FREE VIBRATION ANALYSIS OF CURVED BEAMS WITH VARIABLE CROSS-SECTIONS ON ELASTIC FOUNDATIONS

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ABSTRACT

FREE VIBRATION ANALYSIS OF CURVED BEAMS WITH VARIABLE CROSS-SECTIONS ON ELASTIC FOUNDATIONS

Free out of plane vibration characteristics of curved beams with variable cross-sections on elastic foundations are studied by TMM (Transfer Matrix Method) since the mathematical model of the present system based on the coupled differential eigenvalue problem with variable coefficients which can not be solved easily by exact methods. Vibrations of beams on different elastic foundations are reviewed. Out of plane vibration of curved beams on different elastic foundations are investigated. TMM is detailed with its applications to vibration problems. To solve the vibration problems, TMM is examined with several computer programs developed in Mathematica. The accuracy of the TMM results obtained from the developed program is evaluated by comparing with FEM results found from model created in ANSYS. Finally, the effects of the variation of cross-section of the curved beams and elastic foundation parameters on natural frequencies are investigated.

ÖZET

ELASTİK ZEMİNDEKİ DEĞİŞKEN KESİTLİ EĞRİ ÇUBUKLARIN SERBEST TİTREŞİM ANALİZİ

Elastik zemindeki değişken kesitli eğri çubukların düzlem dışı titreşim karakteristikleri, mevcut sistemin matematiksel modeli kesin metodlarla kolayca çözülemeyen değişken katsayılı bağlaşık diferansiyel özdeğer problemine dayalı olduğundan, TMM (Transfer Matris Metodu) ile incelenmiştir. Çubukların değişik elastik zeminlerdeki titreşimleri gözden geçirilmiştir. Elastik zeminlerdeki eğri çubukların düzlem dışı titreşimleri araştırılmıştır. Titreşim problemleri için TMM detaylandırılmıştır. Titreşim problemlerini çözmek için, Mathematica geliştirilen çeşitli programlar ile TMM denenmiştir. Geliştirilen programdan elde edilen TMM sonuçlarının doğruluğu, ANSYS de oluşturulan modelden elde edilen sonuçlarla karşılaştırılarak değerlendirilmiştir. Son olarak, eğri çubuğun enine kesitinin değişim ve elastik zemin parametrelerinin doğal frekanslara etkileri araştırılmıştır.

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$A(\mathrm{s})$	cross-sectional area of the beam
$b(\mathrm{s})$	breadth function of the beam
b_0	breadth of the beam at root cross-section
b_1	breadth parameter
$B(\mathrm{s})$	angular displacement about z axis as function of s
c_1, c_2	Reissner coefficients
C_β	the ratio of breadth/depth
d	viscous damping coefficient
$\{d\}$	displacement part of state vector
D_0	Hatenyi coefficient
E	modulus of elasticity
f	natural frequency
$\{f\}$	force part of state vector
F_{y}	external force in y direction
G	shear modulus
$h(\mathrm{s})$	depth function of the beam
h_0	depth of the beam at root cross-section
h_1	depth parameter
i	mass polar moment of inertia per unit length
I_{xx}	area moment of inertia of the cross-section about xx axis
$J(\mathrm{s})$	torsional constant of cross-section
k_0	Winkler foundation parameter
k_1	Second foundation parameter
k_G	Pasternak foundation parameter
m	mass per unit length of the beam
M_{x}, M_{z}	bending and twisting moments
N_{z}	internal force along z axis
R	radius
T	Filonenko-Borodich coefficient
T_{z}	external twisting moment about z axis
[T]	transfer matrix

$v(s,t)$	displacement in y direction
V(s)	displacement in y direction as function of s
V_y	shear force in y direction
$\{Z\}$	state vector
α	opening angle
$\beta(s,t)$	angular displacement about z axis
κ	curvature in xz plane
τ	twist about z axis
(`)	differentiation with respect to s
(˙)	differentiation with respect to time

CHAPTER 1

GENERAL INTRODUCTION

Curved beams have many engineering applications. They are in the shape of a space curve or a plane curve. On the other hand, they may have constant or variable curvature and constant or variable cross section. If the cross-section of the curved beam is not symmetrical, the in-plane and out-of-plane vibrations of curved beams are coupled.

Out-of-plane vibrations of the curved beams have been studied by many researchers. However, only a few researchers performed researches for the vibration problem of curved beams with variable cross-section on elastic foundation. The related literature is introduced in the next paragraphs according to subjects. The literature survey on elastic foundations is based on the lecture notes by Yardimoglu (2012).

The models of beam on elastic foundations introduced by numerous investigators are given in the textbook written by Karnovsky (2000). The oldest one is the Winkler (1867) model. It is extensively used because of its simplicity due to one-parameter. In this model, foundation is considered as infinite number of closely spaced unconnected linear elastic vertical springs. However, limitation of this model is the lack of interaction or coupling between adjacent springs. To overcome this weakness, Filonenko-Borodich (1940, 1945) proposed a model connecting the vertical springs by a thin elastic tensioned membrane placed over the springs (Das 2011). The well-known textbook by Hatenyi (1946) presents exact solutions of straight beams on Winkler foundations. Pasternak (1954) modified Winkler model by introducing a second parameter regarding coupling effect of the linear elastic springs, also known as shear interactions. This model sometimes called as two-parameter model. The generalization of Pasternak model is the Reissner model (1958) regarding deflections of plates on a viscoelastic foundation. Kerr (1964) offered three-parameter model. This model consists of a spring bed placed over a Pasternak foundation. Vlasov and Leontiev (1966) considered the shear interactions in a foundation and formulated their problems by using a variational method. Each model may have also viscoelastic properties which can be provided by adding a viscous damping term.

Carefully selected samples of the literature on vibration of beams on different elastic foundations are presented as follows:

Exact analytical vibration characteristics of the aforementioned subject are covered in some textbooks (Hatenyi 1946, Meirovitch 1967, Volterra and Gaines 1971). Yihua et al. (2009) analyzed the vibrations of Timoshenko beams on a nonlinear elastic foundation. A weak form Quadrature Element Method is used for the vibration analysis. The nonlinear foundation parameter stiffness is assumed as:

$$k_f = \alpha + \beta v^2$$

Eisenberger (1994) presented the exact vibration frequencies of beams resting on variable one- and two-parameter elastic foundation. His solution is based on dynamic stiffness matrix for the member including the effects of the variable foundation stiffness. Stiffness of the two-parameter elastic foundation is expressed as follows:

$$k_f(x) = -\frac{\partial}{\partial x}\left[k_1(x)\,v(x)\,\frac{\partial v(x)}{\partial x}\right] + k(x)\,v(x)$$

Avramidis and Morfidis (2006) formulated and analytically solved the bending of a Timoshenko beam resting on a Kerr-type three-parameter elastic foundation.

Out of plane vibration of curved beams on different elastic foundations are studied by following researchers:

Rao (1971) presented three-dimensional vibrations of a ring on elastic foundation. Stiffness parameters of the elastic foundation are based on the bending-torsion motions of the curved beam. For this reason, the first parameter regarding bending motion, the second parameter regarding torsional motion.

Panayotunakos and Theocaris (1980) made an analytical treatment for the determination of the natural frequencies of a circular Timoshenko beam on a Winkler foundation.

Wang and Brannen (1982) studied the effects of Winkler-Pasternak foundations upon natural frequencies of finite circular curved beams vibrating out of their initial plane of curvature.

Issa (1988) and Issa et al. (1990) examined the natural frequencies of curved Timoshenko beams on Winkler- and Pasternak-type foundations, respectively.

Recently, Kim et al (2007) presented the dynamic stiffness matrix for the spatially coupled free vibration analysis of thin-walled curved beams on Winkler- and Pasternak-type foundations. They used the power series method in their solution.

General review of Transfer Matrix Method (TMM) is provided below:

In the middle of the 20th century, several authors have developed Transfer Matrix Methods for the vibration and stability analysis of elastic systems. TMM can be easily used for one-dimensional structures. This structure is also called chain-type structure.

Holzer (1920) used this method for torsional vibrations of shafts. Myklestad (1944) introduced the eigenvalue problem of beam bending vibrations by using this method. Later, Prohl (1945) extended this approach for calculating critical speeds of flexible rotors.

Leckie and Pestel (1960) presented the transfer matrices for chain- and tree-types structures. Their study includes not only the natural vibrations of elastic systems, but also the forced vibrations. Pestel and Leckie (1963) published very comprehensive textbook on transfer matrices for elastomechanical elements up to twelfth order.

Djodjo (1969) interested in Transfer Matrices for beams loaded axially and laid on an elastic foundation.

Recently, He at al. (2012) studied on TMM for natural vibration analysis of tree-type system which is modified chain-type system adding several branches.

In this study, the effects of the variable cross-section of the curved beams and elastic foundation parameters on natural frequencies are investigated. TMM is used to find the natural frequencies numerically. A computer program is developed in Mathematica to determine the natural frequencies depending on cross-sectional and elastic foundation parameters. The accuracy of the TMM results obtained from the developed program is evaluated by comparing with FEM results found from model created in ANSYS. Finally, the effects of the variation of cross-section of the curved beams and elastic foundation parameters on natural frequencies are investigated.

CHAPTER 2

DIFFERENTIAL EQUATIONS AND THEIR SOLUTIONS

2.1. Introduction

This chapter presents the theoretical background with geometrical detail of the current problem considered in this thesis. First of all, title of the thesis is explained shortly. Then, the geometry of the curved is described by introducing the taper functions of breadth and depth of the cross-section of the beam. After providing these backgrounds, Equations of motions of the tapered curved beam are obtained by Newtonian method.

The critical step of the thesis is to be familiarize the elastic foundation model to use the proper one or ones in this thesis. Because of this reason, well-known elastic foundation models existing in the reachable literature are presented in summarized form.

In order to obtain the natural frequencies of the tapered curved beam, TMM (Transfer Matrix Method) is selected. One section in this chapter summarizes the fundamental concepts regarding TMM. Finally, finding the natural frequencies by TMM is given.

2.2. Description of the Problem

The out-of-plane free vibrations of a variable cross-sectioned curved beam on different elastic foundations are considered. The material of the beam is assumed as isotropic. The problems are constructed as a fixed-free beam on Winkler and Pasternak foundations. The cases of variable cross-section and variable foundation parameters are investigated in order to find out the effects of these parameters on natural frequencies.

2.3. Geometry of Curved Beam

A planar tapered curved beam is shown in Figure 2.1.

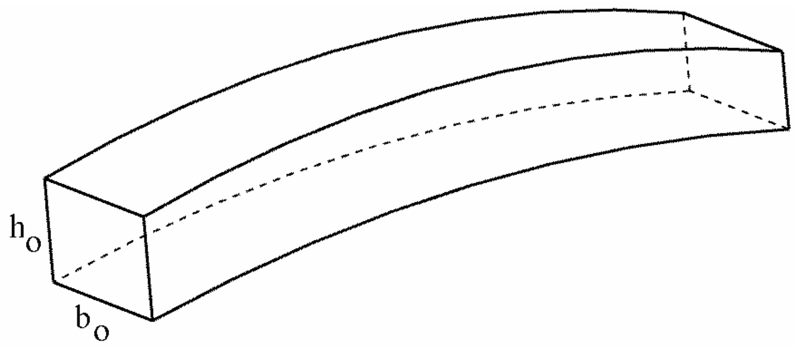


Figure 2.1. A planar tapered curved beam

The breadth and depth functions of the curved beam are selected as follows:

$$b(s) = b_0 - b_1 s \tag{2.1}$$

$$h(s) = h_0 - h_1 s \tag{2.2}$$

where b_0 and h_0 are breadth and depth of the beam at root cross-section, respectively. Also, b_1 and h_1 are breadth and depth parameters, respectively.

2.4. Derivation of the Equations of Motions

Newtonian method is used to derive equations of motions based on the following two vectorial equations:

$$\sum_i \vec{F}_i = m\vec{a} \tag{2.3}$$

$$\sum_i \vec{M}_i = I\vec{\alpha} \tag{2.4}$$

In this method, it is needed to neglect small quantities of higher orders terms in order to obtain linear differential equations. Moreover, expressing the boundary conditions are based on the understanding of the internal forces and moments.

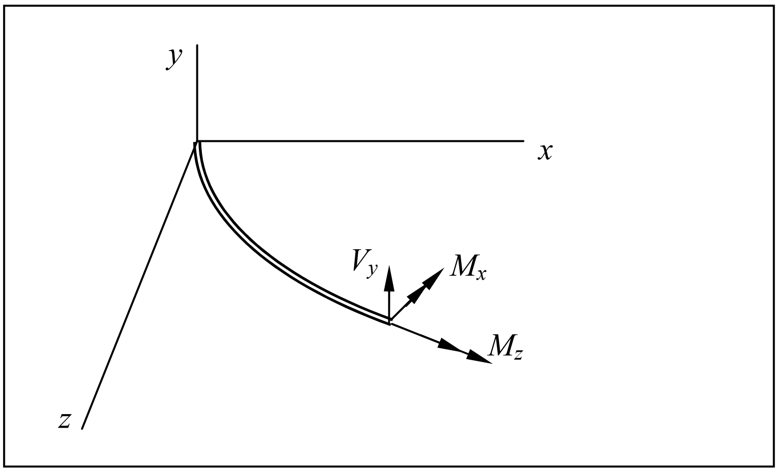


Figure 2.2. A curved beam with internal forces and moments

By using Equations 2.3 and 2.4, force and moment equilibrium equations of the curved beam can be obtained as follows (Love 1944):

$$\frac{dV_y}{ds} + F_y = m\ddot{v} \tag{2.5.a}$$

$$\frac{dM_x}{ds} + \frac{M_z}{\rho_0} - V_y = 0 \tag{2.5.b}$$

$$\frac{dM_z}{ds} - \frac{M_x}{\rho_0} + T_z = i\ddot{\beta} \tag{2.5.c}$$

where $m = \rho A(s)$ is mass per unit length, (2.6.a)

$i = \rho J(s)$ is mass polar moment inertia of unit length. (2.6.b)

It should be noted that external force in y direction F_y and external twisting moment about z axis T_z in Equations 2.5.a-c can be treated as elastic foundation effects. $A(s)$ and $J(s)$ in Equations 2.6.a,b are cross-sectional area and torsional constant of cross-section.

Bending and twisting moments in Equation 2.5.b and 2.5.c are given as

$$M_x = EI_{xx}(s)\,\kappa \tag{2.7.a}$$

$$M_z = GJ(s)\,\tau \tag{2.7.b}$$

where

$$\kappa = \left(\frac{\beta}{\rho_0} - \frac{\partial^2 v}{\partial s^2}\right), \tag{2.8.a}$$

$$\tau = \left(\frac{d\beta}{ds} + \frac{1}{\rho_0}\frac{\partial v}{\partial s}\right) \tag{2.8.b}$$

Geometrical properties are detailed in this paragraph. Area moment of inertia of the cross-section about *xx*-axis is determined by

$$I_{xx}(s) = b(s)\,h(s)^3 / 12 \tag{2.9}$$

Torsional constant for rectangular cross-section is given as (Popov 1998)

$$J(s) = C_\beta\, b(s)\,h(s)^3 \tag{2.10}$$

where the values of parameter C_β depends on the ratio of b/h.

Selecting the state vector as $\{Z\}=\{v, v', \beta, V_y, M_x, M_z\}^T$, transfer matrix of the system is found by the procedure given in Section 2.5.

2.5. Elastic Foundation Models

Elastic foundation models are described by the relation between the reaction of the foundation (or pressure) $p(y,t)$, deflection of the beam and the parameters of foundations. When $p(y,t)$ is not expressed in explicit form, the Differential Equation(s) is given to find the $p(y,t)$. The well-known elastic foundation models with their mathematical expressions are given below:

Winkler foundation (Winkler 1867):

$$p = k_0 y \tag{2.11}$$

Viscoelastic Winkler foundation:

$$p = k_0 y + d\frac{\partial y}{\partial t} \tag{2.12}$$

Filonenko-Borodich Foundation (Filonenko-Borodich 1940):

$$p = k_0 y - T\frac{\partial^2 y}{\partial x^2} \tag{2.13}$$

Hetenyi foundation (Hetenyi 1946):

$$p = k_0 y + D_0 \nabla^2 \nabla^2 y \tag{2.14}$$

Viscoelastic Hetenyi foundation:

$$p = k_0 y + D_0 \nabla^2 \nabla^2 y + d\frac{\partial y}{\partial t} \tag{2.15}$$

Pasternak foundation (Pasternak 1954):

$$p = k_0 y - k_G \frac{\partial^2 y}{\partial x^2} \tag{2.16}$$

Viscoelastic Pasternak foundation:

$$p = k_0 y - k_G \frac{\partial^2 y}{\partial x^2} + d\frac{\partial y}{\partial t} \tag{2.17}$$

Generalized foundation (Pasternak 1954):

$$p = k_0 y \quad , \; m = k_1 \frac{dy}{dn} \tag{2.18}$$

Reissner foundation (Reissner 1958):

$$c_1 y - c_2 \nabla^2 y = p - \frac{c_2}{c_1} \nabla^2 p \tag{2.19}$$

Vlasov and Leontiev (Vlasov and Leontiev 1966):

$$p = k_0 y - 2t \frac{\partial^2 y}{\partial x^2} \tag{2.20}$$

The notations used in Equations 2.11-20 for elastic foundation are listed in "List of Symbols" given at the begining of the thesis.

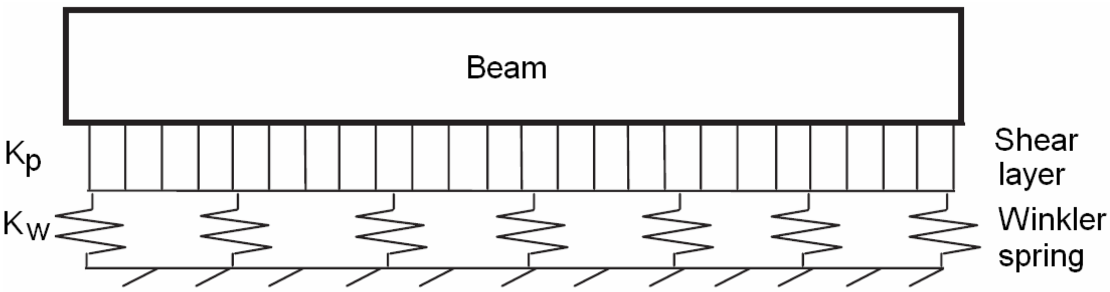


Figure 2.3. A beam on Pasternak foundation

2.5. Transfer Matrix Method (TMM)

There are several methods to derive the transfer matrix for vibration analysis. All these methods can be found in the comprehensive textbook written by Pestel and Leckie (1963). In this thesis, “the solution of n first-order differential equations with variable coefficients” is used. It can be explained briefly as follows:

For the structure shown in Figure 2.4, the following notations are used:

- Left state vector of segment *j* is $\{Z\}_j^L$
- TM for segment *j* is $[T]_j$.

A state vector $\{Z\}_j^L$ having the physical quantities such as displacements and corresponding internal forces regarding the *L* (left) end of segment *j*=1,2,... of elastic domain shown in Figure 2.4 is considered. Transfer matrix of this segment *j* is $[T_j]$ and transfers the state vector from *L* (left) end to *R* (right) end as:

$$\{Z\}_j^R = [T_j]\{Z\}_j^L \qquad j=1,2,3,... \tag{2.21}$$

So, the important step is to derive the transfer matrix of this segment *j* $[T_j]$ by using *n* first-order differential equations with variable coefficients. For this step, the following form of the first-order differential equations can be considered:

$$\frac{d\{Z\}}{ds} = [A(s)]\{Z\} \tag{2.22}$$

By using the matrix [*A*(s)] in given in Equation 2.22 and following the standard procedure given by Pestel and Leckie (1963), transfer matrix is obtained.

In order to explain the obtaining overall transfer matrix, a chain-type structure shown in Figure 2.4 can be considered.

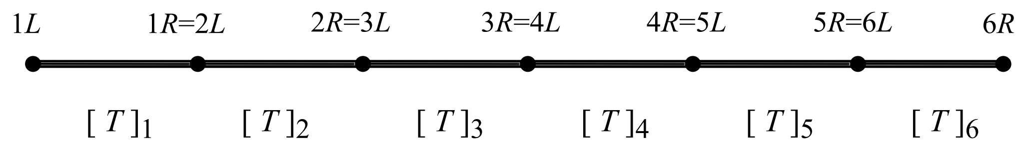


Figure 2.4. A chain-type structure divided into 6 segments

Due to the continuity principle, state vectors have the following properties:

$$\{Z\}_j^L = \{Z\}_{j-1}^R \qquad j=1,2,3,... \tag{2.23}$$

By using Equation 2.23 in Equation 2.21 and considering the state vectors at boundaries, the following general form is obtained:

$$\{Z\}_n^R = \left[\prod_{j=n}^{1}[T_j]\right]\{Z\}_1^L \tag{2.24}$$

Boundary conditions are applied to Equation 2.24.

2.6. Natural Frequencies by TMM

In order to apply the boundary conditions to Equation 2.24, it can be written as follows:

$$\begin{Bmatrix} \{d\} \\ \{f\} \end{Bmatrix}_n^R = \begin{bmatrix} [T_{11}(\omega)] & [T_{12}(\omega)] \\ [T_{21}(\omega)] & [T_{22}(\omega)] \end{bmatrix} \begin{Bmatrix} \{d\} \\ \{f\} \end{Bmatrix}_1^L \tag{2.25}$$

If the left end is fixed and right end is free, $\{d\}_1^L = 0$ and $\{f\}_1^R = 0$. Therefore,

$$\begin{Bmatrix} \{d\} \\ 0 \end{Bmatrix}_n^R = \begin{bmatrix} [T_{11}(\omega)] & [T_{12}(\omega)] \\ [T_{21}(\omega)] & [T_{22}(\omega)] \end{bmatrix} \begin{Bmatrix} 0 \\ \{f\} \end{Bmatrix}_1^L \tag{2.25}$$

By equating the determinant of $[T_{22}(\omega)]$ to zero, natural frequencies are found.

CHAPTER 3

NUMERICAL RESULTS AND DISCUSSION

3.1. Introduction

In this chapter, the numerical investigations for the effects of taper parameters, opening angle and elastic foundation properties of the curved beams for the out-of-plane motion are presented.

The main numerical data used throughout in this chapter are as follows: b_o=h_o=0.01 m, E=200 GPa, G=80 GPa, ρ=7850 kg/m^3, R=0.2 m. Other data used in the modeling of the system are given in table and figure legends. The numerical results obtained by TMM detailed in Section 2.5 and FEM results obtained from the model created in ANSYS are compared. After verifying the computer code developed in Mathematica, parametric study results are given and discussed.

3.2. Validations of the Procedure for Constant Cross-Section

In order to determine the proper number of segment n in TMM, the first natural frequencies are found for a curved beam having 45° opening angle and the results are plotted in Figure 3.1.

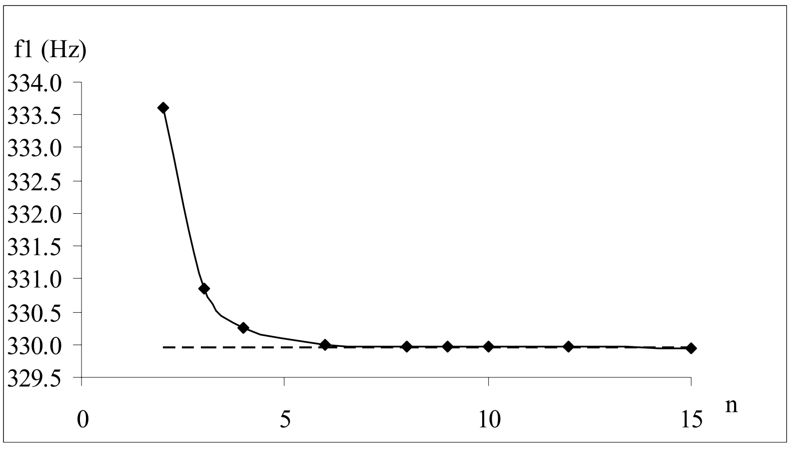


Figure 3.1. Convergence of first natural frequency

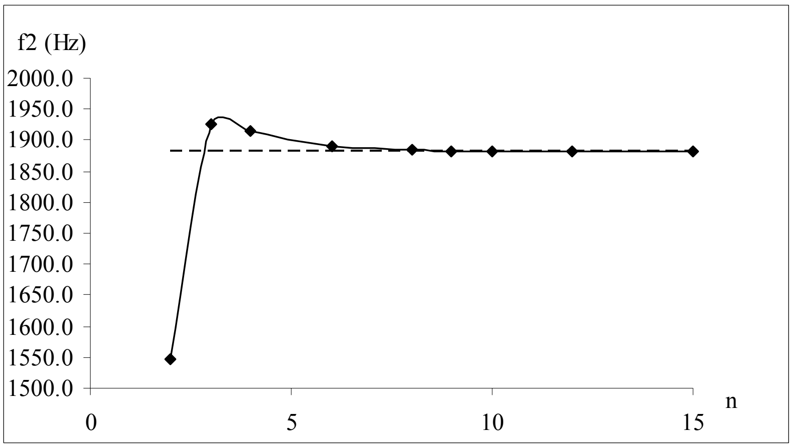


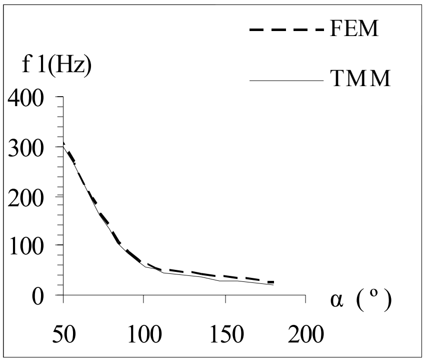
Figure 3.2. Convergence of second natural frequency

It is clear from Figure 3.1 and 2 that the first and second natural frequency does not change significantly after the number of segment n=6. Therefore, n=6 is selected for all cases.

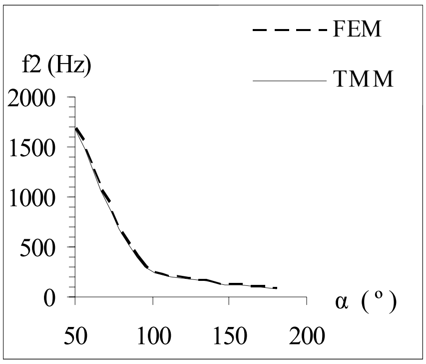
To validate the present model for TMM, FEM is used. For this purpose, finite element models with 30 Beam44 elements are generated by using APDL language in ANSYS. The natural frequencies found for different opening angle α by TMM and FEM are tabulated in Table 3.1 and plotted in Figure 3.3.

Table 3.1. Natural frequencies found by TMM and FEM (b_1=h_1=0)

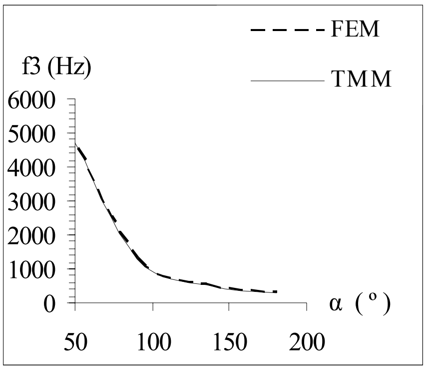
		α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	TMM	330	83	37	22
	FEM	333	85	40	24
f_2 (Hz)	TMM	1890	401	153	76
	FEM	1897	409	156	78
f_3 (Hz)	TMM	5207	1315	533	268
	FEM	5176	1297	529	268
f_4 (Hz)	TMM	6337	2642	1135	600
	FEM	6196	2611	1130	601
f_5 (Hz)	TMM	11360	3649	1678	903
	FEM	11029	3661	1924	1052



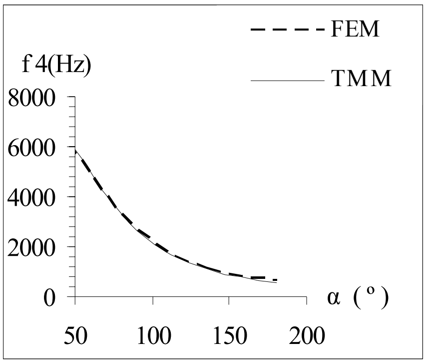
a) First frequency



b) Second frequency

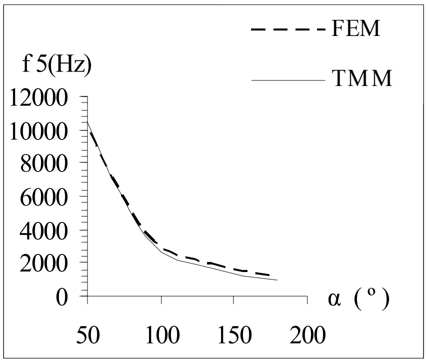


c) Third frequency



d) Fourth frequency

Figure 3.3. Comparisons of natural frequencies found for different *α* by FDM with FEM results.



e) Fifth frequency

Figure 3.3. Comparisons of natural frequencies found for different α by FDM with FEM results (continued).

It is clear from Table 3.1 and Figure 3.3 that developed program based on TMM and the ANSYS results are in good agreement for the constant cross sectioned curved beam without elastic foundation effect for different opening angles.

In order to validate the developed program for the elastic foundation effect, the Winkler model is considered and the results are presented in Tables 3.2-4.

Table 3.2. Comparison of natural frequencies found for different opening angle α by TMM with FEM results ($b_1=h_1=0$, k_0=20 MN/m^2/m, $k_1=0$)

		α =45°	α =90°	α =135°	α =180°
f_1 (Hz)	TMM	340	115	89	83
	FEM	342	117	89.6	84
f_2 (Hz)	TMM	1892	409	172	111
	FEM	1899	417	176	112
f_3 (Hz)	TMM	5207	1318	539	280
	FEM	5176	1299	535	280
f_4 (Hz)	TMM	6337	2643	1138	606
	FEM	6196	2612	1133	607
f_5 (Hz)	TMM	11361	3650	1680	906
	FEM	11029	3661	1926	1055

Table 3.3. Comparison of natural frequencies found for different opening angle α by TMM with FEM results (b_1=h_1=0, k_0=50 MN/m^2/m, k_1=0)

		α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	TMM	354	152	132	129
	FEM	356	153	133	129
f_2 (Hz)	TMM	1895	420	199	148
	FEM	1902	428	201	149
f_3 (Hz)	TMM	5208	1321	548	297
	FEM	5177	1303	544	296
f_4 (Hz)	TMM	6338	2645	1142	614
	FEM	6196	2614	1137	614
f_5 (Hz)	TMM	11361	3650	1683	912
	FEM	11029	3661	1928	1060

Table 3.4. Comparison of natural frequencies found for different opening angle α by TMM with FEM results (b_1=h_1=0, k_0=100 MN/m^2/m, k_1=0)

		α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	TMM	376	198	184	181
	FEM	378	199	184	181
f_2 (Hz)	TMM	1899	439	236	195
	FEM	1906	447	238	196
f_3 (Hz)	TMM	5209	1327	562	323
	FEM	5178	1309	559	322
f_4 (Hz)	TMM	6339	2648	1149	626
	FEM	6197	2617	1144	627
f_5 (Hz)	TMM	11362	3651	1688	920
	FEM	11030	3662	1932	1067

It is clear from Table 3.2-4 that the developed program based on TMM and the ANSYS results are in good agreement for the constant cross sectioned curved beam with elastic foundation effect based on Winkler model for different opening angles.

3.3. Applications for Variable Cross-Section

Numerical applications for the case of variable cross-sectioned curved beams are presented here for various cases. Effects of taper, opening angles and foundation parameters on natural frequencies are studied for Winkler model and the results are presented in Table 3.5-12. The numerical data are given in each table legends.

Table 3.5. Natural frequencies for (b_1=h_1=2/s_L, k_0=20 MN/m^2/m, k_1=0)

	α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	375	132	104	99
f_2 (Hz)	1853	413	181	122
f_3 (Hz)	5203	1240	512	271
f_4 (Hz)	6749	2444	1044	558
f_5 (Hz)	10302	3503	1518	820

Table 3.6. Natural frequencies for (b_1=h_1=4/s_L, k_0=20 MN/m^2/m, k_1=0)

	α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	423	157	127	118
f_2 (Hz)	1803	418	195	141
f_3 (Hz)	4822	1149	483	264
f_4 (Hz)	7826	2191	938	507
f_5 (Hz)	9166	3050	1318	717

Table 3.7. Natural frequencies for (b_1=h_1=2/s_L, k_0=50 MN/m^2/m, k_1=0)

	α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	393	177	157	152
f_2 (Hz)	1856	428	214	167
f_3 (Hz)	5204	1245	524	293
f_4 (Hz)	6749	2446	1050	569
f_5 (Hz)	10302	3505	1522	827

Table 3.8. Natural frequencies for (b_1=h_1=4/s_L, k_0=50 MN/m^2/m, k_1=0)

	α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	447	214	190	176
f_2 (Hz)	1808	440	239	204
f_3 (Hz)	4824	1156	500	296
f_4 (Hz)	7826	2195	947	524
f_5 (Hz)	9167	3053	1325	730

Table 3.9. Natural frequencies for (b_1=h_1=2/s_L, k_0=100 MN/m^2/m, k_1=0)

	α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	420	233	217	211
f_2 (Hz)	1862	453	260	225
f_3 (Hz)	5206	1253	543	326
f_4 (Hz)	6749	2450	1059	587
f_5 (Hz)	10303	3507	1529	840

Table 3.10. Natural frequencies for (b_1=h_1=4/s_L, k_0=100 MN/m^2/m, k_1=0)

	α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	485	284	258	240
f_2 (Hz)	1816	473	304	280
f_3 (Hz)	4827	1169	528	344
f_4 (Hz)	7826	2201	963	551
f_5 (Hz)	9168	3058	1337	751

Effects of taper, opening angles and foundation parameters on natural frequencies are also studied for two-parameter foundation model and the results are presented in Table 3.11-12.

Table 3.11. Natural frequencies for (b_1=h_1=2/s_L, k_0=50 MN/m^2/m, k_1=2000 N)

	α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	388	177	160	158
f_2 (Hz)	1867	438	222	172
f_3 (Hz)	5217	1259	538	307
f_4 (Hz)	6751	2462	1066	586
f_5 (Hz)	10319	3524	1542	848

Table 3.12. Natural frequencies for (b_1=h_1=4/s_L, k_0=50 MN/m^2/m, k_1=2000 N)

	α =45º	α =90º	α =135º	α =180º
f_1 (Hz)	440	215	199	187
f_2 (Hz)	1821	451	245	208
f_3 (Hz)	4844	1176	519	311
f_4 (Hz)	7828	2221	973	549
f_5 (Hz)	9192	3090	1362	765

It can be seen from Table 3.5-3.12 that all natural frequencies decrease when the opening angle increases. Also, when the taper parameter increases (from 2/*sl* to 4/*sl*), namely when the cross-section of the curved beam decreases in the positive *s* direction due to the tapers effect, only the first natural frequencies increase for the considered opening angles and foundation parameters.

The results listed in Table 5.5, 5.7 and 5.9 are plotted in Figures 3.4-3.8 as natural frequency versus opening angle to see the effects of opening angle and Winkler foundation parameter on natural frequencies.

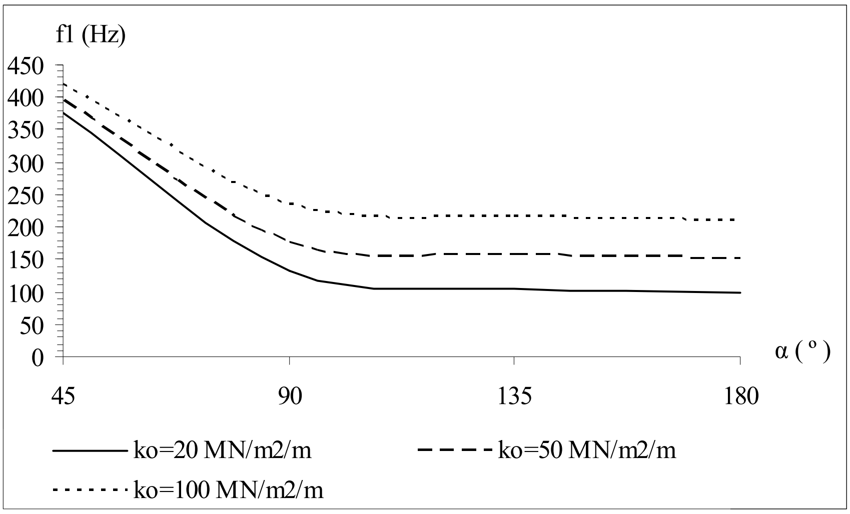


Figure 3.4. First natural frequencies found for b_1=h_1=2/s_L and k_1=0.

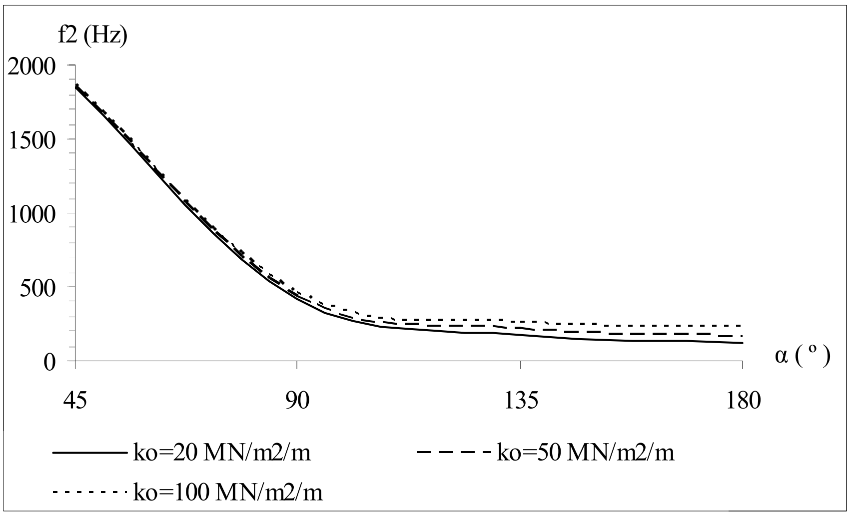


Figure 3.5. Second natural frequencies found for b_1=h_1=2/s_L and k_1=0.

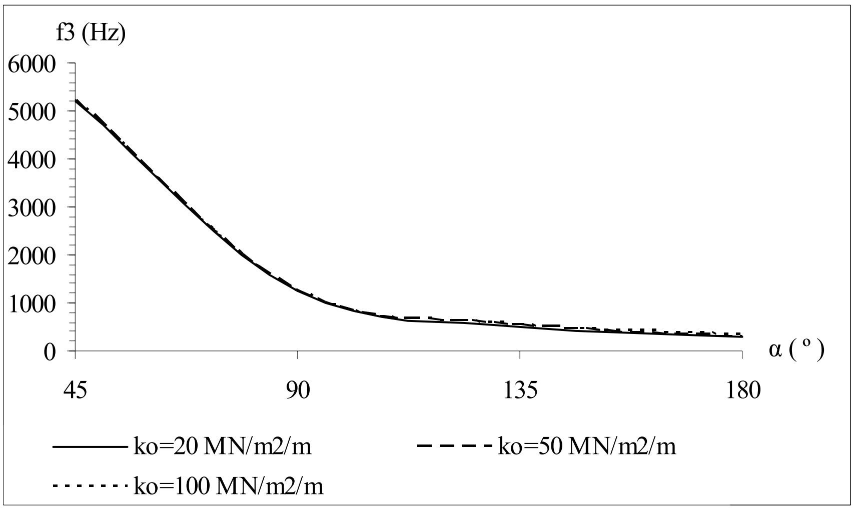


Figure 3.6. Third natural frequencies found for b_1=h_1=2/s_L and k_1=0.

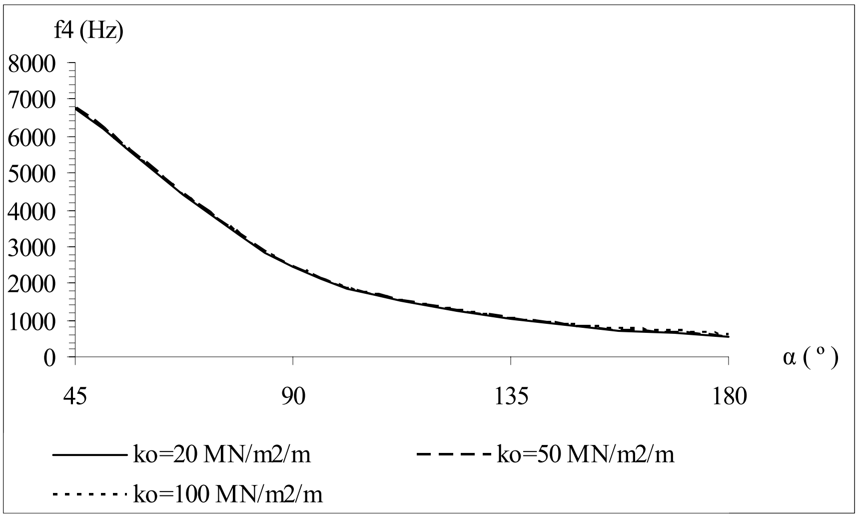


Figure 3.7. Fourth natural frequencies found for b_1=h_1=2/s_L and k_1=0.

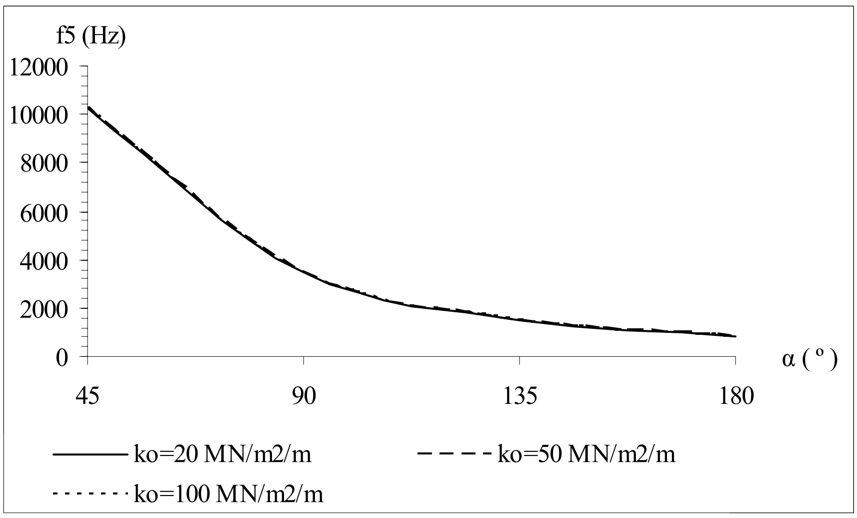


Figure 3.8. Fifth natural frequencies found for b_1=h_1=2/s_L and k_1=0.

Variation of first to fifth natural frequencies found for $b_1=h_1=2/s_L$ and $k_1=0$. are shown in Figure 3.4-3.8. These figures show that Winkler foundation parameter k_0 significantly effects the first natural frequencies for the considered opening angles. When the Winkler foundation parameter k_0 increases, the first natural frequencies of curved beams having opening angles from 45º to 180º increase for the considered taper properties. General tendancy of the all frequencies depending on the opening angle is the same.

CHAPTER 4

CONCLUSIONS

In this study, the differential equations governing the free out-of plane vibrations of curved beams with variable cross-section are presented. The equations of motions are derived by using Newtonian Method. Since the coefficients of the derived differential equations are not constant, it is difficult to express an exact solution.

In order to validate the developed computer program by using TMM to solve the present problem, the models are created in ANSYS. The results found from TMM are compared with the results obtained from ANSYS. Good agreement is obtained for all comparisons.

The effects of taper and curvature parameters on natural frequencies are found for the linearly tapered curved beams.

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