Glashow 1961, Weinberg 1967) provides an elegant theoretical framework and it has successfully passed several precision experiments.

By elementary particles (the point-like constituents of matter) what are meant are those having no known substructure up to the present limits of $10^{-18} - 10^{-19}\, m$. Broadly speaking, there are two types of particles known as matter particles and force carriers. The former refer to fermions of spin $s = 1/2$, and are classified into leptons and quarks. The known leptons are: the electron, e^-, the muon, μ^-, and the tau, τ^- lepton with identical electric charges $Q = -1$. The electrically neutral leptons *i.e.* the neutrinos are the electron neutrino, ν_e, the muon neutrino, ν_μ and the tau neutrino, ν_τ. The known quarks up, *u*, down, *d*, charm, *c*, strange, *s*, top, *t* and bottom, *b* form six different flavors and have fractional electric charges $Q = \frac{2}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$ and $-\frac{1}{3}$, respectively.

The second kind of particles is interaction-mediating particles. By leaving apart the gravitational interactions, in the SM all interactions are mediated by force-carrying spin $s = 1$ bosons. The photon, γ, is the exchanged particle in the electromagnetic interactions, the three weak bosons, $W^\pm$, Z are corresponding intermediate bosons of the weak interactions. The eight gluons g_α ; α = 1,...,8 mediate the strong interactions (Herrero 1998).

The SM of fundamental interactions describes *strong*, *weak* and *electromagnetic* interactions of elementary particles. It is based on a gauge principle, according to which all the forces of Nature are mediated by exchanges of the gauge fields of the corresponding local symmetry group. The symmetry group of the SM is

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \tag{2.1}$$

where each gauge group possesses a number of gauge bosons in accord with their number of generators. The gauge sector of the SM is composed of eight gluons G^a_μ of color $SU(3)_C$ (which has 3^2-1 = 8 generators), B_μ boson of hypercharge $U(1)_Y$ (which has a single generator), W^i_μ bosons of isospin $SU(2)_L$ (which has 2^2-1 = 3 generators).

The scalar sector (the Higgs sector to be discussed below) realizes a spontaneous symmetry breakdown such that the local invariance in (2.1) reduces to

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em} \tag{2.2}$$

so that electromagnetism with gauge invariance $U(1)_{em}$ and color $SU(3)_C$ are exact symmetries of the nature at low energies. The spontaneous breakdown of symmetries in (2.2) gives rise to massive vector bosons *i.e.* $W^{+/-}$ and Z bosons of electroweak theory. These bosons have already been in observed in Large Electron-Positron Collider (LEP) at CERN, Geneva.

The fermion sector of the SM consists of leptons and quarks, which are organized, in three families with identical properties except for their masses. The gauge structure in (2.1) treats left- and right-handed fermions in a completely different fashion. We here note that for massless fermions helicity is physical, and left-handed fermions are assigned positive helicity *i.e.* their momenta and spins are parallel to each other. On the other hand, right-handed fermions do have negative helicity; their momenta and spins are anti-parallel to each other. In other words, SM exhibits a builtin left-right asymmetry. It is different at macroscopic scale in everyday life. In general, left- and right-handed components of a fermion field are defined via

$$e_L^- = \frac{1}{2}(1-\gamma_5)e^-; \qquad e_R^- = \frac{1}{2}(1+\gamma_5)e^- \tag{2.3}$$

where e^- denotes the relativistic Dirac field of electron. Here γ_5 is the usual chirality matrix, which involves multiplication of all four γ matrices.

The left-handed leptons are singlet under $SU(3)_C$ and doublet under $SU(2)_L$ whereas the right-handed ones are singlet under both of these symmetries. In tabular form, we display them as (α =1, 2, and 3 being the generation index):

$$L_{\alpha L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L; \quad E_{\alpha L} = \ e_R, \ \mu_R, \ \tau_R \tag{2.4}$$

showing explicitly their chiral structure. We have a similar structure for quarks:

$$Q_{\alpha L}^i = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L, \begin{pmatrix} c^i \\ s^i \end{pmatrix}_L, \begin{pmatrix} t^i \\ b^i \end{pmatrix}_L; \ U_{\alpha R}^i = u_{iR}, c_{iR}, t_{iR}; \ D_{\alpha R}^i = d_{iR}, s_{iR}, b_{iR} \tag{2.5}$$

so that, for each generation α, the left-handed quarks are $SU(2)_L$ doublets and right-handed quarks are singlets. Clearly, irrespective of chirality and generation each quark flavor is a color triplet: i =1, 2, 3.

The scalar sector of the SM *i.e.* the Higgs sector consists of a single $SU(2)_L$ doublet composed of a neutral (H^0) and charged (H^-) scalar fields:

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} \tag{2.6}$$

where potential energy density of H

$$L_{Higgs} = -V = m^2 H^\dagger H - \frac{\lambda}{2}\left(H^\dagger H\right)^2 \tag{2.7}$$

is such that the neutral component H^0 picks up a non-vanishing vacuum expectation value (VEV) in the energetically-preferred state of the system. This non-vanishing VEV

$$v^2 = \left\langle \left|H^0\right|^2 \right\rangle = -m^2 / \lambda \tag{2.8}$$

triggers the breakdown of electroweak symmetry in (2.1) in the way shown in (2.2). This reduction in symmetry of the system feeds masses to otherwise-massless gauge bosons and fermions. To see how those masses arise it may be useful to review the complete Lagrangian of the SM. As a quantum field theory, the SM. Lagrangian can be examined in terms of field gauge terms and interactions: $L = L_{Gauge} + L_{Yukawa} + L_{Higgs}$ where L_{Higgs} refers to Higgs potential in (2.7) above. Here L_{Gauge} consists gauge terms of gauge fields, Higgs field and fermions:

$$\begin{aligned} L_{gauge} = &-\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} \\ &+ i\bar{L}_\alpha \gamma^\mu D_\mu L_\alpha + i\bar{Q}_\alpha \gamma^\mu D_\mu Q_\alpha + i\bar{E}_\alpha \gamma^\mu D_\mu E_\alpha \\ &+ i\bar{U}_\alpha \gamma^\mu D_\mu U_\alpha + i\bar{D}_\alpha \gamma^\mu D_\mu D_\alpha + \left(D_\mu H\right)^\dagger \left(D_\mu H\right) \end{aligned} \tag{2.9}$$

where

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g \varepsilon^{ijk} W^j_\mu W^k_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{2.10}$$

$$
\begin{aligned}
D_\mu L_\alpha &= \left(\partial_\mu - i\frac{g}{2}\tau^i W_\mu^i + i\frac{g'}{2}B_\mu \right) L_\alpha \\
D_\mu E_\alpha &= \left(\partial_\mu + ig' B_\mu \right) E_\alpha \\
D_\mu \mathsf{Q}_\alpha &= \left(\partial_\mu - i\frac{g}{2}\tau^i W_\mu^i - i\frac{g'}{6}B_\mu - i\frac{g_s}{2}\lambda^a G_\mu^a \right) \mathsf{Q}_\alpha \\
D_\mu U_\alpha &= \left(\partial_\mu - i\frac{2}{3}g' B_\mu - i\frac{g_s}{2}\lambda^a G_\mu^a \right) U_\alpha \\
D_\mu D_\alpha &= \left(\partial_\mu + i\frac{1}{2}g' B_\mu - i\frac{g_s}{2}\lambda^a G_\mu^a \right) D_\alpha
\end{aligned}
\tag{2.11}
$$

with g_s, g, g' being the gauge couplings of $SU(3)_C$, $SU(2)_L, U(1)_Y$, respectively. Moreover, ε^{ijk} and f^{abc} stand for the structure constants of $SU(2)_L$ and $SU(3)_C$, respectively. The kinetic term of the Higgs doublet gives rise to gauge boson mass-squared terms. Indeed, one finds $M_W^2 = \frac{g^2}{2}v^2$ and $M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$ which have been measured rather precisely at LEP experiments.

The second part of the Lagrangian, L_{Yukawa}, refers to interaction terms between the Higgs doublet and fermions (quarks and leptons):

$$
L_{Yukawa} = \mathrm{y}_{\alpha\beta}^L \bar{L}_\alpha E_\beta H + \mathrm{y}_{\alpha\beta}^D \bar{\mathsf{Q}}_\alpha D_\beta H + \mathrm{y}_{\alpha\beta}^U \bar{\mathsf{Q}}_\alpha U_\beta i\tau_2 H^\dagger + h.c. \tag{2.12}
$$

where $\mathrm{y}_{\alpha\beta}^i$ are Yukawa matrices *i.e.* matrices in the space of fermion flavors ($\alpha, \beta = 1, 2, 3$) for the up-type quarks *i=U*, the down-type quarks *i=D* and leptons *i* = *L*. Once H^0 picks up a VEV the quarks and leptons acquire non-vanishing masses proportional to the associated Yukawa couplings. Indeed, one finds

$$
m_{\alpha\beta}^L = \mathrm{y}_{\alpha\beta}^L v,\ m_{\alpha\beta}^U = \mathrm{y}_{\alpha\beta}^{\mathrm{U}} v,\ m_{\alpha\beta}^D = \mathrm{y}_{\alpha\beta}^{\mathrm{D}} v \tag{2.13}
$$

for the masses of charged leptons, up-type quarks and down-type quarks, respectively.

We have given a rather brief summary of what we call the SM – a quantum field theory based on three generations of leptons / quarks with the gauge symmetry (2.1). The SM has passed various precision tests at LEP and many other machines measuring the rare processes. The only experimentally lacking feature is the Higgs boson, h. Its mass-squared $m_h^2 = 2m^2$ (see eq. (2.7)) is expected to lie at the weak scale; however, this boson, which is a vital part of the whole idea of electroweak symmetry breaking, has not shown up in experiments. Discovery of h is the main goal of the Large Hadron Collider to start at CERN, Geneva in September 2007. Despite this good news, there is enough reason to believe that the SM must be extended in structure since even if we find Higgs field to weigh some value at this very day the conceptual questions do not end at all. The reason is that the potential energy density (2.7) of the Higgs doublet exhibits a serious sensitivity to quantum mechanical corrections, and this causes a complete destabilization of the whole idea of electroweak breaking. In the next subsection and onwards we will describe one possible way of stabilizing the Higgs sector: the supersymmetry.

2.2. Why and How Supersymmetry?

In order to appreciate the "bad" quantum behavior of the scalar sector of the SM, let us take a brief look at one-loop corrections in Quantum Electrodynamics (QED), the best understood ingredient of the SM (Drees 1996).

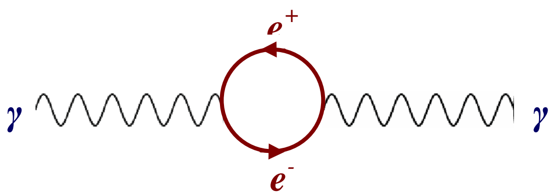

Figure 2.1: The photon self-energy in QED

Let us first investigate photon's two-point function, which receives contributions due to the electron loop diagram of Fig. 2.1:

$$\pi_{\gamma\gamma}^{\mu\nu}(0) = -\int \frac{d^4k}{(2\pi)^4} \; tr \; \left[\left(-ie\gamma^{\mu}\right) \frac{i}{\not{k} - m_e} \left(-ie\gamma^{\nu}\right) \frac{i}{\not{k} - m_e} \right]$$

$$= -4e^2 \int \frac{d^4k}{(2\pi)^4} \frac{2k^{\mu}k^{\nu} - g^{\mu\nu}\left(k^2 - m_e^2\right)}{\left(k^2 - m_e^2\right)} \qquad (2.14)$$

$$= 0$$

which implies that one-loop contribution to photon mass vanishes since two-point functions evaluated at vanishing external momentum gives radiative corrections to mass of the particle under consideration. The result of (2.14) is a consequence of the fact that the photon remains massless at all orders of perturbation theory as a result of the electric charge conservation (electromagnetism can not be saved as a gauge symmetry for a massive photon)

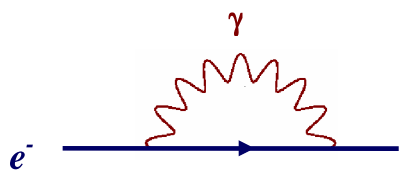

Figure 2.2: The electron self-energy in QED

Next, let us consider the electron self-energy correction shown in Fig. 2.2:

$$\pi_{ee}(0) = \int \frac{d^4k}{(2\pi)^4} \left(-ie\gamma_{\mu}\right) \frac{i}{\not{k} - m_e} \left(-ie\gamma_{\nu}\right) \frac{-ig^{\mu\nu}}{k^2}$$

$$= -e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2\left(k^2 - m_e^2\right)} \gamma_{\mu} \left(\not{k} + m_e\right) \gamma^{\mu} \qquad (2.15)$$

$$= -4e^2 m_e \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2\left(k^2 - m_e^2\right)}$$

This integral has logarithmic divergence at large momenta. However, the corrections to the electron mass are themselves proportional to the electron mass, and if we replace the infinity by the largest scale in particle physics, the Planck scale, we find a correction:

$$\delta m_e \sim 2\frac{\alpha_{em}}{\pi} m_e \log\frac{M_{Planck}}{m_e} \tag{2.16}$$

which is quite modest. At a deeper level, this small correction can be understood from a symmetry: In the limit $m_e \to 0$, the model becomes invariant under chiral rotations $\psi_e \to \exp(i\gamma_5\varphi)\psi_e$ (Wess and Bagger 1992). If this symmetry were exact, the corrections in (2.16) would have to vanish. In reality this symmetry is broken by the electron mass, so the correction must itself be proportional to m_e.

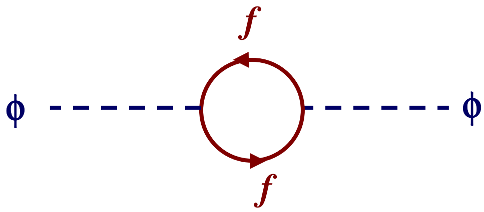

Figure 2.3: Fermion loop contribution to the self-energy of the Higgs boson

Now consider the contribution of a heavy fermion loop correction to the propagator of the Higgs field

$$h = \frac{1}{\sqrt{2}}(H^0 - v) \tag{2.17}$$

as depicted in Fig. 2.3. Let the $hf\bar{f}$ coupling be given by λ_f then

$$\begin{aligned}
\pi_{hh}^{f}(0) &= -N(f)\int\frac{d^4k}{(2\pi)^4} tr\left[i\left(\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}\left(i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}\right] \\
&= -2N(f)\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2+m_f^2}{\left(k^2-m_f^2\right)^2} \\
&= -2N(f)\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2-m_f^2}+\frac{2m_f^2}{\left(k^2-m_f^2\right)^2}\right]
\end{aligned} \tag{2.18}$$

Here, $N(f)$ is a symmetry factor for the Feynman diagram. The first term in the last line of (2.18) is quadratically divergent. To regularize this divergence we cut off high momenta by introducing a scale Λ so that

$$m_h^2 \sim \frac{\lambda_f^2}{4\pi}\Lambda^2 + \text{logarithmic corrections} \tag{2.19}$$

which grows quadratically with Λ. If Λ is replaced by M_{Planck} the resulting correction turns out to be some thirty orders of magnitude larger than the three-level Higgs boson mass (which is expected to weigh a few times the W mass). Note also that the correction itself is independent of m_h. This is related to the fact that setting $m_h = 0$ does not increase the symmetry group of the SM.

This divergence can be renormalized away in the usual way. However, for each order of perturbation theory, an extreme amount of fine-tuning would be needed to cancel the divergences. Additionally, that would still leave us with large finite corrections at each loop order. Furthermore, there would be a contribution similar to (2.19) due to any arbitrarily heavy particle that existed.

Quantity	**Protecting Symmetry**
Photon Mass	Gauge Invariance (Electric Charge Conservation)
Electron Mass	Chiral Symmetry
Higgs Mass	???

Table 2.1. A tabular summary of Sec. 2.2 showing quantities and symmetries that protect them.

We summarize this subsection by tabulating the quantities and symmetries that protect them in Table 2.1. The main goal of this thesis work is to provide a candidate symmetry (to replace questions marks in the table) that protects the Higgs boson mass, and discuss its phenomenological implications.

2.3. Taming the Higgs boson mass

The hierarchy problem that is brought about by the 'Higgs boson mass problem' concerns *protection of a given hierarchy against violent quantum corrections.* Indeed, we do not have an explanation for why W boson weighs 16 orders of magnitude smaller than the Planck mass M_{Pl}. However, we know from (2.19) that Higgs boson mass-squared grows quadratically with the UV scale of the effective theory under concern, and if the theory is valid up to M_{Pl} then all massive vector bosons and fermions of the SM weigh near M_{Pl} in a rather unrealistic way! The gauge hierarchy problem we discuss here is thus related to this quantum theoretic quadratic sensitivity of the Higgs boson mass. Saying differently, we are trying to stabilize the ratio M_W / M_{Pl} rather than explaining why it is such a tiny number. We are about to discuss a formalism that preserves this ratio giving, however, no explanation for its origin.

We now pair Fig. 2.3 by importing a boson S (not necessarily the Higgs boson itself) in the theory such that it possesses an $hhSS$ coupling identical to λ^2, as depicted in the upper line of Fig. 2.4. Clearly, by spin-statistics theorem, the sum of the boson contribution and fermion contribution (similar to one in Fig. 2.3) add up to zero! We call the fermions and bosons with such correlated couplings to Higgs boson as 'partners' by which we mean 'members of a symmetry multiplet'. As shown in the second line of Fig. 2.4 we can design a similar structure for a gauge boson loop by introducing a 'partner' to gauge boson *i.e.* 'gaugino'. In summary, the first line of Fig. 2.4 represents contributions of a scalar S its partner fermion. The second line represents the gauge interaction proportional to the gauge coupling constant g with contribution from the gauge boson and gaugino. In both the cases, the cancellations of the quadratic divergences take place.

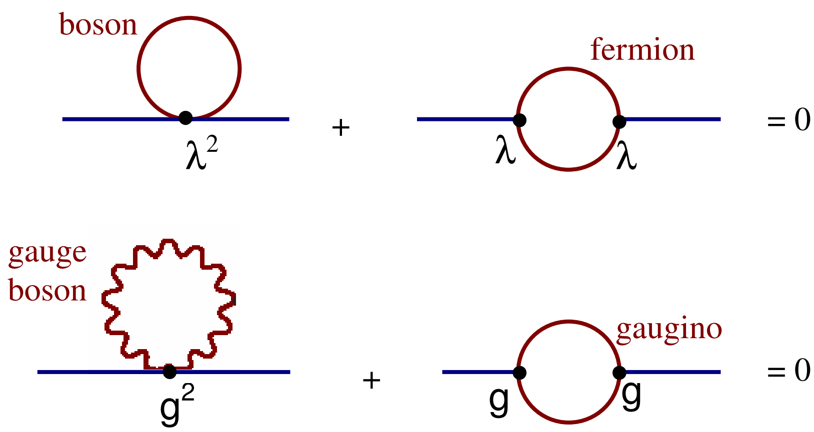

Figure 2.4: Cancellation of the quadratic divergence induced by boson and fermion loops where the boson and fermion exhibit correlated couplings to Higgs, as indicated.

We call the symmetry that relates couplings of bosons and fermions (to the Higgs boson) in the way shown in Fig. 2.4 *supersymmetry or SUSY*, in short. Indeed, for cancellations in Fig. 2.4 to happen the couplings must be related in a precise way. Any imprecision in their equality generates give rise to $O(\Lambda^2)$ contribution to the Higgs boson mass. In the absence of symmetry, these calculations would imply a huge fine-tuning between the boson and fermion couplings. What supersymmetry does is to treat given partners *i.e.* the pair (*fermion, boson*) as a single object in a gauge-invariant way. In SUSY, partners or better *superpartners* do have identical quantum numbers except their spin; it differs by $1/2$ unit between the members of the multiplet. This difference is in the heart of the cancellation mechanism depicted in Fig. 2.4. The other quantum numbers, such as the masses of the members of the multiplet, are identical. This implies, in particular,

$$\sum_{bosons} m^2 = \sum_{fermion} m^2 \tag{2.20}$$

In Nature, SUSY, even if exists, must be a broken symmetry. This is because we have not observed these supersymmetric partners at all: No scalar field having the same mass as electron, no fermion (gaugino) having the same mass as photon! Therefore, symmetry should be a broken symmetry. This breaking can be realized in a simple way by making bosonic and fermionic members of a multiplet to differ from each other not only by their *spins* but also by their *masses.* In other words,

$$\sum_{bosons} m^2 - \sum_{fermions} m^2 = M^2_{SUSY} \tag{2.21}$$

where M_{SUSY} stands for 'the scale at which supersymmetry is broken'. In other words, it for energies much larger than M_{SUSY} that we recover supersymmetry. In the next subsection, we will covert the heuristic arguments of this subsection into a rigorous algebra or symmetry principle.

2.4. Supersymmetry Algebra

The energy-momentum four-vector P_μ and the angular momentum tensor $M_{\mu\nu}$ are the charges carried by the conserved currents corresponding to invariances of physical systems under space-time translation and rotation, respectively. The famous Coleman-Mandula theorem states that there is no other charge with non-trivial transformation properties under Poincaré transformations (the Lorentz transformations plus translations). For instance, one cannot devise a conserved symmetric tensor charge. The charges P_μ and $M_{\mu\nu}$ already facilitate proper scattering processes, and introduction of any further conserved charge over constrains possible interactions among particles. The Coleman-Mandula theorem thus puts stringent constraints on *tensor charges.* However, it does not put any constraint on *spinor charges.* Indeed, *let* Q_a (where a=1, 2 corresponding to two independent components of the spinor Q) be the charge such that

$$\mathsf{Q}_a \left| J \right\rangle = \left| J \pm 1/2 \right\rangle \tag{2.22}$$

for a state with angular momentum J.

Here is the question: can one construct a consistent algebraic scheme in which the fundamental tensor charges P_μ and $M_{\mu\nu}$ combine with the spinorial charge Q_a?

The answer is affirmative (Gol'fand and Likthman 1971). The algebra here, as usual, refers to a set of commutation relations among the charges, which are generators of associated symmetry transformations. Since the spinorial charge Q_a is a symmetry operator, it must commute with the Hamiltonian (which is nothing but the temporal component of energy-momentum four-vector, $H = P^0$) of the system,

$$[\mathsf{Q}_a, H] = 0 \tag{2.23}$$

and so must anticommutator of two different components:

$$[\{\mathsf{Q}_a, \mathsf{Q}_b\}, H] = 0 \tag{2.24}$$

This relation alone guarantees that (Aitchison 2005),

$$\{\mathsf{Q}_a, \mathsf{Q}_b\} \sim P_\mu \tag{2.25}$$

since all components of P_μ commute with each other. The result is that the anti-commutator of the charges must be proportional to the energy-momentum four-vector. Namely, action of $\mathsf{Q}_a\mathsf{Q}_b + \mathsf{Q}_b\mathsf{Q}_a$ on a state vector amounts to a dragging of the state vector.

The statements above can be put into a more concrete form by taking into account the spinorial structure of the charges. First of all, Q_a is a two-component spinor and thus the Dirac matrices should be split into 2x2 sub-matrices. Therefore, it is useful to introduce $\sigma^\mu = (1, \vec{\sigma})$, $\bar{\sigma}^\mu = (1, -\vec{\sigma})$ where $\vec{\sigma}$ are the usual Pauli matrices (See Appendix A.2). Then basic commutation relations among P_μ, $M_{\mu\nu}$ and Q_a can be stated as (See Appendix A.4 for a detailed list of commutation relations):

$$
\begin{aligned}
\left\{Q^i_\alpha, \overline{Q}^j_{\dot\beta}\right\} &= 2\sigma^\mu_{\alpha\dot\beta}P_\mu \\
\left\{Q^i_\alpha, Q^j_\beta\right\} &= \left\{\overline{Q}^i_{\dot\alpha}, \overline{Q}^j_{\dot\beta}\right\} = 0 \\
\left[P_\mu, Q^i_\alpha\right] &= \left[P_\mu, \overline{Q}^i_{\dot\alpha}\right] = 0 \\
\left[M^{\mu\nu}, Q^i_\alpha\right] &= -i\left(\sigma^{\mu\nu}\right)^\beta_\alpha Q^i_\beta \\
\left[M^{\mu\nu}, \overline{Q}^{\dot\alpha i}\right] &= -i\left(\overline{\sigma}^{\mu\nu}\right)^{\dot\alpha}_{\dot\beta}\overline{Q}^{\dot\beta i} \\
\left[M^{\mu\nu}, P_\rho\right] &= i\left[\eta_{\nu\rho}P_\mu - \eta_{\nu\rho}P_\nu\right] \\
\left[M_{\mu\nu}, M_{\rho\sigma}\right] &= -i\left(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}\right)
\end{aligned}
\tag{2.26}
$$

where $\alpha, \beta = 1,2$ are spinorial indices, μ, ν=0,...,3 are spacetime indices, and $i, j = 1,2,...$ are charges satisfying (2.21). In this thesis work we consider only N=1 SUSY so that $i = j = 1$. These commutation relations are added the ones occurring in Poincaré algebra of translations and rotations. The N=1 SUSY corresponds to a single spinor generator Q^i_α and the conjugated one $\overline{Q}^i_{\dot\alpha}$. The algebra (2.26) is called the Super-Poincaré algebra-an algebra that combines tensor and spinor charges in a way consistent with symmetries of the S-matrix (Haag, Lopuszanski and Sohnius 1975).

2.5. Superspace and Supertranslation

In Nature, fields can be divided into bosonic (commuting) fields and fermionic (anti-commuting) fields. This is one of the fundamental discoveries of the quantum theory. The anti-commuting fields are described by spinors and the bosonic fields by tensors according to the spin statistics theorem. One can also introduce in addition to the usual coordinates x^μ anticommuting coordinates θ^α. Then spacetime can be imagined to have been extended via $x^\mu \to (x^\mu, \theta^\alpha, \overline{\theta}_{\dot\alpha})$ by introducing *extra dimensions* composed of anti-commuting (Grassmann coordinates) θ^α and $\overline{\theta}_{\dot\alpha}$. The new spacetime with extra Grassmannian coordinates is called *superspace.* The Grassmann coordinates satisfy (See Appendix A.5 for details of calculation with Grassmann numbers)

$$
\left\{\theta_\alpha, \theta_\beta\right\} = 0, \quad \left\{\overline{\theta}_{\dot\alpha}, \overline{\theta}_{\dot\beta}\right\} = 0 \quad \to \quad \theta^2_\alpha = 0 \text{ and } \overline{\theta}^2_{\dot\alpha} = 0 \tag{2.27}
$$

where $\alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2$. A function in superspace *i.e.* a supersymmetric group element can be constructed in superspace in the same way as ordinary translation in usual spacetime:

$$G\left(x,\theta,\bar{\theta}\right) = \exp\left(i\left\{-x^{\mu}P_{\mu} + \theta \mathrm{Q} + \bar{\theta}\bar{\mathrm{Q}}\right\}\right) \tag{2.28}$$

which induces the translations

$$\begin{array}{lcl} x_{\mu} & \rightarrow & x_{\mu} + i\theta\sigma_{\mu}\bar{\varepsilon} - i\varepsilon\bar{\sigma}_{\mu}\bar{\theta}, \\ \theta & \rightarrow & \theta + \varepsilon, \\ \bar{\theta} & \rightarrow & \bar{\theta} + \bar{\varepsilon}, \end{array} \tag{2.29}$$

where ε and $\bar{\varepsilon}$ are Grassmannian transformation parameters. The supercharge is may then be represented by a differential operator acting in the superspace:

$$\mathrm{Q}_{\alpha} = \partial_{\alpha} - i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\mu} \tag{2.30}$$

where $\partial_{\alpha} = \dfrac{\partial}{\partial\theta_{\alpha}}$, $\partial_{\mu} = \dfrac{\partial}{\partial x^{\mu}}$ with σ_{μ} being Pauli matrices for μ = 1, 2, 3, and the unit matrix for $\mu = 0$. Similarly to charge, the conjugate supercharge is given by

$$\bar{\mathrm{Q}}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} + i\theta_{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu} \tag{2.31}$$

which is a direct hermition conjugation of (2.30). The differential representations (2.30) and (2.31) provide an explicit representation for supercharge, and they satisfy the supersymmetry algebra (See Appendix A for all relativistic notations and representations).

2.6. Superfields

Superfields are objects defined in the superspace. In general, a superfield $\Phi(x,\theta,\bar{\theta})$ is just a scalar function in rigid superspace. It has a finite Taylor expansion in power of $\theta^{\alpha},\bar{\theta}_{\dot{\alpha}}$ as it should according to (2.27). This finite expansion reflects itself in the component expansion of the field:

$$\begin{aligned}\Phi\left(x,\theta,\bar{\theta}\right) &= f\left(x\right)+\theta\phi\left(x\right)+\bar{\theta}\bar{\chi}\left(x\right)+\theta\theta m\left(x\right)+\bar{\theta}\bar{\theta}n\left(x\right) \\ &+\theta\sigma^{\mu}\bar{\theta}V_{\mu}\left(x\right)+\left(\theta\theta\right)\bar{\theta}\bar{\lambda}\left(x\right)+\left(\bar{\theta}\bar{\theta}\right)\theta\psi\left(x\right)+\left(\theta\theta\right)\left(\bar{\theta}\bar{\theta}\right)d\left(x\right)\end{aligned} \tag{2.32}$$

where all higher powers obviously vanish by (2.27). Clearly, the component fields do have varying transformation properties under Poincaré group. Here, $\phi(x),\bar{\chi}(x),\bar{\lambda}(x)$ and $\psi(x)$ are fermionic fields; they are anti-commute with each other and Grassmann coordinates $\theta,\bar{\theta}$. On the other hand, $f(x)$, $m(x)$, $n(x)$ and $d(x)$ are scalar fields, and $V_{\mu}(x)$ is a vector field.

To compute the effect of an infinitesimal supersymmetric transformation on a general scalar superfield, we need the explicit representations of $\mathrm{Q},\bar{\mathrm{Q}}$ as differential operators. For ordinary scalar fields the translation generator P_{μ} is represented by the differential operator $i\partial_{\mu}$. Let ξ^{α} be a constant Grassmann-valued complex Weyl spinor, and consider the effect of multiplication by a supertranslation generator $G(y,\xi)$ on an arbitrary element $\Omega\left(x,\theta,\bar{\theta}\right)$:

$$\begin{aligned}G(y,\xi)\Omega\left(x,\theta,\bar{\theta}\right) &= \exp\left(\mathrm{i}\left[-\mathrm{y}^{\mu}P_{\mu}+\xi\mathrm{Q}+\bar{\xi}\bar{\mathrm{Q}}\right]\right)exp\left(\mathrm{i}\left[-\mathrm{x}^{\mu}P_{\mu}+\theta\mathrm{Q}+\bar{\theta}\bar{\mathrm{Q}}\right]\right) \\ &= exp\left\{i\left[-\left(x^{\mu}+y^{\mu}\right)P_{\mu}+\left(\theta^{\alpha}+\xi^{\alpha}\right)\mathrm{Q}_{\alpha}+\left(\bar{\theta}_{\dot{\alpha}}+\bar{\xi}_{\dot{\alpha}}\right)\bar{\mathrm{Q}}^{\dot{\alpha}}+\frac{i}{2}\left[\xi\mathrm{Q},\bar{\theta}\bar{\mathrm{Q}}\right]+\frac{i}{2}\left[\bar{\xi}\bar{\mathrm{Q}},\theta\mathrm{Q}\right]\right]\right\} \\ &= \Omega\left(\left(x^{\mu}+y^{\mu}-i\xi\sigma^{\mu}\bar{\theta}+i\theta\sigma^{\mu}\bar{\xi}\right),\theta+\xi,\bar{\theta}+\bar{\xi}\right)\end{aligned} \tag{2.33}$$

where the third line is obtained via the commutation relations

$$\left[\xi Q, \bar{\theta}\bar{Q}\right] = 2\xi\sigma^{\mu}\bar{\theta}P_{\mu}$$
$$\left[\bar{\xi}\bar{Q}, \theta Q\right] = -2\theta\sigma^{\mu}\bar{\xi}P_{\mu} \tag{2.34}$$

where use has been made of the differential representations of the charges in (2.30) and (2.31). Equation (2.33) shows that the net effect of supertranslation operation on any supersymmetry element in the superspace is to shift each coordinate by appropriate translation. Under a SUSY transformation, the general scalar superfield in (2.32) changes by an amount:

The supertranslation in (2.33) is generated by a left-multiplication with $G(y,\xi)$. Had we used right-multiplication we would find that the induced motion in superspace is generated by the differential operators:

$$D_{\alpha} = \partial_{\alpha} + i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\mu} \tag{2.35}$$

$$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu} \tag{2.36}$$

which anticommute with the supercharges Q_{α} and $Q_{\dot{\alpha}}$.

Broadly speaking, we will deal exclusively with two kinds of fields: chiral superfields and vector superfields. Having defined a general scalar superfield and its transformation properties in the superspace now we go on defining chiral and vector superfields, which are of important phenomenological relevance.

The chiral superfields are characterized by the condition

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \tag{2.37}$$

which is thus a special class of general scalar superfields. This condition is rather easy to solve since

$$\bar{D}_{\dot{\alpha}}(x^{\mu} + i\theta\sigma^{\mu}\bar{\theta}) = 0 \quad \text{and} \quad \bar{D}_{\dot{\alpha}}\theta = 0 \tag{2.38}$$

so that any function of these two variables satisfy (2.37) automatically. In fact, one finds

$$\Phi = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{2.39}$$

where $y^{\mu} = x^{\mu} + i\theta\sigma^{\mu}\bar{\theta}$. Here A is scalar field, ψ a fermion field and F is an auxiliary field that closes supersymmetry transformations of Φ. It may be instructive to find the transformation properties of the component fields separately. Let us consider the infinitesimal transformations of A and ψ:

$$\delta_{\zeta}A = (\zeta\mathrm{Q} + \bar{\zeta}\bar{\mathrm{Q}})A\ ;\ \delta_{\zeta}\psi = (\zeta\mathrm{Q} + \bar{\zeta}\bar{Q})\psi \tag{2.40}$$

such that

$$(\delta_{\zeta}\delta_{\eta} - \delta_{\eta}\delta_{\zeta})A = -2i(\eta\sigma^{\mu}\zeta - \zeta\sigma^{\mu}\eta)\partial_{\mu}A \tag{2.41}$$

so that transformations close on themselves. For transformations in (2.40) to close on themselves in the sense of (2.41) one finds

$$\delta_{\zeta}F = i\sqrt{2}\bar{\zeta}\bar{\sigma}^{\mu}\partial_{\mu}\psi \tag{2.42}$$

which is a total derivative. In this sense, A is the lowest and F is the highest spin component in a chiral superfield. In general, the highest component in any superfield transforms into a total spacetime derivative (Wess and Bagger 1990). There can not be any component higher than F in a chiral superfield.

One notes that multiplication of any two chiral superfields is again a chiral superfied:

$$\begin{aligned}\Phi_i\Phi_j &= A_i(y)A_j(y)+\sqrt{2}\theta[\psi_i(y)A_j(y)+A_i(y)\psi_j(y)] \\ &+\theta\theta[A_i(y)F_j(y)+F_i(y)A_j(y)-\psi_i(y)\psi_j(y)]\end{aligned} \tag{2.43}$$

whose scalar, fermion and auxiliary components are combinations of those belonging to Φ_i and Φ_j. Notice that now auxiliary component consists of a bilinear of the fermion fields *i.e.* mass term for a fermion.

Unlike (2.43), multiplication of a chiral superfield and hermitian conjugate of another chiral superfield is not a chiral superfield:

$$\begin{aligned}\Phi_i^+\Phi_j &= A_i^*(x)A_j(x) \\ &+\sqrt{2}\theta\psi_j(x)A_i^*(x)+\sqrt{2}\bar{\theta}\bar{\psi}_i(x)A_j(x) \\ &+\theta\theta A_i^*(x)F_j(x)+\bar{\theta}\bar{\theta}F_i^*(x)A_j(x) \\ &+\theta^\alpha\bar{\theta}^{\dot{\alpha}}[i\sigma^\mu{}_{\alpha\dot{\alpha}}(A_i^*(x)\partial_\mu A_j(x)-\partial_\mu A_i^*(x)A_j(x))-2\bar{\psi}_{i\dot{\alpha}}(x)\psi_{j\alpha}(x)] \\ &+\theta\theta\bar{\theta}^{\dot{\alpha}}[\frac{i}{\sqrt{2}}\sigma^\mu{}_{\alpha\dot{\alpha}}(A_i^*(x)\partial_\mu\psi_j^\alpha(x)-\partial_\mu A_i^*(x)\psi_j^\alpha(x))-\sqrt{2}F_j\psi_{\alpha i}(x)] \\ &+\bar{\theta}\bar{\theta}\theta^\alpha[-\frac{i}{\sqrt{2}}\sigma^\mu{}_{\alpha\dot{\alpha}}(\bar{\psi}_i^{\dot{\alpha}}(x)\partial_\mu A_j(x)-\partial_\mu\bar{\psi}_i^{\dot{\alpha}}(x)A_j(x))+\sqrt{2}F_i^*\psi_{\alpha j}(x)] \\ &+\theta\theta\bar{\theta}\bar{\theta}[F_i^*(x)F_j(x)-\frac{1}{2}\partial_\mu A_i^*(x)\partial^\mu A_j(x) \\ &+\frac{1}{4}A_i^*(x)\Box A_j(x)+\frac{1}{4}\Box A_i^*(x)A_j(x) \\ &+\frac{i}{2}\partial_\mu\bar{\psi}_i(x)\bar{\sigma}^\mu\psi_j(x)-\frac{i}{2}\bar{\psi}_i(x)\bar{\sigma}^\mu\partial_\mu\psi_j(x)]\end{aligned} \tag{2.44}$$

which is given in x basis rather than y basis. It is clear that this expression has almost nothing common with (2.42) which is a perfect chiral superfield. One notices that the highest spin component of $\Phi_i^+\Phi_j$ consists of kinetic terms of the scalar, fermion and auxiliary components of the two chiral superfields. Moreover, there are terms involving 0, 1, 2, 3 and 4 powers of θ or $\bar{\theta}$.

In these, expression Φ is a chiral superfield *i.e.* it consists of a fermion with fixed chirality. For instance, left-handed fermions may be assigned into chiral superfields discussed so far. Besides this, the anti-chiral superfields consist of right-handed fermions. In particular, if $\Phi(y,\theta)$ is a chiral superfield, then $\Phi^{\dagger}$ is an antichiral superfield; it satisfies $D_{\alpha}\Phi^{\dagger}=0$ with $\Phi^{\dagger}=\Phi^{\dagger}\left(y^{\dagger},\overline{\theta}\right)$ and $\mathrm{y}^{\dagger}=x^{\mu}-i\theta\sigma^{\mu}\overline{\theta}$. Therefore, interesting thing about $\Phi_i^{+}\Phi_j$ is that it combines chiral superfields of different chirality. In constructing supersymmetric theories, we prefer to work with a single chirality (left-handed), and arrange right-handed superfields into charge-conjugates of left-handed superfields.

The chiral superfields we have discussed above describe spin-0 and spin-1/2 fields. The examples are superfields consisting of leptons, quarks and Higgs bosons. However, we also have to describe the spin-1 gauge bosons of the SM. The vector superfields are defined via the reality condition

$$V=V^{\dagger} \tag{2.45}$$

which should be understood as a power series in θ and $\overline{\theta}$. This reality condition restricts V to have the following form:

$$\begin{aligned} V\left(x,\theta,\overline{\theta}\right) &= C(x)+i\theta\chi(x)-i\overline{\theta}\overline{\chi}(x) \\ &\quad +\frac{i}{2}\theta\theta\left[M(x)+iN(x)\right]-\frac{i}{2}\overline{\theta\theta}\left[M(x)-iN(x)\right] \\ &\quad -\theta\sigma^{\mu}\overline{\theta}v_{\mu}(x)+i\theta\theta\overline{\theta}\left[\overline{\lambda}(x)+\frac{i}{2}\overline{\sigma}^{\mu}\partial_{\mu}\chi(x)\right] \\ &\quad -i\overline{\theta\theta}\theta\left[\lambda(x)+\frac{i}{2}\sigma^{\mu}\partial_{\mu}\overline{\chi}(x)\right]+\frac{1}{2}\theta\theta\overline{\theta\theta}\left[D(x)+\frac{1}{2}\Box C(x)\right] \end{aligned} \tag{2.46}$$

Here, the component fields C,D,M,N and v_{μ} are real and vector scalars and χ,λ are Weyl spinors. The vector field v_{μ} lends its name to the entire multiplet. This particular form of V is dictated by the hermitian quantity $\Phi^{+}+\Phi$ constructed out of the scalar superfields. In fact, one can implement a supersymmetric generalization of gauge invariance by requiring V to be invariant under $V\to V+\Phi^{+}+\Phi$. This special gauge choice is known as Wess-Zumino-Landau gauge. This gauge breaks supersymmetry; however, it reduces vector superfield into a more restricted form with powers

$$V^1\left(x,\theta,\bar{\theta}\right) = -\theta\sigma^\mu\bar{\theta}v_\mu\left(x\right) + i\theta\theta\bar{\theta}\bar{\lambda}\left(x\right) - i\bar{\theta}\bar{\theta}\theta\lambda\left(x\right) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D\left(x\right)$$
$$V^2\left(x,\theta,\bar{\theta}\right) = -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}v_\mu v^\mu \tag{2.47}$$
$$V^3\left(x,\theta,\bar{\theta}\right) = 0$$

form which we see that λ is the lowest spin component in V. We have to construct a chiral and gauge invariant superfield out of V. This can be done by introducing a new superfield via

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V;\ \ \bar{W}_{\dot{\alpha}} = -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V \tag{2.48}$$

which are seen to be chiral superfields since $\bar{D}_{\dot{\beta}}W_\alpha = 0;\ D_\beta\bar{W}_{\dot{\alpha}} = 0$. Since W_α is chiral its $\theta\theta$ component must transform into a total spacetime derivative *i.e.* part of a gauge-invariant Lagrangian. In fact, one finds that

$$\left(W^\alpha W_\alpha\right)_{\theta\theta} = -2i\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \frac{1}{2}v_{\mu\nu}v^{\mu\nu} + D^2 - \frac{i}{4}v_{\mu\nu}\tilde{v}^{\mu\nu} \tag{2.49}$$

where $v_{\mu\nu}$ is field strength tensor of v_μ and $\tilde{v}_\mu$. From (2.49) it is straightforward to construct the Lagrangian density

$$L_{vector} = \frac{1}{4}\left[\left(W^\alpha W_\alpha\right)_{\theta\theta} + \left(\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right)_{\bar{\theta}\bar{\theta}}\right] = -\frac{1}{4}v_{\mu\nu}v^{\mu\nu} - i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + \frac{1}{2}D^2 \tag{2.50}$$

which is nothing but complete Lagrangian for a vector field v_μ and its supersymmetric partner *i.e.* gaugino λ. One notes that $D(x)$ plays the role of interaction potential. It is traditionally called *gauge potential* or D-term contribution. The D-term contribution, like F-term contribution for chiral superfields, represents an auxiliary field needed to close the supersymmetry transformations. It goes to a total space derivative under supersymmetry transformations. The form of the vector superfield in (2.47) is precisely what is expected of a supersymmetric generalization of a gauge field to contain.

2.7. Supersymmetric Lagrangians

The physical systems are described by an action that is extremized by the classical trajectory, which obeys the equations of motion. Consider writing an action for some combinations of the superfields. It will necessarily contain an integration over the usual spacetime x^{μ} and over the anti-commuting coordinates θ and $\overline{\theta}$. Then question is: how to construct a Lagrangian for matter and gauge fields? The remedy comes from the following observation: Given some combination of the superfields then it is that piece which transforms into a total spacetime derivative that can contribute to a field theory defined on spacetime. The reason is that addition of a total derivative to action density does not change physics, and thus, such terms are effectively invariants under both Poincaré and supersymmetry transformations. This can be illustrated by working out a generic and simple model. Given some set of scalar superfields, which exhibit linear, quadratic as well as trilinear couplings. Then their Lagrangian density must have the form:

$$L_{chiral} = \left(\Phi_i^{+}\Phi_i\right)_{\theta\theta\overline{\theta}\overline{\theta}} + \left[\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k + h_i\Phi_i\right]_{\theta\theta} + h.c. \qquad (2.51)$$

where summation of repeated indices is implied. Here subscripts under each term imply extracting that specific component of the superfields contained in that term. For instance,

$$\left(\Phi_i^{+}\Phi_i\right)_{\theta\theta\overline{\theta}\overline{\theta}} = \int d^2\theta d^2\overline{\theta}\ \Phi_i^{+}\Phi_i \qquad (2.52)$$

where one keeps in mind that integration and differentiation are equivalent operations for Grassmann coordinates (See Appendix A.5).

It is instructive to check the explicit form of the Lagrangian (2.48):

$$L_{chiral} = i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i + F_i^* F_i$$
$$+\left[m_{ij}(A_i F_j - \frac{1}{2}\psi_i \psi_j) + g_{ijk}(A_i A_j F_k - \psi_i \psi_j A_k) + h_i F_i + h.c. \right] \tag{2.53}$$

where F_i is clearly an auxiliary field since it has no kinetic term at all. In fact, its equation of motion completely determines it in terms of other fields:

$$\frac{\partial L}{\partial F_k^*} = F_k + h_k^* + m_{ik}^* A_i^* + g_{ijk}^* A_i^* A_j^* = 0 \tag{2.54}$$

so that the Lagrangian (2.50) takes the form

$$L_{chiral} = i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i - \frac{1}{2} m_{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \bar{\psi}_i \bar{\psi}_j$$
$$- g_{ijk} \psi_i \psi_j A_k - g_{ijk}^* \bar{\psi}_i \bar{\psi}_j A_k^* - F_k^* F_k \tag{2.55}$$

which is nothing but complete Lagrangian for a scalar field A_i and a massive fermion ψ_i interacting through the terms in the second line. The explicit form of the last term is read off from (2.51) by eliminating F_i.

The Lagrangian (2.51) can be understood in a more systematic way. Indeed, let us introduce

$$W = h_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \tag{2.56}$$

which is a holomoprhic function of the superfields (it does not contain the hermitian conjugate of Φ_i). Now, it is easy to see that one can organize (2.51) in terms of W as follows:

$$L_{chiral} = i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i - \left[\frac{\partial^2 W}{\partial A_i \partial A_j} \psi_i \psi_j + h.c. \right] - \left| \frac{\partial W}{\partial A_k} \right|^2 \tag{2.57}$$

which implies that W in (2.53) with Φ_i replaced by its scalar component A_i completely determines the interaction terms in the Lagrangian. Here W is called *superpotential* by tradition, and interactions among chiral fields in a supersymmetric theory are entirely governed by giving W. Every W which has to be gauge-invariant and of the mass dimension three defines a supersymmetric theory.

In general, the interaction potential for chiral superfields is given by the F-term contribution in (2.54) and that of vector fields is given by the D-term contribution in (2.50). The results above complete the analysis of supersymmetric Lagrangians to be heavily utilized in the following chapters.

CHAPTER 3

THE MINIMAL SUPERSYMMETRIC STANDARD MODEL (MSSM)

In this chapter, we present a basic introduction to the minimal supersymmetric standard model (MSSM) which is nothing but a direct supersymmetrization of the SM field content. We will first describe interactions, which respect supersymetry by introducing the Lagrangian in Sec. 3.2. Then we will consider supersymmetry breaking by introducing the relevant Lagrangian in Sec. 3.3.

The MSSM is defined to be the minimal supersymmetric extension of the SM, and hence is an $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ supersymmetric gauge theory with a general set of supersymmetry-breaking terms. The known matter and gauge fields of the SM are promoted to superfields in the MSSM.

3.1. Particle content and Superpotential

The MSSM is the minimal supersymmetric extension of the SM. It is introduced with the minimal number of new particles. In a supersymmetric extension of the SM, each of the known fundamental particles is assigned into either a chiral or a vector superfield, and has a superpartner with spin differing by $1/2$ units. All of the SM fermions are members of the (left-handed or right-handed) chiral supermultiplets; because chiral supermultiplets can contain fermions whose left-handed parts transform differently under the gauge group than their right-handed parts. The spin-0 partners of the quarks and leptons are constructed by adding an "s", which is short for 'scalar'. Thus, generically they are called *squark* and *slepton*. The left-handed and right-handed components of quarks and leptons are separate two component Weyl fermions with different gauge transformation properties. For example, the superparners of the left-handed and right-handed parts of electron are called left-handed selectron, $\tilde{e}_L$ and right-handed selectron $\tilde{e}_R$, respectively. The neutrinos that are shown in Table 3.1 are always left-handed, so the snuetrinos are denoted by $\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$. A similar nomenclature applies for smuons and stuas: $\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R$. As seen in Table 3.1, the Higgs fields are also included in a chiral multiplet, and their partners are called higgsinos with spin $1/2$.

In the Standard Model, each fermion must have its own complex scalar partner, but these partners are not included in the SM spectrum; they need new additional states making up a supersymmetric field theory.

Table 3.1: Chiral superfields in the MSSM

Superfields		**Spin 0**	**Spin 1/2**	$SU(3)_C, SU(2)_L, U(1)_Y$
(squark, quark) (× 3 families)	Q $\bar{u}$ $\bar{d}$	$(\tilde{u}_L, \tilde{d}_L)$ $\tilde{\bar{u}}_L(\tilde{u}_R)$ $\tilde{\bar{d}}_L(\tilde{d}_R)$	(u_L, d_L) $\bar{u}_L \sim (d_R)^c$ $\bar{d}_L \sim (d_R)^c$	**3, 2,** 1/3 $\bar{\mathbf{3}}$**, 1,** -4/3 $\bar{\mathbf{3}}$**, 1,** 2/3
(slepton, lepton) (×3 families)	L $\bar{e}$	$(\tilde{\nu}_{eL}, \tilde{e}_L)$ $\tilde{\bar{e}}_L(\tilde{e}_R)$	(ν_{eL}, e_L) $\bar{e}_L \sim (e_R)^c$	**1, 2,** -1 **1, 1,** 2
(higgs,higgsino)	H_u H_d	(H_u^+, H_u^0) (H_d^0, H_d^-)	$(\tilde{H}_u^+, \tilde{H}_u^0)$ $(\tilde{H}_d^0, \tilde{H}_d^-)$	**1, 2,** 1 **1, 2,** -1

The vector bosons of the SM are placed in gauge supermultiplets. By now, we know that their fermionic superpartners are generically referred to as gauginos. The $SU(3)_C$ color gauge interactions of QCD are mediated by the gluon, whose supersymmetric partner is the gluino with spin $1/2$. The gauge bosons of the electroweak gauge symmetry $SU(2)_L \otimes U(1)_Y$ have spin 1, and their superpartners are called winos and bino which have spin $1/2$ (Martin 1999). They are all shown on Table 3.2.

Table 3.2: Gauge superfields in the MSSM

Superfield	**Spin 1/2**	**Spin 1**	$SU(3)_C, SU(2)_L, U(1)_Y$
(gluino, gluon)	$\tilde{g}$	g	**8,** **1,** 0
(wino, W boson)	$\tilde{W}^{\pm}, \tilde{W}^{0}$	$W^{\pm}, W^{0}$	**1,** **3,** 0
(bino, B boson)	$\tilde{B}$	B	**1,** **1,** 0

As for any supersymmetric theory, the Lagrangian is based on the "superpotential"- a holomorphic function of the chiral superfields. It has mass dimension three. It provides the Yukawa interactions, and the so-called F-term contributions in the Lagrangian. The renormalizable interactions of the MSSM are encoded as terms of two and three dimensions in the superpotential of the theory. The superpotential terms include the Yukawa couplings of the quarks and leptons to the Higgs doublets, as well as a mass term which couples H_u and H_d. Explicitly:

$$W = \varepsilon_{\alpha\beta}\left[-\hat{H}_u^{\alpha}\hat{Q}_i^{\beta}Y_{uij}\hat{U}_j^c + \hat{H}_d^{\alpha}\hat{Q}_i^{\beta}Y_{dij}\hat{D}_j^c + \hat{H}_d^{\alpha}\hat{L}_i^{\beta}Y_{eij}\hat{E}_j^c - \mu\hat{H}_d^{\alpha}\hat{H}_u^{\beta}\right] \tag{3.1}$$

where i and j are family indices, and α and β are $SU(2)_L$ indices. Here $\varepsilon_{\alpha\beta} = (i\sigma_2)_{\alpha\beta}$ is the usual 'metric' in superspace; it is necessary to contract $SU(2)_L$ doublets into singlets.

The superpotential of the MSSM dictates all of the supersymmetric couplings of the theory, aside from the gauge couplings. The superpotential and gauge couplings thus dictate the couplings of the Higgs potential of the theory – a feature not found in the SM. This would appear to reduce the number of independent parameters of the MSSM.

With the exception of the Higgs sector, the MSSM particle content, which is listed in Table 3.1 and Table 3.2, includes only the known SM fields and their superpartners. Supersymmetric theories with additional matter and gauge content can of course easily be constructed.

3.2. Lagrangian of the MSSM

In the exact supersymmetry it is dictated that every superpartner is degenerate in mass with its corresponding Standard Model (SM) particle, which is completely ruled by experimental results, because no superparticle has found near the energy scales of its SM partners. That is the reason why the supersymmetry is thought to be “broken” at low energies. However, this breaking cannot occur in a arbitrary fashion because there is always a danger of regenerating the quadratic divergences, discussed in Chapter 2, in the Higgs boson mass. In general, the supersymmetric Lagrangian consists of two different parts: the ‘supersymmetric part’ plus ‘softly-broken supersymmetric part’:

$$L_{MSSM} = L_{SUSY} + L_{Soft} \tag{3.2}$$

where supersymmetric Lagrangian respects supersymmetry transformations whereas the soft Lagrangian violates supersymmetry them.

3.2.1. Supersymmetric Part

In this section, we will describe the construction of supersymmetric Lagrangians. Our aim is to arrive at a sort of recipe, which will allow us to write down the allowed interactions of a general supersymmetric theory. The full supersymmetric Lagrangian is

$$L_{SUSY} = L_{Kinetic} + L_{Gauge} + L_{Yukawa} - V_{\bar{G}\psi\bar{\psi}} - V_D - V_F \tag{3.3}$$

The Yukawa interactions, which is obtained from the superpotential, just by replacing two of the superfields by their fermionic components setting the third to its scalar component:

$$\begin{aligned} L_{Yukawa} = \varepsilon_{ij} &\left[E_i^c Y_{eij} L_j^\alpha H_d^\beta + D_i^c Y_{dij} \mathrm{Q}_j^\alpha H_d^\beta + U_i^c Y_{uij} \mathrm{Q}_j^\alpha H_u^\beta + \mu \tilde{H}_u^\alpha \tilde{H}_d^\beta \right] \\ &+ \varepsilon_{ij} \left[\tilde{E}_i^c Y_{eij} L_j^\alpha \tilde{H}_d^\beta + \tilde{D}_i^c Y_{dij} \mathrm{Q}_j^\alpha \tilde{H}_d^\beta + \tilde{U}_i^c Y_{uij} \mathrm{Q}_j^\alpha \tilde{H}_u^\beta \right] \\ &+ \varepsilon_{ij} \left[\tilde{E}_i^c Y_{eij} L_j^\alpha \tilde{H}_d^\beta + \tilde{D}_i^c Y_{dij} \mathrm{Q}_j^\alpha \tilde{H}_d^\beta + \tilde{U}_i^c Y_{uij} \mathrm{Q}_j^\alpha \tilde{H}_u^\beta \right] \\ &+ \text{h.c.} \end{aligned} \tag{3.4}$$

The second type of interactions are obtained by the first computing the F-terms, $F = \dfrac{\partial W}{\partial \varphi_i}$; and squaring:

$$V_F = \sum_i \left| \frac{\partial W(\varphi)}{\partial \varphi_i} \right|^2 \tag{3.5}$$

φ_i being the scalar components of the superfields.

The gauge interactions introduce two kinds of interaction terms in (3.3). The first is related to gaugino-matter-smatter interactions:

$$V_{G\psi\bar{\psi}} = i\sqrt{2} g_a \varphi_k \bar{\lambda}^a \left(T^a\right)_{kl} \bar{\psi}_l + \text{h.c.} \tag{3.6}$$

where (φ, ψ) are the (spin-0, spin-1/2) components of the chiral superfields, respectively. T^a is generator of the gauge group, λ^a is the gaugino field and g_a its coupling constant. This structure is repeated for each gauge group.

Next, one has D-term contributions coming from gauge sector. This contribution does not involve gauge bosons, instead gauge quantum numbers of scalars are involved:

$$V_D = \frac{1}{2} \sum D^a D^a \tag{3.7}$$

with

$$D^a = g^a \varphi_i^* \left(T^a\right)_{ij} \varphi_j \tag{3.8}$$

where again φ_i are the scalar components of the superfields.

From the Lagrangian (3.3) we can obtain the full supersymmetric Lagrangians of the MSSM, as well as their interactions which contain the usual gauge interactions, the fermion-Higgs interactions (Gunion et al. 1990), and the pure SUSY interactions. A very detailed treatment of this Lagrangian and the process of derivation of the fortcoming results can be found in (Simonsen 1995).

3.2.2. Soft Supersymmetry Breaking

In general, in unbroken SUSY theory, the masses of the particles contained in the Standard Model and their superpartners (sparticles) do have identical masses. This is not realistic. In fact, no superpartners have been discovered until now. This defines a lower limit for the masses of the superpartners to be ~ TeV. Thus, SUSY must be a broken symmetry of Nature. This implies the appearance of supersymmetry-breaking terms in the Lagrangian. An immediate question is wether such terms spoil supersymmetry's elegant solution to the hierarchy problem or not. As generic quantum field theories with scalars generally have hierarchy problem, if all supersymmetry breaking terms consistent with other symmetries of the theory are allowed the dangerous UV divergences may indeed be reintroduced.

Fortunately, such dangerous divergences are not generated at any order of perturbation theory if only a certain subset of supersymmetry breaking terms are present in the theory. Such operators are said to *break supersymmetry softly*, and their couplings are collectively denoted as *soft parameters*. The part of the Lagrangian which contains these terms is generically called the soft supersymmetry breaking Lagrangian L_{Soft} , or simply the soft Lagrangian.

$$L_{Soft} = L_{Soft}^{Higgs} + L_{Soft}^{gaugino} + L_{Soft}^{sfermion} \tag{3.9}$$

The soft supersymmetry breaking is parameterized by various soft terms belonging to Higgs, gaugino and scalar fermion sectors:

$$
\begin{aligned}
-L_{Soft}^{Higgs} &= m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + \left[\mu B H_u H_d + \text{h.c.}\right] \\
-L_{Soft}^{gaugino} &= \frac{1}{2}\sum_{a=3,2,1}\left[M_a \lambda^a \lambda^a + \text{h.c.}\right] \\
-L_{Soft}^{sfermion} &= \tilde{Q}^\dagger m_Q^2 \tilde{Q} + \tilde{U} m_U^2 \tilde{U}^\dagger + \tilde{D} m_D^2 \tilde{D}^\dagger + \tilde{L} m_L^2 \tilde{L}^\dagger + \tilde{E} m_E^2 \tilde{E}^\dagger \\
&\quad + \left[\tilde{U} Y_u^A \tilde{Q} H_u + \tilde{D} Y_d^A \tilde{Q} H_d + \tilde{E} Y_e^A \tilde{L} H_d + \text{h.c.}\right]
\end{aligned} \tag{3.10}
$$

where $Y_{u,d,e}^A$, like Yukawa themselves, are non-hermitian flavor matrices whereas the sfermion mass-squareds $m_{Q,\ldots,E}^2$ are all hermitian. These matrices span a 3x3 flavor space.

The interactions contained in (3.10) exhibits mixing of various flavors in soft terms. We focus only on the flavor diagonal interactions due to the fact that flavor mixings generically prohibit the construction of RG invariants except for those parameters which depend on traces or determinants of the flavor matrices. Consequently, we switch off flavor mixings in all soft parameters to obtain

$$
Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_t \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_b \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_\tau \end{pmatrix}
$$

$$
Y_u^A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_t A_t \end{pmatrix}, \quad Y_d^A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_b A_b \end{pmatrix}, \quad Y_e^A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h_\tau A_\tau \end{pmatrix}
$$

$$
m_Q^2 = \begin{pmatrix} m_{\tilde{u}_L}^2 & 0 & 0 \\ 0 & m_{\tilde{c}_L}^2 & 0 \\ 0 & 0 & m_{\tilde{t}_L}^2 \end{pmatrix}, \quad m_L^2 = \begin{pmatrix} m_{\tilde{e}_L}^2 & 0 & 0 \\ 0 & m_{\tilde{\mu}_L}^2 & 0 \\ 0 & 0 & m_{\tilde{\tau}_L}^2 \end{pmatrix}
$$

$$
m_U^2 = \begin{pmatrix} m_{\tilde{u}_R}^2 & 0 & 0 \\ 0 & m_{\tilde{c}_R}^2 & 0 \\ 0 & 0 & m_{\tilde{t}_R}^2 \end{pmatrix}, \quad m_D^2 = \begin{pmatrix} m_{\tilde{d}_R}^2 & 0 & 0 \\ 0 & m_{\tilde{s}_R}^2 & 0 \\ 0 & 0 & m_{\tilde{b}_R}^2 \end{pmatrix}, \quad m_E^2 = \begin{pmatrix} m_{\tilde{e}_R}^2 & 0 & 0 \\ 0 & m_{\tilde{\mu}_R}^2 & 0 \\ 0 & 0 & m_{\tilde{\tau}_R}^2 \end{pmatrix} \tag{3.11}
$$

where $m_{\tilde{u}_L}^2 = m_{\tilde{d}_L}^2$, $m_{\tilde{c}_L}^2 = m_{\tilde{s}_L}^2$ and $m_{\tilde{t}_L}^2 = m_{\tilde{b}_L}^2$ by gauge invariance. Note that light fermion Yukawa couplings are totally neglected. This reduction scheme for flavor mixing sets up the notation and framework for the fermion sector.

3.2.2.1. Soft Supersymmetry Breaking: Holomorphic Case

Supersymmetry is broken because these terms contribute explicitly to masses and interactions of winos or squarks but not their superpartners. How supersymmetry breaking is transmitted to the superpartners is encoded in the parameters of L_{Soft}. All of the quantities in L_{Soft} receive radiative corrections and thus are scale-dependent, satisfying known renormalization group equations.

The soft supersymmetry breaking Lagrangian is defined to include all the allowed terms that do not introduce quadratic divergences in the theory: all gauge invariant and Lorentz invariant terms of dimensions two and three. The terms of L_{Soft} can be categorized as follows:

- Soft *trilinear scalar* interactions: $\frac{1}{3!}\tilde{A}_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$.
- Soft *bilinear scalar* interactions: $\frac{1}{2}b_{ij}\phi_i\phi_j + \text{h.c.}$.
- Soft *scalar mass-squares*: $m_{ij}^2\phi_i^+\phi_j$.
- Soft *gaugino masses*: $\frac{1}{2}M_a\lambda^a\lambda^a + \text{h.c.}$.

Finally, the soft supersymmetry breaking Lagrangian L_{Soft} takes the form

$$\begin{aligned}
-L_{Soft} &= \frac{1}{2}\left[M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B}\right] \\
&+\varepsilon_{\alpha\beta}\left[-bH_d^\alpha H_u^\beta - H_u^\alpha\tilde{Q}_i^\beta\tilde{A}_{uij}\tilde{U}_j^c + H_d^\alpha\tilde{Q}_i^\beta\tilde{A}_{dij}\tilde{D}_j^c + H_d^\alpha\tilde{L}_i^\beta\tilde{A}_{eij}\tilde{E}_j^c + \text{h.c.}\right] \\
&+m_{H_d}^2\left|H_d\right|^2 + m_{H_u}^2\left|H_u\right|^2 + \tilde{Q}_i^\alpha m_{Qij}^2\tilde{Q}_j^{\alpha*} \\
&+\tilde{L}_i^\alpha m_{Lij}^2\tilde{L}_j^{\alpha*} + \tilde{U}_i^{c*}m_{Uij}^2\tilde{U}_j^c + \tilde{D}_i^{c*}m_{Dij}^2\tilde{D}_j^c + \tilde{E}_i^{c*}m_{Eij}^2\tilde{E}_j^c
\end{aligned} \tag{3.12}$$

in the MSSM. The soft parameters here are to be taken as strictly diagonal matrices in the flavor space, as mentioned in the previous section.

The soft terms in (3.12) are said to be 'holomorphic' by which we mean that all trilinear interactions are nothing but the replica of the superpotential in that they do not contain hermitian conjugates of any scalar fields. This is a holomorphic structure in that all fields (complex numbers) are taken without hermition conjugation (complex comjugation). Indeed, as we will see in the next section, one may devise additional terms in L_{soft} which respect gauge invariance but violate holomorphicity.

3.2.2.2. Soft Supersymmetry Breaking: Non-Holomorphic Case

The MSSM Lagrangian is usually claimed to include all possible soft supersymmetry breaking terms, terms which split the masses and couplings of particles and their superpartners, but which do not remove the supersymmetric protection against large radiative corrections to scalar masses.

However, the statements above are not exhaustive at all. Indeed, one can consider adding to (3.12) additional terms which

- respect the gauge symmetry
- are soft
- are non-holomorphic.

Such additional terms are known to be perfectly soft (non-dangerous) as longs as the model under concern does not contain pure singlets under gauge group (like MSSM). In this sense for a complete understanding of the MSSM phenomenology (as well as its astrophysical and cosmological implications) one has to resort to non-holomorphic structures. Possible non-homomorphic structures consist of, as its name shows, hermitian conjugate of at least one MSSM scalar: These non-analytic terms include novel trilinear couplings as well as Dirac mass terms for Higgsinos:

$$L^{non-hol} = C_{ij}^{u} H_{u}^{*} Q_{i} U_{j} + C_{ij}^{d} H_{d}^{*} Q_{i} D_{j} + C_{ij}^{e} H_{d}^{*} L_{i} E_{j} + \tilde{\mu} \tilde{H}_{u} \tilde{H}_{d} + c.c. \qquad (3.13)$$

where i, j are family indices, and weak isospin, spinor, and colour indices are suppressed. C_{ij}^{f} and $\tilde{\mu}$ are new trilinear and μ-like soft supersymmetry breaking couplings.

The non-holomorphic SUSY breaking terms to be added to the MSSM Lagrangian are given in (3.13). Using the usual formula for extracting the full Lagrangian from the superpotential, the following terms are those involving $m_{H_{u/d}}^{2}$, the Higgs soft masses, μ, the Higgs bilinear superpotential term, and the non-holomorphic soft supersymmetry breaking couplings:

$$\begin{aligned} L = \ldots &+ (\tilde{\mu}-\mu) H_u H_d + \left(\mu^2 + m_{H_u}^2\right)\left|H_u\right|^2 + \left(\mu^2 + m_{H_d}^2\right)\left|H_d\right|^2 \\ &+ \left(C_{ij}^{u} - Y_{uij}\mu\right) H_d^{*} Q_i U_j + \left(C_{ij}^{d} - Y_{dij}\mu\right) H_d^{*} Q_i D_j + \left(C_{ij}^{e} - Y_{eij}\mu\right) H_d^{*} L_i E_j \\ &+ c.c. \end{aligned} \tag{3.14}$$

where $Y_{uij}, Y_{dij}, Y_{eij}$ are the Yukawa couplings. One notices that the usual structures $\mu Y^{ij}{}_{u,d,e}$ are modified by the corresponding non-holomorphic couplings. It is interesting that the F-term contributions assume a certain form of independence from rest of the soft-breaking Lagrangian due to these non-holomorphic contributions.

3.3. Renormalization Group Equations (RGEs)

In this section, we will illustrate derivation of the renormzalization group equations (RGEs) for a general supersymmetric theory with soft supersymmetry breaking terms. In general, RGEs govern the evolution of a Lagrangian parameter or field with the energy scale. The RGEs are first order differential equations in the scale parameter, and typically exhibit a coupled nature depending on the gauge charges of the fields.

As mentioned in the previous section, the scalar potential of a softly-broken supersymmetric theory has form $V = V_{SUSY} + V_{Soft}$. These can be expanded as

$$V_{SUSY} = \left|f_a\right|^2 + \frac{1}{2} D^A D^A$$

$$V_{Soft} = m_b^{2a} z^b z_a + \left(\eta + \text{h.c.}\right) \tag{3.15}$$

where the scalar components of the chiral fields ϕ^a are denoted by z^a with the convention

$$z_a = \left(z^a\right)^* \text{ and } f_a = \frac{\partial f}{\partial z^a} \tag{3.16}$$

where f stands for superpotential (with superfields are replaced by their scalar components)

$$f = l_a z^z + \frac{1}{2} \mu_{ab} z^a z^b + \frac{1}{6} f_{abc} z^a z^b z^c \tag{3.17}$$

D is the D-term contribution or the gauge potential

$$D^A = g_A z^a T_a^{Ab} z_b \tag{3.18}$$

and η stands for soft-breaking terms which are replica of the superpotential:

$$\eta = L_a z^a + \frac{1}{2} M_{ab} z^a z^b + \frac{1}{6} \eta_{abc} z^a z^b z^c \tag{3.19}$$

where (l_a, L_a), (μ_{ab}, M_{ab}) and (f_{abc}, η_{abc}) are, respectively, the linear, bilinear and trilinear couplings in the superpotential and soft-breaking lagrangian (Grisaru and Girardello 1982).

The crucial input for the derivation of RGEs is the one-loop renormalized effective potential (Weinberg 2005). The one-loop effective potential, specifically in the Wess-Zumino-Landau gauge, is given by

$$V_{1\,Loop} = V + hSTrM^{2} - kSTrM^{4} \tag{3.20}$$

where $h = \dfrac{\Lambda^2}{32\pi^2}$ and $k = \dfrac{\ln \Lambda^2}{64\pi^2}$ with Λ being the UV cutoff. Here Λ independent finite terms like $\dfrac{1}{64\pi^2} STrM^4 \ln M^2$ has been omitted since they are irrelevant for RGEs which derive from nothing but derivative of a given quantity with respect to Λ. All one has to do is to first form the mass matrix M of all the fields present in the spectrum, and then take its square and fourth power and plug in (3.20). In (3.20) we have defined $STr\mathrm{Q} = \sum_p (2j_p + 1)\,(-1)^{(2j_p+1)} Tr(\mathrm{Q})$ as a trace operation over a mass matrix with appropriate spin weighting. Generically, the mass-matrices of different spin multiplets are given by

$$\begin{gathered}
\left(M_1^2\right)^{AB} = D^{Aa} D_a^B + D_a^A D^{Ba}\,;\quad M_{\frac{1}{2}} = \begin{pmatrix} f^{ab} & i\sqrt{2} D^{Ba} \\ i\sqrt{2} D^{Ab} & \mu_A \delta^{AB} \end{pmatrix} \\
M_0^2 = \begin{pmatrix} f^{ac} f_{cb} + D^{Ba} D_b^B + D_b^{Ba} D^B + m_b^{2a} & f^{abc} f_c + D^{Ba} D^{Bb} + \eta^{ab} \\ f_{abc} f^c + D_a^B D_b^B + \eta_{ab} & f_{ac} f^{cb} + D_a^B D^{Bb} + D_a^{Bb} D^B + m_b^{2a} \end{pmatrix}
\end{gathered} \tag{3.21}$$

Using these mass matrices, we get

$$\begin{aligned}
STrM^4 = STrM^4_{SUSY} &+ 4m_a^{2b}\left[g_A^2 C_A(a) z^a z_b + D_b^{Aa} D^A\right] \\
&+ \left[-2g_A^2 C_A(a)\eta_a z^a + 8\mu_A g_A^2 C_A(a) f_a z^a + 2f^{abc} f_c \eta_{ab} + \text{h.c.}\right] \\
&+ 4m_b^{2a} f_{ac} f^{cb} + 2\eta_{ab}\eta^{ab} - 16 g_A^2 C_A(a)\mu_A^2 \left|z^a\right|^2 \\
&- 2\sum_A \mu_A^4 + 4Trm^4
\end{aligned} \tag{3.22}$$

and

$$\begin{aligned} STrM^4_{SUSY} &= 2g^2_A C_A(a)\left[-4f^a f_a - f_{ac}f^c z^a - z_a f_c f^{ac}\right] \\ &\quad + \left[4g^3_A C_A(a) z_a T^{Aa}_b z^b - g_A z_a \left\{X, T^A\right\}^a_b z^b\right] D^A \\ &\quad + 2f_a X^a_b f^b + 2g^2_A\left[T_A(z) - 3C_A(Adj.)\right] D^B D^B \end{aligned} \tag{3.23}$$

where Adj. is mean of Adjoint representation.Then we have introduced

$$\begin{aligned} &X^a_b = f^{acd} f_{bcd} \\ &g^2_A T_A(z)\,\delta^{AB} = D^{Aa}_b D^{Bb}_a = g^2_A Tr\left(T^A T^B\right) \\ &g^2_A C_A(a)\,\delta^a_b = D^{Aa}_c D^{Ac}_b = g^2_A\left[\left(T^A\right)^2\right]^a_b \end{aligned} \tag{3.24}$$

where derivations of various quantities are detailed in Appendix B.

Renormalizability means that the bare and radiatively-corrected Lagrangians are of the same form, and divergences can be included in redefinitions of the fields and parameters. These renormalizations of fields and parameters are found by solving the following equation:

$$\hat{V}(\hat{z}) = V_{1\,Loop}(z) \tag{3.25}$$

where $\hat{V}$ is the tree-level potential V with bare parameters are replaced by the renormalized ones. Let us consider first the supersymmetric part of the Lagrangian. The renormalization constants are found by solving (Barbieri 1982):

$$\hat{V}_{SUSY}(\hat{z}) = V_{SUSY}(z) - kSTrM^4_{SUSY} \tag{3.26}$$

with the result:

$$
\begin{aligned}
\hat{z}^{a} &= \left[1-k\left(2g_A^2C_A\left(a\right)1-X\right)\right]_{a'}^{a} z^{a'} \\
\hat{l}_a &= \left[1+k\left(4g_A^2C_A\left(a\right)1-X\right)\right]_{a}^{a'} l_{a'} \\
\hat{\mu}_{ab} &= \left[1+k\left(4g_A^2C_A\left(a\right)1-X\right)\right]_{a}^{a'}.\left[1+k\left(4g_B^2C_B\left(b\right)1-X\right)\right]_{b}^{b'} \mu_{a'b'} \\
\hat{f}_{abc} &= \left[1+k\left(4g_A^2C_A\left(a\right)1-X\right)\right]_{a}^{a'}.\left[1+k\left(4g_B^2C_B\left(b\right)1-X\right)\right]_{b}^{b'} \\
&\quad \left[1+k\left(4g_C^2C_C\left(c\right)1-X\right)\right]_{c}^{c'} f_{a'b'c'} \\
\hat{g}_A &= \left[1+2kg_A^2\left(3C_A\left(Adj.\right)-T\left(A\right)\right)\right]g_A
\end{aligned}
\tag{3.27}
$$

where again renormalized parameters are denoted by a hat a top. The remaining task is the solution of $\hat{V}_{Soft}$ which can be done by a comaprison of the coefficients of the powers of z and z^*:

$$
\hat{V}_{Soft}\left(\hat{z}\right) = V_{Soft} - k\left(STrM^4 - STrM_{SUSY}^4\right) \tag{3.28}
$$

The first order gives:

$$
\begin{aligned}
\hat{L}_a &= \left[1+k\left(4g_A^2C_A\left(a\right)1-X\right)\right]_{a}^{a'} L_{a'} - 2k\{4\mu_A g_A^2 C_A\left(a\right) l_a \\
&\quad +\eta_{acd} f^{cdb} l_b + 2m_c^{2b} f_{abd}\mu^{cd} + f^{bcd}\mu_{ab}M_{cd} + \eta_{abc}M^{bc}\}
\end{aligned}
\tag{3.29}
$$

The second order gives:

$$
\begin{aligned}
\hat{m}_a^{2b} &= m_a^{2b} - k\left\{X_a^{a'}m_{a'}^{2b} + X_{b'}^{b}m_a^{2b'} + 4m_c^{2d} f_{ade}f^{bce}\right. \\
&\quad \left. +4g_A D_a^{Ab} Tr\left(T^2m^2\right) + 2\eta_{acd}\eta^{bcd} - 16g_A^2\mu_A^2 C_A\left(a\right)\right\} \\
\hat{M}_{ab} &= \left[1+k\left(4g_A^2C_A\left(a\right)1-X\right)\right]_{a}^{a'}.\left[1+k\left(4g_B^2C_B\left(b\right)1-X\right)\right]_{b}^{b'} \\
&\quad -k\left\{8\mu_{ab}\mu_A g_A^2\left[C_A\left(a\right)+C_A\left(b\right)\right] + 2\left(f_{abc}M_{de} + \mu_{ac}\eta_{bde} + \mu_{bc}\eta_{ade}\right)f^{cde}\right\}
\end{aligned}
\tag{3.30}
$$

And third order gives:

$$\hat{\eta}_{abc} = \left[1 + k\left(4g_A^2 C_A(a) 1 - X\right)\right]_a^{a'} . \left[1 + k\left(4g_B^2 C_B(b) 1 - X\right)\right]_b^{b'}$$
$$\left[1 + k\left(4g_C^2 C_C(c) 1 - X\right)\right]_c^{c'} \eta_{a'b'c'} - k\left\{8\mu_A g_A^2 \left[C_A(a) + C_A(b) + C_A(c)\right] f_{abc}\right. \tag{3.31}$$
$$\left. + 2\left(f_{abf}\eta_{cde} + f_{acf}\eta_{bde} + f_{bcf}\eta_{ade}\right) f^{def}\right\}$$

The RGEs are obtained by differentiating the relations (3.27) - (3.31) with respect to Λ and requiring that the renormalized parameters are independent of Λ. Thus, all RGEs are first order differential equations

$$\frac{d}{dt} = \frac{1}{32\pi^2} \frac{d}{dk} \equiv \frac{d}{d \ln \Lambda} \tag{3.32}$$

We now list down RGEs for various Lagrangian parameters. All these differential equations are obtained by following the procedure outlined above. Following the conventions of (Falck 1985) they are given by:

1. Superpotential Parameters

$$\begin{aligned}
\frac{dl_a}{dt} &= \frac{1}{32\pi^2} X_a^{a'} l_{a'} \\
\frac{d\mu_{ab}}{dt} &= \frac{1}{32\pi^2} \left\{X_a^{a'} \mu_{a'b} + X_b^{b'} \mu_{ab'} - 8g_A^2 C_A(a) \mu_{ab}\right\} \\
\frac{df_{abc}}{dt} &= \frac{1}{32\pi^2} \left\{X_a^{a'} f_{a'bc} + X_b^{b'} f_{ab'c} + X_c^{c'} f_{abc'}\right. \\
&\quad \left. - 4g_A^2 \left[C_A(a) + C_A(b) + C_A(c)\right] f_{abc}\right\}
\end{aligned} \tag{3.33}$$

2. Soft Breaking Parameters

$$
\begin{aligned}
\frac{dL_a}{dt} &= \frac{1}{32\pi^2}\Big\{X_a^{a'} L_{a'} + 2\eta_{acd} f^{bcd} l_b + 4m_c^{2b} f_{abd}\mu^{cd} \\
&\quad +2\mu_{ab} f^{bcd} M_{cd} + 2\eta_{abc} M^{bc}\Big\} \\
\frac{dM_{ab}}{dt} &= \frac{1}{32\pi^2}\Big\{X_a^{a'} M_{a'b} + X_b^{b'} M_{ab'} - 8g_A^2 C_A(a) M_{ab} \\
&\quad +16\mu_{ab}\mu_A g_A^2 C_A(a) + 2\left(f_{abc} M_{de} + \mu_{ac}\eta_{bde} + \mu_{bc}\eta_{ade}\right) f^{cde}\Big\} \\
\frac{d\eta_{abc}}{dt} &= \frac{1}{32\pi^2}\Big\{X_a^{a'} \eta_{a'bc} + X_b^{b'} \eta_{ab'c} + X_c^{c'} \eta_{abc'} \\
&\quad +4g_A^2\left(2\mu_A f_{abc} - \eta_{abc}\right)\left[C_A(a) + C_A(b) + C_A(c)\right] \\
&\quad +2\left(f_{abf}\eta_{cde} + f_{acf}\eta_{bde} + f_{bcf}\eta_{ade}\right) f^{def}\Big\} \\
\frac{dm_a^{2b}}{dt} &= \frac{1}{32\pi^2}\Big\{X_a^{a'} m_{a'}^{2b} + X_{b'}^{b} m_a^{2b'} + 4g_A^2\, D_a^{Ab} Tr\left(T^A m^2\right) \\
&\quad +4m_c^{2d} f_{ade} f^{bce} + 2\eta_{acd}\eta^{bcd} - 16g_A^2\mu_A^2 C_A(a)\Big\}
\end{aligned}
\tag{3.34}
$$

3. Gauge Couplings and Gaugino Masses

$$
\begin{aligned}
\frac{dg_A}{dt} &= \frac{1}{16\pi^2}\left[T(A) - 3C_A(Adj.)\right] g_A^3 \\
\frac{d\mu_A}{dt} &= \frac{1}{8\pi^2}\left[T(A) - 3C_A(Adj.)\right] g_A^2 \mu_A
\end{aligned}
\tag{3.35}
$$

where one notes that the last equation cannot be achieved by using the effective potential alone, since scalar potential does not contain gaugino mass terms. Therefore, the RGEs of gaugino masses follow from diagrammatic calculations, and of course they exhibit a strong similarity to those of the gauge couplings. Finally, we note that the linear terms involving (l_a, L_a) can exist only in those models which contain a gauge singlet. However, non-holomorphic terms in such models are dangerous as they cause destabilization of the Higgs sector.

CHAPTER 4

RENORMALIZATION GROUP INVARIANTS

By Renormalization Group Invariants (RG invariants) we mean those quantities Q_i for which

$$\frac{dQ_i}{dt} = 0 \tag{4.1}$$

with one-loop accuracy. In other words, we take one-loop RGEs (eventually for the MSSM with and without non-holomorphic soft terms) computed in Chapter 3, and construct certain quantities Q_i whose derivatives with respect to the scale variable t vanishes.

Our motivation for studying RGEs is as follows. The RGEs, which can be used to relate measurements at the electroweak scale to physics at ultra high energies, provide important information about high-scale physics due to the scale invariance of the quantities. Since the coupled nature of the RGEs disturbs analytical solutions it would be beneficial to know if one can construct certain invariants that give relations among the spectrum of supersymmetric particles. Indeed, RG invariants may provide a direct and accurate way of testing the internal consistency of the model and determine the mechanism which breaks the supersymmetry. Such quantities prove highly useful not only for projecting the experimental data to high energies but also for deriving certain sum rules which enable fast consistency checks of the model. Let us suppose that there is a measurement, which reveals a specific relation between some of the soft masses then with the help of scale-independent relations coming from RG invariants one can arrive at certain inferences about the mechanism that breaks SUSY.

In interpreting the RGEs, we neglect modifications in the particle spectrum and RGEs coming from decoupling of the heavy fields. In other words, we assume that all soft masses are approximately equal to $M_{SUSY} \sim$ 1TeV in logarithmic sense. This scale sets the infrared (IR) boundary for exact superymmetric RG flow. The UV boundary lies just beneath the scale of string territory, and we will take it to the scale of gauge coupling unification in the MSSM: $M_{GUT} \sim 10^{16}$ GeV. Therefore, in our framework the RG invariance of a given quantity means its scale independence in between IR and UV scales above. Here, we combine the RGEs of individual quantities until we arrive at a RG invariant observable within one loop accuracy. In general, there is no guarantee of maintaining RG invariance of a given quantity at higher loop levels. Moreover, we note that flavor mixings prohibit construction of RG invariants, and therefore we restrict ourselves to limiting case of no flavor violation. In other words, we work in the basis in which equation (3.11) holds.

In the following sections, we will discuss RG-invariant observables in supersymmetry with *holomorphic* and *non-holomorphic* soft terms (The methods of finding these RG-invariants and associated examples are given in Appendix C).

4.1. RG Invariants in the MSSM with Holomorphic Soft Terms

The RGEs of the MSSM, as follows from the results of Chapter 3, are listed in Appendix B.1. By using the RGEs for gauge couplings, we derive an invariant

$$I_1 = \frac{c_1}{g_1^2} + \frac{c_2}{g_2^2} + \frac{33c_1 + 5c_2}{15g_3^2} \tag{4.2}$$

where c_1 and c_2 are arbitrary constants. These constants may be related by using values of gauge couplings at M_{SUSY} and M_{GUT} where g_0 is the common value of the gauge couplings at the unification scale M_{GUT}:

$$\frac{c_1}{c_2} = -\frac{5}{3}\frac{g_1(M_{SUSY})^2}{g_2(M_{SUSY})^2}\left(\frac{g_0^2 g_2(M_{SUSY})^2 + 3g_0^2 g_3(M_{SUSY})^2 - 4g_2(M_{SUSY})^2 g_3(M_{SUSY})^2}{11g_0^2 g_1(M_{SUSY})^2 + 5g_0^2 g_3(M_{SUSY})^2 - 16g_1(M_{SUSY})^2 g_3(M_{SUSY})^2}\right) \tag{4.3}$$

From the RGEs of Yukawa couplings and the μ parameter, we obtain the invariant:

$$I_2 = \mu\left(\frac{g_2^9 g_3^{256/3}}{h_t^{27} h_b^{21} h_\tau^{10} g_1^{73/33}}\right)^{1/61} \tag{4.4}$$

where exponents of gauge and Yukawa couplings follow from group-theoretic factors in their RGEs. This invariant enables one to determine the value of, say, μ at any scale in terms of Yukawa and gauge couplings. Indeed, for any scale $Q \in [M_{SUSY}, M_{GUT}]$ we find

$$\begin{aligned}\mu(Q_2) = \mu(Q)\left(\frac{h_t(Q_2)}{h_t(Q_1)}\right)^{\frac{27}{61}}\left(\frac{h_b(Q_2)}{h_b(Q_1)}\right)^{\frac{21}{61}}\left(\frac{h_\tau(Q_2)}{h_\tau(Q_1)}\right)^{\frac{10}{61}} \\ \times\left(\frac{g_3(Q_1)}{g_3(Q_2)}\right)^{\frac{256}{183}}\left(\frac{g_2(Q_1)}{g_2(Q_2)}\right)^{\frac{9}{61}}\left(\frac{g_1(Q_2)}{g_1(Q_1)}\right)^{\frac{73}{2013}}\end{aligned} \tag{4.5}$$

which makes it manifest that μ at any scale Q depends on the strong coupling g_3 although its RGE does not exhibit such a direct dependence at all. This exemplifies one interesting aspect of the RG invariants: They establish manifest direct relations among otherwise unrelated or uncorrelated model parameters. This aspect makes them quite important for consistency check of the model. By putting $Q_2 = M_{SUSY}$ and $Q_1 = M_{GUT}$ one finds that the ratio $\mu(M_{SUSY})/\mu(M_{GUT})$, which is one of the most crucial factors that determine the amount of fine tuning needed to achieve the correct value of the Z boson mass, is entirely determined by the interplay between the IR and UV values of the rigid parameters *i.e.* gauge and Yukawa couplings. In fact, (4.5) shows that the strongest dependence is on the strong coupling constant not on the isospin and hypercharge ones.

The RGEs of the gauge couplings and gaugino masses admit a further invariant

$$I_3 = \frac{M_a}{g_a^2}, (a = 1,2,3) \tag{4.6}$$

which shows that the ratio of the gaugino mass to fine structure constant of the same group is an RG invariant. This invariance property guarantees that

$$M_a(Q_2) = M_a(Q_1)\left(\frac{g_a(Q_2)}{g_a(Q_1)}\right)^2 \tag{4.7}$$

so that knowing two of the gaugino masses at a scale Q suffices to know the third if gauge coupling unification holds. This very relation also shows that $M_3(M_{SUSY})/M_3(M_{GUT})$ is much larger $M_{1,2}(M_{SUSY})/M_{1,2}(M_{GUT})$ due to asymptotic freedom. In fact, in minimal supergravity for instance, typically gluino is the first superpartner to decouple from the light spectrum.

From the RGEs of the trilinear couplings and Higgs bilinear mass B we arrive at another invariant:

$$I_4 = B - \frac{27}{61}A_t - \frac{21}{61}A_b - \frac{10}{61}A_\tau - \frac{256}{183}M_3 - \frac{9}{61}M_2 + \frac{73}{2013}M_1 \tag{4.8}$$

by which one can express B at any scale Q in terms of other dimension–one soft masses after using (4.7):

$$\begin{aligned} B(Q_2) = B(Q_1) &+ \frac{27}{61}\left(A_t(Q_2) - A_t(Q_1)\right) + \frac{21}{61}\left(A_b(Q_2) - A_b(Q_1)\right) \\ &+ \frac{10}{61}\left(A_\tau(Q_2) - A_\tau(Q_2)\right) + \frac{256}{183}M_3(Q_1)\left(\frac{g_3(Q_2)^2}{g_3(Q_1)^2} - 1\right) \\ &+ \frac{9}{61}M_2(Q_1)\left(\frac{g_2(Q_2)^2}{g_2(Q_1)^2} - 1\right) - \frac{73}{2013}M_1(Q_1)\left(\frac{g_1(Q_2)^2}{g_1(Q_1)^2} - 1\right) \end{aligned} \tag{4.9}$$

which clearly shows how strong the dependence on the gluino mass.

The RGE of B parameter is independent of the gluino mass; however, the expression in (4.8) does. This revealing of M_3 dependence is again a consequence of the RG invariants that give rise to explicit dependences in a otherwise implicit relation among parameters.

Having completed the discussion of the rigid and dimension-one soft parameters of the theory, we now start analyzing the scale invariant combinations of the scalar mass-squareds. They are given by

$$
\begin{aligned}
I_5 &= m^2_{\tilde{\tau}_R} - 2m^2_{\tilde{\tau}_L} - 3|M_2|^2 + \frac{1}{11}|M_1|^2 \\
I_6 &= m^2_{H_u} - \frac{3}{2}m^2_{\tilde{t}_R} + \frac{4}{3}|M_3|^2 + \frac{3}{2}|M_2|^2 - \frac{5}{66}|M_1|^2 \\
I_7 &= m^2_{H_d} - \frac{3}{2}m^2_{\tilde{b}_R} - m^2_{\tilde{\tau}_L} + \frac{4}{3}|M_3|^2 - \frac{1}{33}|M_1|^2 \\
I_8 &= m^2_{\tilde{t}_R} + m^2_{\tilde{b}_R} - 2m^2_{\tilde{t}_L} - 3|M_2|^2 + \frac{1}{11}|M_1|^2 \\
I_9 &= m^2_{\tilde{u}_L} - \frac{8}{9}|M_3|^2 + \frac{3}{2}|M_2|^2 + \frac{1}{198}|M_1|^2 \\
I_{10} &= m^2_{\tilde{u}_R} - \frac{8}{9}|M_3|^2 + \frac{8}{99}|M_1|^2 \\
I_{11} &= m^2_{\tilde{d}_R} - \frac{8}{9}|M_3|^2 + \frac{8}{99}|M_1|^2 \\
I_{12} &= m^2_{\tilde{e}_L} - \frac{3}{2}|M_2|^2 + \frac{1}{22}|M_1|^2 \\
I_{13} &= m^2_{\tilde{e}_R} + \frac{2}{11}|M_1|^2 \\
I_{14} &= m^2_{H_u} + m^2_{H_d} - 3m^2_{\tilde{t}_L} - m^2_{\tilde{\tau}_L} + \frac{8}{3}|M_3|^2 - 3|M_2|^2 + \frac{2}{11}|M_1|^2 \\
I_{15} &= m^2_{H_d} - \frac{3}{2}m^2_{\tilde{b}_R} - \frac{3}{2}m^2_{\tilde{\tau}_L} + \frac{1}{4}m^2_{\tilde{\tau}_R} + \frac{4}{3}|M_3|^2 - \frac{3}{4}|M_2|^2 - \frac{1}{132}|M_1|^2 \\
I_{16} &= 2m^2_{\tilde{u}_L} + m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} - \frac{32}{9}|M_3|^2 + 3|M_2|^2 + \frac{1}{9}|M_1|^2 \\
I_{17} &= m^2_{\tilde{e}_L} + \frac{1}{2}m^2_{\tilde{e}_R} - \frac{3}{2}|M_2|^2 + \frac{3}{22}|M_1|^2
\end{aligned}
\tag{4.10}
$$

All these invariants are combinations of the scalar masses (squarks, sleptons and Higgses) and gaugino masses. The gauge and Yukawa couplings *i.e.* the rigid parameters of the theory do not appear in these invariants. They are purely formed by the soft masses of scalars and gauginos. Each invariant puts forward a specific relation among masses of scalars and gauginos. For instance, the invariant I_{13} fixes the mass-squared parameter of selectron in terms of the hypercharge gaugino mass up to a constant. On the other hand, invariant I_8 expresses the difference between the mass-squareds of left-handed and right-handed top and bottom squarks in terms of the isospin and hypercharge gaugino masses.

More explicitly, I_8 gives

$$
\begin{aligned}
m_{\tilde{t}_R}^2(Q_2)+m_{\tilde{b}_R}^2(Q_2)-2m_{\tilde{t}_L}^2(Q_2) = 3\left|M_2(Q_1)\right|^2 & \left[\left(\frac{g_2(Q_2)}{g_2(Q_1)}\right)^4-1\right] \\
-\frac{1}{11}\left|M_1(Q_1)\right|^2 & \left[\left(\frac{g_1(Q_2)}{g_1(Q_1)}\right)^4-1\right]
\end{aligned}
\qquad (4.11)
$$

which shows that mass-splitting between left- and right- handed squarks in third generation is a function only of the isospin and hypercharge gauginos. The evolution of these masses are milder than the strong coupling so that one does not expect a large hierarchy between these two left-and right-handed sectors. Indeed, by taking $Q_2 = M_{SUSY}$ and $Q_1 = M_{GUT}$ one finds $-0.97M_2\left(M_{GUT}\right)^2+0.08M_1\left(M_{GUT}\right)^2$ for the right-hand side of (4.11). This result shows that the quantity in (4.11) is mainly governed by the isospin gaugino mass at the GUT scale.

Each invariant in (4.10) provides a relation among soft masses of scalars and gauginos. As for any RG invariant, their combinations are also invariants. In this sense, one can for instance, construct invariants, which involve only the scalar mass-squareds:

$$m_{H_u}^2(Q) = \frac{7}{12}m_0^2 + \frac{5}{12}m_{\tilde{e}_R}^2(Q) + m_{\tilde{u}_L}^2(Q) - \frac{21}{12}m_{\tilde{u}_R}^2(Q) - \frac{3}{4}m_{\tilde{d}_R}^2(Q) + \frac{3}{2}m_{\tilde{t}_R}^2(Q)$$

$$m_{H_d}^2(Q) = -\frac{15}{4}m_0^2 - \frac{1}{4}m_{\tilde{e}_R}^2(Q) - 3\left(m_{\tilde{u}_L}^2(Q) - \frac{1}{2}m_{\tilde{u}_R}^2(Q)\right)$$

$$-3\left(m_{\tilde{t}_L}^2(Q) - \frac{1}{2}m_{\tilde{t}_R}^2(Q) - \frac{3}{4}m_{\tilde{b}_R}^2(Q)\right) + \frac{3}{2}\left(m_{\tilde{\tau}_L}^2(Q) - \frac{1}{6}m_{\tilde{\tau}_R}^2(Q)\right)$$

(4.12)

where m_0 is the common scalar mass at the unification scale M_{GUT}. These expressions correlate Higgs soft mass-squareds with the soft mass-squareds of squarks and sleptons. Since W and Z masses are eventually fixed by the masses of the Higgs doublets, (4.12) is an expression of how sizes of various soft mass-squareds be tuned to generate their experimentally observed values.

Another phenomenologically useful observation is that

$$m_{\tilde{u}_R}^2 - m_{\tilde{d}_R}^2 - \frac{1}{3}m_{\tilde{e}_R}^2 \qquad (4.13)$$

which puts a correlation between the mass-squared of the right-handed selectron and right-handed up and down squark masses. These, seemingly unrelated sectors are tied up by this invariant.

The RG invariants above can be useful for various purposes:

- They reveal correlations among otherwise unrelated quantities such as equation (4.5) which correlates μ and strong coupling g_3. Normally, the RGE of μ is independent of g_3. However, RGEs of the Yukawa couplings transmit g_3 dependence, and invariant (4.4) thus exhibits an explicit dependence on g_3.
- They serve as consistency checks of the underlying model. For instance, a measurement of masses of top and bottom squarks of either chirality must satisfy (4.11) otherwise model turns out to be some other model possibly an extension of the MSSM involving new gauge groups and thus new gauginos.
- They serve as finding certain undetermined parameters in terms of the measured ones in a scale-independent way as long as the underlying model keeps becoming MSSM. For example, right-handed up squark mass can be computed in terms of

the right-handed selectron and down squark masses via (4.13) provided that we know the integration constant in the equation.

In the next section, we will discuss RG invariants in the MSSM with non-holomorphic soft terms.

4.2. RG Invariants in the MSSM with Non-Holomorphic Soft Terms

In this section, we will discuss RG invariants in the MSSM with non-holomorphic soft terms given in equation (3.13). We base discussions on the same assumptions made in Sec.4.1 above, that is, the Lagrangian parameters are scale dependent objects obeying the RGEs in Appendix B.2 such that the non-holomorphic MSSM holds in between the IR scale $(Q = Q_2)$ and the UV scale $(Q = Q_1)$. These IR and UV scales may be taken to be M_Z and M_{GUT} for applications relating measurements at the electroweak scale to high-scale models at the unification (or string) scale.

The RGEs for rigid parameters *i.e.* gauge and Yukawa couplings can be combined with that of the μ' parameter (a seemingly-hard actually-soft mass parameter for Higgsinos) to find

$$I'_1 = \mu' \left(\frac{g_2^9 g_3^{256/3}}{h_t^{27} h_b^{21} h_\tau^{10} g_1^{73/33}} \right)^{1/61} \tag{4.14}$$

as the one-loop RG invariant corresponding to I_1 in (4.4). In fact, this is identical to the RG invariant (4.4) with obvious implication given in (4.5). This direct similarity stems from the fact that gauge and Yukawa structures are kept unchanged when going to non-holomorphic MSSM.

We continue our analysis with the construction of the RG invariants of the soft parameters of the theory. Of this sector, a well-known RG invariant is the ratio of the gaugino masses to fine structure constants

$$I'_2 = \frac{M_a}{g_a^2} \tag{4.15}$$

which is identical to the RG invariant I_2 in equation (4.6) of the previous section. The reason is that non-holomorphic soft terms do not influence the running of the gaugino masses.

Another invariant of mass dimension-one is related to the B parameter for which we obtain:

$$
\begin{aligned}
I'_3 = B - \frac{27}{61}A_t - \frac{21}{61}A_b - \frac{10}{61}A_\tau - \frac{256}{183}M_3 - \frac{9}{61}M_2 + \frac{73}{2013}M_1 \\
+c_1 A'_t + c_2 A'_b + c_3 A'_\tau - (c_1 + c_2 + c_3)\mu'
\end{aligned}
\tag{4.16}
$$

with arbitrary coefficients c_i such that in the limit $A'_{t,b,\tau}, \mu' \to \mu$ it reproduces the well- known MSSM invariant in (4.8). One notices that the 'soft' nature of μ' makes it getting involved with the RG invariants of the soft masses like B in (4.16).

Concerning scalar mass-squareds, we obtain a general invariant

$$
\begin{aligned}
I'_4 = &\left(\frac{c_1}{6} + \frac{9c_2}{16} + \frac{c_3}{2} + \frac{c_4}{2}\right)m^2_{H_u}(t) + \left(\frac{-c_1}{6} + \frac{3c_2}{16} - \frac{c_3}{2} - \frac{c_4}{2}\right)m^2_{H_d}(t) \\
&+ \left(\frac{c_1}{6} - \frac{9c_2}{16} - \frac{c_3}{2} + \frac{3c_4}{2}\right)m^2_{t_L}(t) + \left(\frac{-c_1}{6} - \frac{9c_2}{16} - \frac{c_3}{2} - \frac{3c_4}{2}\right)m^2_{t_R}(t) \\
&+ \left(\frac{c_1}{6} - \frac{3c_2}{16} + \frac{c_3}{2} - \frac{3c_4}{2}\right)m^2_{l_L}(t) + c_3 m^2_{b_R}(t) + c_4 m^2_{l_R}(t) \\
&- \left(\frac{c_1}{33} + \frac{c_2}{44}\right)M_1^2(t) + c_1 M_2^2(t) + c_2 M_3^2(t) \\
&+ c_5 A'^2_t(t) + c_6 A'^2_b(t) + c_7 A'^2_\tau(t) - \left(\frac{3c_2}{4} + c_5 + c_6 + c_7\right)\mu'^2(t)
\end{aligned}
\tag{4.17}
$$

where c_i are arbitrary real parameters. In the limit all parameters but $c_{5,6,7}$ are non-zero we obtain:

$$
c_5 A'_t(t) + c_6 A'_b(t) + c_7 A'_\tau(t) - (c_5 + c_6 + c_7)\mu'^2(t) \tag{4.18}
$$

which is obviously invariant in the limit $A'_{t,b,\tau}, \mu' \to \mu$. By varying coefficients of various soft masses in (4.18) we obtain several forms of RG invariants. Let us consider, for example, setting all coefficients to zero except $c_1 = -3, c_4 = 1$. Then we find

$$I'_5 = -2m^2_{l_L}(t) + m^2_{l_R}(t) + 3|M_2(t)|^2 - \frac{1}{11}|M_1(t)|^2 \tag{4.19}$$

which correlates left-and right-handed selectron mass-squareds with those of the isospin and hypercharge gaugino. Furthering this kind of analysis, we find more invariants that combine mass-squareds of scalars and gaugino masses:

$$\begin{aligned}
I'_6 &= m^2_{H_u}(t) - \frac{3}{2}m^2_{t_R}(t) + \frac{4}{3}|M_3(t)|^2 + \frac{3}{2}|M_2(t)|^2 - \frac{5}{66}|M_1(t)|^2 - |\mu'(t)|^2 \\
I'_7 &= m^2_{H_d}(t) - \frac{3}{2}m^2_{b_R}(t) - m^2_{l_L}(t) + \frac{4}{3}|M_3(t)|^2 - \frac{1}{33}|M_1(t)|^2 - |\mu'(t)|^2 \\
I'_8 &= m^2_{t_R}(t) + m^2_{b_R}(t) - 2m^2_{t_L}(t) - 3|M_2(t)|^2 + \frac{1}{11}|M_1(t)|^2 \\
I'_9 &= m^2_{H_u}(t) + m^2_{H_d}(t) - 3m^2_{t_L}(t) - m^2_{l_L}(t) \\
&\quad + \frac{8}{3}|M_3(t)|^2 - 3|M_2(t)|^2 + \frac{1}{33}|M_1(t)|^2 - 2|\mu'(t)|^2 \\
I'_{10} &= m^2_{H_d}(t) - \frac{3}{2}m^2_{b_R}(t) - \frac{3}{2}m^2_{l_L}(t) + \frac{1}{4}m^2_{l_R}(t) \\
&\quad + \frac{4}{3}|M_3(t)|^2 - \frac{3}{4}|M_2(t)|^2 + \frac{1}{132}|M_1(t)|^2 - |\mu'(t)|^2
\end{aligned} \tag{4.20}$$

which should be contrasted with the RG invariants of the previous section. One lesson to be inferred from these is that μ' parameter is consistently involved in all invariants with appropriate coefficient. This, compared to ones in equation (4.10) of previous section, shows that μ' is a soft parameter and its evolution influences those of the other soft terms. The difference between (4.10) and (4.20) is a striking example of showing how RG invariants depend on the underlying model. The comments at the end of Sec. 4.1 above are also valid for this chapter. One, however, keeps in mind that

μ is a perfect rigid parameter in the MSSM with holomorphic soft terms, μ' exhibits a soft nature as can be seen from (4.20).

CHAPTER 5

CONCLUSION

In this thesis work, we have analyzed RG invariants in the MSSM with and without non-holomorphic soft terms. In Chapter 2 we have provided a brief summary of the SM followed by an introduction to phenomenological necessity and basic structure of the supersymmetry. Basic concepts of a generic supersymmetric field theory *i.e.* superspace, superfields, supersymmetric Lagrangians and superpotential are all included in Chapter 2.

In Chapter 3 we have discussed the MSSM by giving its particle spectrum, gauge structure and superpotential. We have investigated the manner in which supersymmetry breaking takes place, and examined structures of the soft supersymmetry breaking terms. We have, therein, also discussed holomorphic and non-holomorphic soft terms in a comparative fashion. We have also discussed RGEs, and gave their complete derivation in a softly broken supersymmetric theory (which is directly applicable to all models such as the MSSM).

In Chapter 4 we have discussed construction of various RG invariants and their implications for phenomenology of the underlying model.

We have listed several basic relations, definitions as well as RGEs in appendices.

Before concluding this thesis work, we emphasize that RG invariants can be utilized to:

i. test the internal consistency of the model while fitting to the experimental data,

ii. rehabilitate poorly known parameters by supplementing the well-measured ones,

iii. have clues on what kind if supersymmetry breaking mechanism is in operation,

iv. separate analysis of trilinear couplings (in both holomorphic and non-holomorphic cases) from scalar mass-squareds.

v. determine if the model assumed is self-consistent. Indeed, one may find that correlations suggested by certain invariants are indeed satisfied by experimental values of the parameters in a future collider environment like LHC; however, certain parameters may not match to any invariant relation at all. Then one can infer that the assumptions about the underlying model are false, and goes on correcting it. The correction may require non-holomorphic structures or more extended gauge structures. The interesting aspect is that all such invariants differ model to model (compare, for instance, the RG invariants (4.9) and (4.19)) in a distinctive fashion.

vi. However, as mentioned in the introduction, the power of RG invariants is limited by their sensitivity to higher loop corrections (in which case certain expressions get modified by additional terms involving at least one loop factor) and flavor mixings. Indeed, when flavor mixings are switched on the mass-squared parameters pertaining to slepton and squark sectors develop non-trivial flavor structures, and their RGEs do not admit any RG invariants even at one loop level.

However, experimental data collected so far about rare processes have already started to imply that flavor violation effects must be limited if not forbidden by some yet-to-be found symmetry.

This thesis work, mainly based on two publications (Demir 2005, Çakir 2005), is intended to give a further impetus to phenomenological relevance of the RG invariants.

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APPENDIX A

BASICS

A.1. Relativistic Notation

The notation and conventions used in this thesis work are taken from (Simonsen 1995).

In this report we will adopt standard relativistic units, i.e.

$$\hbar = c = 1 \tag{A. 1}$$

A general contravariant and covariant four-vector will be denoted by

$$\begin{aligned} A^{\mu} &= \left(A^0; A^1, A^2, A^3\right) = \left(A^0; A\right) \\ A_{\mu} &= \left(A_0; A_1, A_2, A_3\right) = \left(A_0, -A\right) \end{aligned} \tag{A. 2}$$

The compact “Feynman slash” notation

$$\not{A} = \gamma^{\mu} A_{\mu} \tag{A. 3}$$

will be used. The metric tensor $g^{\mu\nu}$, which connects A^{μ} and A_{μ}, is defined by

$$g^{\mu\nu} = diag\left(1, -1, -1, -1\right) \tag{A. 4}$$

Moreover, we will use the (relativistic) summation convention which states that repeated Greek indices $\mu, \nu, \rho, \sigma, \tau$, are summed from 0 to 3 and Latin indices run from 1 to 3 unless specifically indicated to the contrary.

The Minkowski product (the four-product) will be denoted by AB and defined as

$$AB \equiv A^{\mu} B_{\mu} = A^0 B^0 - \mathrm{AB} \qquad \text{(A. 5)}$$

Practical notation for the four-gradients, ∂^{μ} and ∂_{μ}, will be used

$$\begin{aligned} \partial^{\mu} &\equiv \frac{\partial}{\partial x_{\mu}} = \left(\frac{\partial}{\partial t}; -\nabla \right) \\ \partial_{\mu} &\equiv \frac{\partial}{\partial x^{\mu}} = \left(\frac{\partial}{\partial t}; \nabla \right) \end{aligned} \qquad \text{(A. 6)}$$

The totally antisymmetric Levi-Civita tensors in three and four dimensions are respectively defined by

$$\varepsilon_{ijk} = \begin{cases} +1, & \text{for even permutations of } 1\,2\,3 \\ -1, & \text{for odd permutations} \\ 0, & \text{otherwise} \end{cases} \qquad \text{(A. 7)}$$

$$\varepsilon_{\mu\nu\rho\sigma} = \begin{cases} +1, & \text{for even permutations of } 0\ 1\ 2\ 3 \\ -1, & \text{for odd permutaions} \\ 0, & \text{otherwise} \end{cases} \qquad \text{(A. 8)}$$

where

$$\begin{aligned} \varepsilon_{ijk} &= -\varepsilon^{ijk} \\ \varepsilon_{\mu\nu\rho\sigma} &= -\varepsilon^{\mu\nu\rho\sigma} \end{aligned} \qquad \text{(A. 9)}$$

A.2. Pauli Matrices

The well known Pauli Matrices are defined by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A. 10}$$

and satisfy the commutator relation

$$\left[\sigma^i, \sigma^j\right] = 2i\varepsilon^{ijk}\sigma^k, \qquad i, j, k, = 1, 2, 3 \tag{A. 11}$$

From this definition it is evident that

$$\begin{aligned} \left(\sigma^i\right)^t &= \sigma^i, \qquad i = 1, 2, 3 \\ \left(\sigma^i\right)^2 &= 1 \\ Tr\left(\sigma^i\right) &= 0 \end{aligned} \tag{A. 12}$$

For later use, we also introduce

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A. 13}$$

and a useful arrangement of these matrices is

$$\sigma^\mu = \left(\sigma^0; \vec{\sigma}\right) = \left(\sigma^0; \sigma^1, \sigma^2, \sigma^3\right) \tag{A. 14}$$

The index structure of the σ - matrices is given by

$$\sigma^\mu = \left[\sigma^\mu_{\alpha\dot{\alpha}}\right] \tag{A. 15}$$

We now introduce some “Pauli related” matrices defined by

$$\bar{\sigma}^{\mu\alpha\dot{\alpha}} \equiv \sigma^{\mu\alpha\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}\sigma^{\mu}_{\beta\dot{\beta}} \tag{A. 16}$$

where the “matrices” ε and $\bar{\varepsilon}$ have been used. By direct computations, one can establish the following relations

$$\begin{aligned} &\bar{\sigma}^0 = \sigma^0 \\ &\bar{\sigma}^i = -\sigma^i, \qquad i = 1,2,3 \end{aligned} \tag{A. 17}$$

Moreover, the following relations are true

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\beta}\beta}_{\mu} = 2\delta^{\beta}_{\alpha}\delta^{\dot{\beta}}_{\dot{\alpha}} \tag{A. 18}$$

$$Tr\left(\sigma^{\mu}\bar{\sigma}^{\nu}\right) = 2g^{\mu\nu} \tag{A. 19}$$

$$\left(\sigma^{\mu}\bar{\sigma}^{\nu} + \sigma^{\nu}\bar{\sigma}^{\mu}\right)_{\alpha}^{\beta} = 2g^{\mu\nu}\delta^{\beta}_{\alpha} \tag{A. 20}$$

$$\left(\bar{\sigma}^{\mu}\sigma^{\nu} + \bar{\sigma}^{\nu}\sigma^{\mu}\right)_{\dot{\beta}}^{\dot{\alpha}} = 2g^{\mu\nu}\delta^{\dot{\alpha}}_{\dot{\beta}} \tag{A. 21}$$

$$\left(\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\rho} + \sigma^{\rho}\bar{\sigma}^{\nu}\sigma^{\mu}\right) = 2\left(g^{\mu\nu}\sigma^{\rho} + g^{\nu\rho}\sigma^{\mu} - g^{\mu\rho}\sigma^{\nu}\right) \tag{A. 22}$$

$$\left(\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\sigma}^{\rho} + \bar{\sigma}^{\rho}\sigma^{\nu}\bar{\sigma}^{\mu}\right) = 2\left(g^{\mu\nu}\bar{\sigma}^{\rho} + g^{\nu\rho}\bar{\sigma}^{\mu} - g^{\mu\rho}\bar{\sigma}^{\nu}\right) \tag{A. 23}$$

$$Tr\left(\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\rho}\bar{\sigma}^{\sigma}\right) = 2\left(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma} - i\varepsilon^{\mu\nu\rho\sigma}\right) \tag{A. 24}$$

Most of the above relations are easily proved by direct computations. Besides, they (Müller-Kristen and Wiedemann 1987) have proved most of them and in particular (A. 24) which is the most difficult one.

$$\begin{aligned} \sigma^{\mu\nu} &= \frac{i}{4}\left(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}\right) \\ \bar{\sigma}^{\mu\nu} &= \frac{i}{4}\left(\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}\right) \end{aligned} \tag{A. 25}$$

By utilizing the index structure of the σ- matrices, it is easily seen that $\sigma^{\mu\nu}$ and $\bar{\sigma}^{\mu\nu}$ must have the index structure $\sigma^{\mu\nu} = \left[\left(\sigma^{\mu\nu} \right)_{\alpha}^{\beta} \right]$ and $\bar{\sigma}^{\mu\nu} = \left[\left(\bar{\sigma}^{\mu\nu} \right)_{\dot{\alpha}}^{\dot{\beta}} \right]$. In fact $\sigma^{\mu\nu}$ and $\bar{\sigma}^{\mu\nu}$ are the generators of $SL(2,C)$ in the spinor representations $\left(\frac{1}{2}, 0 \right)$ and $\left(0, \frac{1}{2} \right)$ respectively. The proofs together with the establishment of the below formulae can be found in (Ramond 1990):

$$\sigma^{\mu\nu t} = \bar{\sigma}^{\mu\nu} \tag{A. 26}$$

$$\sigma^{\mu\nu} = \frac{1}{2i} \varepsilon^{\mu\nu\rho\sigma} \sigma_{\rho\sigma} \tag{A. 27}$$

$$\bar{\sigma}^{\mu\nu} = -\frac{1}{2i} \varepsilon^{\mu\nu\rho\sigma} \bar{\sigma}_{\rho\sigma} \tag{A. 28}$$

$$Tr\left(\sigma^{\mu\nu} \right) = Tr\left(\bar{\sigma}^{\mu\nu} \right) = 0 \tag{A. 29}$$

$$Tr\left(\sigma^{\mu\nu} \sigma^{\rho\sigma} \right) = \frac{1}{2} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) + \frac{i}{2} \varepsilon^{\mu\nu\rho\sigma} \tag{A. 30}$$

$$Tr\left(\bar{\sigma}^{\mu\nu} \bar{\sigma}^{\rho\sigma} \right) = \frac{1}{2} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) - \frac{i}{2} \varepsilon^{\mu\nu\rho\sigma} \tag{A. 31}$$

A.3. Dirac Matrices

The Dirac γ- matrices are defined by the anticommutation (Cliford Algebra) relations

$$\left\{ \gamma^{\mu}, \gamma^{\nu} \right\} = 2g^{\mu\nu} \tag{A. 32}$$

From the four γ-matrices above, it is possible to define a “fifth- γ- matrix” by

$$\gamma_5 \equiv \gamma^5 \equiv i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \tag{A.33}$$

It possesses the following properties which follows easily from the definitions (A.32) and (A.33)

$$\{\gamma^5, \gamma^\mu\} = 0 \tag{A. 34}$$

$$(\gamma^5)^2 = 1 \tag{A. 35}$$

We will now state three explicit represented of the γ- matrices, namely the so-called Dirac representation, the Majorana representation, and finally the Chiral representation.

A.3.1. Representations

The lowest non-trivial representation of these matrices is of dimension four, and we will concentrate on this representation. From now on, we will assume that a four dimensional representation is used.

A.3.1.1. The Dirac Representation or Canonical Basis

In this particular representation the γ- matrices read

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A. 36}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \bar{\sigma}^i & 0 \end{pmatrix} \qquad i = 1, 2, 3 \tag{A. 37}$$

$$\gamma^5 = \begin{pmatrix} 0 & \sigma^0 \\ \bar{\sigma}^0 & 0 \end{pmatrix} \tag{A. 38}$$

Where 1 denotes the 2×2 identity matrix and σ^μ and $\bar{\sigma}^\mu$ are the Pauli matrices defined in the previous section.

A.3.1.2. The Majorana Representation

In this representation all γ- matrices are pure imaginary and have the explicit form:

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \tag{A. 39}$$

$$\gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix} \tag{A. 40}$$

$$\gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ -\bar{\sigma}^2 & 0 \end{pmatrix} \tag{A. 41}$$

$$\gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix} \tag{A. 42}$$

and finally

$$\gamma^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} \tag{A. 43}$$

A.3.1.3. The Chiral Representation or Weyl Basis

The basis is of particular interest to persons doing SUSY. In this representation the γ- matrices take on the explicate form

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{A. 44}$$

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A. 45}$$

A.4. SUSY Algebra

In this section, notation and conventions conform to those of (Wess and Bagger 1990).

General 4-dimensional SUSY algebra:

$$\left\{ \mathsf{Q}_{\alpha}^{A}, \bar{\mathsf{Q}}_{\dot{\beta}B} \right\} = 2\sigma_{\alpha\dot{\beta}}^{\mu} P_{\mu} \delta_{B}^{A} \tag{A. 54}$$

$$\left\{ \mathsf{Q}_{\alpha}^{A}, \mathsf{Q}_{\beta}^{B} \right\} = \varepsilon_{\alpha\beta} a^{lAB} B_{l} \tag{A. 55}$$

$$\left\{ \bar{\mathsf{Q}}_{\dot{\alpha}A}, \bar{\mathsf{Q}}_{\dot{\beta}B} \right\} = -\varepsilon_{\dot{\alpha}\dot{\beta}} a_{lAB}^{*} B^{l} \tag{A. 56}$$

$$\left[\mathsf{Q}_{\alpha}^{A}, P_{\mu} \right] = \left[\bar{\mathsf{Q}}_{\dot{\alpha}}^{A}, P_{\mu} \right] = 0 \tag{A. 57}$$

$$\left[\mathsf{Q}_{\alpha}^{A}, M_{\mu\nu} \right] = \sigma_{\mu\nu\alpha}{}^{\beta} \mathsf{Q}_{\beta}^{A} \tag{A. 58}$$

$$\left[\bar{\mathsf{Q}}_{A}^{\dot{\alpha}}, M_{\mu\nu} \right] = \sigma_{\mu\nu\dot{\beta}}^{\dot{\alpha}} \bar{\mathsf{Q}}_{A}^{\dot{\beta}} \tag{A. 59}$$

$$\left[P_{\mu}, P_{\nu} \right] = 0 \tag{A. 60}$$

$$\left[M_{\mu\nu}, P_{\rho} \right] = i \left[\eta_{\nu\rho} P_{\mu} - \eta_{\mu\rho} P_{\nu} \right] \tag{A. 61}$$

$$\left[M_{\mu\nu}, M_{\rho\sigma} \right] = -i \left[\eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho} \right] \tag{A. 62}$$

$$\left[\mathsf{Q}_{\alpha}^{A}, B_{l} \right] = S_{l}{}^{A}{}_{B} \mathsf{Q}_{\alpha}^{B} \tag{A. 63}$$

$$\left[\bar{\mathsf{Q}}_{\dot{\alpha}A}, B_{l} \right] = -S_{lA}^{*}{}^{B} \bar{\mathsf{Q}}_{\dot{\alpha}B} \tag{A. 64}$$

$$\left[B_{l}, B_{k} \right] = i C_{lk}{}^{j} B_{j} \tag{A. 65}$$

$$\left[P_{\mu}, B_{l} \right] = \left[M_{\mu\nu}, B_{l} \right] = 0 \tag{A. 66}$$

Where P_{μ} is the energy-momentum four-vector, $M_{\mu\nu}$ is the angular momentum tensor and B_{l} are the Ableian generators. Here, the a^{l} are antisymmetric matrices, and S_{l}, a_{l} must satisfy the intertwining relation:

$$S_{l}{}^{A}{}_{C} a^{CBk} = -a^{ACk} S_{C}{}^{*lB} \tag{A. 67}$$

Note also the perverse but essential conception implicit in (Wess and Bagger 1990),

$$a^{lAB} = -a_{AB}^{l} \tag{A. 68}$$

$N=1$ SUSY algebra in 4 dimensions

$$\left\{Q_{\alpha},\bar{Q}_{\dot{\beta}}\right\}=2\sigma^{\mu}_{\alpha\dot{\beta}}P_{\mu} \qquad \text{(A. 69)}$$

$$\left\{Q_{\alpha},Q_{\beta}\right\}=\left\{\bar{Q}_{\dot{\alpha}},\bar{Q}_{\dot{\beta}}\right\}=0 \qquad \text{(A. 70)}$$

$$\left[Q_{\alpha},P_{\mu}\right]=\left[\bar{Q}_{\dot{\alpha}},P_{\mu}\right]=0 \qquad \text{(A. 71)}$$

$$\left[Q_{\alpha},M_{\mu\nu}\right]=\sigma_{\mu\nu\alpha}{}^{\beta}Q_{\beta} \qquad \text{(A. 72)}$$

$$\left[\bar{Q}^{\dot{\alpha}},M_{\mu\nu}\right]=\sigma^{\dot{\alpha}}_{\mu\nu\dot{\beta}}\bar{Q}^{\dot{\beta}} \qquad \text{(A. 73)}$$

$$\left[P_{\mu},P_{\nu}\right]=0 \qquad \text{(A. 74)}$$

$$\left[M_{\mu\nu},P_{\rho}\right]=i\left[\eta_{\nu\rho}P_{\mu}-\eta_{\mu\rho}P_{\nu}\right] \qquad \text{(A. 75)}$$

$$\left[M_{\mu\nu},M_{\rho\sigma}\right]=-i\left(\eta_{\mu\rho}M_{\nu\sigma}-\eta_{\mu\sigma}M_{\nu\rho}-\eta_{\nu\rho}M_{\mu\sigma}+\eta_{\nu\sigma}M_{\mu\rho}\right) \qquad \text{(A. 76)}$$

$$\left[Q_{\alpha},R\right]=rQ_{\alpha} \qquad \text{(A. 77)}$$

$$\left[\bar{Q}_{\dot{\alpha}},R\right]=-R\bar{Q}_{\dot{\alpha}} \qquad \text{(A. 78)}$$

$$\left[P_{\mu},R\right]=\left[M_{\mu\nu},R\right]=0 \qquad \text{(A. 79)}$$

where the R is the $U(1)$ generator.

A.5. Anti-commuting Coordinates

Grassmann numbers θ and η are anti-commuting objects:

$$\theta\eta=-\eta\theta \qquad \text{(A.80)}$$

and hence $\theta^2=0$ (Berezin 1987). Because of this, the most general function of a single Grassmann variable θ is given uniquely by:

$$f(\theta)=A+B\theta \qquad \text{(A.81)}$$

with $A,B\in C$. Integral over Grassmann variables are defined by

$$\int d\theta[A+B\theta]:=B \tag{A.82}$$

which is called the Berezin integral.

a) Defining the derivative

$$\frac{d}{d\theta}\theta=1, \qquad \frac{d}{d\theta}A=0 \qquad (A\in C) \tag{A.83}$$

These show that Berezin integral of total derivative is zero and that the Berezin integral is translation invariant, i.e.

$$\int d\theta \frac{d}{d\theta}f(\theta)=0 \tag{A.84}$$

$$\int d\theta f(\theta+a)=\int d\theta f(\theta) \tag{A.85}$$

for $a\in C$. These properties of ordinary integrals of the type $\int_{\infty}^{\infty} dxf(x)$ is the motivation for the unusual definition (A.84). For the Grassmann variables, integration and differential are equivalent operations.

b) If one has several linearly independent Grassmann variables $\theta_i\,(i=1,\ldots,n)$, where

$$\theta_i\theta_j=-\theta_j\theta_i \qquad \forall \text{i,j} \tag{A.86}$$

then

$$\int d\theta_1 ... d\theta_n f(\theta_i)=c \tag{A.87}$$

where c is the coefficient in front of the $\theta_n\theta_{n-1}\ldots\theta_1$ -term in $f(\theta_i)$:

$$f=\ldots+c\theta_n\theta_{n-1}\ldots\theta_1 \tag{A.88}$$

APPENDIX B

RENORMALIZATION GROUP EQUATIONS

B.1. Renormalization Group Equations in the MSSM with Holomorphic Soft Terms

In this Appendix we list down the RGEs in the MSSM by extending to cases with finite bottom and tau Yukawas in a way including all three generations of sfermions. The one-loop RGEs of the gauge couplings are given by

$$\begin{aligned}
\frac{dg_3}{dt} &= \left(2N_F - 9\right) g_3^3 \\
\frac{dg_2}{dt} &= \left(2N_F - 5\right) g_2^3 \\
\frac{dg_1}{dt} &= \left(2N_F + \frac{3}{5}\right) g_1^3
\end{aligned} \tag{B. 1}$$

where $t \equiv \left(4\pi\right)^{-2} \ln \mathrm{Q}/M_{GUT}$, $N_F = 3$.

The evolutions of the superpotential parameters are given by

$$\begin{aligned}
\frac{dh_t}{dt} &= h_t\left(6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right) \\
\frac{dh_b}{dt} &= h_b\left(6h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right) \\
\frac{dh_\tau}{dt} &= h_\tau\left(4h_\tau^2 + 3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right) \\
\frac{d\mu}{dt} &= \mu\left(3h_t^2 + 3h_b^2 + h_\tau^2 - 3g_2^2 - \frac{3}{5}g_1^2\right)
\end{aligned} \tag{B. 2}$$

The gaugino masses envolve as

$$\begin{aligned}\frac{dM_3}{dt} &= (4N_F - 18)\, g_3^2 M_3 \\ \frac{dM_2}{dt} &= (4N_F - 10)\, g_2^2 M_2 \\ \frac{dM_1}{dt} &= \left(4N_F + \frac{6}{5}\right) g_1^2 M_1.\end{aligned} \tag{B. 3}$$

The RG evolutions of the trilinear couplings are given by

$$\begin{aligned}\frac{dA_t}{dt} &= 2\left(6h_t^2 A_t + h_b^2 A_b\right) + 2\left(\frac{16}{3} g_3^2 M_3 + 3g_2^2 M_2 + \frac{13}{15} g_1^2 M_1\right) \\ \frac{dA_b}{dt} &= 2\left(6h_b^2 A_b + h_t^2 A_t + h_\tau^2 A_\tau\right) + 2\left(\frac{16}{3} g_3^2 M_3 + 3g_2^2 M_2 + \frac{7}{15} g_1^2 M_1\right) \\ \frac{dA_\tau}{dt} &= 2\left(4h_\tau^2 A_\tau + 3h_b^2 A_b\right) + 2\left(3g_2^2 M_2 + \frac{9}{5} g_1^2 M_1\right) \\ \frac{dB}{dt} &= 2\left(3h_t^2 A_t + 3h_b^2 A_b + h_\tau^2 A_\tau\right) + 2\left(3g_2^2 M_2 + \frac{3}{5} g_1^2 M_1\right)\end{aligned} \tag{B. 4}$$

where B is the Higgs bilinear coupling in its potential. The scalar soft mass-squared parameters evolve according to

$$\begin{aligned}\frac{dm_{H_u}^2}{dt} &= 6\left(m_{H_u}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + |A_t|^2\right) h_t^2 - 8\left(\frac{3}{4} g_2^2 |M_2|^2 + \frac{3}{20} g_1^2 |M_1|^2\right) \\ \frac{dm_{H_d}^2}{dt} &= 2\left(m_{H_d}^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 + |A_\tau|^2\right) h_\tau^2 + 6\left(m_{H_d}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{b}_R}^2 + |A_b|^2\right) h_b^2 \\ &\quad -8\left(\frac{3}{4} g_2^2 |M_2|^2 + \frac{3}{20} g_1^2 |M_1|^2\right)\end{aligned} \tag{B.5}$$

$$
\begin{aligned}
\frac{dm^2_{\tilde{t}_L}}{dt} &= 2\left(m^2_{\tilde{t}_L} + m^2_{H_d} + m^2_{\tilde{b}_R} + |A_b|^2\right)h_b^2 + 2\left(m^2_{\tilde{t}_L} + m^2_{H_u} + m^2_{\tilde{t}_R} + |A_t|^2\right)h_t^2 \\
&\quad -8\left(\frac{4}{3}g_3^2|M_3|^2 + \frac{3}{4}g_2^2|M_2|^2 + \frac{1}{60}g_1^2|M_1|^2\right) \\
\frac{dm^2_{\tilde{t}_R}}{dt} &= 4\left(m^2_{\tilde{t}_L} + m^2_{H_u} + m^2_{\tilde{t}_R} + |A_t|^2\right)h_t^2 - 8\left(\frac{4}{3}g_3^2|M_3|^2 + \frac{4}{15}g_1^2|M_1|^2\right) \\
\frac{dm^2_{\tilde{b}_R}}{dt} &= 4\left(m^2_{\tilde{t}_L} + m^2_{H_d} + m^2_{\tilde{b}_R} + |A_b|^2\right)h_b^2 - 8\left(\frac{4}{3}g_3^2|M_3|^2 + \frac{1}{15}g_1^2|M_1|^2\right) \\
\frac{dm^2_{\tilde{\tau}_L}}{dt} &= 2\left(m^2_{\tilde{\tau}_L} + m^2_{H_d} + m^2_{\tilde{\tau}_R} + |A_\tau|^2\right)h_\tau^2 - 8\left(\frac{3}{4}g_2^2|M_2|^2 + \frac{3}{20}g_1^2|M_1|^2\right) \\
\frac{dm^2_{\tilde{\tau}_R}}{dt} &= 4\left(m^2_{\tilde{\tau}_L} + m^2_{H_d} + m^2_{\tilde{\tau}_R} + |A_\tau|^2\right)h_\tau^2 - \frac{24}{5}g_1^2|M_1|^2 \\
\frac{dm^2_{\tilde{u}_L}}{dt} &= -8\left(\frac{4}{3}g_3^2|M_3|^2 + \frac{3}{4}g_2^2|M_2|^2 + \frac{1}{60}g_1^2|M_1|^2\right) \\
\frac{dm^2_{\tilde{u}_R}}{dt} &= -8\left(\frac{4}{3}g_3^2|M_3|^2 + \frac{4}{15}g_1^2|M_1|^2\right) \\
\frac{dm^2_{\tilde{d}_R}}{dt} &= -8\left(\frac{4}{3}g_3^2|M_3|^2 + \frac{1}{15}g_1^2|M_1|^2\right) \\
\frac{dm^2_{\tilde{e}_L}}{dt} &= -8\left(\frac{3}{4}g_2^2|M_2|^2 + \frac{3}{20}g_1^2|M_1|^2\right) \\
\frac{dm^2_{\tilde{e}_R}}{dt} &= -\frac{24}{5}g_1^2|M_1|^2
\end{aligned}
\tag{B. 6}
$$

When writing the RGEs for scalar mass-squareds we assumed that they unify at some scale, preferably, at the GUT scale.

B.2. Renormalization Group Equations in the MSSM with non-Holomorphic Soft Terms

For the non-holomorphic soft terms one-loop RGEs can be found from (Jack, Jones and Kord 2004). We also present them here for the sake of completeness.

$$\frac{dm_{H_d}^2}{dt} = 2h_\tau^2\left(m_{H_d}^2 + A_\tau^2 + m_{l_L}^2 + m_{l_R}^2\right) + 6h_b^2\left(m_{H_d}^2 + A_b^2 + m_{t_L}^2 + m_{b_R}^2\right)$$

$$+ 6h_t^2 A_{t'}^2 - 8\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right)\mu'^2 - 6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2$$

$$\frac{dm_{H_u}^2}{dt} = 6h_t^2\left(m_{H_u}^2 + A_t^2 + m_{t_L}^2 + m_{t_R}^2\right) + 2h_\tau^2 A_{\tau'}^2 + 6h_b^2 A_{b'}^2$$

$$- 8\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right)\mu'^2 - 6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2$$

$$\frac{dm_3^2}{dt} = \left(h_\tau^2 + 3h_b^2 + 3h_t^2\right)m_3^2 + 2h_\tau^2 A_{\tau'} A_\tau + 6h_b^2 A_{b'} A_b + 6h_t^2 A_{t'} A_t$$

$$- 4\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right)m_3^2 + 6g_2^2 \mu' M_2^2 + \frac{6}{5}g_1^2 \mu' M_1^2$$

$$\frac{d\mu'}{dt} = \left(h_\tau^2 + 3h_b^2 + 3h_t^2 - 3g_2^2 - \frac{3}{5}g_1^2\right)\mu'$$

$$\frac{dA_{\tau'}}{dt} = \left(h_\tau^2 - 3h_b^2 + 3h_t^2\right)A_{\tau'} + 6h_b^2 A_{b'} + \left(4A_{\tau'} - 8\mu'\right)\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right)$$

$$\frac{dA_\tau}{dt} = 8h_\tau^2 A_\tau + 6h_b^2 A_b + 6g_2^2 M_2 + \frac{18}{5}g_1^2 M_1$$

$$\frac{dA_{b'}}{dt} = \left(-h_\tau^2 + 3h_b^2 + 3h_t^2\right)A_{b'} + 2h_\tau^2 A_{\tau'} - 2h_t^2\left(A_{t'} - 2\mu'\right)$$

$$+ \left(4A_{b'} - 8\mu'\right)\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right) \qquad \text{(B. 7)}$$

$$\frac{dA_b}{dt} = 2h_\tau^2 A_\tau + 12h_b^2 A_b + 2h_t^2 A_t + \frac{32}{3} g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{5} g_1^2 M_1$$

$$\frac{dA_{t'}}{dt} = \left(h_\tau^2 + 3h_b^2 + 3h_t^2\right) A_{t'} - 2h_b^2 A_{b'} + 4h_b^2 \mu' + \left(4A_{t'} - 8\mu'\right)\left(\frac{3}{4} g_2^2 + \frac{3}{20} g_1^2\right)$$

$$\frac{dA_t}{dt} = 2h_b^2 A_b + 12h_t^2 A_t + \frac{32}{3} g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15} g_1^2 M_1$$

$$\frac{dm_{t_L}^2}{dt} = 2h_b^2 \left(m_{t_L}^2 + m_{b_R}^2 + m_{H_d}^2 + A_{b'}^2 + A_b^2 - 2\mu'\right)$$

$$+ 2h_t^2 \left(m_{t_L}^2 + m_{t_R}^2 + m_{H_u}^2 + A_{t'}^2 + A_t^2 - 2\mu'\right)$$

$$- \frac{32}{3} g_3^2 M_3 - 6g_2^2 M_2 - \frac{2}{15} g_1^2 M_1$$

$$\frac{dm_{t_R}^2}{dt} = 4h_t^2 \left(m_{t_L}^2 + m_{t_R}^2 + m_{H_u}^2 + A_{t'}^2 + A_t^2 - 2\mu'\right) - \frac{32}{3} g_3^2 M_3 - \frac{32}{15} g_1^2 M_1$$

$$\frac{dm_{b_R}^2}{dt} = 4h_b^2 \left(m_{t_L}^2 + m_{b_R}^2 + m_{H_d}^2 + A_{b'}^2 + A_b^2 - 2\mu'\right) - \frac{32}{3} g_3^2 M_3 - \frac{8}{15} g_1^2 M_1$$

$$\frac{dm_{l_L}^2}{dt} = 2h_\tau^2 \left(m_{l_L}^2 + m_{l_R}^2 + m_{H_d}^2 + A_{\tau'}^2 + A_\tau^2 - 2\mu'\right) - 6g_2^2 M_2 - \frac{6}{5} g_1^2 M_1$$

$$\frac{dm_{l_R}^2}{dt} = 4h_\tau^2 \left(m_{l_L}^2 + m_{l_R}^2 + m_{H_d}^2 + A_{\tau'}^2 + A_\tau^2 - 2\mu'\right) - \frac{24}{5} g_1^2 M_1$$

$$\frac{dM_i}{dt} = 2b_i M_i g_i^2 \qquad \text{(B.8)}$$

here $b_{1,2,3} = \left(\frac{33}{5}, 1, -3\right)$ for hypercharge, isospin and color gauge groups, respectively.

APPENDIX C

RENORMALIZATION GROUP INVARIANTS

We determine RG invariants by combining the RGEs of various parameters. We here provide some examples with certain sample calculations.

Exp.1) We know from RGEs of the gauge couplings that

$$\frac{dg_1}{dt} = \frac{33}{5} g_1^3$$

$$\frac{dg_2}{dt} = g_2^3 \qquad \text{(C.1)}$$

$$\frac{dg_3}{dt} = -3 g_3^3$$

The above equations can be written as;

$$\frac{1}{g_1^3}\frac{dg_1}{dt} = \frac{33}{5} \qquad \Rightarrow \qquad \frac{d}{dt}\left(\frac{1}{g_1^2}\right) = -\frac{66}{5}$$

$$\frac{1}{g_2^3}\frac{dg_2}{dt} = 1 \qquad \Rightarrow \qquad \frac{d}{dt}\left(\frac{1}{g_2^2}\right) = -2 \qquad \text{(C.2)}$$

$$\frac{1}{g_3^3}\frac{dg_3}{dt} = -3 \qquad \Rightarrow \qquad \frac{d}{dt}\left(\frac{1}{g_3^2}\right) = 6$$

If we combine them to find invariant,

$$\frac{d}{dt}\left(\frac{c_1}{g_1^2} + \frac{c_2}{g_2^2} + \frac{c_3}{g_3^2}\right) = -\frac{66}{5}c_1 - 2c_2 + 6c_3 \qquad \text{(C.3)}$$

Equation (C.3) is invariant, when the right part is equal to zero.

$$-\frac{66}{5}c_1 - 2c_2 + 6c_3 = 0 \qquad \Rightarrow \qquad c_3 = \frac{33c_1 + 5c_2}{15} \tag{C.4}$$

$$\frac{d}{dt}\left(\underbrace{\frac{c_1}{g_1^2} + \frac{c_2}{g_2^2} + \frac{33c_1 + 5c_2}{15g_3^2}}_{In\,\mathrm{var}\,iant\;\; I_2}\right) = 0 \tag{C.5}$$

$$\frac{dI_2}{dt} = 0$$

Exp.2) We know from RGEs of the scalar soft mass-squared parameters that

$$\frac{dm_{H_d}^2}{dt} = 2\left(m_{H_d}^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 + |A_\tau|^2\right)h_\tau^2 + 6\left(m_{H_d}^2 + m_{\tilde{t}_L}^2 + m_{\tilde{b}_R}^2 + |A_b|^2\right)h_b^2$$
$$-8\left(\frac{3}{4}g_2^2|M_2|^2 + \frac{3}{20}g_1^2|M_1|^2\right) \tag{C.6}$$

$$\frac{dm_{\tilde{b}_R}^2}{dt} = 4\left(m_{\tilde{t}_L}^2 + m_{H_d}^2 + m_{\tilde{b}_R}^2 + |A_b|^2\right)h_b^2 - 8\left(\frac{4}{3}g_3^2|M_3|^2 + \frac{1}{15}g_1^2|M_1|^2\right) \tag{C.7}$$

$$\frac{dm_{\tilde{\tau}_L}^2}{dt} = 2\left(m_{\tilde{\tau}_L}^2 + m_{H_d}^2 + m_{\tilde{\tau}_R}^2 + |A_\tau|^2\right)h_\tau^2 - 8\left(\frac{3}{4}g_2^2|M_2|^2 + \frac{3}{20}g_1^2|M_1|^2\right) \tag{C.8}$$

where, after forming a combination of the form

$$(\mathrm{C.6}) - \frac{3}{2}(\mathrm{C.7}) - (\mathrm{C.8})$$

we find

$$\frac{dm_{H_d}^2}{dt} - \frac{3}{2}\frac{dm_{\tilde{b}_R}^2}{dt} - \frac{dm_{\tilde{\tau}_L}^2}{dt} = 16g_3^2|M_3|^2 + \frac{4}{5}g_1^2|M_1|^2 \tag{C.9}$$

However, we know from RGEs of the gaugino masses that

$$\frac{d|M_3|^2}{dt} = -12 g_3^2 |M_3|^2$$
$$\frac{d|M_1|^2}{dt} = \frac{132}{5} g_1^2 |M_1|^2 \tag{C.10}$$

and if we replace (C.10) in (C.9) we can find a new invariant

$$\frac{d}{dt}\left(m_{H_d}^2 - \frac{3}{2} m_{\tilde{b}_R}^2 - m_{\tilde{\tau}_L}^2 \right) = -\frac{4}{3}\frac{d|M_3|^2}{dt} + \frac{1}{33}\frac{d|M_1|^2}{dt} \tag{C.11}$$

which yields the invariant I_7, after using (C.10),

$$\frac{d}{dt}\left(\underbrace{m_{H_d}^2 - \frac{3}{2} m_{\tilde{b}_R}^2 - m_{\tilde{\tau}_L}^2 + \frac{4}{3}|M_3|^2 - \frac{1}{33}|M_1|^2}_{In\,\mathrm{var}\,iant\ I_7} \right) = 0$$
$$\frac{dI_7}{dt} = 0 \tag{C.12}$$

These sample calculations can be extended to all other parameters of the model to find invariants (Demir 2005).