SPECTRUM SENSING TECHNIQUES FOR COGNITIVE RADIO SYSTEMS WITH MULTIPLE ANTENNAS

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by
Refik Fatih ÜSTOK

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We approve the thesis of **Refik Fatih ÜSTOK**

Assist. Prof. Dr. Berna ÖZBEK
Supervisor

Assist. Prof. Dr. Mustafa Aziz ALTINKAYA
Committee Member

Assist. Prof. Dr. Türker INCE
Committee Member

25 June 2010

Prof. Dr. M. Barıs ÖZERDEM
Head of the Department of
Electrical - Electronics Engineering

Assoc. Prof. Dr. Talat YALÇIN
Dean of the Graduate School of
Engineering and Sciences

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ABSTRACT

SPECTRUM SENSING TECHNIQUES FOR COGNITIVE RADIO SYSTEMS WITH MULTIPLE ANTENNAS

The aim of this study is to focus on spectrum sensing in cognitive radio which is a recently introduced technology in order to increase the spectrum efficiency. Increasing efficiency of the spectrum usage is an urgent need as an intrinsic result of the rapidly increasing number of wireless users and also the conversion of voice oriented applications to multimedia applications. Static allocation of the frequency spectrum does not meet the needs of current wireless technology that is why dynamic spectrum usage is required for wireless networks. Cognitive radio is considered as a promising candidate to be employed in such systems as they are aware of their operating environments and can adjust their parameters. Cognitive radio can sense the spectrum and detect the idle frequency bands, thus secondary users can be allocated in those bands when primary users do not use those in order to avoid any interference to primary user by secondary user. There are several spectrum sensing techniques proposed in literature for cognitive radio based systems. In this thesis, energy detection and cyclostationary feature detection based spectrum sensing systems for cognitive radios with and without multiple antenna are examined in detail and comparative performance results are obtained in wireless communication channels.

ÖZET

ÇOKLU ANTENLİ BİLİŞSEL RADYO SİSTEMLERİ İÇİN SPEKTRUM SEZİNLEME TEKNİKLERİ

Bu çalışmanın amacı, spektrumun kullanım verimliliğini arttırmak amacıyla yeni ortaya çıkan bilişsel radyo sistemlerinde spektrum sezinleme üzerine odaklanmaktır. Spektrum verimliliğinin arttırılması, hızla artan kablosuz kullanıcı sayısı ile sadece ses odaklı sistemlerden multimedya sistemlerine dönüşümünün doğal bir sonucu olarak acil bir ihtiyaç olarak karşımıza çıkmaktadır. Frekans spektrumunun durağan tahsisi günümüzün kablosuz teknolojilerinin ihtiyaçlarını karşılamamaktadır bu nedenle kablosuz ağlar için spektrumunun dinamik kullanımı gereklidir. Bilişsel radyo, çevrelerini algılayabilmeleri ve parametrelerini buna göre değiştirebilmeleri nedeniyle dinamik frekans kullanımlı sistemler için umut verici aday sistemler olarak görülmektedir. Bilişsel radyo spektrumu sezinleyebilir, boş bantları algılayabilir ve ikincil kullanıcılar bu kullanılmayan boş bantlara birinci kullanıcıya girişim yapmayacak şekilde yerleştirilebilirler. Bilişsel radyo tabanlı sistemler için literatürde bir çok spektrum sezinleme yöntemi önerilmektedir. Bu tezde tekli ve çoklu antenli bilişsel radyo sistemleri için enerji algılama ve döngüsel-durağan öznitelik algılama tekniklerine dayanan spektrum sezinleme sistemleri kablosuz iletişim kanallarında detaylı bir şekilde incelenmiş, karşılaştırma performans sonuçları çıkartılmıştır.

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LIST OF ABBREVIATIONS

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Bipolar Phase Shift Keying
CP	Cyclic Prefix
CR	Cognitive Radio
CSI	Channel State Information
COL	Collaborative
DAB	Digital Audio Broadcasting
DFT	Discrete Fourier Transform
DVB-T	Digital Video Broadcasting-Terresterial
EGC	Equal Gain Combining
FFT	Fast Fourier Transform
ICI	Intercarrier Interference
IFFT	Inverse Fast Fourier Transform
IID	Independent and Identically Distributed
ISI	Intersymbol Interference
MIMO	Multiple Input Multiple Output
ML	Maximum-likelihood
MRC	Maximum Ratio Combining
OFDM	Orthogonal Frequency Division Multiplexing
PU	Primary User
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
SISO	Single Input Single Output
SIMO	Single Input Multiple Output
SC	Selection Combining
SLC	Square Law Combining
SLS	Square Law Selection
SU	Secondary User

CHAPTER 1

INTRODUCTION

1.1. Overview

Efficient usage of the limited natural resources is one of society's greatest challenges. Just like petroleum and coal, the natural frequency spectrum is limited and needed to be used more efficiently in order not to use up all. It is apparent that current static frequency allocation schemes can not accommodate demands of the rapidly increasing number of higher data rate devices (Arslan and Yücek 2007) . Therefore; dynamic usage of the spectrum must be distinguished from the static usage to increase the availability of frequency spectrum. For this purpose, cognitive radio is proposed as a new technology that provides optimum satisfaction of user requirements like effective spectrum usage.

Inconsistent transmission scheme of primary users with not fully occupied licensed bands in frequency, time and space led to a more sophisticated and structured manner - cognitive radio that is aware of its environmental, internal state, and location, and autonomously adjusts its operations to achieve designed objectives (Mitola 2000). With another way of saying, cognitive radio first senses its spectral environment over a wide frequency band, then adapts the parameters to maximize spectrum efficiency while co-existing with legacy wireless networks (Haykin 2005).

This thesis only focuses on the spectrum sensing in cognitive radio. Several Spectrum Sensing Methods proposed in the literature are theoretically analysed and interpreted in the sense of advantages and drawbacks. There are generally five signal detection methods that are proposed in the literature:

1. Matched Filtering
2. Energy Detection
3. Cyclostationary Feature Detection
4. Other Methods:

-Higher Order Statistics

-Waveform based sensing

The aim of this thesis is to study on multiple antenna techniques applied to various spectrum sensing techniques and to find the least complex and the most feasible method to detect the signals with low probability of missing. Low missing probability indicates low probability of the interference caused by secondary users to primary users which is the main problem that must be solved using cognitive radio based systems.

1.2. Thesis Outline

In chapter 2, background information and literature review on Cognitive Radios is provided.

In chapter 3, an overview is given for spectrum sensing for Cognitive Radio systems. Implementation of spectrum sensing for systems using OFDM modulation is given. Following that, spectrum sensing techniques such as Matched-Filtering, Energy Detection, Cyclostationary Detection, Waveform based detection and higher order statistics detection are defined. Finally performances are compared and interpreted for single-input-single-output (SISO) systems.

In chapter 4, Multiple Antenna Techniques are applied to the spectrum sensing systems using energy detection and cyclostationary feature detection in order to mitigate the shadowing effects caused by the channel and thus improve the detection probability and increase the performance. Some multiple antenna techniques already proposed in the literature are studied and some multiple antenna techniques are proposed with giving derivations and performance results to compare with SISO systems.

In chapter 5, a conclusion of the work presented in this thesis is given and some possible future works are suggested to extend this research.

CHAPTER 2

BACKGROUND

This chapter provides background on some of the fundamental concepts. The characteristics of the wireless communication channels are described in section 2.1, followed by an overview of Multiple Antenna Techniques which are used to mitigate the fading effects of the channels. The correlation model between the antennas is also defined in this section. In Section 2.3 multi-carrier systems are mentioned and Orthogonal Frequency Division Multiplexing (OFDM) is explained in detail. The paradigm of cognitive radios and its physical architecture are given in the following section. Finally, the literature on Spectrum Sensing for cognitive radio systems is reviewed in section 2.5.

2.1. Wireless Channel Characteristics

The characteristics of a wireless communication channel between transmitter and receiver controls the performance of the overall system. Thus, in order to understand the effects of wireless channel on the performance of spectrum sensing, we first need to understand its characteristics. In this section, the characteristics of the mobile radio environment used in this thesis are introduced.

Communication systems operate in diverse environments, including those prone to fading. That's why, spectrum sensing must be analysed in fading environments. There are two types of fading effects called as large-scale fading and small-scale fading that characterize mobile communications (Rappaport 1999). The propagation models that characterize signal strength over large transmitter receiver separation distances (several hundreds or thousands of meters) are called large-scale propagation models. On the other hand, propagation models that characterize the rapid fluctuations of the received signal strength causing multiple versions of the transmitted signal to arrive at the receiver, each distorted in amplitude, phase and angle of arrival over very short travel distances (a few wavelengths)or short time durations (on the order of seconds) are called small-scale fading models. A relative motion between the transmitter and receiver causes significant attenuations of the signal power within a short period of time which is called Doppler shift. Considering all, it is very challenging to overcome this time varying nature of the multipath wireless channel.

Fading radio channels have been classified considering two ways. The first type of classification discusses whether the fading is flat (frequency non-selective) or frequency selective, while the second classification is based on the rate at which the wireless channel is changing (or in other words, the rate of change of the impulse response of channel), i.e. whether the fading is fast or slow. In connection with these characterizations of fading channels, it is useful to note the following quantities:

- **Coherence bandwidth:** Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered "flat" (i.e. frequency non-selective, or in other words a channel which passes all spectral components with equal gain and phase). It may also be defined as the range of frequencies over which any two frequency components have a strong potential for amplitude correlation. It has been shown that:

$$B_c \propto \frac{1}{\sigma_\tau} \tag{2.1}$$

  where σ_τ is the RMS delay spread. Also, if we define the coherence bandwidth B_c as that bandwidth over which the frequency correlation function is above 0.5 (i.e. the normalized cross-correlation coefficient is higher than 0.5 for all frequencies) then $B_c \propto \frac{1}{5\sigma_\tau}$. Note that if the signal bandwidth is greater than B_c, then the different frequency components in the signal will not be faded the same way. The channel then appears to be "frequency-selective" to the transmitted signal.

- **Doppler spread and Coherence time:** While σ_τ and B_c describe the time dispersive nature of the channel in an area local to the receiver, they do not offer any information about the time-variations of the channel due to relative motion between the transmitter and the receiver. The doppler spread B_D, defined as a measure of spectral broadening caused by the time-rate of change of the channel (related to the doppler frequency). The coherence time T_c is a statistical measure of the time duration over which two received signals have a strong potential for amplitude correlation. Thus if the inverse bandwidth of the basebad signal is greater than the coherence time of the channel then the channel changes during transmission of the baseband message. This will cause a distortion at the receiver. It is shown that:

$$T_c \propto \frac{1}{B_D} \tag{2.2}$$

Multipath fading has a significant impact on the fragility of wireless links. It is considered as a small-scale phenomenon in the sense that the level of attenuation of the signal changes substantially if the position of the receiver or the transmitter is varied by about half a wavelength. Another physical phenomenon of interest is shadowing; it is considered a large-scale effect, as it corresponds to substantial deviations of the RF signal from its mean due to large obstacles, which create shadow zones that cause deep fades if a receiver happens to enter them. In built up areas like big cities, reflections,refractions and diffractions occur as a result of the fact that mobile antennas are lower than the height of the surrounding buildings. In Figure 2.1, it is seen that the signal arrives at the receiver through different paths.

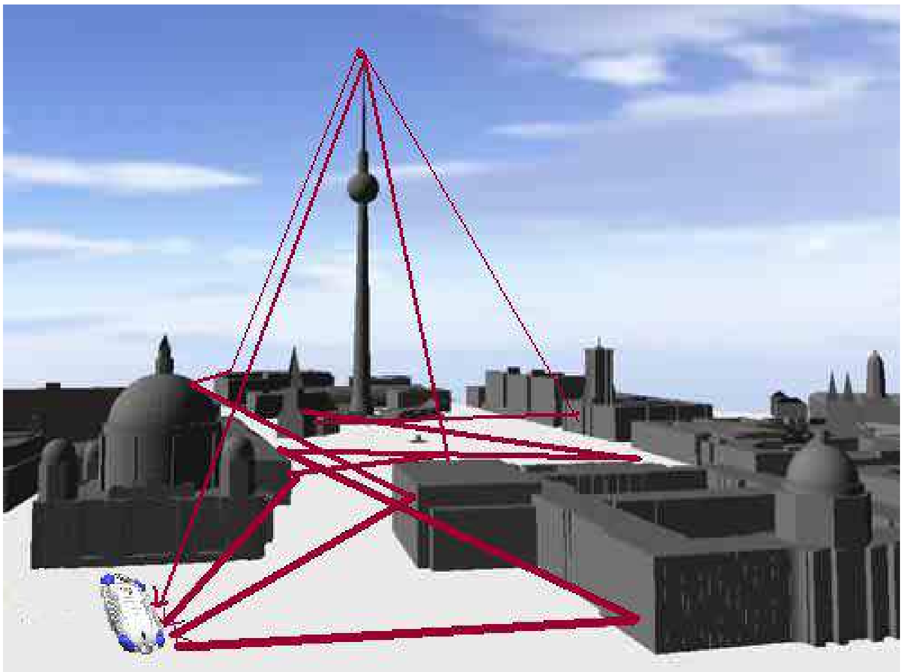

Figure 2.1. Multipath Propagation

Multipath fading channels have been modelled and simulated since 1950s and early 1960's. The multipath channel is a summation of the transmitted signal replicas with different amplitudes, propagation delays, phases and angles of arrival. The channel impulse response (CIR) of the multipath channel can be modelled as follows;

$$h(t,\tau) = \sum_{l=0}^{L-1} \alpha_l(t) e^{j\theta_l(t)} \delta(\tau - \tau_l(t)) \tag{2.3}$$

where $\alpha_l(t)$ is the real amplitude and $\theta_l(t)$ is the phase value of the lth multipath component at time t, τ_l is the excess time that belongs to lth path. L is the total number of multipath components and δ is the unit impulse function that determines the specific excess delay of a multipath at time t.

If the channel impulse response is assumed to be constant over the transmission, it is simplified as follows,

$$h(\tau) = \sum_{l=0}^{L-1} \alpha_l e^{-j\theta_l} \delta(\tau - \tau_l). \tag{2.4}$$

The amplitudes of paths, which arrive at the receiver at the same time delay with different phases, could add constructively or destructively. Also, within a short period of time the phases of these paths may change. Thus, the resulting amplitude of the channel at a particular time delay could vary within a short time interval. When there are many paths, having independent amplitudes and phases, the channel impulse response $h(t,\tau)$ can be modelled as a complex Gaussian random process based on the Central limit Theorem. Furthermore, if there is no LOS component from the transmitter to the receiver, the amplitude of the channel can be modelled as Rayleigh fading channel. However, if there is a dominant LOS component, it can be modelled as Ricean fading channel. In this thesis, we will focus on Rayleigh fading channels.

If a channel has a constant response for a bandwidth greater than the transmitted signal bandwidth, then the channel is said to be a flat fading channel. The conditions for a flat fading channel are:

$$B_s \ll B_c \tag{2.5}$$
$$T_s \gg T_c \tag{2.6}$$

where B_s and T_s are the signal bandwidth and the symbol duration respectively. In the frequency domain, a flat fading channel has a constant amplitude and linear phase response over the transmitted signal bandwidth. The spectral characteristic of the transmitted signal is preserved at the receiver for this fading type (Rappaport 1999). In a flat fading channel, the CIR can be written as:

$$h(t) = \alpha(t)e^{j\theta(t)} \tag{2.7}$$

where $\alpha(t)$ is Rayleigh distributed for the channel without LOS path, and is Ricean distributed when LOS path exists and $\theta(t)$ is uniformly distributed.

A channel is said to be frequency selective if the signal bandwidth is greater than the coherence bandwidth of the channel. In such a case, different frequency components of the transmit signal undergo fading to different extents (Rappaport 1999). For a frequency-selective fading situation:

$$B_s > B_c \tag{2.8}$$
$$T_s < T_c \tag{2.9}$$

From the time domain perspective, the symbol period is shorter than the rms delay spread. The channel will spread the signal beyond the symbol period and induce intersymbol interference (ISI) onto the next transmitted symbol. Then the frequency-selective fading channels can be represented as:

$$h(t,\tau) = \sum_{l=0}^{L_t-1} h_l(t)\delta(\tau - lT_s) \tag{2.10}$$

where $h_l(t)$ is the complex path coefficient of the lth tap at time t whose amplitude is Rayleigh and phase is uniformly distributed, T_s is the symbol period. Frequency-selective fading channels are much more difficult to model than flat-fading channels since each multipath signal must be modelled.

2.2. Multiple Antenna Techniques

Multiple Antenna techniques provide a promising solution for enabling better wireless communications because they mitigate problems inherent in ground-to-ground links, which are the most common links used by wireless devices, including cell phones and WiFi. Typically, ground-to-ground links are not line of sight. The electromagnetic waves transmitted from the antennas are faded along the wireless channel in some forms that are mentioned previously. Multiple Antenna systems provide a number of advantages over single-input-single-output (SISO) communication. Sensitivity to fading is reduced by the spatial diversity provided by multiple spatial paths.

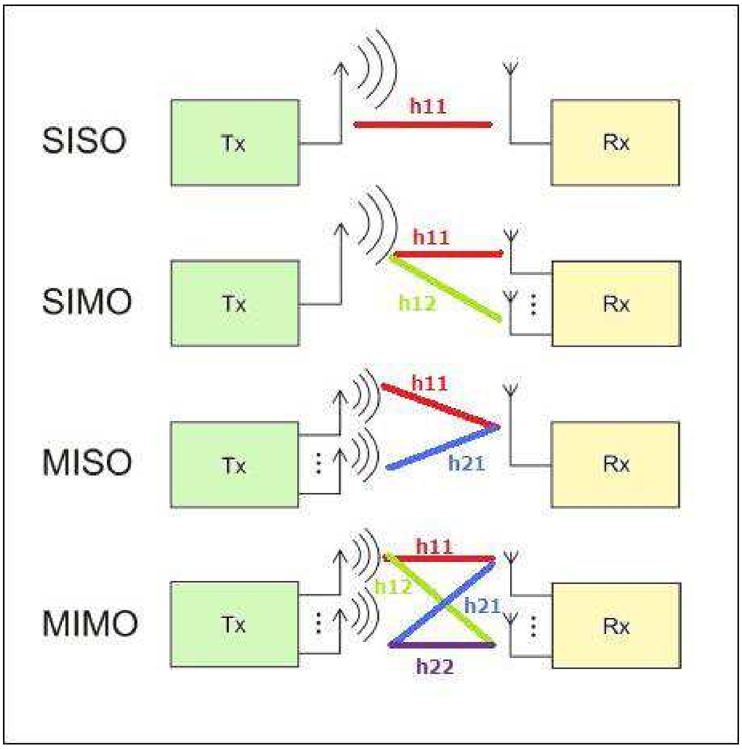

Figure 2.2. Multiple Antenna Techniques

Up to now, multi-antenna technology has been mainly developed and is implemented in some standards with different types. Multiple antenna techniques have been classified into three types as single-input-multiple-output (SIMO), multiple-input-single-output (MISO) and multiple-input-multiple-output (MIMO) as seen in Figure 2.2.

Hidden Primary user problem defined in (Akyıldız, et al. 2006) is one of the most important challenges encountered during spectrum sensing in cognitive radio based systems due to the multipath and shadowing effects of the communication channels. If the secondary user can not sense primary user due to an obstacle or deep fade effect, then probability of interference to primary user increases, therefore hidden primary user problem must be mitigated. To reduce the multipath and shadowing effects of the wireless channels, multiple antenna techniques provide promising solutions as proposed for improving BER performances. In chapter 4, the multiple antenna techniques for spectrum sensing are examined in detail.

2.2.1. Antenna Correlation Model

For independent Rayleigh channels the channel coefficients provided by multiple antennas scales linearly as the number of antennas under some conditions. However, some impairments of the radio propagation channel may lead to a substantial degradation in multiple antenna performance. A degradation on the capacity with multiple antenna techniques can occur due to the correlation between individual channels of the matrix channel. Increase in the correlation coefficient results in capacity decrease and, finally, when the correlation coefficient equals to unity, no advantage is provided by the multiple antenna architecture.

In this thesis, the correlation in the wireless channel is modelled by a correlation model given in (Loyka 2001) as;

$$h_i = R^{\frac{1}{2}} h_{x,i} \tag{2.11}$$

where $h_{x,i}$ has identically independent complex Gaussian distribution with zero mean and unit variance and the elements of the correlation matrix is given by,

$$R_{ij} = \begin{cases} \rho^{j-i}, & i \leq j \\ R_{ij}^*, & i > j, \end{cases} \quad (2.12)$$

where $i, j = 1, 2, ...N_r$, N_r is the antenna number, ρ is a complex value and $0 \leq |\rho| \leq 1$. The phase of the correlation coefficient is generated is uniformly distributed over 2π. Similarly, 120^o refers to the case where the phase of ρ is uniformly generated between -60^o and 60^o. The cases of $|\rho| = 0$ and $|\rho| = 0.9$ indicate spatially independent and strongly correlated channels respectively.

2.3. Orthogonal Frequency Division Multiplexing (OFDM)

The tremendous evolution of wireless communication systems leads to a rising demand for high performance and high capacity in multiuser communication systems within the boundaries of limited resources such as spectrum, bandwidth etc... The bandwidth provided by GSM is too small to allocate the high data rate applications (e.g. multimedia applications). Therefore OFDM is proposed as a candidate modulation scheme with multiple carries, allowing high data rate transmissions with the advantages of a lower data rate and minimizing the effects of the inter symbol interference (ISI)(Engels 2002). Another saying; the idea behind OFDM is to convert a frequency selective channel into a collection of frequency-flat subchannels with partially overlapping spectra.

Generally, higher data rates lead to severe ISI encountered in the received signals. To avoid the ISI, a large number of closely- spaced orthogonal subcarriers are used to carry data in OFDM systems. The data are divided into several parallel data streams or channels, one for each sub-carrier. Each sub-carrier is modulated with a conventional modulation scheme (such as QAM, PSK...) at a low symbol rate, maintaining total data rates similar to conventional single-carrier modulation schemes in the same bandwidth.

In OFDM, the entire channel is divided into many narrow subchannels that are used in parallel transmissions. Increasing a symbol period to an OFDM period that becomes way too much larger than the channel delay spread and thus reducing the effect of interblock interference (IBI) caused by the dispersive Rayleigh-fading environment (Engels 2002). Therefore, OFDM is an effective method for combating multipath fading and for high-bit-rate transmission over mobile wireless channels. In addition, OFDM can achieve adaptive allocation of transmission load in different subchannels to achieve optimum entire transmission rate. Furthermore, OFDM can diminish the effect of impulse noise because of the increased duration of the OFDM symbol. An OFDM carrier signal is the sum of a number of orthogonal sub-carriers, with baseband data on each sub-carrier being independently modulated commonly using some type of quadrature amplitude modulation (QAM) or phase-shift keying (PSK). This composite baseband signal is typically used to modulate a main RF carrier.

The message to transmit is represented as s[n] that is a serial stream of binary digits. By inverse multiplexing, these are first demultiplexed into N parallel streams and each one mapped to a symbol stream using some modulation constellation (QAM, PSK, etc.). It is also possible that using adaptive modulation; OFDM gives the opportunity to use different modulation schemes for each channel.

In the next step; inverse FFT is computed on each set of symbols , giving a set of complex time-domain samples. These samples are then quadrature-mixed to passband in the standard way. The real and imaginary components are first converted to the analogue domain using digital-to-analogue converters (DACs); the analogue signals are then used to modulate cosine and sine waves at the carrier frequency, fc, respectively. These signals are then summed to give the transmission signal, s(t).

For the implementation as seen in Figure 2.3, a message stream consisting of binary digits is formed and then demultiplexed into N parallel streams using reshape function. Next, parallel streams are modulated IFFT size must be equal to the number of subchannels. After IFFT transformation, the result is then transformed into a serial stream by using reshape function again. Finally cyclic prefix is inserted into the serial stream.

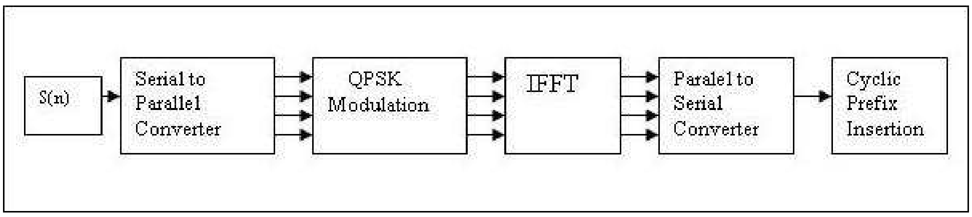

Figure 2.3. OFDM transmitter

One of the main principle of OFDM modulation scheme is that since low bit rate modulation schemes suffer ISI caused by multipath less rather than high data rate modulation schemes, its very useful to send low data bits in parallel streams instead of high data rate serial streams (Engels 2002). Addition of a guard prefix to every OFDM symbol is a very common way of combating the delay spread . For a successful OFDM process; the length of the cyclic prefix to insert must be 1/2 or 1/4 of the number of subchannels. As long as there exists a cyclic prefix period longer than the expected delay spread, ISI will be avoided. But if the length of the cyclic prefix can not be chosen accurately, this can lead to Inter-carrier interference (ICI) due to the operational nature of the FFT.

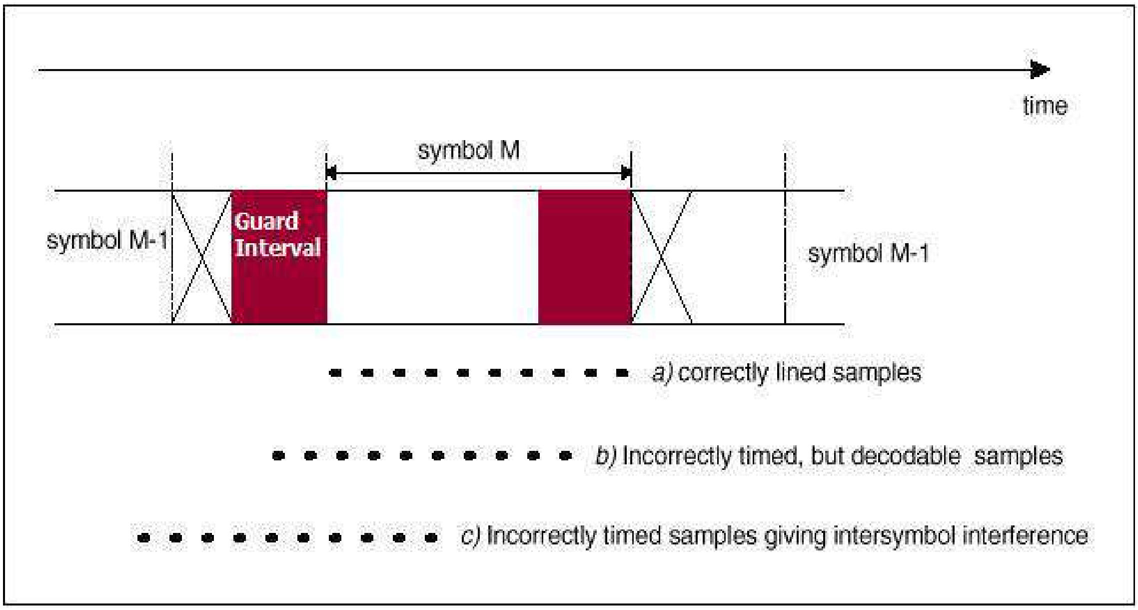

Figure 2.4. Cyclic Prefix Insertion

The serial data stream is sent from transmitter to receiver through a channel. The receiver picks up the signal r(t). The received signal is then quadrature-mixed down to baseband using cosine and sine waves at the carrier frequency. This also creates signals centered on 2fc, so low-pass filters are used to reject these. Later, the baseband signals are sampled and digitised using analogue-to-digital converters (ADCs), and then a forward FFT is used to convert back to the frequency domain. This returns N parallel streams, each of which is converted to a binary stream using an appropriate symbol detector. These streams are then re-combined into a serial stream, s[n], which is an estimate of the original binary stream at the transmitter.

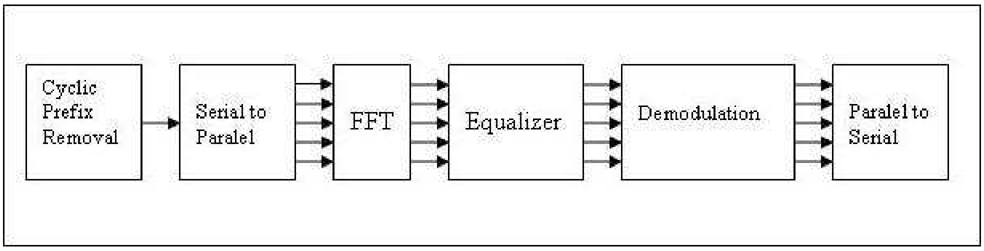

Figure 2.5. OFDM receiver

In the receiver side as seen in Figure 2.5, cyclic prefix inserted in transmitter is removed and then the serial data is demultiplexed into N parallel streams. In the next step FFT is computed on each parallel stream and then equalizer is applied to reduce the channel effects. Later the parallel stream in frequency domain is demodulated (for PSK,QAM etc.) and then transformed from parallel to serial data stream.

For spectrum sensing systems, we do not follow the steps from equalizer block in receiver side as we aim to do the detection without any prior knowledge on primary signal.

2.4. Cognitive Radios

Cognitive Radio (CR) is a paradigm that enables an xG network to use spectrum in a dynamic manner. The term, cognitive radio, can formally be defined as follows (FCC Report 2002):

”Cognitive Radio is a radio for wireless communications in which either a network or a wireless node changes its transmission or reception parameters based on the interaction with the environment to communicate efficiently without interfering with licensed users.”

This changing of parameters is based on the active monitoring of external and internal radio environment such as radio frequency spectrum, user behaviour trends and network state.

The idea of cognitive radio is first proposed in Joseph Mitola III’s PhD dissertation (Mitola 2000) as a promising solution for the dynamic usage of the spectrum. Mitola described it as ”the point in which wireless Personal Digital Assistants (PDAs) and the related networks are sufficiently and computationally intelligent about radio resources and related computer to computer communications to detect user communications needs as a function of use context and to provide radio resources and wireless services most appropriate to those needs".

Simply, the idea behind the cognitive radio paradigm is the utilization of the idle frequency bands allocated to primary users or licensed users by the secondary users or unlicensed users without any interference to primary users' communications. If we scan the portions of radio spectrum, we would find that some frequency bands are largely unoccupied most of the time while some other frequency bands are only partially occupied and the remaining frequency bands are heavily used (Haykin 2005),(FCC Report 2002). Hence cognitive radio aims to detect those idle bands and allocate the secondary users to those. It is assumed that cognitive radio senses the environment in a period of time and if primary user needs to start communicating, secondary users are moved to another idle band. In doing so the cognitive radios change the power level so that to minimize the interference that may caused due to the secondary users. Thus, we see that knowing the availability of the idle spectrum band is not enough to decide the usage of unused spectrum bands. Many factors like frequency selection, modulation schemes and power level should be considered to capture the variation in radio environment so as to avoid possible interference to other users.

The Federal Communication Commission (FCC) has identified in (FCC Report 2002) the following features that cognitive radios can incorporate to enable a more efficient and flexible usage of the spectrum.

- **Frequency Agility** The cognitive radio is able to change its operating frequency for its adaptation to the environment.

- **Dynamic Frequency Selection** The cognitive radio senses signals from nearby transmitters to choose an optimal environment to work in.

- **Adaptive Modulation** The transmission characteristics and waveforms can be reconfigured to exploit all opportunities for the usage of spectrum in an efficient way.

- **Transmit Power Control** The transmission power is adapted to full power limits when necessary on the one hand and to lower levels on the other hand to allow greater sharing of spectrum.

- **Location Awareness** The cognitive radio is able to determine its location and the location of other devices operating in the same spectrum to optimize transmission parameters for increasing spectrum re-use.

- **Negotiated Use** The cognitive radio may have algorithms enabling the sharing of spectrum in terms of prearranged agreements between a licensee and a third party or on an ad-hoc/real time basis.

2.4.1. Physical Architecture

Before going further issues on the cognitive radio, it'd be better to know its architecture simply in order to understand them better. As seen in Figure 2.8, the main components of a cognitive radio transceiver are the radio front-end and the baseband processing unit. Reconfigurability of each component is provided by a control bus to able to adapt the time varying radio environment (Akyıldız, et al. 2006). The RF front-end consists of a amplifier where the received signal is amplified and then mixed and analog to digital (A/D) converted by A/D unit. In the baseband processing unit, the digitized signal is modulated/demodulated and encoded/decoded. The novel part of the cognitive radio is the RF front-end since the baseband processing unit of a cognitive radio is essentially similar to existing transceivers. That is why the RF front-end of the cognitive radio must be mentioned in detail. In a cognitive radio, the RF front-end has a wideband sensing capability. RF hardware for the cognitive radio should be capable of tuning to any part of a large range of frequency spectrum in order to exploit all opportunities of the frequency spectrum. Also such spectrum sensing enables real-time measurements of spectrum information from radio environment. Generally, a wideband front-end architecture for the cognitive radio has the following structure as shown in Figures 2.6 and 2.7.

The components of a cognitive radio RF front-end according to (Akyıldız, et al. 2006) are as follows:

- **RF filter:** The RF filter filters the received RF signal with a band-pass filter and selects the desired band.

- **Low noise amplifier (LNA):** The LNA amplifies the desired signal while minimizing noise component at the same time.

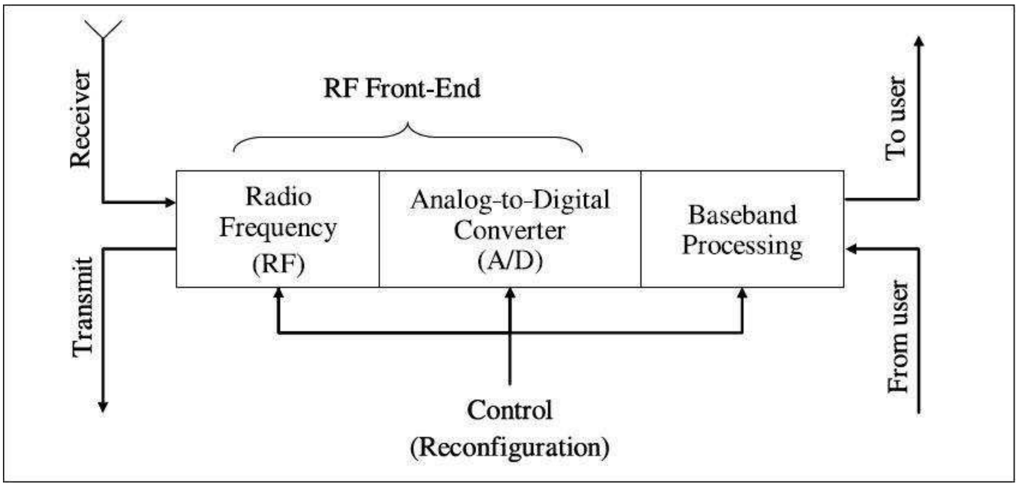

Figure 2.6. Cognitive Radio Transceiver

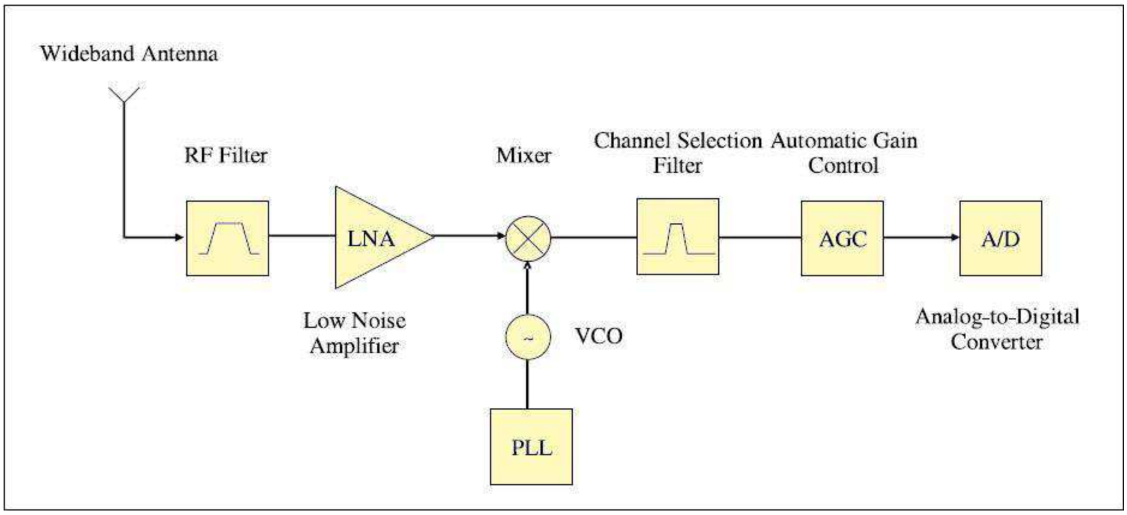

Figure 2.7. Wideband RF/ Analog Front-end

- **Mixer:** In the mixer, the received signal is mixed with locally generated RF frequency and converted to the baseband or the intermediate frequency (IF).

- **Voltage-controlled oscillator (VCO):** The VCO generates a signal at a specific frequency for a given voltage to mix with the incoming signal. This procedure converts the incoming signal to baseband or an intermediate frequency.

- **Phase locked loop (PLL):** The PLL ensures that a signal is locked on a specific frequency and can also be used to generate precise frequencies with fine resolution.

- **Channel selection filter:** The channel selection filter is used to select the desired channel and to reject the adjacent channels. There are two types of channel selection filters. The direct conversion receiver uses a low-pass filter for the channel selection. On the other hand, the super heterodyne receiver adopts a band pass filter.

- **Automatic gain control (AGC):** The AGC maintains the gain or output power level of an amplifier constant over a wide range of input signal

The wideband signals are received through the RF front end and then are sampled using the high speed analog-to-digital (A/D) converter and furthermore different measurements are done for detection of licensed user signal. But in real applications, RF antenna receives signal from various transmitters operating at different power levels, bandwidths and locations which makes it hard to detect weak signals in that kind of range. So there should be multi-GHz speed A/D converter with high resolution but it is practically infeasible to implement. Moreover this need of multi-GHz speed A/D converter requires the dynamic range of the signal to be reduced before A/D conversion. This reduction can be achieved by filtering strong signals which can be located anywhere in the wide spectrum range and using tunable notch filters. Another approach can be the usage of multiple antennas. As explained previously, the key challenge of the physical architecture of the cognitive radio is an accurate detection of weak signals of licensed users over a wide spectrum range. Hence, the implementations of RF wideband front-end and A/D converter are critical issues in xG networks (Akyıldız, et al. 2006).

2.4.2. Characteristics of Cognitive Radio

The dictionary meaning of word cognition means becoming acquainted with, mental process of knowing through perception or reasoning or intuition or knowledge or we can simply say learning by understanding. Considering these ideas together there are two main characteristics of cognitive radio which is worth mentioning. According to (Akyıldız, et al. 2006) they are cognitive capability and reconfigurability which is describe in detail as follow:

2.4.2.1. Cognitive Capability

Cognitive capability is the capability to get the information about the unused spectrum in the radio environment so as to provide the cognitive users with best operating parameters to use the spectrum efficiently without any interference to the primary users. This capability makes it versatile and efficient to interact with the real radio environment in order to detect appropriate communication parameters that are necessary for cognitive users to use. Thus, cognitive radio has to perform some tasks which is referred as cognitive cycle and is shown in Figure 2.8 (Akyıldız, et al. 2006).

In this thesis we only focus on spectrum sensing which is discussed in detail in Chapter 3.

2.4.2.2. Reconfigurability

The reconfigurability of a cognitive radio is the capability of programming the radio dynamically without any modification in hardware components. Cognitive radio can be programmed to be used as transmitter or receiver, also in different frequency or cognitive radio can use different modulation techniques with variable transmission power with respect to the communication link. This capability is realizable as an intrinsic result of the development of software-defined radio (SDR) platform which is a fully reconfigurable wireless device that is able to adjust its communication parameters in response to either network or user demands. A software-defined radio (SDR) system is a radio communication system which can tune to any frequency band and receive any modulation

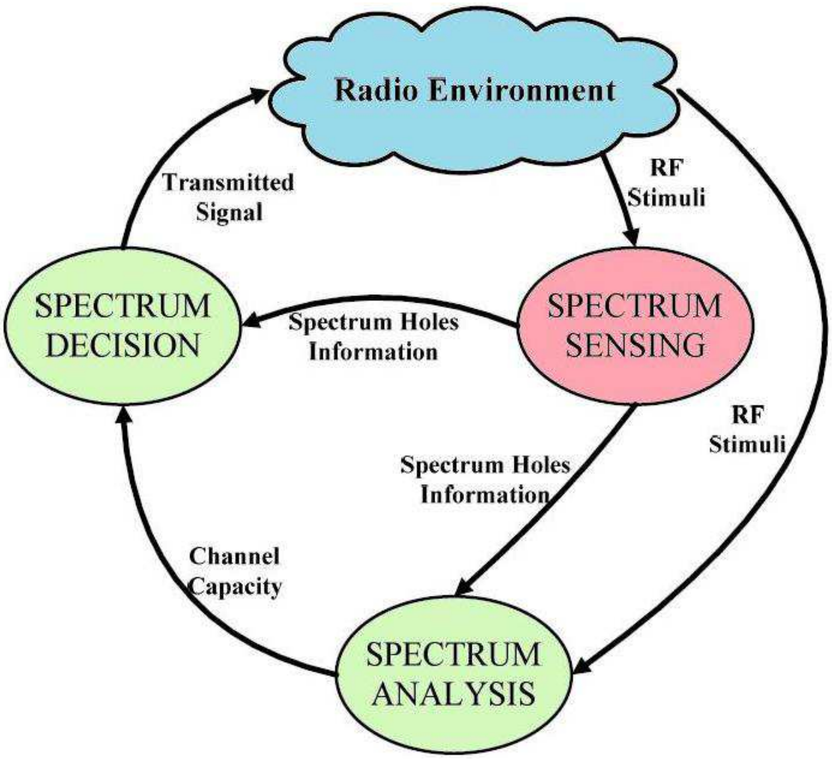

Figure 2.8. Cognitive Cycle

over a large frequency spectrum by means of programmable hardware which is controlled by software. Another saying, a software-defined radio can easily switch among multiple wireless protocols or move to different frequencies, waveforms, protocols, or applications, but the user must command it to do so. A complete hardware based radio system has limited utility since parameters for each of the functional modules are fixed. On the other hand, a software-defined radio extends the utility of the system for a wide range of applications that use different link-layer protocols and modulation/demodulation techniques. Thus this software based approach provides reconfigurability to the cognitive radio systems. It is noted earlier that reconfigurability means to able to reconfigure the transmission parameters during the transmission. This fact indicates that the cognitive radio is capable of configuring both transmitter and receiver parameters in order to switch to different spectrum bands by using appropriate protocols and modulation schemes with assigning appropriate power level of the signal.

2.5. Literature Review

Being the focus of (Arslan and Yücek 2007), spectrum sensing is the most important task among others for the establishment of cognitive radio. Several articles have been published on spectrum sensing so far. In (Arslan and Yücek 2007); spectrum sensing is defined and various aspects of spectrum sensing task are mentioned and discussed. Some challenges encountered in spectrum sensing is also given in this paper. Most common spectrum sensing methods such as Matched Filtering, Energy Detection and Cyclostationary Feature Detection are also studied and compared in terms of advantages and drawbacks.

(Akyıldız, et al. 2006) is the one of the most detailed papers written on Cognitive Radios. First part of this paper focuses on cognitive radio and its architecture. In the next part, the present and future of the frequency spectrum is discussed, followed by a detailed analysis of spectrum sensing task. Some spectrum sensing methods are studied and compared. In the following section, spectrum sharing is discussed and its challenges are mentioned.

Srinavasa and Jafar focuses in their paper published in 2007 on different interpretations of cognitive radio that underlay, overlay, and interweave the transmissions of the cognitive user with those of licensed users. Underlay approach allows concurrent primary and secondary transmissions and aims to protect primary (licensed) users by enforcing a spectral mask on the secondary (unlicensed) users' signals so that unlicensed users' interference to licensed users is below the acceptable noise threshold set for safer communication of licensed users. Overlay approach allows concurrent primary and secondary transmissions as well but contrary to underlay approach, secondary users can use part of their power to communicate and remained power to assist primary user. If the power split is chosen carefully, the increase in a primary user's SNR due to the assistance from secondary user is exactly offset by the decrease in primary users' SNR as a result of interference. Interweave approach is based on Opportunistic Communication (Mitola 2000) which is about using the spectrum holes that indicate to the fact that they are not in use by primary users. Interweave approach is also known as Interference Avoidance approach. In our study Interweave approach is considered for implementation.

In the papers of (Tian and Giannakis 2006), (Hur, et al. 2006) and

(Quan, et al. 2008); wideband spectrum sensing task is examined. However, more research on wideband sensing is needed especially when the center frequencies and bandwidths of the primary signals are unknown within the frequency range of interest.

(Urkowitz 1967) focuses on the Energy Detection of Unknown Signals. It's one of the most important and cited paper in the literature on Energy Detection. In this paper; although the signal is actually unknown in detail, it's considered deterministic. Deterministic signal is assumed in this paper as the input with signal present is Gaussian but not zero mean. The spectral region is known and the noise is assumed to be Gaussian and additive with zero mean. It explains detection in white noise considering whether the noise is a Low-pass or Bandpass random process and operating receiver characteristics are given theoretically.

In (Cabric, et al. 2004), (Cabric, et al. 2006) and (Digham, et al. 2003), cooperation among cognitive radios is proposed in order to mitigate the hidden node problem. On the other hand, in (Ashish and Linnartz 2007), (Digham, et al. 2007), (Kuppusamy and Mahapatra 2008), it is proposed to implement some multiple antenna techniques in order to overcome some spectrum sensing challenges.

(Gardner, et al. 2006) is a review on Cyclostationary which has been studied by scientists for more than 50 years in many research areas. They first mentioned the general properties of cyclostationary processes and then gave information on implementation in many applications.

Zhang and Xu focuses on the implementation of a cyclic periodogram detection for CR environments. The algorithm is implemented on Agilent VEE Pro.

(Sadeghi and Azmi 2008) and (Rajarshi and Krusheel 2008) propose some multiple antenna techniques for cyclostationary based spectrum sensing systems

CHAPTER 3

SPECTRUM SENSING

Spectrum sensing for cognitive radios is still an ongoing development and the techniques for the primary signal detection are limited in the present literature (Arslan and Yücek 2007). One of the most distinguished features of cognitive radio networks will be an ability to switch between radio access technologies, transmitting in different parts of the radio spectrum as idle frequency band slots arise (Haykin 2005),(Akyıldız, et al. 2006). This dynamic spectrum access which was proposed in (Mitola 2000) for the first time, is one of the fundamental requirements for transmitters to adapt to varying channel quality, network congestion, interference and service requirements. Cognitive radio networks, assumed to be secondary users, will also need to coexist with primary users, which have the right to use the spectrum and thus must have a guarantee not to be interfered by secondary users.

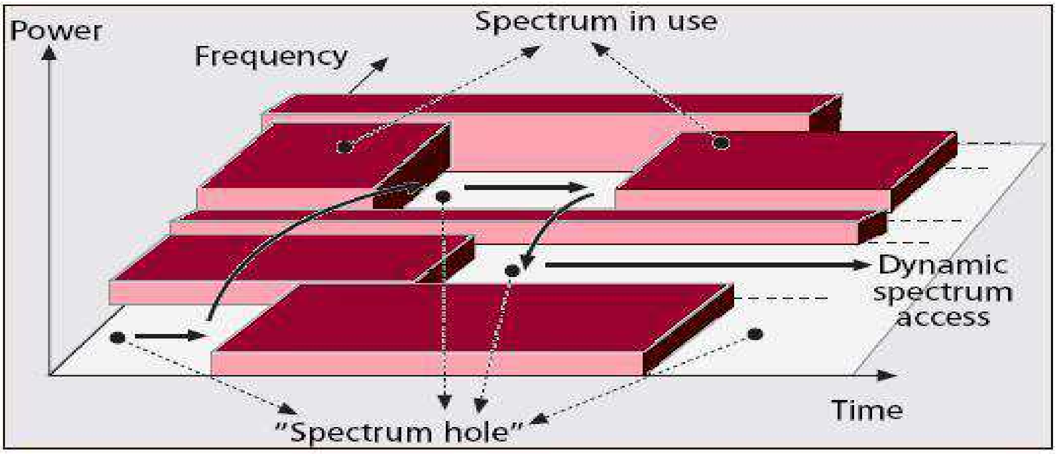

Figure 3.1. Spectrum Sensing

As a result of these facts, under-utilization of the current spectrum and the need to increase the spectrum efficiency is motivating researchers to exploit the wireless medium. In this direction, the Federal Communications Commission (FCC) Spectrum Policy Task Force has published (FCC Report 2002) in 2002, in which it investigates the under utilization of the radio spectrum. While the FCC is responsible of determining the spectrum usage and its policies, the Whitespace Coalition, formed by companies such as Microsoft, Google, Dell, HP and Intel have research groups studying to exploit the spectrum vacancies in the television band. Cognitive radio networks are envisioned to be able to opportunistically exploit those ”spectrum holes”, by awareness of the environment and cognition capability, in order to adapt their radio parameters accordingly. Spectrum sensing is the main step that will enable cognitive radio networks to achieve this goal.

Fundamentally, a spectrum sensing device must be able to give a general idea on the medium over the entire radio spectrum. This allows the cognitive radio network to analyse all degrees of freedom (time, frequency and space) in order to predict the spectrum usage. Wideband spectrum sensing works are also available in the literature and studied in (Tian and Giannakis 2006), (Hur, et al. 2006) and (Quan, et al. 2008). However, equipment able to perform wide-band sensing all at once is prohibitively difficult to build with today’s technology. Feasible spectrum sensing device can quickly sweep the radio spectrum, analysing one narrowband segment at a time. In this section, we have emphasized the importance of the spectrum sensing technique for cognitive radio networks. In the next section, we aim at understanding the underlying characteristics of the spectrum sensing problem, which will enable us to develop the approaches presented further in this chapter.

3.1. Characteristics of Spectrum Sensing

Spectrum sensing is based on a well known technique called signal detection. Signal detection can be described as a method for identifying the presence of a signal in a noisy environment. Signal detection has been thoroughly studied for radar purposes since the fifties (Arslan and Yücek 2007). Analytically, signal detection can be reduced to a simple identification problem, formalized as a hypothesis test (Poor 1994), (Urkowitz 1967).

$$\begin{aligned} H_1 : x(n) &= s(n)h + w(n) \\ H_0 : x(n) &= w(n) \end{aligned} \tag{3.1}$$

where x(n) is the received signal by secondary users, s(n) is the transmitted signal of the primary user, h is the channel coefficient; and w(n) is additive white Gaussian noise with variance σ_w^2.

H_0 and H_1 are the sensing states for absence and presence of signal respectively. Another saying; H_0 is the null hypothesis which indicates that primary user does not communicate and H_1 is the alternative hypothesis that indicates the existence of the primary user. We can define four possible cases for the detected signal:

1. declaring H_1 under H_1 hypothesis which leads to *Probability of Detection*(P_d)
2. declaring H_0 under H_1 hypothesis which leads to *Probability of Missing* (P_m)
3. declaring H_1 under H_0 hypothesis which leads to *Probability of False Alarm* (P_f)
4. declaring H_0 under H_0 hypothesis

If H_0 is decided under H_1 hypothesis, then it leads to probability of missing, P_m, that is probability of deciding that there's no primary signal while primary signal actually exists. Another saying it's the probability of signal missing. If H_1 is decided while H_0 is observed then it refers to find the probability of false alarm which indicates to decide primary signal exists while there's actually no primary user communicating. Thus false alarm error leads to inefficient usage of the spectrum.

Literally, the aim of the signal detector is to achieve correct detection all of the time, but this can never be perfectly achieved in practice because of the statistical nature of the problem. Therefore signal detectors are designed to operate within prescribed minimum error levels. Missed detections are the biggest issue for spectrum sensing, as it means possibly interfering with the primary system. Nevertheless, it is desirable to keep false alarm probability as low as possible, so that the system can exploit all possible transmission opportunities.

In a wireless radio network, since it is reasonable to assume that the spectrum sensing device does not know the location of the transmitter, two options arise:

- A low h is solely due to the path loss (distance) between the transmitter and the sensing device meaning that the latter is out of range and can safely transmit;

- A low h is due to shadowing or multipath, meaning that the sensing device might be within the range of the transmitter and can cause harmful interference.

Not only fading effects, but there are also some other challenges that cognitive radio networks may face while sensing the spectrum. Some of these challenges are summarized in the following section:

3.2. Spectrum Sensing Challenges

3.2.1. Hidden Node Problem

The fading effects of the wireless channel plays an especially negative role in the well known 'hidden node' problem (Fullmer and Garcia-Luna-Aceves 1997) which also refers to hidden primary user. In this problem, the spectrum sensing terminal is deeply faded with respect to the transmitting node while having a good channel to the receiving node. The spectrum sensing node then senses a free medium and initiates its transmission, which produces interference on the primary transmission. Thus, fading here introduces uncertainty regarding the estimation problem. To solve this issue, cooperative sensing has been proposed (Akyıldız, et al. 2006).

3.2.2. Limited Sensing Ability

In (Hötyä, et al. 2007) it's underlined that cognitive radio has only a basic 'sense of hearing' to detect the spectrum holes that's why its ability is limited. That indicates, a cognitive radio has to detect its multidimensional environment with only a single sense. For instance; considering a blind person trying to go across in a busy traffic only uses hearing sense just like a cognitive radio does. Many open questions are related to the sensing ability and performance in wide bandwidths. Advanced techniques are needed to overcome this problem and sense very wide bandwidths reliably and rapidly.

3.2.3. Wideband Sensing

One of the main concerns in spectrum sensing is how to set the boundaries of spectrum to sense. Instead of very wide band detection, limited spectrum can be used for spectrum sensing. Working in limited spectrum; received signal can be sampled at or above Nyquist rate with current technology. Furthermore; the computational difficulty encountered in wide band detection can be restricted to a reasonable level (Hötyä, et al. 2007). Expensive analog front-end required for a very wide spectrum can also be avoided. Taking everything into consideration; regulatory agencies should allocate spectrum bands for different types of cognitive radios depending on the spectrum range that they work (Hötyä, et al. 2007).

3.2.4. Spectrum Sensing in Multi-Dimensional Environment

The environment that cognitive radio based systems work in generally consists of multiple unlicensed users with multiple licensed users (Hötyä, et al. 2007). The existence of multiple secondary users causes some challenges in spectrum sensing as interference to other secondary users in the environment that is expected to make it difficult to sense primary users reliably. To overcome the problem, a few aspects of cooperation in spectrum sensing must be considered such as the distributed information (transmitted power, frequency of other users, location ...), the way to cooperate with other unlicensed users, the need of primary users involving in the cooperation (Hötyä, et al. 2007).

3.2.5. Sensing Time

Using cognitive radios; it's guaranteed that licensed users can use their frequency bands any time and to increase the capacity of the spectrum and avoid interference; spectrum holes must be detected as quickly as possible to accommodate the secondary users. In (Hötyä, et al. 2007), it's underlined that spectrum sensing algorithm must be performed within a limited time duration. It also must be taken into account that how often cognitive radio sense the spectrum. It needs to sense very frequently in order not to miss any opportunity.

3.2.6. Spectrum Sensing for OFDM Systems

Recently, multi-carrier methods have been recognized as potential candidates for the physical layer of Cognitive Radio systems. By assigning secondary users assumed to be cognitive radios, to the subcarriers that coincide with the idle parts of the spectrum which are not used by primary users, multicarrier methods provide much flexibility to fill in the spectral holes and thus to best harness the available resources. Moreover, orthogonal frequency-division multiplexing (OFDM), the most popular multicarrier method, has been introduced as the first candidate for this purpose (Weiss and Jondral 2004).

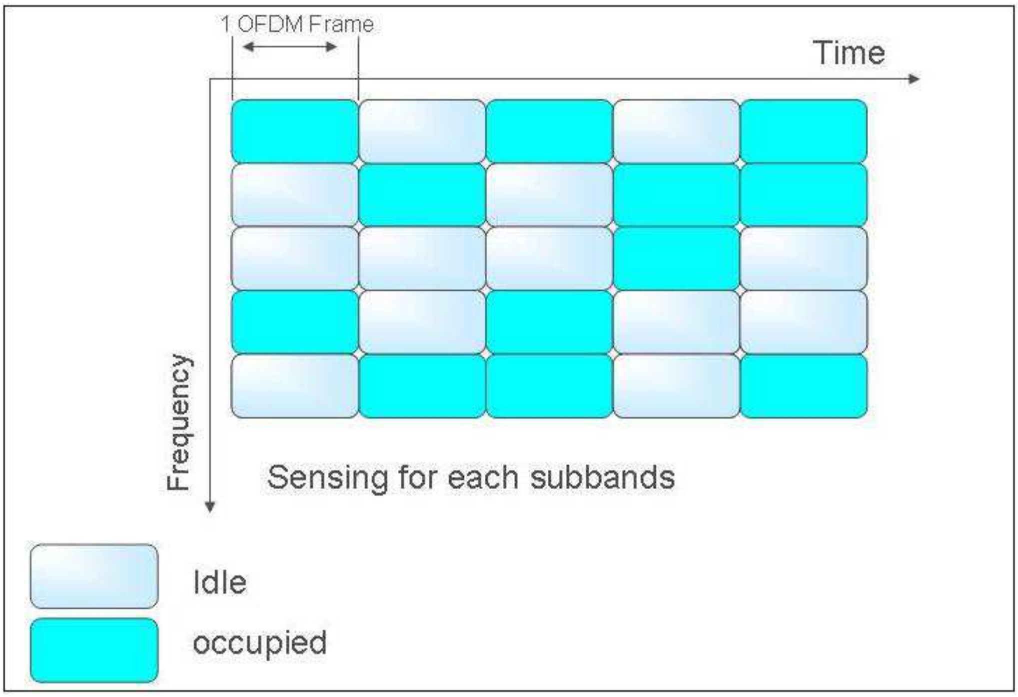

Figure 3.2. OFDM Band Usage

Because of the operation of primary networks, cognitive radio users cannot obtain a reliable communication channel for a long time period. Moreover, CR users may not detect any single spectrum band to meet the user's requirements. Therefore, multiple spectrum subbands can be simultaneously used for transmission.

For the systems using OFDM, the spectrum is analysed subband by subband as given in the Figure 3.2. and secondary users data can be allocated into these idle subbands.

3.3. Spectrum Sensing Methods

In the literature generally 5 spectrum sensing approaches in some of which the characteristics of the identified transmission are detected for deciding the signal transmission as well as identifying the signal type are proposed and in this section;some of these methods are explained in detail.

3.3.1. Matched Filtering

The optimal way for signal detection is a matched filtering since it maximizes received SNR (Proakis 2001) and also requires short time to achieve a certain probability of false alarm or probability of miss detection as compared to other methods (Hötyä, et al. 2007). On the other hand matched filtering requires perfect a *priori* knowledge of licensed users' features such as bandwidth, frequency, modulation type, etc. to demodulate received signals. Therefore it needs dedicated signal receivers for each signal type that leads to the implementation complexity and large power consumption as various receiver algorithms need to be executed for detection (Hötyä, et al. 2007).

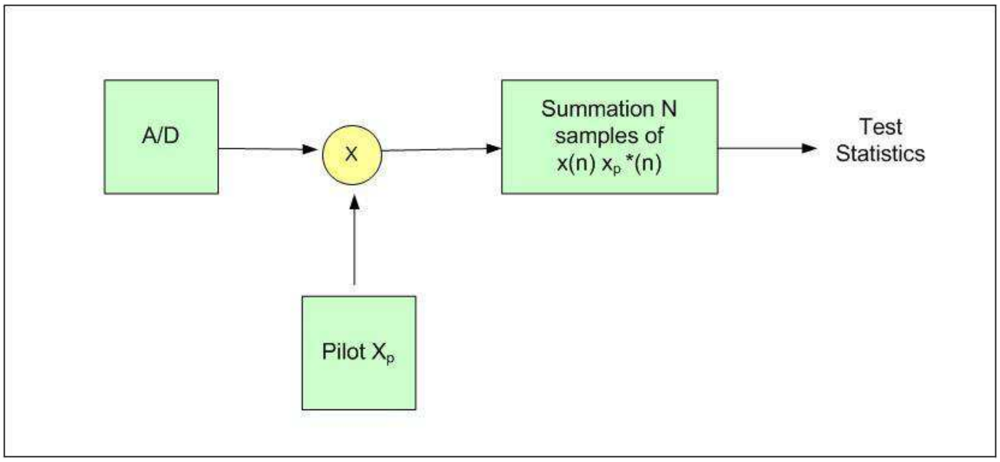

Figure 3.3. Pilot Detection via Matched Filtering

As it is illustrated in Figure 3.3., matched filter projects the received signal in direction of the pilot data via the equation given below:

$$\Lambda_{MF} = \sum_N x(n)x_p(n)^* \tag{3.2}$$

where $x(t)$ is the received signal in additive white Gaussian Noise and $x_p(t)^*$ is the conjugate of the known pilot data. Commonly, the pilot signal is chosen as orthogonal to data and can be considered independently. Under Neyman-Pearson detection method; the likelihood ratio tests leads to an optimal solution (Hötyä, et al. 2007) therefore Λ_{MF} can be compared with a threshold found by N-P test and thus primary signals' existence can be decided. The theoretical analysis shows that coherent processing can transform low SNR into high SNR, thus giving adequate sample; very weak signals can be detected.

Considering the fact that it requires perfect knowledge on signal features such as bandwidth, operating frequency, modulation type and order, pulse shaping and frame format, and it also needs large power consumption it's very impractical to implement in cognitive radios

3.3.2. Energy Detection

Energy detection is an optimal way to detect primary signals when priori information of the primary signal is unknown to secondary users. It measures the energy of the received waveform over a specified observation time (Urkowitz 1967), (Hötyä, et al. 2007), (Digham, et al. 2003), (Lethomaki, et al. 2005).

In the literature, we come across various algorithms indicating that energy detection can be implemented both in time and also frequency domain using Fast Fourier Transform(FFT). Energy Detector simply needs a band-pass filter; an analog to digital converter, square law device and an integrator. First the input signals bandwidth is limited to focus through a band-pass filter. Then the filtered signal is squared and integrated over an observation interval T. Finally the output of the integrator is compared with a threshold to decide whether primary signal exists or not (Urkowitz 1967). The block diagram of energy detection is shown in Figure 3.4.

We first apply $x(n)$ to a band pass filter (BPF) and then to square law device, followed by an integrator.

The output of the energy detector can be given in time domain using the block

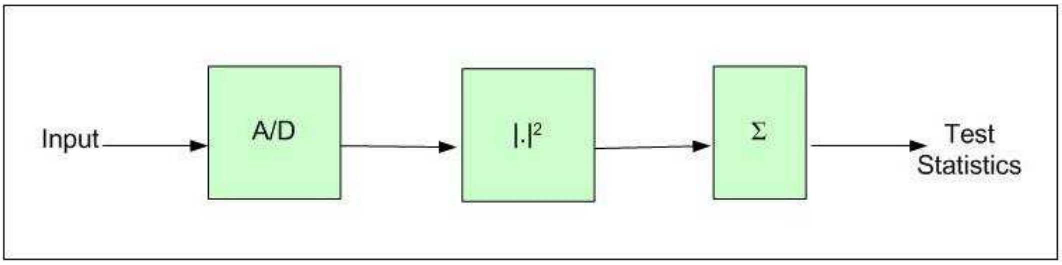

Figure 3.4. Energy Detection

diagram as follows:

$$\Xi_{time} = \sum_{n=1}^{N} |x(n)|^2 \tag{3.3}$$

Finally, this output signal Ξ_{time} is compared to the threshold λ in order to decide whether a signal is present or not in that frequency band.

$$\Xi_{time} \lesseqgtr \lambda \tag{3.4}$$

The energy detection can also be implemented in frequency domain using periodograms.

3.3.2.1. Periodogram

The periodogram method is a DFT based method to estimate power spectral density (PSD). The name of the periodogram comes from the fact that it was first used in determining possible hidden periodicities in time series (Stoica and Moses 1997). For simulations, FFT is used instead of DFT.

$$X(k) = \sum_{n=1}^{N} x(n)e^{-j2\pi(k-1)(n-1)/N} \qquad \text{where} \qquad 1 \leq k \leq N \tag{3.5}$$

where $x(n)$ is the discrete received signal, N is the FFT size. Then we apply $X(k)$ to an energy detector as follows:

$$\Xi_{periodogram} = \frac{1}{N}\sum_{k=1}^{N}|X(k)|^2 \tag{3.6}$$

As seen in this equation, we sum N components of the output of square law device where $X(k)$ is applied to, hence the variance of the statistics fluctuates with respect to FFT size. In order to mitigate this fluctuation, we divide the statistics with FFT number in order to hold the variance constant.

The Welch's method, described in the following Section, is one of the several modified periodogram-based methods which attempt to improve the statistical properties of the periodogram method.

3.3.2.2. Welch's Periodogram

The idea of the Welch's periodogram is to divide the data sequence into segments with windowing. In the Welch's method these data segments can be overlapping and non-overlapping (Stoica and Moses 1997).

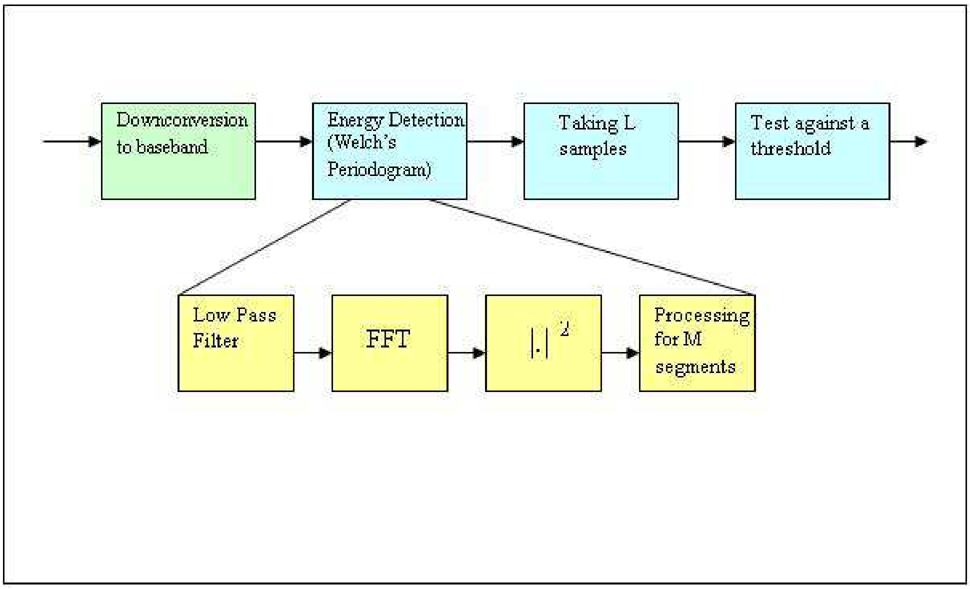

Figure 3.5. Energy Detection with Welch's Periodogram

The block diagram of the Welch's periodogram is shown in Figure 3.5. First, the input data sequence is down-converted and lowpass filtered. After that, the data sequence is partitioned into M non-overlapping or overlapping segments as given in Figure 3.6. and the segments are processed with FFT. After FFT, the samples are sent to the square-law device.Then L samples are taken from these M segments, followed by a summation of the L samples. Finally, the output values in the band of interest are compared to the threshold and the decision whether the signal is present or not is done (Matinmikko 2009).

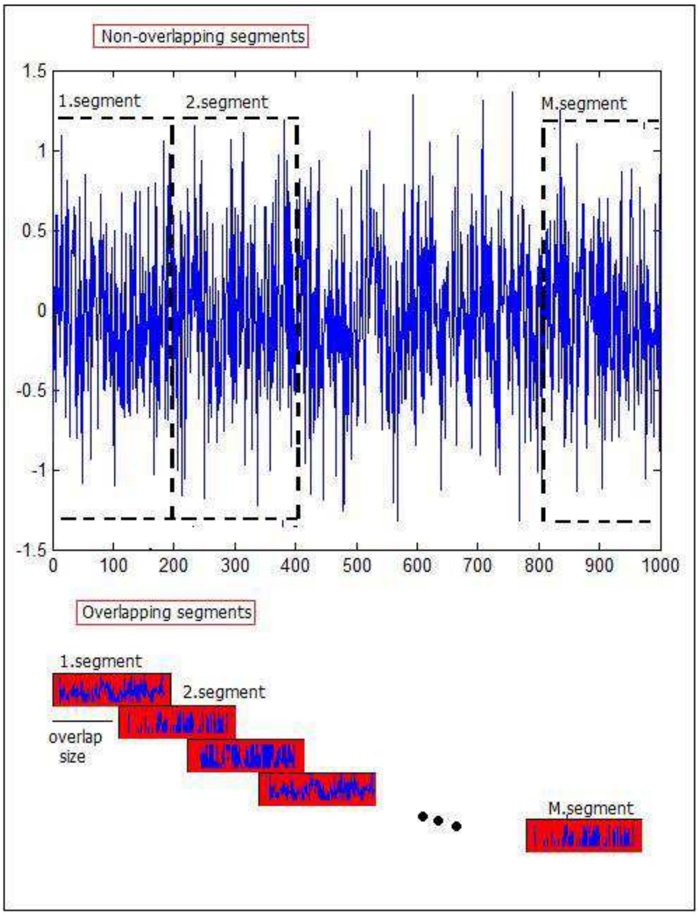

Figure 3.6. Overlapping and non-overlapping segments

In the papers using Welch's periodogram used non-overlapping segments. Actually overlapping segments can not offer gain compared to non-overlapping segments as its performance is only dependent on ML product.

3.3.2.3. Receiver Operating Characteristics

Next we derive the analytical receiver operating characteristics for Energy Detection algorithms. Probability of False Alarm is first derived for energy detection in time domain and then rewritten for frequency domain and Welch's periodogram.

False Alarm Probability

In most of papers about spectrum sensing, cognitive radio is aimed to achieve a certain level of *False Alarm Probability* which indicates the efficiency of the spectrum usage. High probability of false alarm leads to inefficient usage of spectrum because it means many idle spectrum bands are detected as used by primary user, that's why cognitive radio can not use these spectrum bands, thus the efficiency of the spectrum usage decreases. For this reason, it is important to analyse the detection performance of a system for a given efficiency level.

We know that False Alarm Probability is the probability that cognitive radio decides in favour of H_1 hypothesis while H_0 hypothesis exists which means cognitive radio detects the idle band as an occupied band by primary user.

$$x(n) = w(n); \text{under } H_0 \text{ hypothesis} \tag{3.7}$$

where $w(n)$ is a zero-mean $AWGN$ noise with random variables $\{w_n\}_{n \in N}$ distributed as

$$f(w_n) = \frac{1}{\sqrt{2\pi\sigma_w^2}} e^{-w_n^2/2\sigma_w^2} \tag{3.8}$$

Then it is applied to an A/D converter and then energy detector as follows :

$$\Xi_{time} = \sum_{n=1}^{N} |x(n)|^2 \tag{3.9}$$

$$\Xi_{time} = \sum_{n=1}^{N} |w(n)|^2 \qquad \text{under } H_0 \text{ hypothesis} \tag{3.10}$$

where N is the symbol length.

Under H_0 hypothesis, the output of the energy detector is summation of N squares of Gaussian variables. Thus the distribution of the output becomes a chi-square pdf with

N degrees of freedom as follows; (The derivation of central Chi-Square distribution is given in Appendix A)

$$f_{\Xi_{time}}(\xi) = \frac{1}{\sigma_w^N 2^{N/2}\Gamma(\frac{1}{2}N)}\xi^{(N/2)-1}e^{-\xi/2\sigma_w^2}, \text{for } \xi \geq 0 \tag{3.11}$$

From the definition of False Alarm Probability, the cognitive radio decides in favour of H_1 hypothesis while the band is idle, hence if we integrate the output of the energy detector under H_0 hypothesis from threshold the infinity, we can derive the $False\ Alarm\ Probability(P_{FA})$ as follows:

$$\Xi_{time} > \lambda, \qquad \text{under } H_0 \text{ hypothesis} \tag{3.12}$$

Hence, the false alarm probability in time domain can be derived as given below:

$$\begin{aligned} P_{FA(time)} &= Pr\{\Xi_{time} > \lambda | H_0\} \qquad (3.13)\\ &= \int_{\lambda}^{\infty} f_{\Xi_{time}}(\xi)d\xi \\ &= \int_{\lambda}^{\infty} \frac{1}{\sigma_w^N 2^{N/2}\Gamma(\frac{1}{2}N)}\xi^{(N/2)-1}e^{-\xi/2\sigma_w^2}d\xi \\ &= \frac{1}{\sigma_w^N 2^{N/2}\Gamma(\frac{1}{2}N)}\int_{\lambda}^{\infty}\xi^{(N/2)-1}e^{-\xi/2\sigma_w^2}d\xi \end{aligned}$$

To solve this equation, we apply some variable conversions as follows ;

$$\begin{aligned} \xi/2\sigma_w^2 &= t \qquad (3.14)\\ \xi &= 2\sigma_w^2 t \qquad (3.15)\\ d\xi &= 2\sigma_w^2 dt \qquad (3.16)\\ \lambda_1 &= \frac{\lambda}{2\sigma_w^2} \qquad (3.17) \end{aligned}$$

Then we rewrite the equation (3.12) as follows ;

$$
\begin{aligned}
P_{FA(time)} &= \frac{1}{\sigma_w^N 2^{N/2}\Gamma(\frac{1}{2}N)} \int_{\lambda_1}^{\infty} (2\sigma_w^2)^{(N/2)-1} \quad t^{(N/2)-1} e^{-t} 2\sigma_w^2 dt \qquad (3.18) \\
&= \frac{1}{\sigma_w^N 2^{N/2}\Gamma(\frac{1}{2}N)} \int_{\frac{\lambda}{2\sigma_w^2}}^{\infty} 2^{N/2}\sigma_w^N t^{(N/2)-1} e^{-t} dt
\end{aligned}
$$

We know that incomplete gamma function; $\Gamma(\varsigma, a) = \int_{\varsigma}^{\infty} t^{a-1}e^{-t}dt$. So we can rewrite last equation as follows;

$$
P_{FA(time)} = \frac{\Gamma(N/2, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N/2)} \qquad (3.19)
$$

We can derive the false alarm probability equations for frequency domain as well. In this case, received signal under H_0 hypothesis can be written as follows.

$$
\begin{aligned}
x(n) &= w(n) \qquad (3.20) \\
W(k) &= \sum_{n=1}^{N} w(n)e^{-j2\pi(k-1)(n-1)/N} \qquad (3.21) \\
\Xi_{periodogram} &= \frac{1}{N}\sum_{k=1}^{N} |W(k)|^2 \qquad (3.22)
\end{aligned}
$$

where N is the FFT size and assumed to be equal to symbol length. We divide the summation with N in order to keep the variance constant after FFT process. We know that Fourier Transform of a Gaussian is also a Gaussian, therefore the output of the energy detector $\Xi_{periodogram}$ is the summation of squares of Gaussian variables. Hence, $\Xi_{periodogram}$ has a pdf of central chi-square distribution with N degree of freedom. Applying same operations as in from *(3.11)* to *(3.21)*, we again get the same equation of False Alarm Probability as in time domain, but in frequency domain, N is the fft size.

$$
P_{FA(Periodogram)} = \frac{\Gamma(N/2, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N/2)}
$$

We also need to derive the false alarm probability equations for Welch's periodogram. In Welch periodogram, M segments are taken and applied to Fourier Transform and then L samples are taken from these segments.

$$\begin{aligned} w(n) &= [w_1(p)w_2(p) \quad ... \quad w_M(p)] && (3.23)\\ w_l(\gamma) &= \sum_{p=1}^{N} w_l(p)e^{-j2\pi(\gamma-1)(p-1)/N} \text{where } l\text{=1,2...M} && (3.24)\\ \Xi_{welch} &= \sum_{\gamma=1}^{L}\sum_{l=1}^{M}\frac{1}{N}|w_l(\gamma)|^2 && (3.25) \end{aligned}$$

If the received signal is applied to an energy detector using Welch's Periodogram, the output again becomes the summation of ML squares of Gaussian variables as a result of the fact that fourier transform of a Gaussian is again a Gaussian. Hence the distribution of the output becomes central chi square distribution with ML degree of freedom (Appendix A).

$$P_{FA(Welch)} = \frac{\Gamma(ML/2, \frac{\lambda}{2\sigma_w^2})}{\Gamma(ML/2)} \tag{3.26}$$

The Probability of False Alarm can be also derived for *Complex Gaussian noise* systems. Lets define $\{w_n\}_{n\in N}$ as complex Gaussian noise $CN(0, \sigma_w^2)$

$$w_n = a_n + b_n i \tag{3.27}$$

where a and b are Gaussian variables; $N(0, \sigma_w^2/2)$. Using the definition of absolute value;

$$|w_n|^2 = a_n^2 + b_n^2 \tag{3.28}$$

Therefore we can get the output of the energy detector under H_0 hypothesis as follows;

$$\Xi_{time}^{c} = \sum_{n=1}^{N} |w(n)|^2 = \sum_{n=1}^{N} a(n)^2 + \sum_{n=1}^{N} b(n)^2 \tag{3.29}$$

N is the number of samples per either real and imaginary components (We finally have 2N terms to sum over). Hence the summation of 2N Gaussian squares lead to the chi square distribution with 2N independent degree. Considering this the pdf of the energy detector output under H_0 hypothesis can be derived as follows;

$$f_{\Xi_{time}^{c}}(\xi) = \frac{1}{\sigma_w^{2N} 2^N \Gamma(N)} \xi^{N-1} e^{-\xi/2\sigma_w^2}, \text{for } \xi \geq 0 \tag{3.30}$$

Using this pdf, False Alarm Probability can be easily derived for complex Gaussian noise systems as follows;

$$\Xi_{time}^{c} > \lambda, \qquad \text{under } H_0 \text{ hypothesis} \tag{3.31}$$

Under H_0 hypothesis, we derive the false alarm probability as given below;

$$\begin{aligned} P_{FA_{(time)}^{c}} &= Pr\{\Xi_{time}^{c} > \lambda | H_0\} \\ &= \int_{\lambda}^{\infty} f_{\Xi_{time}^{c}}(\xi) d\xi \\ &= \int_{\lambda}^{\infty} \frac{1}{\sigma_w^{2N} 2^N \Gamma(N)} \xi^{N-1} e^{-\xi/2\sigma_w^2} d\xi \\ &= \frac{1}{\sigma_w^{2N} 2^N \Gamma(N)} \int_{\lambda}^{\infty} \xi^{N-1} e^{-\xi/2\sigma_w^2} d\xi \end{aligned} \tag{3.32}$$

Likewise in from (3.15) to (3.20); we can derive the False Alarm Probability for complex gaussian noise.

$$P_{FA_{(time)}^{c}} = \frac{\Gamma(N, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N)} \tag{3.33}$$

We can rewrite the probability of false alarm of periodogram and Welch's periodogram in Gaussian noise in (3.22) and (3.27) to complex Gaussian noise following the analysis for energy detection in time domain for complex gaussian noise. As the independent degrees of the distributions in Gaussian noise are multiplied with 2 for complex Gaussian noise, the false alarm probabilities can be derived as follows ;

$$P_{FA^c_{(Periodogram)}} = \frac{\Gamma(N, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N)} \tag{3.34}$$

where N is the FFT size

$$P_{FA^c_{(Welch)}} = \frac{\Gamma(ML, \frac{\lambda}{2\sigma_w^2})}{\Gamma(ML)} \tag{3.35}$$

where M is the segment number and L is the number of samples which are taken in each segment.

3.3.2.4. Simulation Results of Energy Detection for SISO Cognitive Radios

In our simulations we first evaluate the systems using BPSK modulation scheme and one tap Rayleigh channel.In the first simulation, we evaluated false alarm probability vs. detection probability. Therefore we fix SNR at 2 dB and evaluate the simulation for different false alarm probabilities, thus different thresholds. 20 samples are taken into the process.

Figure 3.7. indicates that higher false alarm probability leads to higher detection probability and vice versa. In order to achieve high performance of detection, we need high false alarm probability, on the other hand higher false alarm probability refers to less efficiency. That's why we need to decide the efficiency level that our system can tolerate. In the following simulations we assume that our system can tolerate $P_f = 0.01$.

In the following simulation given in *Figure 3.8*, we employ energy detector using Welch's periodogram for different M segment number and L number of samples taken

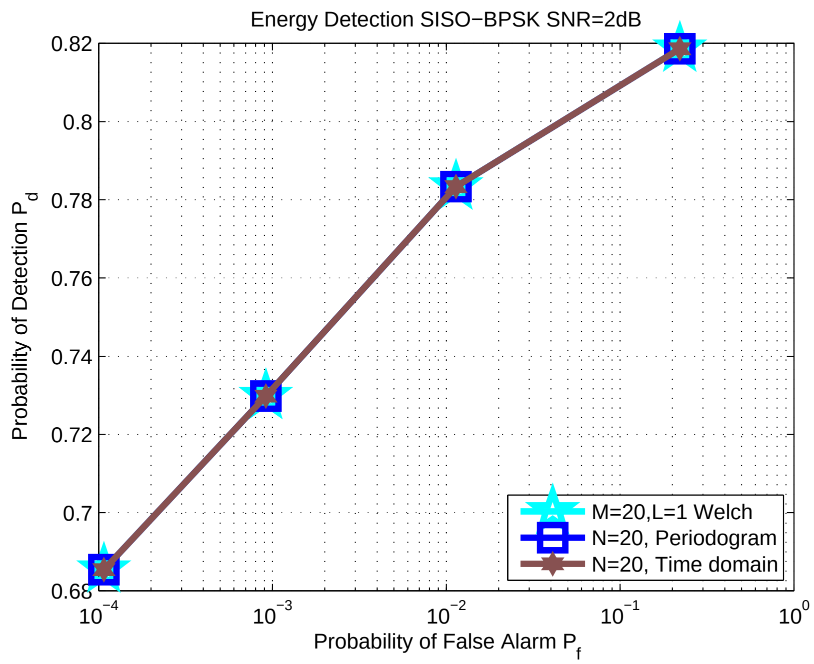

Figure 3.7. Probability of Detection vs. Probability of False alarm - BPSK

after FFT. FFT size is taken as same as the segment size.We set the threshold for the system to achieve False Alarm Probability P_f =0.01. We have 30 segments and 1 sample from each segment after FFT and applied it to an energy detector. Then we evaluated the same system for 10 segments and 3 samples from each segment. The simulation results in Figure 3.8 indicate that both scenarios give the same performance in terms of Probability of Missing and Probability of False Alarm, because in both scenarios, the output of the energy detector has the same independent degree,$M.L = 30$.

As seen in the this figure, they both have the same performance. Detection probability increases as SNR increases. We see that for this scenario -4dB is like a SNR wall to detect the primary user. Under -4dB, energy detector can not perform good.

In Figure 3.9, we compare different energy detection algorithms for different sample numbers by setting the thresholds to achieve $P_f = 0.01$ for each sample numbers.

As seen in the Figure 3.9 that all energy detection algorithms have almost the same performance. The graph indicates that as N increases, the detection probability increases. On the other hand as the number of samples used increases, the sensing time increases. So the main aim of spectrum sensing is to achieve better detection probability during lower sensing time.

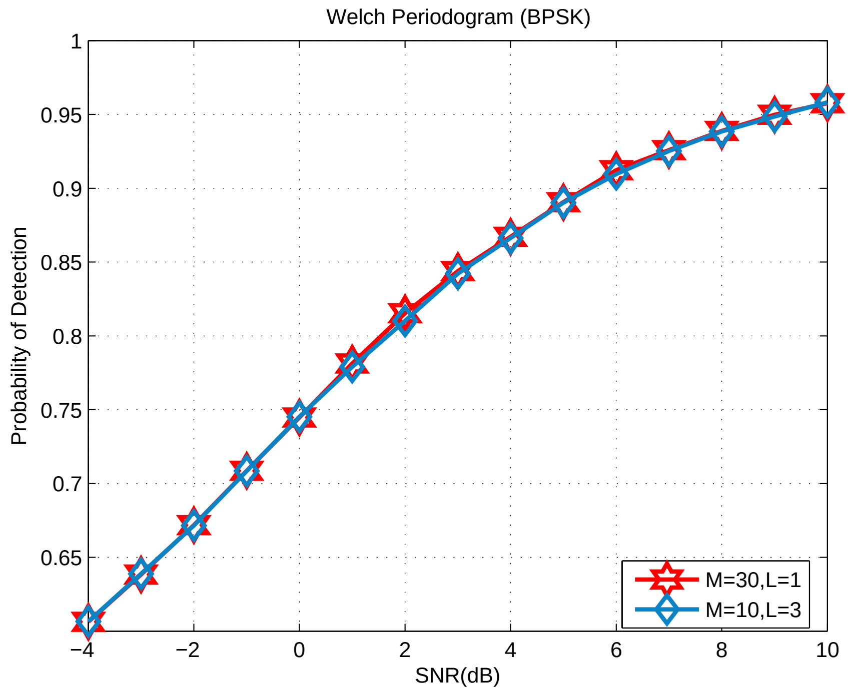

Figure 3.8. P_d vs. SNR using Welch Periodogram with different M,L values - BPSK

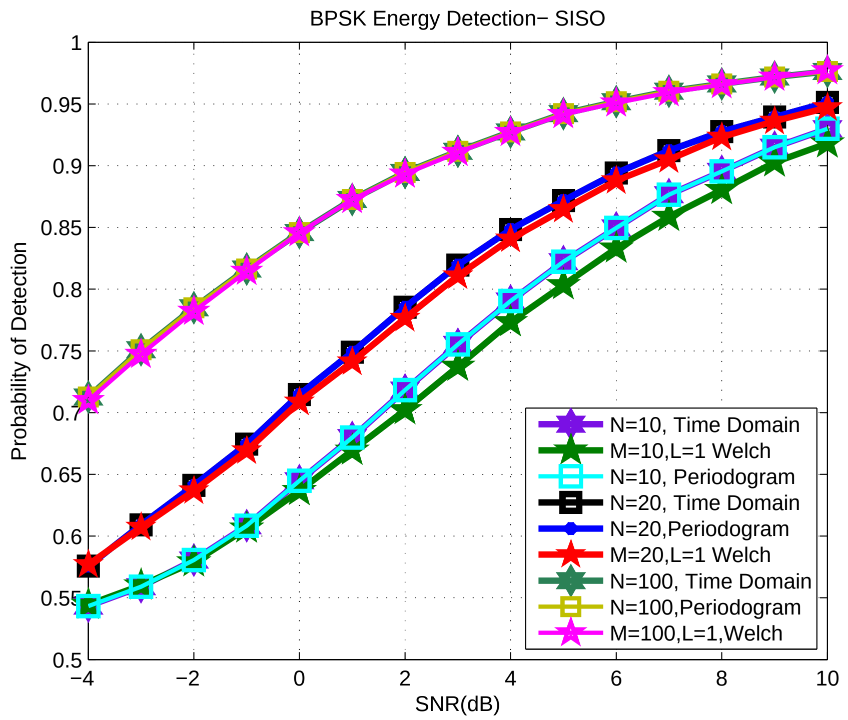

Figure 3.9. P_d vs. SNR for SISO-BPSK using different energy detection algorithms

Figure 3.10, we show the performance of energy detectors using same parameters in terms of Probability of Missing. As seen in the figure energy detectors have almost same performance again and as SNR increases, P_m decreases. Furthermore, if we use more samples, we get less missing probability as expected.

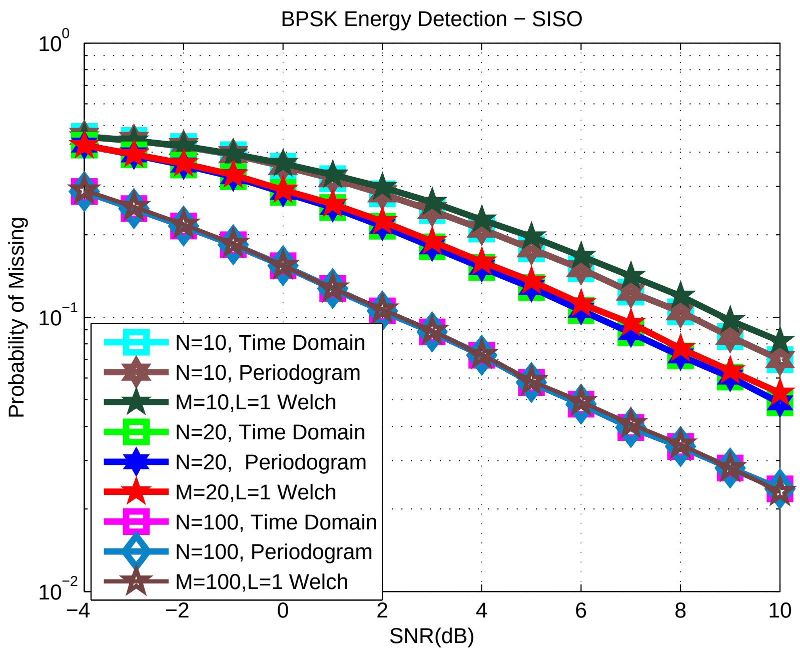

Figure 3.10. P_m vs. SNR for SISO-BPSK using different energy detection algorithms

In the next step of the simulations given in Figure 3.11, we perform a system employing QPSK modulation scheme. First we investigate how probability of detection changes with respect to probability of false alarm. Thus we evaluate our simulations for different false alarm probabilities while SNR is 2 dB and 20 samples are applied into energy detector.

In this figure, it is straightforwardly seen that less false alarm probability leads more detection probability. We then decide to keep our following simulations for $P_f = 0.01$.

In the next step given in Figure 3.12; we again set the thresholds for achieving $P_f = 0.01$. We know from the previous simulations that more samples taken leads to better detection performance, that's why we take 110 samples and evaluate our simulations for different SNR values. It is investigated if any energy detector scheme performs better

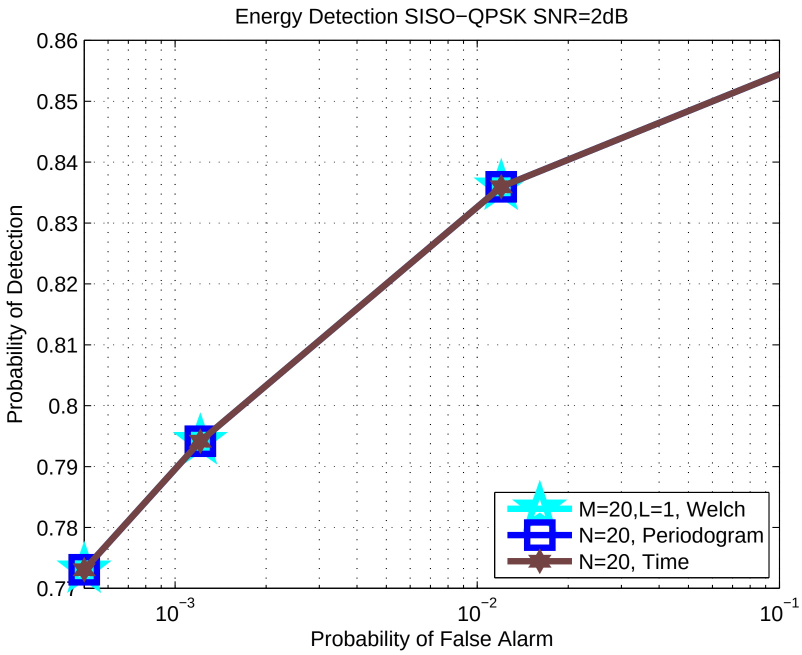

Figure 3.11. Probability of Detection vs. Probability of False alarm - QPSK

than others for QPSK modulation scheme.

The results indicate that all algorithms have same performance again. Again as SNR increases, the detection probability increases.

In Figure 3.13, we investigate the performances of energy detectors with respect to the different amount of samples taken. We again set the thresholds to achieve $P_f = 0.01$.

In this figure, it's straightforward to see that energy detectors perform slightly better for QPSK modulation scheme than BPSK modulation scheme since degree of freedom increases. As it is seen more samples taken will lead a better performance.

In Figure 3.14, we compare the performances in terms of missing probabilities. It is seen that as SNR and sample number increase, the missing probability decreases which will have a lower probability of interference caused by secondary users to primary users.

In the following step, we evaluate the simulations for SISO systems using OFDM. We consider primary user transmitting OFDM signal with 16 subcarriers having 10 OFDM symbols each. In OFDM transmission the symbol passed through IFFT transmission block with the size of subcarrier number, 16, and cyclic prefix is added after parallel to serial transmission. Then OFDM symbols are sent to the receiver via 1-tap,2-tap and 3-tap Rayleigh fading channel. The noise is assumed to be complex gaussian noise and

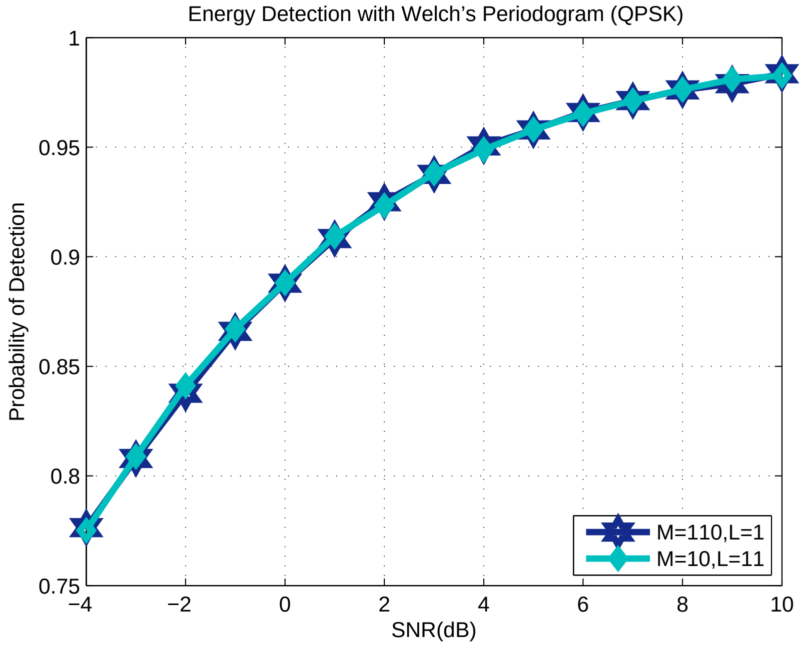

Figure 3.12. P_d of Welch Periodogram using different M,L values -QPSK

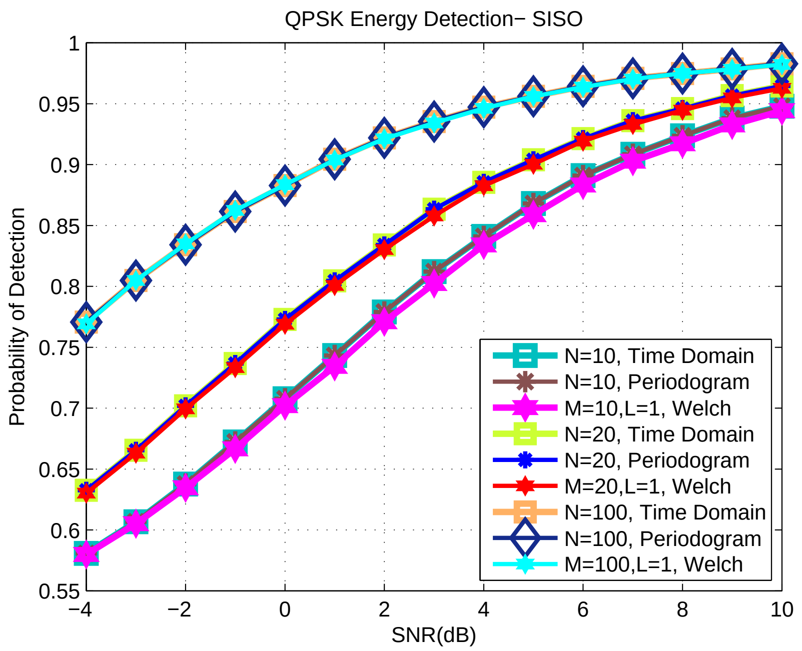

Figure 3.13. P_d vs. SNR for SISO-QPSK using different energy detection algorithms

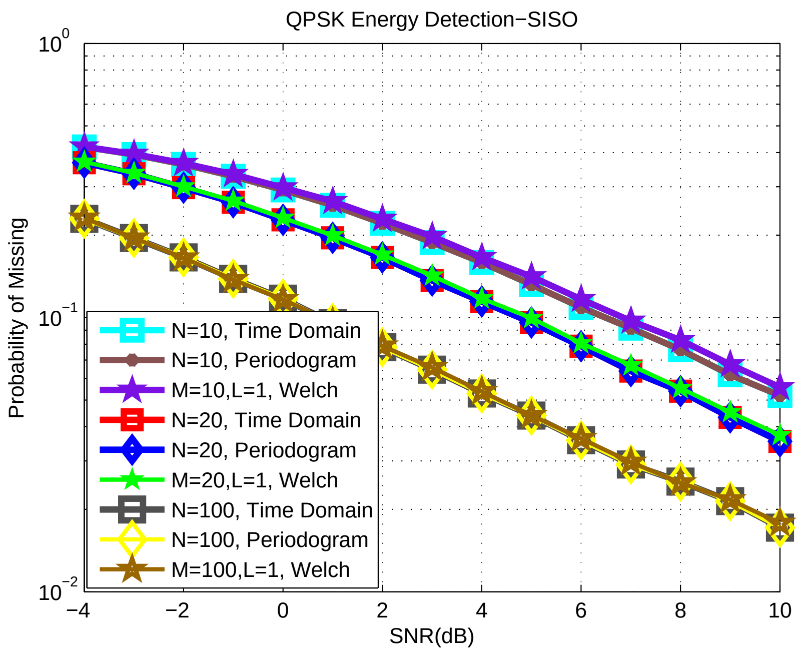

Figure 3.14. P_m vs. SNR for SISO-QPSK using different energy detection algorithms

OFDM scheme employs QPSK modulation. Then cognitive radio receives the data and start sensing each sub-band. The decision statistics are compared with a threshold set to achieve $P_f = 0.01$ false alarm probability. The decision making block marks the sub-band as idle when the decision statistics is less than threshold value. This procedure is repeated for all sub-carriers and subsequently, the number of sub-bands available for use by cognitive radio is determined.

Figure 3.15 shows the dependence of P_m on SNR and tap number of the channel. It is seen that multi-tap channel provides a gain in the performance and degrades the probability of missing.

In the Figure 3.16, Probability of Detection is plotted vs. SNR. Again multi-tap channel provides a gain and increases the probability of detection. It's possible to achieve approximately 85 percent of detection probability at SNR=-10dB via a 3-tap channel while it is very difficult to detect the primary user via a 1-tap channel at that level of SNR.

3.3.2.5. Advantages and Drawbacks

Taking everything into consideration; the drawbacks of the energy detection can be concluded as;

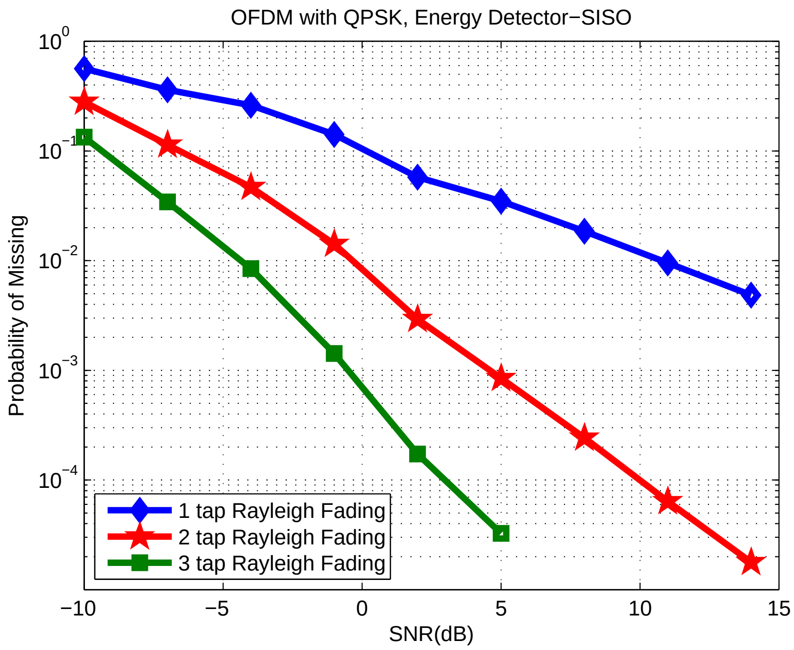

Figure 3.15. P_m vs. SNR for SISO -OFDM with QPSK

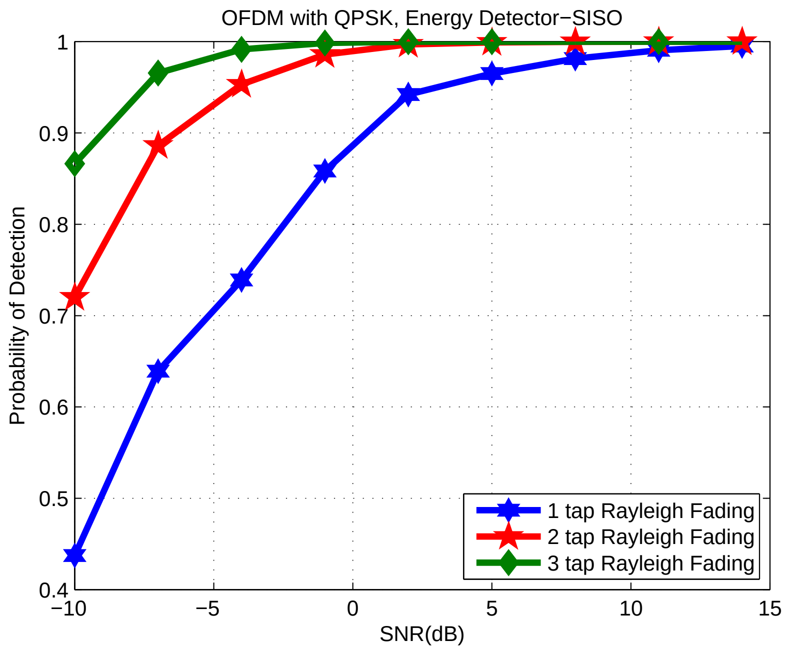

Figure 3.16. P_d vs. SNR for SISO-OFDM with QPSK

- The energy detection performance is very susceptible to changing noise level.
- It requires longer time than matched filter detection.
- This technique can not distinguish modulated signals, noise and interference.
- The detection of spread spectrum signals is not possible with energy detection techniques.

On the other hand, energy detection approach has several advantages that motivate us to study:

- It is more generic (as compared to methods given in this section) as receivers do not need any knowledge on the primary user's signal.
- It's much more simple to implement.
- The signals can be detected at low SNRs, provided the detection interval is adequately long and noise power spectral density is known.

3.3.3. Cyclostationary Feature Detection

Another detection method used in literature is the cyclostationary feature detection which depends on the fact that modulated signals are generally coupled with sine wave carriers, pulse trains, repeating spreading, hopping sequences or cyclic prefixes which result in periodicity and their statistics, mean and autocorrelation, exhibit periodicity in wide sense (Gardner, et al. 2006). This periodicity trend is used for analysing various signal processing tasks such as detection, recognition and estimation of the received signals. Despite having a drawback of high computationally complexity, cyclostationary feature detection performs satisfyingly well under low SNR regimes due to its robustness against unknown level of noise. Another saying; it is not susceptible to noise levels as energy detection.

Free bands in the spectrum is detected due to the following hypotheses testing problem on the received signal x(t) (Gardner, et al. 2006):

$$x(t) = s(t)h + w(t) \tag{3.36}$$

where s(t) is the modulated signal, h is channel coefficient and w(t) is AWGN noise.

- Under H_0 x(t) is not cyclostationary and thus the band is considered free (null hypotheses)
- Under H_1 x(t) is cyclostationary and thus the band is considered congested

A modulated signal $x(t)$ is considered to be a periodic signal or a cyclostationary signal in wide sense if its mean and autocorrelation exhibit periodicity as follows (Tengyi, et al. 2009):

$$m_x(t + T_0) = m_x(t) \tag{3.37}$$

$$R_x(t + T_0, u + T_0) = R_x(t, u) \tag{3.38}$$

where the period of mean and autocorrelation is T_0. If we replace t and u in autocorrelation equation with $t+\tau/2$ and $t-\tau/2$, we can further express (3.46) in Fourier series as;

$$R_x(t + \tau/2, t - \tau/2) = \sum_{\alpha} R_x^{\alpha}(\tau)e^{j2\pi\alpha t} \tag{3.39}$$

where R_x^{α} denotes the *Cyclic Autocorrelation* (CA) function and α denotes the cyclic frequency. Cyclic frequency is assumed to be known in the receiver. The CA can be obtained by

$$R_x^{\alpha}(\tau) = \frac{1}{T}\int_{-\frac{1}{T}}^{\frac{1}{T}} R_x(t + \tau/2, t - \tau/2)e^{-j2\pi\alpha t}dt \tag{3.40}$$

The Fourier transform of the cyclic autocorrelation function is defined as the *Cyclic Spectral Density* (CSD) function, given by

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau) e^{-j2\pi f\tau} d\tau \tag{3.41}$$

Cyclic Spectral Density is also named as *Spectral Correlation Function* (SCF). *SCF* can be measured by the normalized correlation between two spectral components of $x(t)$ at frequencies $(f + \alpha/2)$ and $(f - \alpha/2)$ over an interval of length Δt. Taking those into consideration SCF can be express as follows:

$$S_x^\alpha(f) = \lim_{T\to\infty} [\lim_{\triangle t\to\infty} \frac{1}{\triangle t} \int_{-\triangle t}^{\triangle t} \frac{1}{T} X_T(t, f + \alpha/2) X_T^*(t, f - \alpha/2) dt] \tag{3.42}$$

where finite time Fourier transform is $X_T(t, u) = \int_{t-T/2}^{t+T/2} x(u) e^{-j2\pi u} du$.

Contrary to power spectrum density which is real-valued one dimensional transform; spectral correlation function is a two dimensional transform, in general complex-valued and the parameter α is called cyclic frequency. It obviously can be said that **power spectral density** is a special form of spectral correlation function for $\alpha = 0$.

3.3.3.1. Implementation of Cyclostationary Feature Detection

Generally, the analysis of the stationary random signals is based on autocorrelation function and spectral density. On the other hand; as it is mentioned in (Gardner 1988), cyclostationary signals exhibit correlation function between widely separated spectral components due to the spectral redundancy caused by periodicity. In this section, cyclostationary feature detection is given to implement in spectrum sensing for cognitive radios step by step.

- **Step 1** Determine the points of cyclic frequency, carrier frequency, overlap number, window size and fftsize.

  In our simulations we define some parameters as follows :

$$
\begin{aligned}
\text{n} &= \text{message length} \\
\text{noverlap} &= \text{overlap number} \\
\text{nfft} &= \text{fft size} \\
\text{nwind} &= \text{window size} \\
\text{noverlap} &= \frac{2}{5}n \\
\text{nwind} &= \frac{3}{2}\text{noverlap}
\end{aligned}
$$

- **Step 2** Compute the *Spectral Correlation Function (SCF)* for each frame.

Step 2.1 We first shift the received signal,$x(t)$, by $\alpha/2$ and $-\alpha/2$ in time domain.

$$
\begin{aligned}
x_1(t) &= x(t).e^{-j2\pi\frac{\alpha}{2}t} \\
x_2(t) &= x(t).e^{j2\pi\frac{\alpha}{2}t}
\end{aligned}
\tag{3.43}
$$

Step 2.2 Windowing

Sliding windows are very useful in the analysis of dominant cyclic features. In our simulations we decided to use Hanning window defined as follows:

Probability Mass Function of Hanning window can be written as;

$$
window(n) = \begin{cases} 0.5(1 - cos(\frac{2\pi n}{nwind-1})), & 0 \leq n \leq nwind, \\ 0, & \text{otherwise} \end{cases}
\tag{3.44}
$$

In Figure 3.17. Hanning window function is illustrated in both time domain and frequency domain.

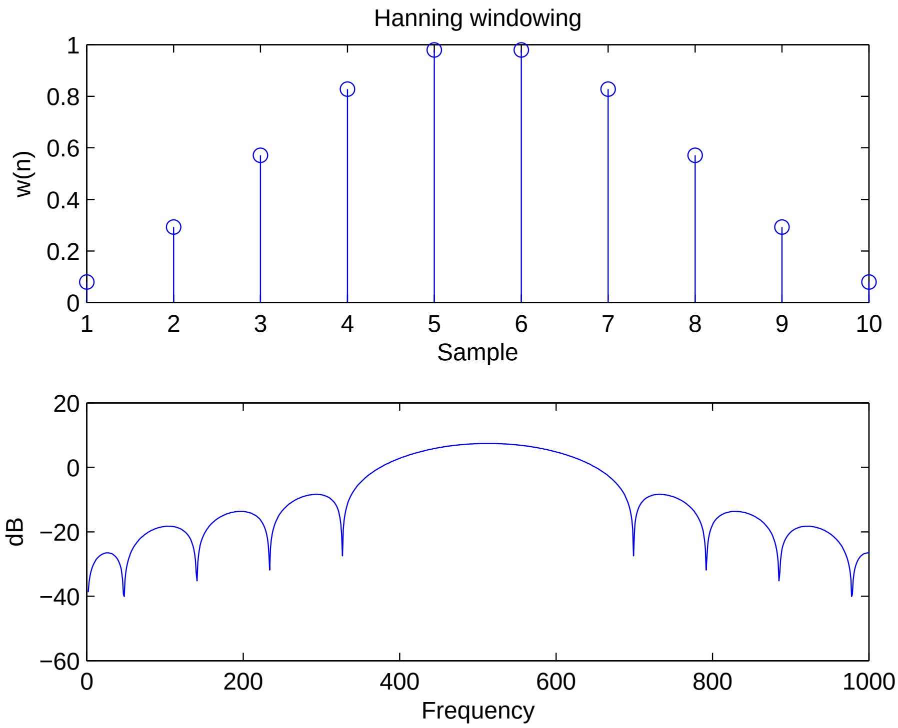

Figure 3.17. Hanning window.

The windowed signals can be shown as follows;

$$\begin{aligned} \text{window} &= \text{hanning}(nwind) \\ x_{1i}(t) &= x_1(t).\text{window} \\ x_{2i}(t) &= x_2(t).\text{window} \end{aligned}$$

Step 2.3 Then we take Fourier Transform of these windowed signals to continue the computations in frequency domain.

$$\begin{aligned} \text{window} &= \text{hanning}(nwind) \\ X_{1i}(f) &= \text{fft}(x_{1i}(t),\text{fftsize}) \\ X_{2i}(f) &= \text{fft}(x_{2i}(t),\text{fftsize}) \end{aligned}$$

Step 2.4 Finally we compute the *Spectral Correlation Function* for each frame, then normalize it by taking its mean.

$$\begin{aligned} S^{\alpha}_{X_i}(f) &= X_{1i}(f).conj(X_{2i}(f)) \\ \widehat{S^{\alpha}_{X}(f)} &= \frac{1}{K.W}\sum_{i=1}^{K} S^{\alpha}_{X_i}(f) \end{aligned}$$

where K is the frame size and $W = \|window\|^2$.

Then the maximum statistics is found and compared with a threshold as follows;

$$I(\alpha) = \max|\widehat{S^{\alpha}_{X}(f)}| \tag{3.45}$$

$$I(\alpha) \lesseqgtr \lambda \tag{3.46}$$

The threshold, λ, can be determined by plotting the histogram of the statistics for various False Alarm probabilities (Kim, et al. 2007).

In order to give an implementation of the spectral correlation function on BPSK modulated signal with a carrier at 125 MHz. bandwidth 20 MHz square wave, sampling frequency 4 GHz; Firstly we compute the cyclic spectrum of the BPSK modulated signal which is corrupted by AWGN noise at SNR 6dB. After that we computed the cyclic spectrum of H_0 hypothesis under which the cognitive radio receives only AWGN noise.

It is straightforwardly seen in Figure 3.18 and Figure 3.19 that the cyclic spectrum of modulated signal and Gaussian noise are very different. Modulated signal has peaks at $f = \mp f_c$ for $\alpha = 0$ and $f = 0$ for $\alpha = \mp 2f_c$. On the other hand noise has maximum components only for $\alpha = 0$ which refers to power spectrum density as mentioned before. Noise does not have any maximum component for $\alpha = \mp 2f_c$. Therefore if we compute the spectral correlation function at $\alpha = \mp 2fc$ we can detect the primary user easily.

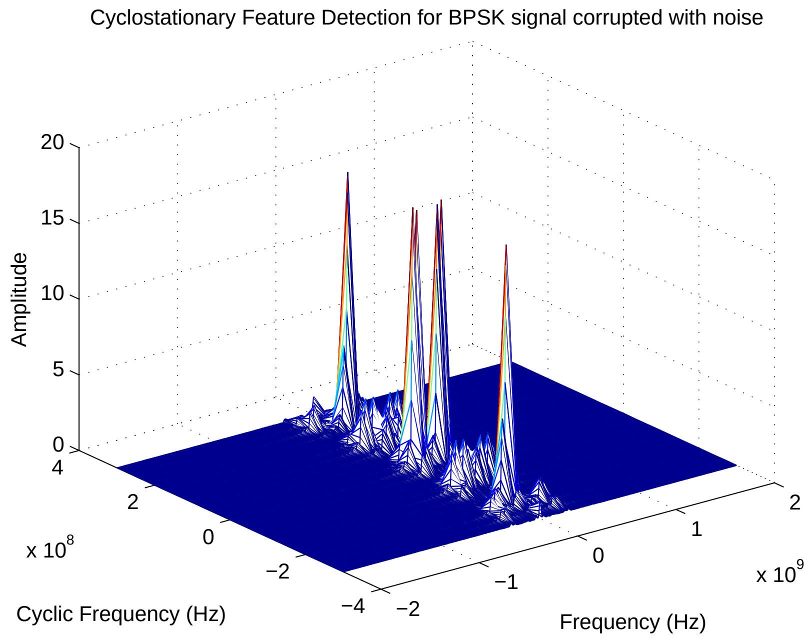

Figure 3.18. Cyclic Spectrum of BPSK signal with AWGN Noise

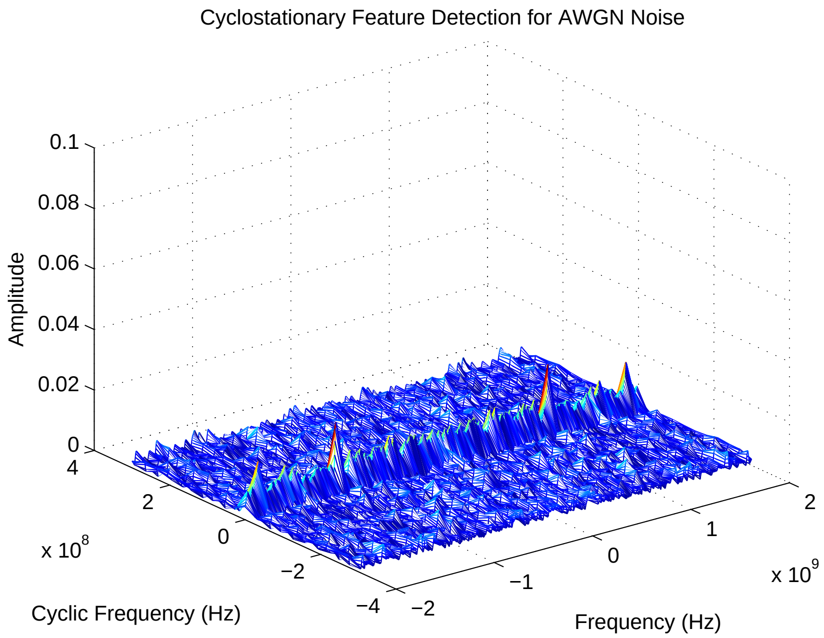

Figure 3.19. Cyclic Spectrum of AWGN Noise

3.3.3.2. Simulation Results of Cyclostationary Feature Detection for SISO Cognitive Radios

We evaluated BPSK signals which are applied to Cyclostationary Feature Detector in both AWGN and Rayleigh Fading Channel. Window is again assumed to be Hanning window. The bandwidth is assumed to be 20MHz, carrier frequency is 125Mhz and sampling frequency is 4GHz. The performance of the system is given in the Figure 3.20. Considering the results,cyclostationary feature detection gives better performance for AWGN channel compared to Fading channel. Cyclostationary detector for fading channel achieves to detect the primary user under 0.01 False alarm probability at very low SNR such as $-15dB$ with approximately 55 percent of detection probability. For -4dB, it achieves the detection probability of over 90 percent for $P_f = 0.01$.

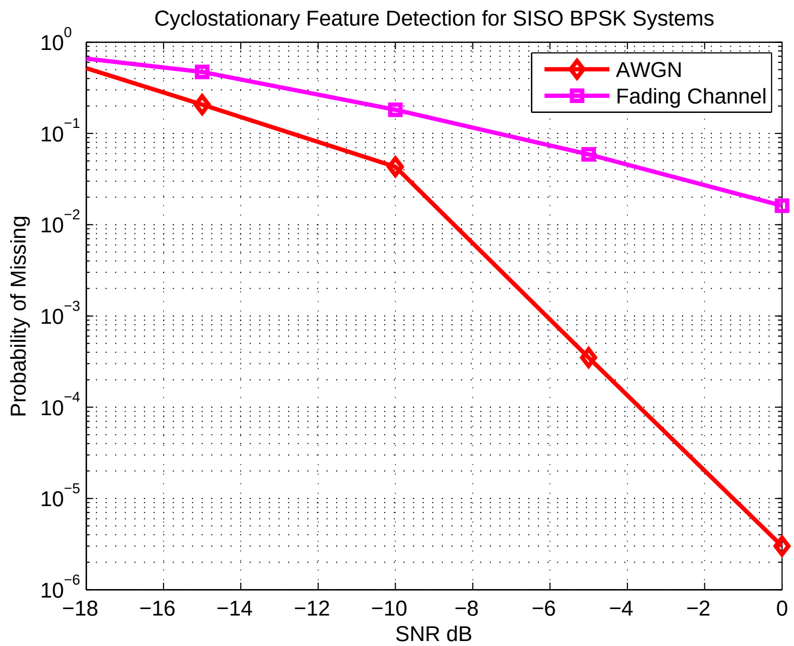

Figure 3.20. P_m under $P_f = 0.01$ for SISO-BPSK

In Figure 3.21, the simulation results are given in terms of Missing probability. Therefore the probability of interference caused by secondary user to primary user is considerably low at lower SNRs compared to energy detector.

In the next step, we evaluated the simulations for both energy detection and cyclostationary feature detection. We used the same signal that we generated in the previous

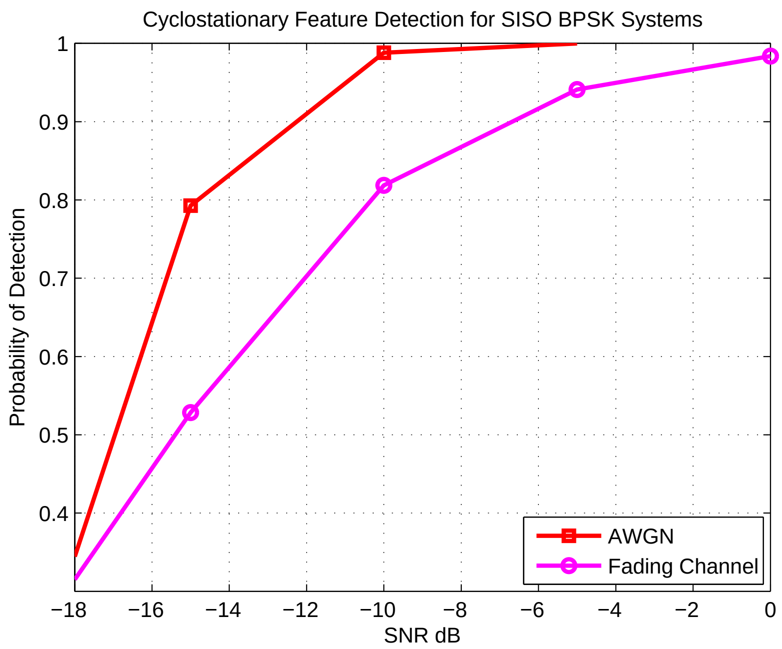

Figure 3.21. P_d under $P_f = 0.01$ for BPSK modulated symbols

step and it is assumed to be transmitted through a fading channel. We again set the thresholds to achieve False Alarm Probability of 1 percent.

As it is seen in Figure 3.22 and 3.23, there is approximately $5dB$ difference between two methods. Cyclostationary Feature Detection achieves 0.80 detection probability at -10dB while energy detector achieves the same detection probability at -5dB.

3.3.3.3. Advantages and Drawbacks

Consequently; cyclostationary feature detection has several advantages such as; (Enserik and Cochran 1995)

- Cyclostationary feature detection is more robust to changing noise level than energy detection.
- Cyclostationary detectors can work in lower SNR compared to energy detection because feature detectors exploit information embedded in the received signal.
- Feature detectors can achieve a huge processing gain compared to energy detectors.

Finally drawbacks of the cyclostationary feature detection can be summarized as;

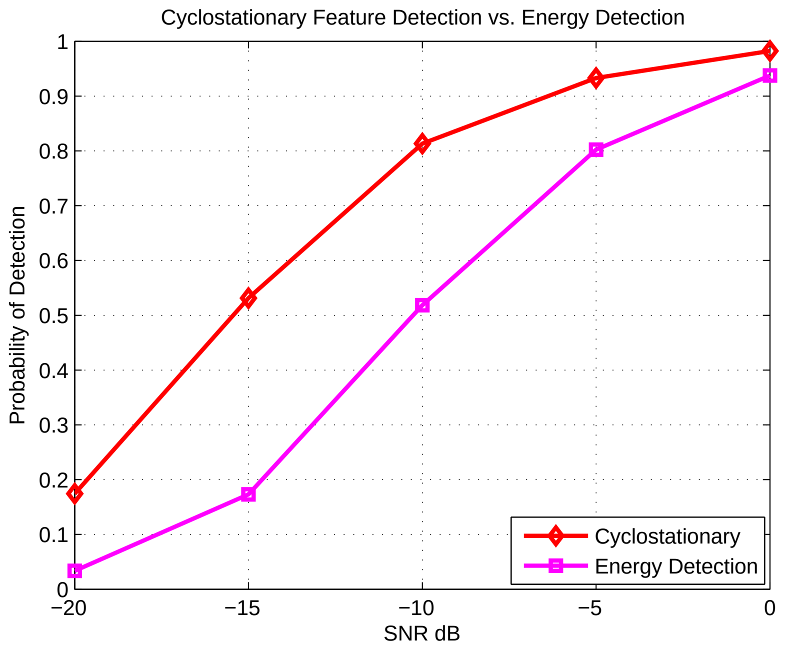

Figure 3.22. Probability of Detection, Cyc. Feature Detection vs. Energy Detection

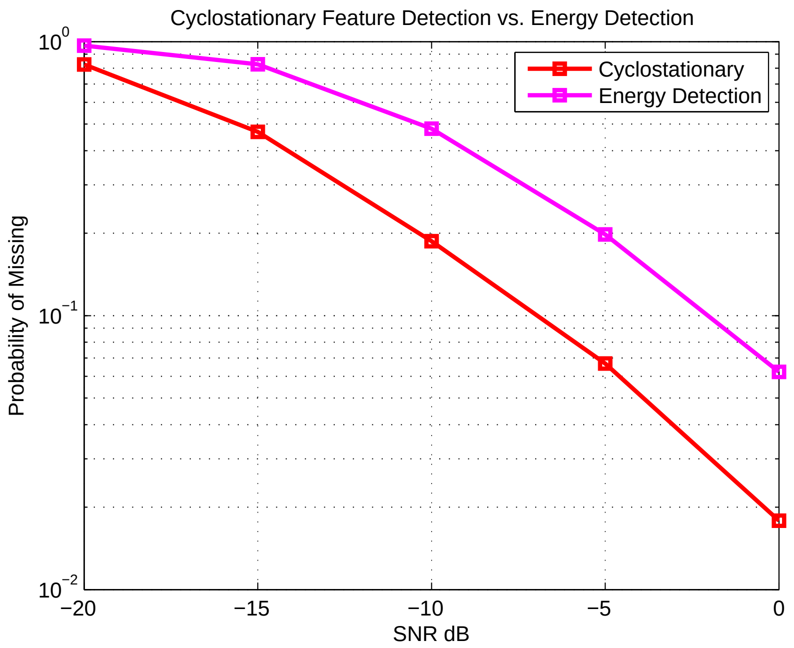

Figure 3.23. Probability of Missing, Cyc. Feature Detection vs. Energy Detection

- It requires prior knowledge on the primary user's signal.
- If the secondary user does not have the knowledge of the cyclic frequencies giving the peak values of CSF, it needs to compute CSF for all possible cyclic frequencies and find the peak value. Therefore the implementation cost extremely increases.
- Cyclostationary feature detection is very complex to implement rather than energy detectors.

3.3.4. Other Methods

There are several spectrum sensing techniques recently proposed in literature. Some research groups have been working on these techniques and publishing papers.

-Higher Order Statistics

Throughout the last decades; the need for a more efficient and flexible communication system has motivated people to implement higher order statistics into signal detection operations because these statistics are very useful in problems where non-gaussian, colored noise have to be considered (Mendel 1991). These statistics, known as cumulants, and their fourier transforms, known as polyspectra, not only exhibit amplitude information but also phase information. Cumulant based detectors can handle colored Gaussian noise measurements. Another saying, higher order statistics are applicable in non-Gaussian or non-linear processes (Nikias and Mendel 1993).

As a result of the boom in the interest of spectral analysis for signal detection; non-parametric and parametric polyspectral methods have become an alternative to address signal detection problem. The former are subject to the same problems that plague nonparametric spectral methods, namely high variances and low resolution. The latter first estimate the parameters of an underlying data-generating model and then use the classes of moving average (MA), autoregressive (AR) or autoregressive moving average (ARMA) processes (Mendel 1991).

For zero mean real random variables, the second-, third-, and fourth-order cumulants are given by

$$cum(X_1, X_2) = E\{X_1X_2\} \quad (3.47)$$

$$cum(X_1, X_2, X_3) = E\{X_1X_2X_3\} \quad (3.48)$$

$$\begin{aligned} cum(X_1, X_2, X_3, X_4) = & E\{X_1X_2X_3X_4\} - E\{X_1X_2\}E\{X_3X_4\} \\ & - E\{X_1X_3\}E\{X_2X_4\} - E\{X_1X_4\}E\{X_2X_3\} \end{aligned} \quad (3.49)$$

The drawbacks of the parametric and non-parametric polyspectral analysis can be concluded as;

- The performance of cumulant-based non-Gaussian processes are dependent on the degree of non-Gaussianity of the waveforms, which indicates that cumulant based algorithms may exhibit poor performance if the sources of interest are near-Gaussian.
- The performance of cumulant-based algorithms depend on data length and SNR results when other source parameters are fixed.
- Its complicated to implement into signal detection systems.

-Waveform Based Sensing

As it is mentioned before; the energy detection method is very prone to false detections since its susceptible to changing noise levels. In (Arslan and Yücek 2007) and (Yücek and Arslan 2009); a new approach named waveform based sensing is proposed. This method is only applicable to systems with known signal patterns. In (Yücek and Arslan 2009), it is underlined that waveform-based sensing outperforms energy detectors in reliability and convergence time. It is also shown that the performance of the sensing algorithms increases as the length of the known signal pattern increases

In this approach, it is assumed that the known time-domain signal pattern contains N_B signal samples. Consider the following waveform sensing metric:

$$S = Re[\sum_{n=1}^{N_B} x(n)s^*(n)] \quad (3.50)$$

when the null hypothesis is considered (signal is absent) the sensing metric is;

$$S = S_0 = Re[\sum_{n=1}^{N_B} w(n)s^*(n)] \tag{3.51}$$

when the primary signal is present, the sensing metric is;

$$S = S_1 = \sum_{n=1}^{N_B} |s(n)|^2 + Re[\sum_{n=1}^{N_B} w(n)s^*(n)] \tag{3.52}$$

The sensing metric can be approximated as a Gaussian random variable when NB is large[8]. In [8], sensing error floor for waveform-based sensing to compare the sensing metric with is computed as;

$$SEF = Q(\sqrt{N_B}\frac{\sqrt{SNR}}{\sqrt{(\alpha-1)SNR + (1/2)} + \sqrt{1/2}}) \tag{3.53}$$

where

$$\alpha = E\{|s(n)|^4\}/[E\{|s(n)|^2\}]^2 \tag{3.54}$$

Simulation results in (Yücek and Arslan 2009) show the advantage of waveform based sensing as;

- Waveform-based sensing can achieve good performance even at low SNR as long as N_B is sufficiently large.

Finally the drawback of waveform sensing method can be concluded as;

- It requires perfect knowledge on the primary user's signal.
- Computing error floor for waveform-based sensing requires the knowledge of noise and detected powers (Arslan and Yücek 2007). The noise power can be estimated

but it is very difficult to estimate the signal power as it changes depending on on-going transmission characteristics and the distance between the cognitive radio and the user.

CHAPTER 4

MULTIPLE ANTENNA METHODS FOR SPECTRUM SENSING

Multiple Antenna techniques are known in reference (Haykin and Moher 2004) in the context of MIMO communication systems and have been used to improve the system performance in terms of bit error rates. However, we aim in this chapter to use the processing in conjunction with energy detectors and cyclostationary feature detectors in order to improve primary user detection performance in a cognitive radio using both single carrier and multi carrier modulation techniques.

Hidden Primary user problem defined in (Fullmer and Garcia-Luna-Aceves 1997) is one of the most important challenges encountered during spectrum sensing in cognitive radio based systems due to the multipath and shadowing effects of the communication channels. If the secondary user can not sense primary user due to an obstacle or deep fade effect, then probability of interference to primary user increases, therefore hidden primary user problem must be mitigated. To reduce the multipath and shadowing effects of the wireless channels, multiple antenna techniques provide promising solutions as to improve BER performances. When we search the literature for multiple antenna techniques in cognitive radio based spectrum sensing systems, we get various techniques employing one or several detectors only considered at receiver side. In (Digham, et al. 2003),(Digham, et al. 2007), (Kuppusamy and Mahapatra 2008), (Po, et al. 2008), multiple antenna techniques for energy detectors are studied. In (Digham, et al. 2003) and (Ashish and Linnartz 2007); the multiple antenna methods proposed require only one energy detector, in meanwhile authors of (Digham, et al. 2007), (Kuppusamy and Mahapatra 2008) and (Po, et al. 2008) propose methods which require one energy detector for each antenna. (Kuppusamy and Mahapatra 2008) and (Po, et al. 2008) also focus on detecting OFDM signals with multiple antennas.

This chapter consists of four sections. First section discusses the multiple antenna techniques in literature for energy detection based systems. Then our proposed multiple antenna technique for energy detection based systems is given with its analytical derivations. We also give the simulation results for the systems based on energy detection. In the following section, multiple antenna techniques in literature for cyclostationary feature based systems are investigated. Then our proposed method for the systems based on cyclostationary feature detection is explained in detail. Simulation results for the systems based on cyclostationary feature detection are given in the last section.

4.1. Multiple Antennas Methods for Energy Detection Based Systems

4.1.1. Selection Combining

In Ref.(Digham, et al. 2003) and (Ashish and Linnartz 2007), selection combining method is applied to an energy detection based spectrum sensing system. In this method cognitive radio is assumed to have the knowledge of channel state information and thus chooses the branch with the highest channel gain and receives the data from that antenna. Then simply the received data is applied to an Energy Detector.

One of the most promising advantages of this method can be the fact that the threshold does not change as the noise level remains constant. As cognitive radio chooses one of the branches, it acts like a SISO system, that's why the threshold remains exactly as same as it is set for SISO systems.

$$x_j(t) = s(t)h_j + w(t)\text{where j=1,2,....}N_r \tag{4.1}$$

where N_r is the number of antenna at receiver side, $x_j(t)$ is the received signal at jth antenna, h_j is the channel gain for jth antenna which is assumed to be Rayleigh fading channel.

In this method, cognitive radio is assumed to know the channel state information and thus chooses the branch with the highest gain.

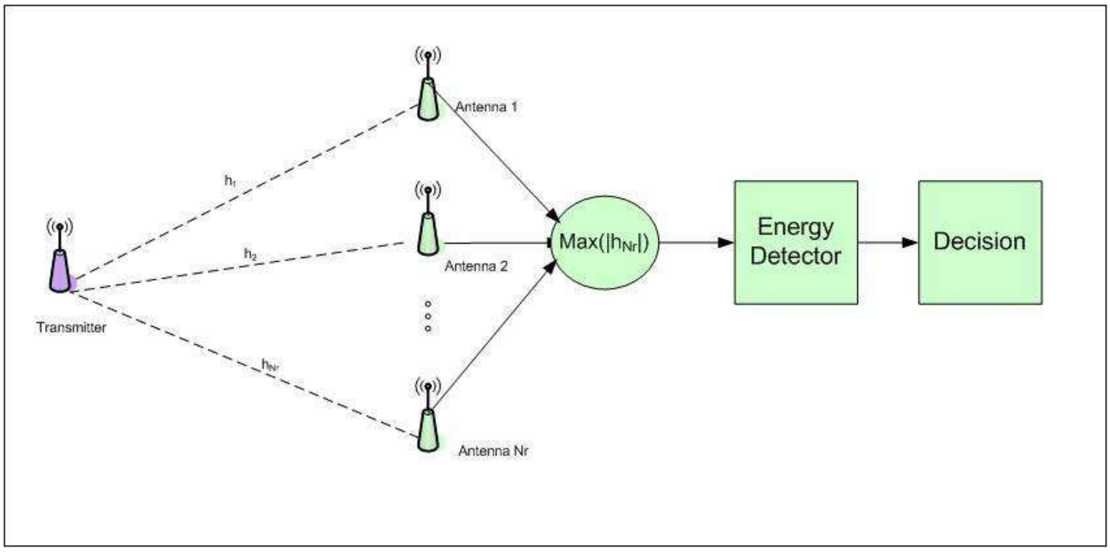

Figure 4.1. Selection Combining with Channel State Information

$$j_{max} = \max_{j}(|h_j|) \tag{4.2}$$

Then the received signal at j_{max}th antenna is applied to the energy detector.

$$\Xi_{SC} = \sum_{k=1}^{N} |x_{j_{max}}(k)|^2 \tag{4.3}$$

where N is the symbol length to be sensed by cognitive radio.

Likewise in SISO systems, we evaluate the simulations for a threshold set to achieve a specific certain level of false alarm. Under H_0 hypothesis,

$$\Xi_{SC} = \sum_{k=1}^{N} |w_{j_{max}}(k)|^2 \tag{4.4}$$

where $w_{j_{max}}(k)$ is additive white gaussian noise with zero mean and σ_w^2 variance as it is in SISO systems. Therefore Ξ_{SC} has a central chi square distribution with N degree of freedom. Considering the operations in Chapter 3, it is straightforward to see that False alarm equation can be written as follows:

$$P_{FA(SC)} = \frac{\Gamma(N/2, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N/2)} \tag{4.5}$$

For the case of a *complex gaussian noise* $w_{j_{max}}(k)$ has totally $2N$ elements. That's why the distribution of Ξ_{SC} is a chi square with $2N$ degree of freedom. Hence False alarm equation can be written as

$$P_{FA_{(SC)}^{c}} = \frac{\Gamma(N, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N)} \tag{4.6}$$

Although this method seems very promising as the noise level does not change, it requires perfect channel knowledge as a result of choosing the branch with the highest channel gain, that's why it can not be an optimum solution.

4.1.2. Maximum Ratio Combining

This method focuses on Maximum Ratio Combining (MRC) proposed in (Ashish and Linnartz 2007) to increase spectral efficiency for cognitive radios. It depends on the known maximum ratio combining introduced in (Haykin and Moher 2004) in order to increase BER. In this method, cognitive radio is assumed to have perfect channel state information. It receives data from each antenna and multiplies them with the conjugate of each channel gain. Then it sums all the multiplied data and applies it to an energy detector.

The received data at each antenna is multiplied with the conjugate of the channel gain and summation of all is applied to an energy detector as follows:

$$\Xi_{MRC} = \sum_{k=1}^{N} |(\sum_{j=1}^{N_r} x_j(k).h_j^*)|^2 \tag{4.7}$$

The MRC provides some gain compared to SISO systems but it's difficult to implement this method in spectrum sensing systems as it requires perfect channel knowledge.

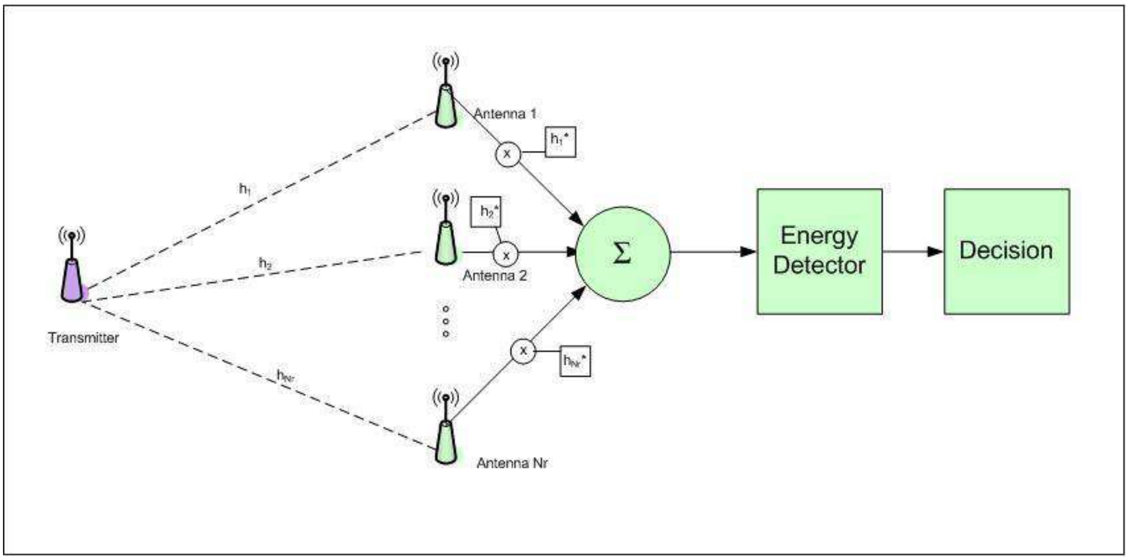

Figure 4.2. Maximum Ratio Combining

Contrary to the *Selection Combining* method, threshold changes, because the energy levels changes as we multiply the received data with the conjugate of the channel gains and then combine them. As an intrinsic result of this threshold changes due to the channel. Thus to find an accurate threshold to achieve a certain level of false alarm, the perfect channel knowledge is required.

As we multiply the received signal under H_0 hypothesis with the conjugate of the channel gain, the noise variance changes with the square of the each channel gain.

$$f_{\Xi_{MRC}}(\xi) = \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2})^N \, 2^{N/2}\Gamma(\frac{1}{2}N)} \xi^{(N/2)-1} e^{-\xi/(2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2})}, \text{for } \xi \geq 0 \quad (4.8)$$

Then we compare the statistics that we get at the output as follows:

$$\Xi_{MRC} > \lambda, \qquad \text{under } H_0 \text{ hypothesis} \qquad (4.9)$$

Under H_0 hypothesis, we derive the false alarm probability as follows:

$$P_{FA(MRC)} = Pr\{\Xi_{MRC} > \lambda | H_0\} = \int_{\lambda}^{\infty} f_{\Xi_{MRC}}(\xi) d\xi \quad (4.10)$$

$$= \int_{\lambda}^{\infty} \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2}^{N} 2^{N/2}\Gamma(\frac{1}{2}N)} \xi^{(N/2)-1} e^{-\xi/(2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2}) d\xi$$

$$= \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2}^{N} 2^{N/2}\Gamma(\frac{1}{2}N)} \int_{\lambda}^{\infty} \xi^{(N/2)-1} e^{-\xi/(2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2}) d\xi$$

To solve this equation, we apply some variable conversions as follows ;

$$t = \xi/(2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2) \quad (4.11)$$

$$\xi = (2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2)t \quad (4.12)$$

$$d\xi = (2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2)dt \quad (4.13)$$

$$\lambda_1 = \frac{\lambda}{(2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2)} \quad (4.14)$$

Then we rewrite the equation (4.10) as follows ;

$$P_{FA(MRC)} = \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2}^{N} 2^{N/2}\Gamma(\frac{1}{2}N)} ... \quad (4.15)$$

$$... \int_{\lambda_1}^{\infty} (2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2)^{(N/2)-1} \quad t^{(N/2)-1} e^{-t} 2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2 dt$$

$$= \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2}^{N} 2^{N/2}\Gamma(\frac{1}{2}N)} ...$$

$$... \int_{\frac{\lambda}{2\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2)}}^{\infty} 2^{N/2} \sqrt{(\sum_{j=1}^{N_r}(h_j^{*2})\sigma_w^2))}^{N} \quad t^{(N/2)-1} e^{-t} dt$$

We know that incomplete gamma function; $\Gamma(\varsigma, a) = \int_{\varsigma}^{\infty} t^{a-1}e^{-t}dt$. So we can rewrite last equation as follows;

$$P_{FA(MRC)} \quad = \quad \frac{\Gamma(N/2, \frac{\lambda}{2\sum_{j=1}^{Nr}(h_j^{*2})\sigma_w^2)})}{\Gamma(N/2)} \tag{4.16}$$

For the case of a *complex gaussian noise* ;

$$P_{FA}{}^{c}_{(MRC)} = \frac{\Gamma(N, \frac{\lambda}{2\sum_{j=1}^{Nr}(h_j^*)^2\sigma_w^2})}{\Gamma(N)} \tag{4.17}$$

To sum up then, MRC can provide a somewhat gain but likewise *Selection Combining Method*,it is not an optimal solution as it requires perfect channel knowledge and also we need to change the thresholds with respect to the channel.

4.1.3. Equal Gain Combining

One of the other multiple antenna techniques proposed for spectrum sensing systems is *Equal Gain Combining*. In the literature we encounter a few methods proposed under the name of *Equal Gain Combining* but some of them are quiet different from the others. In some papers the method Square Law Combining method is studied under the name of Equal Gain Combining. We'll study that method as *Square Law Combining* as in (Digham, et al. 2007).

In *Equal Gain Combining* proposed in (Digham, et al. 2003), likewise it is used in (Haykin and Moher 2004) in order to improve BER performance, normalized data which is multiplied with the conjugate of the channel gain and then divided by its absolute value is applied to only one energy detector.

The equations for EGC can be drived as follows;

$$\Xi_{EGC} = \sum_{k=1}^{N} |(\sum_{j=1}^{N_r} x_j(k).h_j^*/|h_j|)|^2 \tag{4.18}$$

Likewise MRC, in EGC method, channel gain is required to be known perfectly by the cognitive radio because the threshold depends on the channel.

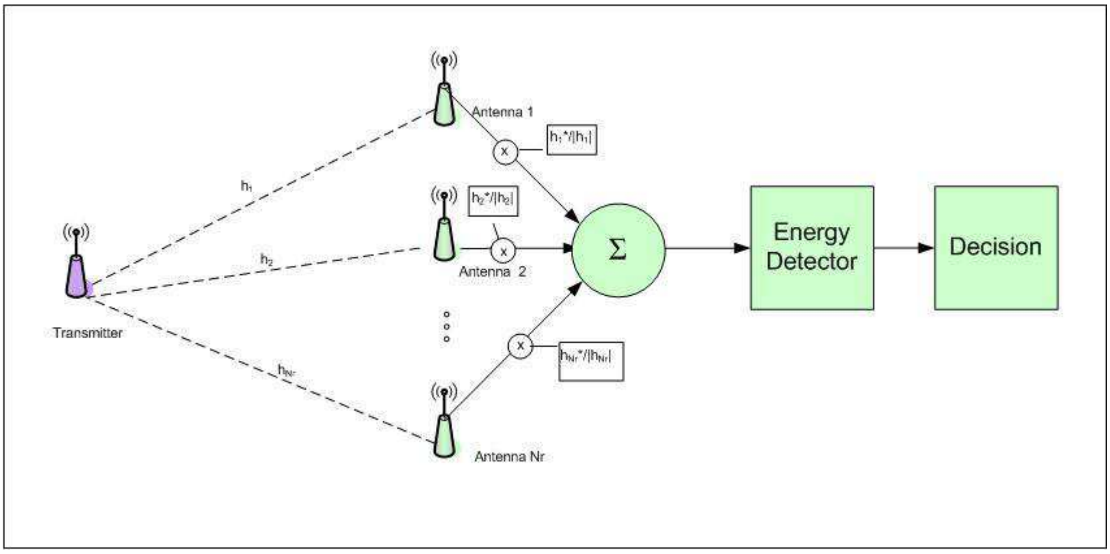

Figure 4.3. Equal Gain Combining

Under H_0 hypothesis the received signal is AWGN but differently from SISO system the variance of the noise becomes $\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2$ and is dependent on the wireless channel . Hence the distribution of the output of energy detector under H_0 hypothesis can be derived as follows:

$$f_{\Xi_{EGC}}(\xi) = \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)}^N 2^{N/2}\Gamma(\frac{1}{2}N)} \xi^{(N/2)-1} e^{-\xi/(2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2} \tag{4.19}$$

$$\Xi_{EGC} > \lambda, \qquad \text{under } H_0 \text{ hypothesis} \tag{4.20}$$

$$\begin{aligned} P_{FA(EGC)} &= Pr\{\Xi_{EGC} > \lambda | H_0\} \\ &= \int_\lambda^\infty f_{\Xi_{EGC}}(\xi) d\xi \end{aligned} \tag{4.21}$$

$$= \int_\lambda^\infty \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)}^N 2^{N/2}\Gamma(\frac{1}{2}N)} \xi^{(N/2)-1} e^{-\xi/(2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2}) d\xi$$

$$= \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)}^N 2^{N/2}\Gamma(\frac{1}{2}N)} \int_\lambda^\infty \xi^{(N/2)-1} e^{-\xi/(2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2}) d\xi$$

To solve this equation, we apply some variable conversions as follows ;

$$\xi/(2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2) = t \tag{4.22}$$

$$\xi = (2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)t \tag{4.23}$$

$$d\xi = (2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)dt \tag{4.24}$$

$$\lambda_1 = \frac{\lambda}{(2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)} \tag{4.25}$$

Then we rewrite the equation (4.21) as follows ;

$$P_{FA(EGC)} = \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)}^N 2^{N/2}\Gamma(\frac{1}{2}N)} ... \tag{4.26}$$

$$... \int_{\lambda_1}^{\infty} (2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)^{(N/2)-1} \quad t^{(N/2)-1}e^{-t}2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2 dt$$

$$P_{FA(EGC)} = \frac{1}{(\sqrt{\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)}^N 2^{N/2}\Gamma(\frac{1}{2}N)} ... \tag{4.27}$$

$$... \int_{\frac{\lambda}{2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)}}^{\infty} 2^{N/2}\sqrt{(\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2))}^N \; t^{(N/2)-1}e^{-t}dt$$

We know that incomplete gamma function; $\Gamma(\varsigma, a) = \int_{\varsigma}^{\infty} t^{a-1}e^{-t}dt$. So we can rewrite last equation as follows;

$$P_{FA(EGC)} = \frac{\Gamma(N/2, \frac{\lambda}{2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)})}{\Gamma(N/2)} \tag{4.28}$$

For the case of a *complex gaussian noise* ;

$$P_{FA^c_{(EGC)}} = \frac{\Gamma(N, \frac{\lambda}{2\sum_{j=1}^{N_r}(h_j^{*2}/|h_j|^2)\sigma_w^2)})}{\Gamma(N)} \tag{4.29}$$

As a conclusion, EGC is expected to provide diversity gain in detection performance but likewise *Maximum Ratio Combining*, threshold changes with respect to channel gain that's why it can not be an optimal solution as it requires perfect channel knowledge.

4.1.4. Square-Law Combining

In this scheme proposed in (Digham, et al. 2007), the outputs of the square-law devices (energy detectors) are combined and compared with a threshold to set a certain level of False Alarm probability.

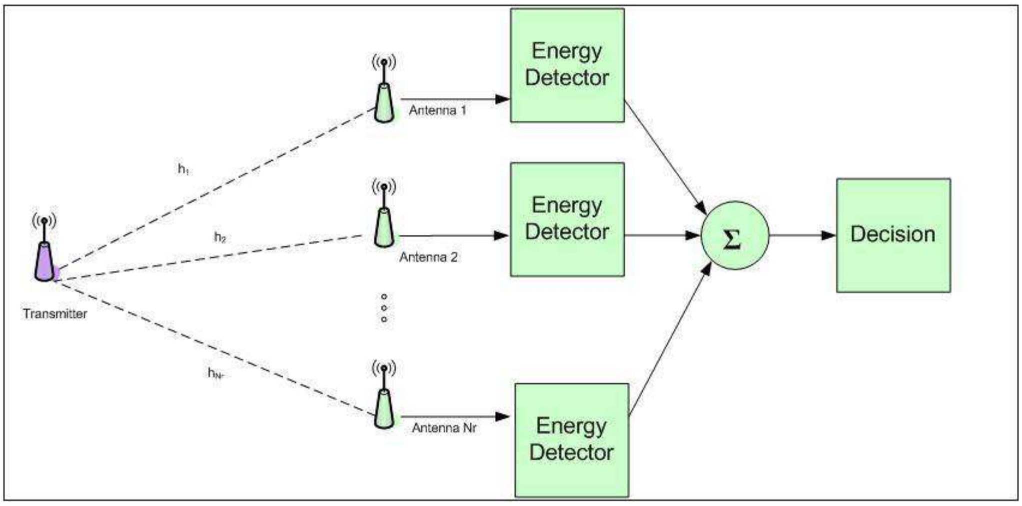

Figure 4.4. Square Law Combining

The equations for *Square Law Combining* can be derived as follows;

$$\Xi_j = \sum_{k=1}^{N} |x_j(k)|^2 \tag{4.30}$$

$$\Xi_{(SLC)} = \sum_{j=1}^{N_r} \Xi_j \tag{4.31}$$

It's straightforwardly seen that an energy detector must be allocated to each antenna.

As we mentioned in SISO systems , under H_0 hypothesis, the received signal *x(k)* is AWGN of which variance is assumed to be σ_w^2 . In this scheme we sum the statistics coming from multiple detectors and compare with a threshold. If the symbol length is assumed to be N and number of antenna used is N_r then we get $N.N_r$ Gaussian noise with zero mean and σ_w^2 variance at the output to compare with a threshold. Hence the distribution of the output becomes central chi square distribution with $N.N_r$ degree of freedom which can be formulated as follows:

$$f_{\Xi_{SLC}}(\xi) = \frac{1}{\sigma_w^{N_r.N} 2^{N_r.N/2} \Gamma(\frac{1}{2}N_r.N)} \xi^{(N_r.N/2)-1} e^{-\xi/2\sigma_w^2}, \text{for } \xi \geq 0 \tag{4.32}$$

To find the probability of False Alarm, we set a threshold and integrate the pdf of $f_{\Xi_{SLC}}(\xi)$ under H_0 hypothesis from threshold to infinity as follows:

$$\Xi_{SLC} > \lambda, \qquad \text{under } H_0 \text{ hypothesis} \tag{4.33}$$

$$\begin{aligned} P_{FA(SLC)} &= Pr\{\Xi_{SLC} > \lambda | H_0\} \qquad (4.34) \\ &= \int_\lambda^\infty f_{\Xi_{SLC}}(\xi) d\xi \\ &= \int_\lambda^\infty \frac{1}{\sigma_w^{N_r.N} 2^{N_r.N/2} \Gamma(\frac{1}{2}N_r.N)} \xi^{(N_r.N/2)-1} e^{-\xi/2\sigma_w^2} d\xi \\ &= \frac{1}{\sigma_w^{N_r.N} 2^{N_r.N/2} \Gamma(\frac{1}{2}N_r.N)} \int_\lambda^\infty \xi^{(N_r.N/2)-1} e^{-\xi/2\sigma_w^2} d\xi \end{aligned}$$

To solve this equation, we apply some variable conversions as follows ;

$$\xi/2\sigma_w^2 = t \tag{4.35}$$

$$\xi = 2\sigma_w^2 t \tag{4.36}$$

$$d\xi = 2\sigma_w^2 dt \tag{4.37}$$

$$\lambda_1 = \frac{\lambda}{2\sigma_w^2} \tag{4.38}$$

Then we rewrite the equation as follows ;

$$
\begin{aligned}
P_{FA(SLC)} &= \frac{1}{\sigma_w^{N_rN}2^{N_rN/2}\Gamma(\frac{N_rN}{2})}\int_{\lambda_1}^{\infty}(2\sigma_w^2)^{(N_rN/2)-1}t^{(N_rN/2)-1}e^{-t}2\sigma_w^2 dt \quad (4.39)\\
&= \frac{1}{\sigma_w^{N_rN}2^{N_rN/2}\Gamma(\frac{1}{2}N_rN)}\int_{\frac{\lambda}{2\sigma_w^2}}^{\infty}2^{N_rN/2}\sigma_w^{N_rN}t^{(N_r.N/2)-1}e^{-t}dt
\end{aligned}
$$

We know that incomplete gamma function; $\Gamma(\varsigma, a) = \int_{\varsigma}^{\infty} t^{a-1}e^{-t}dt$. So we can rewrite last equation as follows;

$$P_{FA(SLC)} = \frac{\Gamma(N_r.N/2, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N_r.N/2)} \quad (4.40)$$

Using this formula, we can find thresholds to set to achieve various false alarm probabilities for SLC.

For the case of $w(k)$ is a complex gaussian, the $P_{FA(SLC)}$ can be derived as follows with the operations mentioned in Chapter 3.

$$P_{FA_{(SLC)}^{c}} = \frac{\Gamma(N_r.N, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N_r.N)} \quad (4.41)$$

Underlining the advantages of this scheme, it does not require channel knowledge like SC, MRC and EGC do and also it is very easy to find the threshold set to achieve a certain level of False Alarm. On the other hand, it requires several energy detectors which refer to an increase in the implementation costs.

4.1.5. Square Law Selection

In Square Law Selection (SLS) method proposed in (Digham, et al. 2007); each energy detector gets data from antennas, then cognitive radio collects and chooses the branch having the maximum energy, then it compares the statistics with a threshold and makes a final decision whether primary user exists or not.

The equations for *Square Law Selection* technique can be derived as follows;

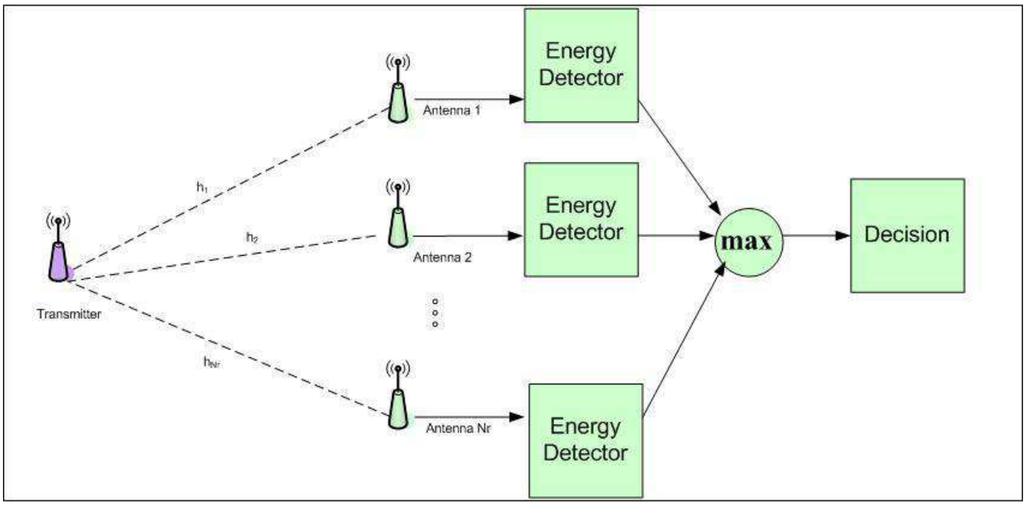

Figure 4.5. Square Law Selection

$$\Xi_j = \sum_{k=1}^{N} |x_j(k)|^2 \tag{4.42}$$

where $j = 1...N_r$

$$\Xi_{SLS} = \max_j \Xi_j \tag{4.43}$$

where k=1,2,.....N and j=1,2,.....N_r.

As an intrinsic result of selecting a maximum statistics in a set having N_r elements, the false alarm equation changes as follows although the mean and the variance remains constant like SISO systems.

False Alarm Probability for this scheme is given in (Digham, et al. 2007) as follows;

$$P_{FA(SLS)} = 1 - \left[1 - \frac{\Gamma(N/2, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N/2]}\right]^{N_r} \tag{4.44}$$

For the case of $w(k)$ is a complex gaussian, the $P_{FA(SLS)}$ can be derived as follows:

$$P_{FA^{c}_{(SLS)}} = 1 - \left[1 - \frac{\Gamma(N, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N)}\right]^{N_r} \tag{4.45}$$

Therefore it's very easy to set a threshold to achieve a certain level of False Alarm. To sum up then, it's very promising as threshold does not depend on channel information and also it's very easy to find the threshold. On the other hand, it requires several energy detectors which refer to an increase in the implementation costs like SLC.

4.1.6. Collaborative Technique - Hard Decision with Logic tables

In order to improve the performance of the spectrum sensing systems, some researchers proposed the collaboration among the relays. In this scheme, all antennas have a detector which makes a decision individually then cognitive radio combines these decisions according to a logical table and makes a final decision.

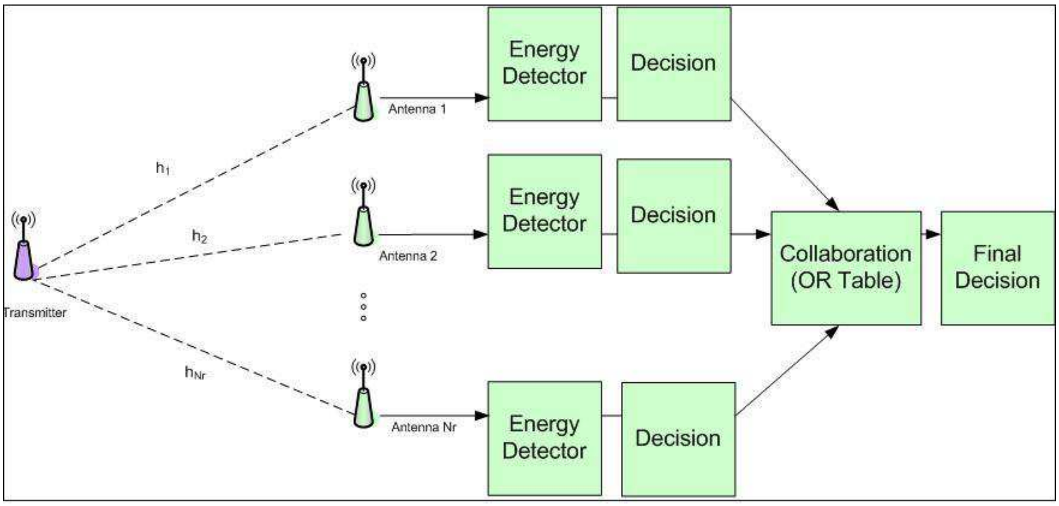

Figure 4.6. Collaborative Spectrum Sensing

Each detector in cognitive radio make their own decision and then send information whether they decide that band is occupied or idle. (Ghasemi and Sousa 2005) and (Visser, et al. 2008) focuses on Collaboration Technique using OR table.

According to OR table given in Table 4.1, cognitive radio decides in favour of H_1 hypothesis if one of the detectors decides the primary user exists. Hence missing

Table 4.1. OR Logical Table

Detector1	Detector2	Detector3	Detector4	Final Decision
H_0	H_0	H_0	H_0	H_0
H_0	H_0	H_0	H_1	H_1
H_0	H_0	H_1	H_0	H_1
H_0	H_0	H_1	H_1	H_1
H_0	H_1	H_0	H_0	H_1
H_0	H_1	H_0	H_1	H_1
H_0	H_1	H_1	H_0	H_1
H_0	H_1	H_1	H_1	H_1
H_1	H_0	H_0	H_0	H_1
H_1	H_0	H_0	H_1	H_1
H_1	H_0	H_1	H_0	H_1
H_1	H_0	H_1	H_1	H_1
H_1	H_1	H_1	H_0	H_1
H_1	H_0	H_1	H_1	H_1

probability is kept as low as possible. Other logical tables also can be applied to the collaborative sensing systems. In (Lee, et al. 2008) a sample collaborative sensing with a different logical table. With this method cognitive radio decides that primary user exists if number of H_1 hypothesis decided in energy detectors is more than the number of H_0 hypothesis decided. On the other hand it's straightforwardly seen that it gives worse performance than OR table because with this table cognitive radio decides in favour of H_0 hypothesis even if some of the detectors decides in favour of H_1 hypothesis. That's why it's expected to miss primary user's signal more probably than OR table.

Since we know this table does not provide gain as much as OR table, we won't give simulation results of the evaluation of this system.

4.1.7. The Proposed Technique

We have studied some multiple antenna techniques in literature so far. We know that sensing with multiple antenna techniques having only one energy detector such as SC, MRC and EGC require perfect channel knowledge. For MRC and EGC it is also very difficult to set a threshold to achieve a certain *False Alarm Probability* as the noise level changes due to the channel. When we look at the sensing techniques with multiple antennas requiring an energy detector for each antenna such as SLC, SLS and Collaboration Techniques, we see that it's not very difficult to obtain a global formula to set the threshold but on the other hand; they require several detectors which will of course increase the implementation costs.

Considering the drawbacks of the methods proposed in literature, we investigate a method which requires only one energy detector and does not need any prior knowledge on the wireless channels, and thus has a global threshold formula which does not depend on the channel.

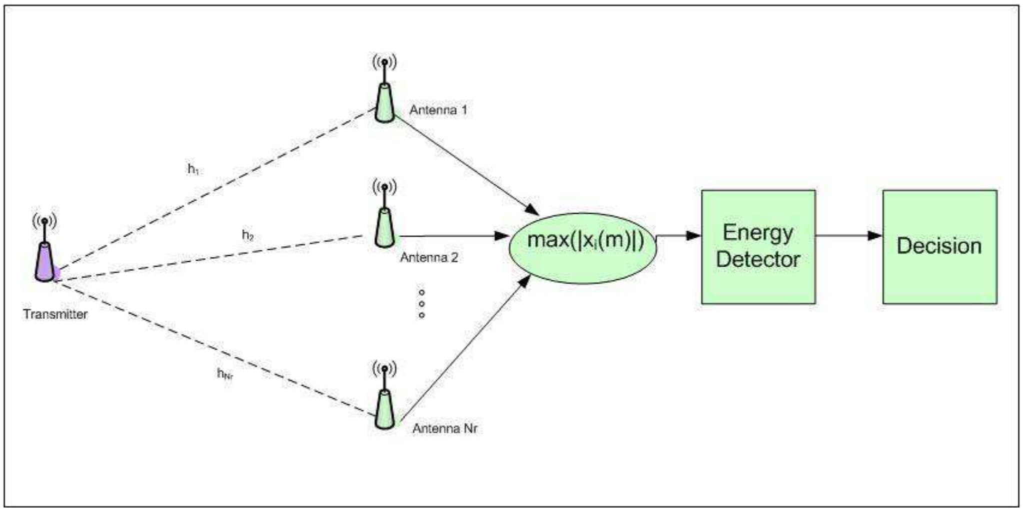

Figure 4.7. Proposed Maximum Selection

As it is seen in the Figure 4.7 of the proposed method; cognitive radio receives data from each antenna and chooses the branch having the maximum absolute value of data for each symbol. After choosing the branch, it applies chosen data to an energy detector symbol by symbol and then make a decision comparing the overall energy of the data applied to the energy detector. Therefore this method does not require any prior knowledge in choosing the branch, that's why threshold set to achieve a certain level of false alarm probability does not depend on the channel.

$$x_j(n) = s(n)h_j + w(n); \quad k\text{=1,...,}N \text{ and } j\text{=1,...}N_r \tag{4.46}$$

$$j_{max}(n) = arg \max_j(|x_j(n)|) \tag{4.47}$$

$$\Xi_{(Proposed_{Max})} = \sum_{n=1}^{N} |x_{j_{max}(n)}(n)|^2 \tag{4.48}$$

We first have to consider the received data under H_0 hypothesis as we need to compute the energy level in order to obtain the threshold set to achieve certain level of False Alarm Probability.

Under H_0 hypothesis; cognitive radio only receives AWGN noise. Hence considering this, we can derive False Alarm equations if we integrate the energy of the noise from threshold to infinity .

First we have to decide how variance changes under H_0 hypothesis for the proposed scheme for 2 antennas in the receiver side.

Let $X_{j_{max_{(n)}}}$ be the maximum of two absolute values, then it can be written as follows;

$$j_{max}(n) = arg \max_{1,2}(|x_1(n)|, |x_2(n)|) \tag{4.49}$$

$$x_{j_{max}(n)}(n) = \begin{cases} \max(x_1(n), x_2(n)) & \text{if}\{x_1(n), x_2(n)\} \in A, \\ \min(x_1(n), x_2(n)) & \text{if}\{x_1(n), x_2(n)\} \in B. \end{cases} \tag{4.50}$$

where

$$A : \{x_1(n) > 0, x_2(n) > 0 \quad \text{or} \quad x_1(n) < 0 < x_2(n), |x_2(n)| > |x_1(n)| \tag{4.51}$$
$$\text{or} \quad x_2(n) < 0 < x_1(n), |x_1(n)| > |x_2(n)|\}$$
$$B : \{x_1(n) < 0, x_2(n) < 0 \quad \text{or} \quad x_1(n) < 0 < x_2(n), |x_2(n)| < |x_1(n)|$$
$$\text{or} \quad x_2(n) < 0 < x_1(n), |x_1(n)| < |x_2(n)|\}$$

Therefore we can write for the region A $\cup B$;

$$x_{j_{max(n)}}(n) : \max_{1,2}(x_1(n), x_2(n)) \cup \min_{1,2}(x_1(n), x_2(n)) \tag{4.52}$$

We need to know how variance of the received signal under H_0 hypothesis changes with the proposed scheme in order to derive false alarm probability. We know that for the union of the regions A and B, received signal becomes the maximum and minimum of two Gaussian variables. Hence we need to find the variance for both maximum and minimum of two Gaussian variables. Authors of (Nadarajah and Kotz 2008) derived the mean and variance of the maximum and minimum of Gaussian variables. Using these derivations;

For Maximum;

$$X = \max(X_1, X_2) \tag{4.53}$$

$$E(X) = \mu_1 \Phi(\frac{\mu_1 - \mu_2}{\theta}) + \mu_2 \Phi(\frac{\mu_2 - \mu_1}{\theta}) + \theta\phi(\frac{\mu_1 - \mu_2}{\theta}) \tag{4.54}$$

$$E(X^2) = (\sigma_1^2 + \mu_1^2)\Phi(\frac{\mu_1 - \mu_2}{\theta}) + (\sigma_2^2 + \mu_2^2)\Phi(\frac{\mu_2 - \mu_1}{\theta}) + (\mu_1 + \mu_2)\theta\phi(\frac{\mu_1 - \mu_2}{\theta}) \tag{4.55}$$

For Minimum;

$$Y = \min(X_1, X_2) \tag{4.56}$$

$$E(Y) = \mu_1 \Phi(\frac{\mu_1 - \mu_2}{\theta}) + \mu_2 \Phi(\frac{\mu_2 - \mu_1}{\theta}) - \theta\phi(\frac{\mu_1 - \mu_2}{\theta}) \tag{4.57}$$

$$E(Y^2) = (\sigma_1^2 + \mu_1^2)\Phi(\frac{\mu_1 - \mu_2}{\theta}) + (\sigma_2^2 + \mu_2^2)\Phi(\frac{\mu_2 - \mu_1}{\theta}) - (\mu_1 + \mu_2)\theta\phi(\frac{\mu_1 - \mu_2}{\theta}) \tag{4.58}$$

where;

$$\theta = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{4.59}$$

We know σ_1^2,σ_2^2 = σ_w^2 and $\rho = 0$ because we assume that noise is iid Gaussian variables.

$$\Phi(x) = \frac{1}{2}(1 + erf(\frac{x}{\sqrt{2}})) \tag{4.60}$$

$$\theta = \sqrt{\sigma_w^2 + \sigma_w^2 - 2\rho\sigma_1\sigma_2} = \sqrt{2\sigma_w^2} \tag{4.61}$$

$$\phi = \frac{1}{\sqrt{2\pi}} e^{-\rho/2} = 1/\sqrt{2\pi} \tag{4.62}$$

Therefore we can find mean and variance of maximum of two Gaussians which is the first part of our distribution as follows;

$$E(\max(X_1, X_2)) = \sqrt{2\sigma_w^2}.\frac{1}{\sqrt{2\pi}} = 0.5642\sigma_w \tag{4.63}$$

$$\begin{aligned} E((max(X_1, X_2))^2) = (\sigma_1^2 + \mu_1^2)\Phi(\frac{\mu_1 - \mu_2}{\theta}) + (\sigma_2^2 + \mu_2^2)\Phi(\frac{\mu_1 - \mu_2}{\theta}) \\ +(\mu_1 + \mu_2)\theta\phi(\frac{\mu_1 - \mu_2}{\theta}) \\ = \sigma_w^2\Phi(0) + \sigma_w^2\Phi(0) = 2\sigma_w^2\Phi(0) = \sigma_w^2(1 + erf(0)) = \sigma_w^2 \end{aligned} \tag{4.64}$$

Then we can obtain variance for maximum as follows;

$$var((max(X_1,X_2))) = E((max(X_1,X_2)^2)) - [E(max(X_1,X_2))]^2 \tag{4.65}$$
$$= \sigma_w^2 - (0.5642\sigma_w)^2 = 0.6817\sigma_w^2$$

Then we will do the same operations for minimum of two Gaussians.

$$E(min(X_1,X_2)) = \mu_1\Phi(\frac{\mu_2-\mu_1}{\theta}) + \mu_2\Phi(\frac{\mu_1-\mu_2}{\theta}) - \theta\phi(\frac{\mu_2-\mu_1}{\theta}) \tag{4.66}$$
$$= -\theta\phi(\frac{\mu_1-\mu_2}{\theta})$$
$$= -\sqrt{2\sigma_w^2}.\frac{1}{\sqrt{2\pi}} = -0.5642\sigma_w$$

$$E((min(X_1,X_2))^2) = (\sigma_1^2+\mu_1^2)\Phi(\frac{\mu_2-\mu_1}{\theta}) + (\sigma_2^2+\mu_2^2)\Phi(\frac{\mu_2-\mu_1}{\theta}) \tag{4.67}$$
$$-(\mu_1+\mu_2)\theta\phi(\frac{\mu_2-\mu_1}{\theta})$$
$$= \sigma_w^2\Phi(0) + \sigma_w^2\Phi(0) = 2\sigma_w^2\Phi(0) = \sigma_w^2(1+erf(0)) = \sigma_w^2$$

Variance;

$$var((min(X_1,X_2))) = E((min(X_1,X_2)^2)) - [E(min(X_1,X_2))]^2 \tag{4.68}$$
$$= \sigma_w^2 - (-0.5642\sigma_w)^2 = 0.6817\sigma_w^2$$

For the statistics of the exact distribution of maximum of the absolute value of two Gaussian can be obtained for the region $A \cup B$ as follows;

$$E(max(|x_1|,|x_2|)) = E(max(X_1,X_2)) + E(min(X_1,X_2)) \tag{4.69}$$
$$= 0.5642\sigma_w - 0.5642\sigma_w = 0$$

$$var(max(|X_1|,|X_2|)) = var(max(X_1,X_2)) + var(min(X_1,X_2)) \tag{4.70}$$
$$= 0.6817\sigma_w^2 + 0.6817\sigma_w^2 = 1.3634\sigma_w^2$$

So choosing the maximum of two absolute gaussian variables, variance becomes $1.3634\sigma_w^2$, that indicates if we choose the data from two antennas, the variance of the noise becomes $1.3634\sigma_w^2$. Therefore if we normalize the received data dividing by $\sqrt{1.3634}$, the variance of the received data under H_0 hypothesis becomes σ_w^2. In the same paper (Nadarajah and Kotz 2008); it is mentioned that maximum and minimum of two gaussian are actually not gaussian distributed but often assumed to be Gaussian in many papers. The authors show how poor this approximation is if standard derivations of two variables are not equal. But on the other hand, the result given in (Nadarajah and Kotz 2008) shows that it can be assumed as Gaussian for the variables having the same standard derivations. In our system it is assumed that the variance of the noise is same for all antennas thus we can assume that it is Gaussian and then we have the same distribution with SISO systems after normalization.

Also it is showed in (Nadarajah and Kotz 2008) that the equations can also be applied for more than two gaussian variables, thus we can extend these equations for the case of N_r antennas. In Appendix B, we calculated the variances as $1.6112\sigma_w^2$, $1.7802\sigma_w^2$, $1.8952\sigma_w^2$ and $1.9758\sigma_w^2$ for 3,4,5 and 6 antennas respectively.

We denote these normalizing coefficients as C where $C_j = [C_1; C_2; C_3; ...C_{N_r}]$ where j is the number of antenna used in the system. C_1 refers to the coefficient for SISO systems (1 antenna) therefore it is 1. We computed $C_2 = 1.3634$, $C_3 = 1.6112$, $C_4 = 1.7802$, $C_5 = 1.8952$ and $C_6 = 1.9758$.

Then we can derive the False Alarm Probability equation for the Proposed Scheme as follows;

Under H_0 hypothesis the received signal is;

$$x_j(n) = w_j(n) \quad \text{where} \quad n = 1...N, \quad j = 1, ...N_r. \tag{4.71}$$

where $w_j(n)$ is AWGN noise with zero mean and σ_w^2 variance, N is the symbol length to be sensed and j is the antenna number. We apply the proposed selection process as;

$$x(n) = w_{jmax(n)}(n) \tag{4.72}$$

where the variance of the received signal $x(n)$ under H_0 hypothesis becomes $C_j\sigma_w^2$. So if we normalize the received data dividing by $\sqrt{C_j}$ as follows:

$$\widehat{x}(n) = \sqrt{\frac{1}{C_j}}x(n) \tag{4.73}$$

The variance of the normalized received data $\widehat{x}(k))$ becomes σ_w^2.

$$\widehat{\Xi}_{Proposed_{Max}} = \sum_{n=1}^{N} |\widehat{x}(n)|^2 \tag{4.74}$$

Under H_0 hypothesis, the output of the energy detector is summation of N squares of Gaussian variables having σ_w^2 variance. Thus the distribution of the output becomes a chi-square pdf with N degrees of freedom just like SISO systems as follows; (Chi-Square Derivation, Appendix A)

$$f_{\widehat{\Xi}_{Proposed_{Max}}}(\widehat{\xi}) = \frac{1}{\sigma_w^N 2^{N/2}\Gamma(\frac{1}{2}N)}\widehat{\xi}^{(N/2)-1}e^{-\widehat{\xi}/2\sigma_w^2}, \text{for } \widehat{\xi} \geq 0 \tag{4.75}$$

if we integrate the output of the energy detector under H_0 hypothesis; we can derive the $False\ Alarm\ Probability(P_{FA})$ as derived in Chapter 3:

$$P_{FA_{Proposed_{Max}}} = \frac{\Gamma(N/2, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N/2)} \tag{4.76}$$

which is exactly as same as P_{FA} equation for SISO system.

For the case of complex gaussian noise we apply our proposed system to the real and imaginary parts of the received signals independently as given in the figure. Then we combine the real and imaginary parts and apply them to an energy detector.

Considering this scheme, the outputs of the real and imaginary parts which are normalized with C coefficients mentioned before are central chi squares with N degree of freedom. After combining the real and imaginary outputs, the final distribution becomes

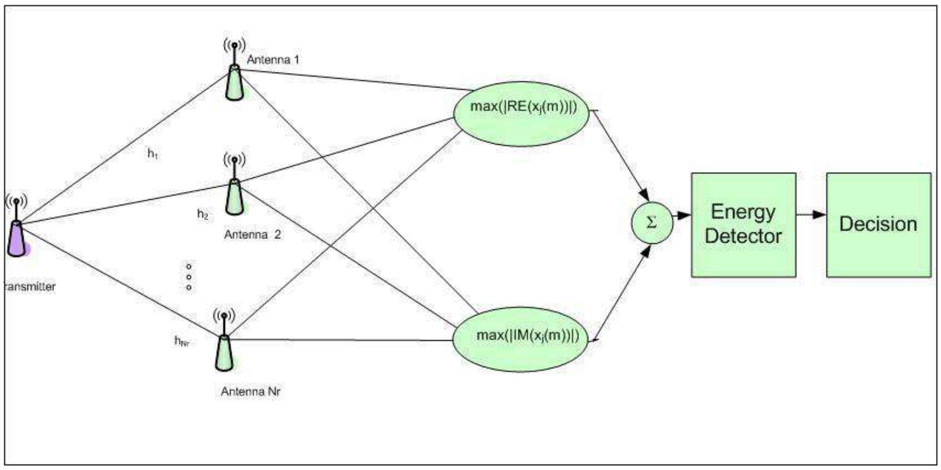

Figure 4.8. Proposed Maximum Selection

$2N$ degree of freedom. That's why the False Alarm Probability equation for the proposed scheme when the noise is complex gaussian can be written as follows:

$$P^c_{FA_{Proposed_{Max}}} = \frac{\Gamma(N, \frac{\lambda}{2\sigma_w^2})}{\Gamma(N)} \tag{4.77}$$

Taking everything into consideration, the proposed scheme offers threshold independence on the channel and a considerable gain in sense of missing probability with a less implementation cost as it requires only one energy detector independent on how many antennas implemented in the system.

4.2. Simulation Results for Multiple Antenna Methods Based on Energy Detection

In this section, we give the results of the simulations evaluated in which we implemented multiple antenna techniques into the spectrum sensing system based on energy detection. In the first simulations we compare the performances of multiple antenna schemes with respect to SNR changes. BPSK modulation is first employed for the simulations. Symbol length to be sensed,N, is assumed to be 10. Energy Detection is applied in time domain. The number of antenna used in cognitive radio,N_r, is taken as 4.

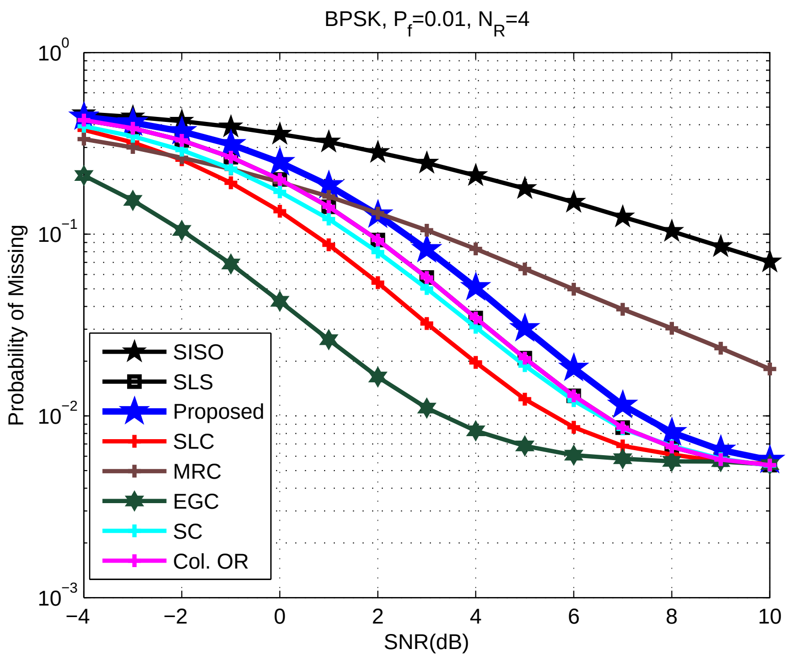

Figure 4.9. P_m vs. SNR for BPSK SIMO systems

We set the thresholds in order to achieve $P_f = 0.01$ and then evaluated the simulation. The results indicate that the best performance is provided by Equal Gain Combining which requires perfect channel knowledge. Square Law Combining and Square Law Selection gives considerably better results than SISO systems. On the other hand Maximum Ratio Combining provides somewhat gain but worse than equal gain combining. Proposed scheme provides gain approximately as much as Square Law Selection. Considering the fact that Proposed scheme needs only one energy detector while SLS requires a detector for each antenna, proposed scheme performs really well and can be shown as a promising multiple antenna technique candidate to be implemented in spectrum sensing systems.

In the Figure 4.10, Probability of detection vs. SNR is given. Again $N_r = 4$ antenna is used and False Alarm Probability is set as 0.01. 10 symbols of the received signal are sensed. As seen in the figure, EGC and SLC give the best performances. When we consider the fact that EGC requires perfect channel knowledge, then it is safe to say that employing SLC is more sensible.

It is also seen that Selection Combining, Square Law Selection, Collaborative with OR table and Proposed Scheme provides very close detection performance. Hence Proposed scheme is more sensible to implement as it requires only one energy detector

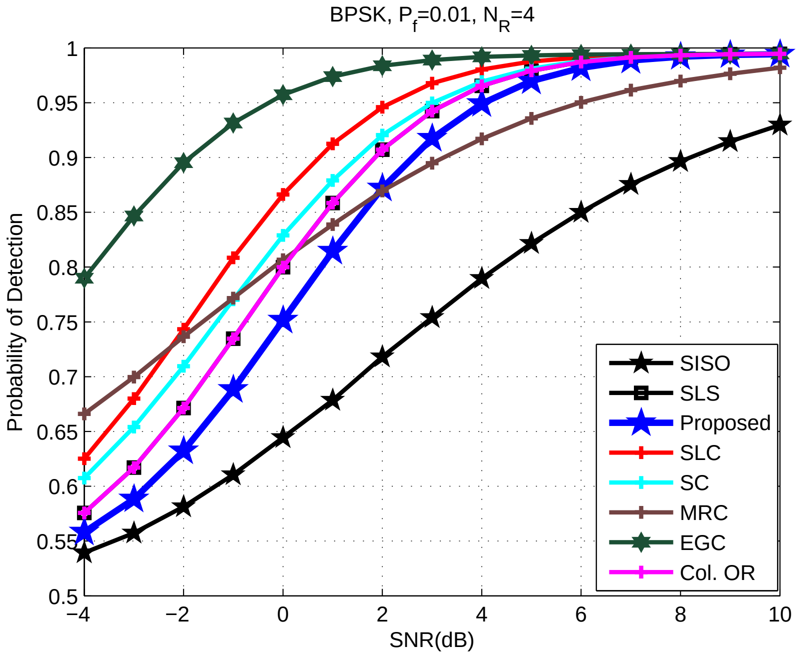

Figure 4.10. P_d vs. SNR for BPSK SIMO systems

and does not require any prior information about the channel.

In the following simulations given in Figure 4.11 we analysed how detection probabilities change with respect to the antenna numbers. We again set the False Alarm Probability as 0.01. We evaluated the simulations for $SNR = 5dB$ and symbol length to be sensed,$N = 10$ symbols. We evaluated the simulations for N_r is 2,3,4,5 and 6.

As seen in the figure that Probability of Detection increases with Antenna numbers. It's also seen that more antennas used indicate closer results for all multiple antenna schemes. Hence this graph also leads to the fact that proposed scheme provides a good performance of signal detection.

In the Figure 4.12, we evaluated the system for QPSK modulation scheme. We again set the False Alarm Probability as 0.01 and symbol length to be sensed is decided as 10. 4 antennas are used in cognitive radio and all multiple antenna techniques mentioned earlier are implemented.

According to the simulation results given in this figure; EGC again gives the best performance. Again we see in the results that MRC gives a somewhat gain but it is not worth to be implemented in the spectrum sensing system as it requires the perfect channel knowledge. On the other hand proposed scheme gives better performance for QPSK

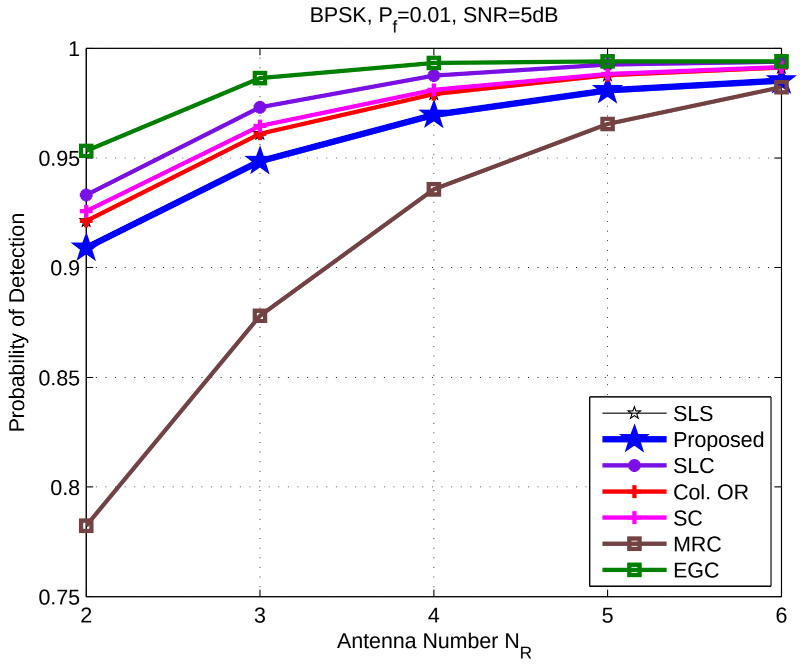

Figure 4.11. P_d vs. Antenna Number for BPSK SIMO systems

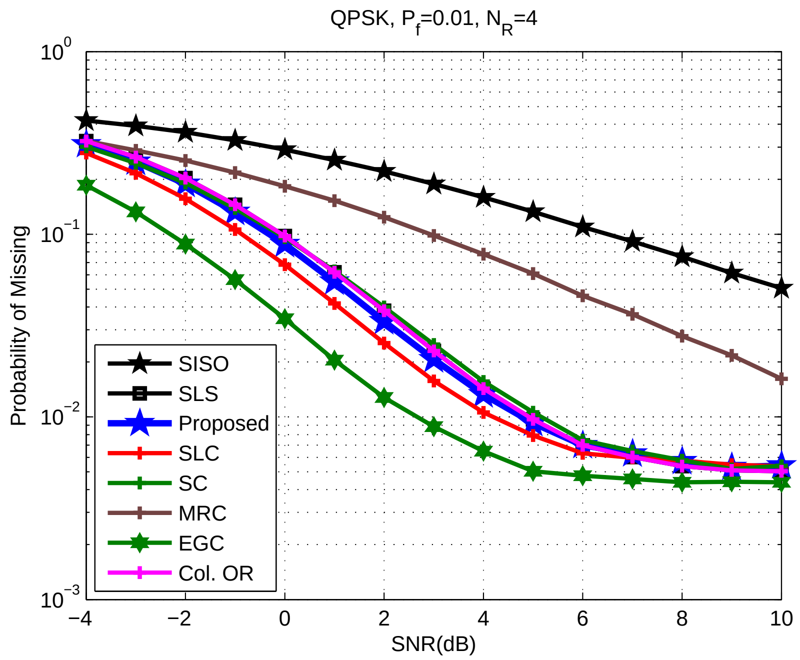

Figure 4.12. P_m vs. SNR for QPSK SIMO systems

scheme rather than BPSK scheme, and also provides gain almost as much as SLC does. There's only $0.5dB$ difference between SLC and proposed method for low SNRs. For higher SNRs, the difference approximates to $0dB$.

In the Figure 4.13; we draw probability of detection versus SNR for QPSK modulated systems. As seen in the figure, EGC gives the best performance while SLC catches its performance at $SNR = 2dB$. Proposed scheme gives slightly more gain than it is in BPSK modulated schemes. It also provides better detection probability rather than Selection Combining, Square Law Selection and Collaboration techniques with *OR* table.

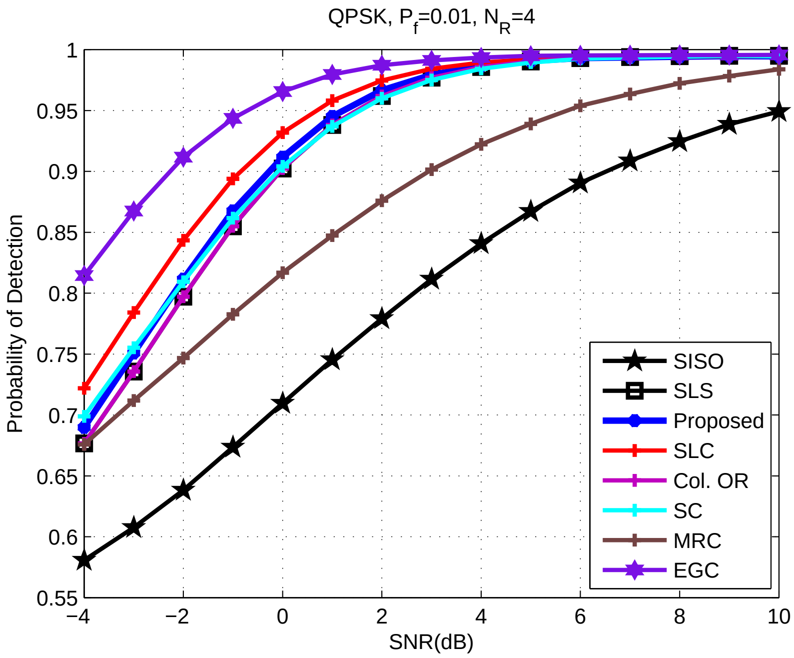

Figure 4.13. P_d vs. SNR for QPSK SIMO systems

The results also indicate that the performance of all multiple antenna techniques except MRC approximate to each other from SNR 4dB. Therefore our proposed scheme is seemed to be a promising and advantageous solution for spectrum sensing employing QPSK modulation scheme.

In the next step of the simulations, we evaluate the system for QPSK modulation in order to analyse how detection probabilities change with respect to the antenna numbers. We again set $P_f = 0.01$. We evaluate the simulations for $SNR = 5dB$, symbol length to be sensed,$N = 10$ symbols and N_r is 2,3,4,5 and 6.

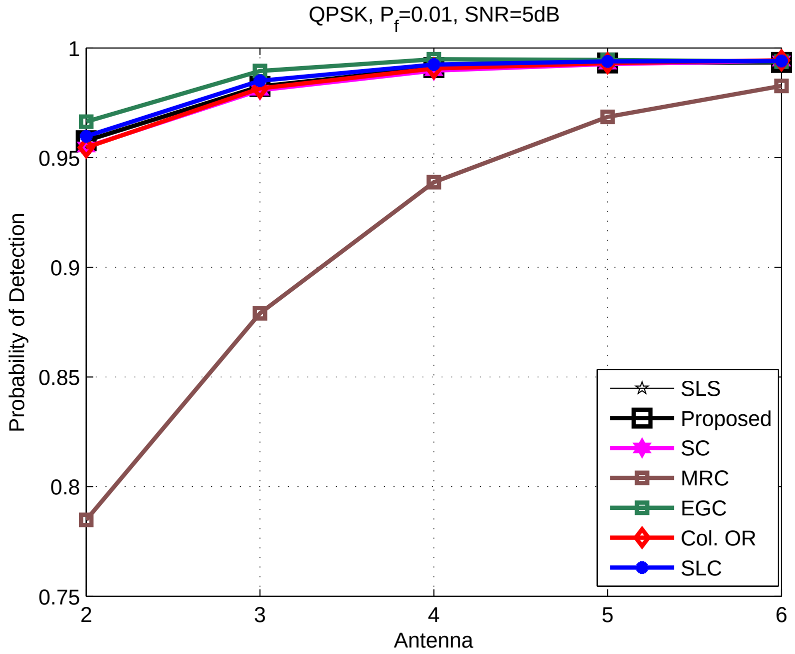

Figure 4.14. P_d vs. Antenna Number for QPSK SIMO systems

As seen in Figure 4.14, proposed scheme performance is very close to SLC,SLS and collaboration methods which do not require any prior information on the channel.

In the next simulations, we evaluated the system for OFDM modulation scheme. For OFDM cognitive radio is assumed to sense each sub-channels and decide whether the sub-channel is idle or not. We again set the False Alarm Probability as 0.01 and symbol length to be sensed is decided as 10 in each 16 sub-channels. 4 antennas are used in cognitive radio and all multiple antenna techniques mentioned earlier are implemented.

Considering the simulation results given in Figure 4.15; SLC gives the best performance but on the other hand,the performances of SLS, Col. OR and proposed scheme are very close to the performance of SLC. From SNR=$0dB$, the difference between SLC, SLS,Collaborative and Proposed Scheme approximates to zero.

The Figure 4.16 refers to detection performance with respect to SNR for OFDM modulated systems. Again it is straightforwardly seen that *Proposed Scheme* provides gain approximately as much as SLC does.

As seen in this figure, performance of proposed scheme in terms of detection probability is very close to SLC,SLS and collaboration methods which do not require any prior information on the channel. From $-4dB$ to higher SNRs, there's almost no gain

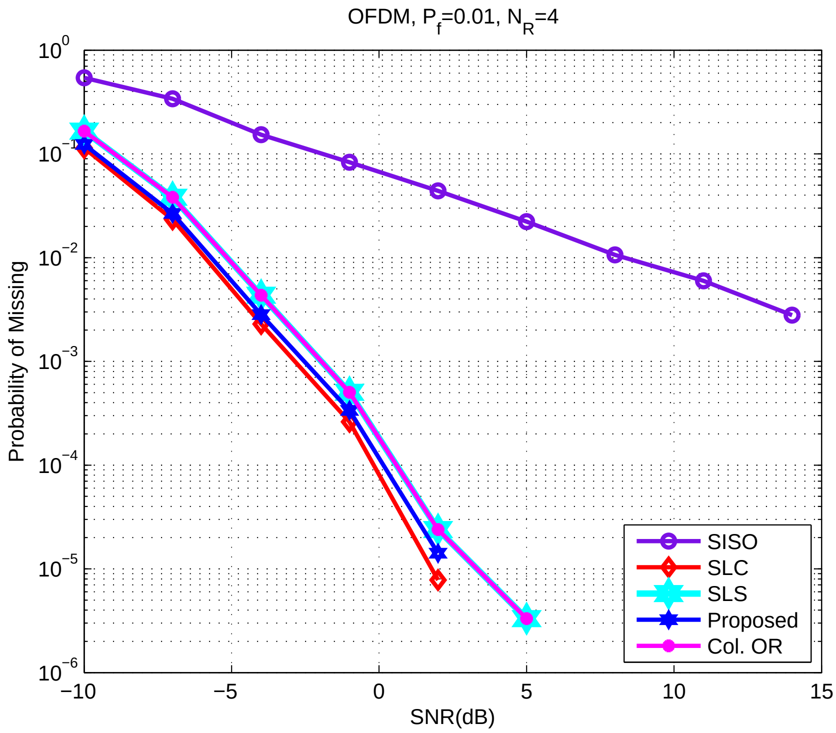

Figure 4.15. P_m vs. SNR for OFDM SIMO systems

difference between SLC, SLS, Collaborative with OR table and Proposed scheme.

In the next step of the simulations, OFDM modulation is employed in the system. The simulation is evaluated in order to analyse how detection probabilities change with respect to the antenna numbers. We again set the False Alarm Probability as 0.01. We evaluated the simulations for $SNR = -5dB$ because the minimum SNR value to detect for OFDM systems is lower than BPSK and QPSK modulation schemes. Symbol length to be sensed in each sub-channel is assumed to be,$N = 10$ symbols. We evaluated the simulations for Nr is 2,3,4,5 and 6.

As seen in the Figure 4.17 for $SNR = -5dB$ the performances of all multiple antenna schemes are very close to each other. Hence this result also indicates that it is more sensible to use the proposed scheme rather than other methods as it provides gain as much as other methods although it does not require either any prior knowledge on channel or detectors dedicated to each antenna.

In the following simulations we considered the antennas are correlated with a correlation model defined in Chapter 2. We didn't evaluate SC, MRC and EGC because they require perfect channel knowledge and implementation of these methods would not be

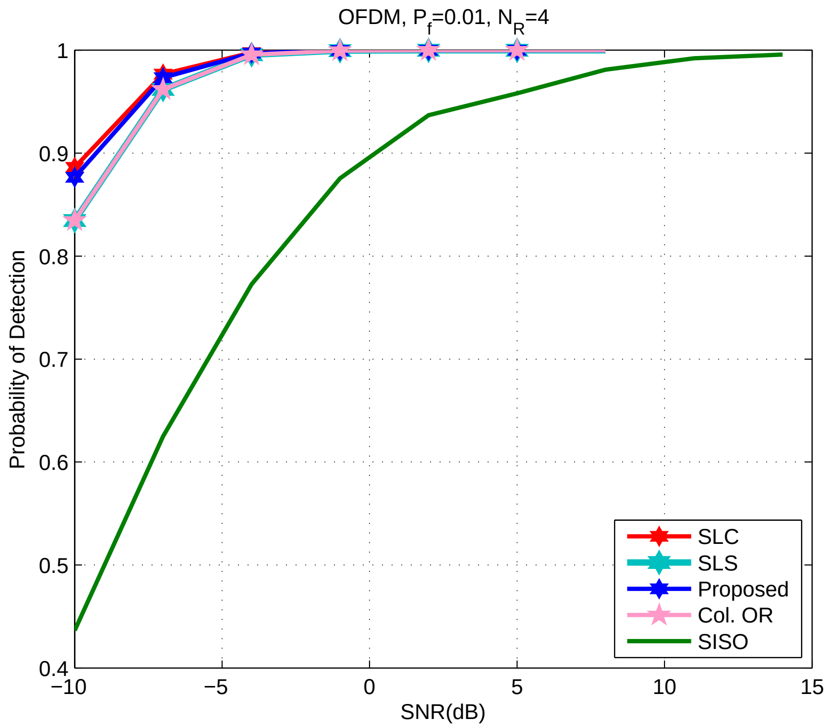

Figure 4.16. P_d vs. SNR for OFDM SIMO systems with QPSK

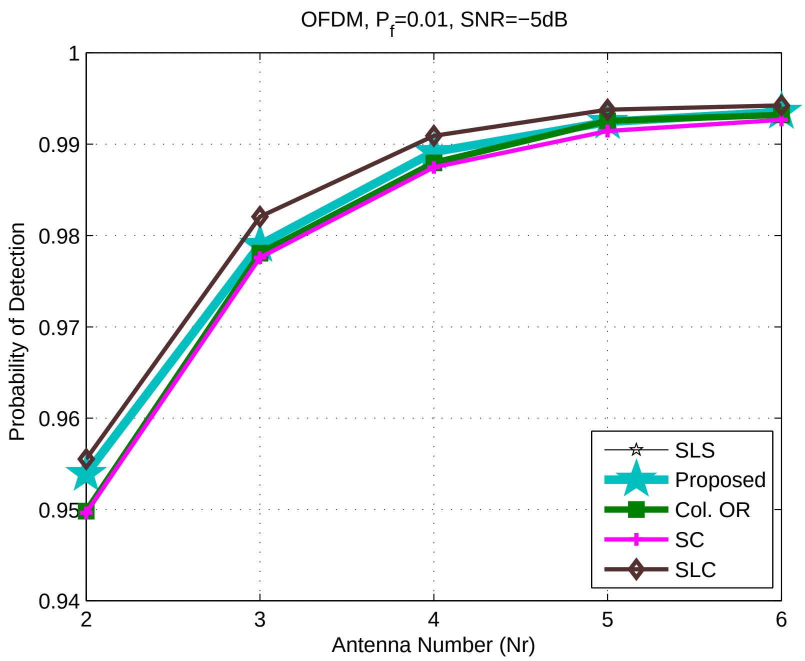

Figure 4.17. P_d vs. Antenna Number for OFDM SIMO systems

realistic. First we evaluated the simulation for BPSK system with a correlation $|\rho| = 0.6$ and $|\rho| = 0.9$. When we look at the results, the detection performance slightly degraded compared to no correlation scheme but the performance order of the multiple antenna methods remained the same. It's also shown that the performance degrades as the correlation between antennas increases.

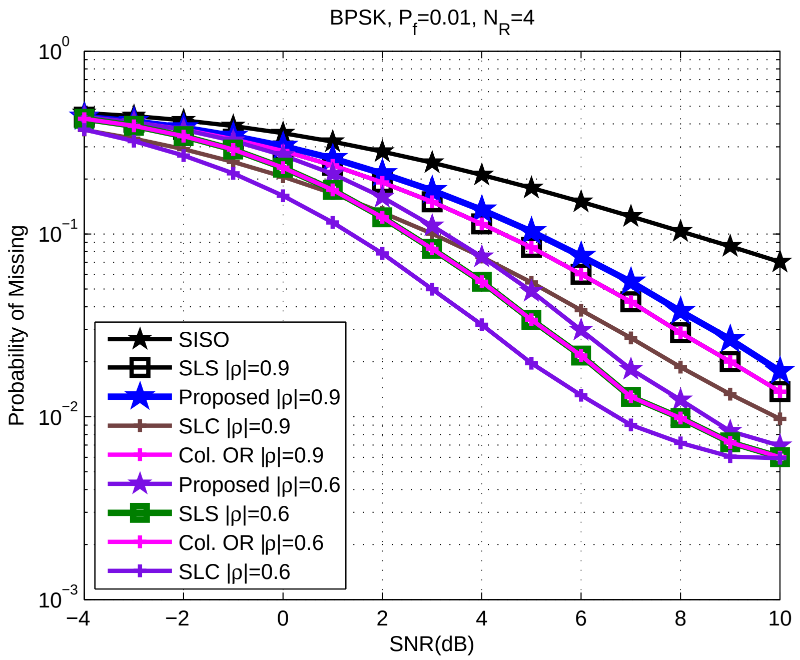

Figure 4.18. P_m vs. SNR for BPSK correlated SIMO systems

It's again seen Figure 4.18 that SLC gives the best performance among all, while the performance of the other methods are pretty close to its.

Figure 4.19 shows the relation between detection probability and SNR changes. As seen in the figure; detection probability slightly decreased compared to non-correlation scheme.

It also indicates that the difference between proposed scheme and both SLS and *collaboration with OR table* is only $0.5dB$ while the difference between proposed scheme and both SLC is 2 dB for only low SNRs. As it is mentioned before, the difference degrades as SNR increases. Likewise in Missing Probability figure it is seen that the difference between SLS and Proposed scheme degrades as the correlation coefficient increases for low SNRs. The results also indicate that the difference between SLC and proposed scheme decreased with the correlation.

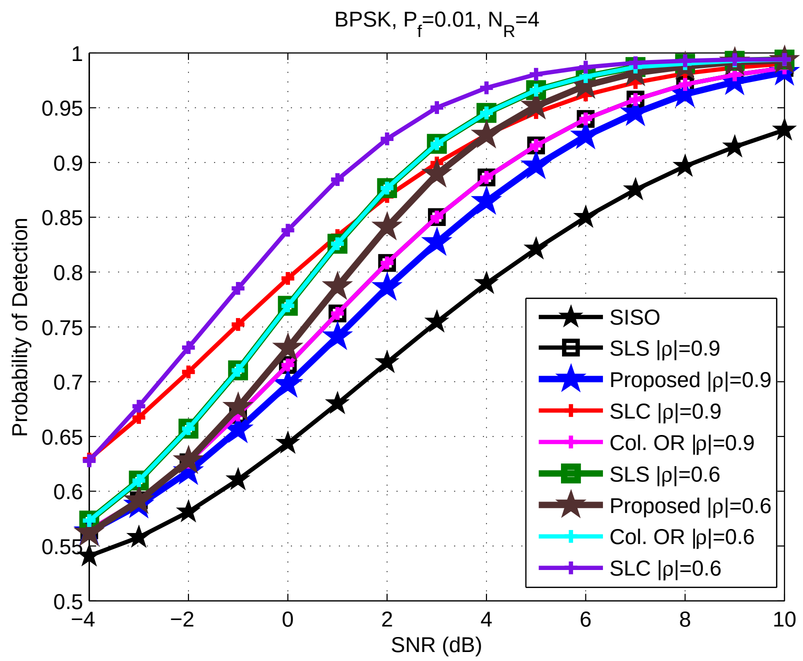

Figure 4.19. P_d vs. SNR for BPSK correlated SIMO systems

In the following step of the simulations given in Figure 4.20, we again considered the antennas are correlated for QPSK modulated systems. We again evaluated the simulation with a correlation of $|\rho| = 0.6$ and $|\rho| = 0.9$. When we look at the results, the detection performance slightly degraded compared to the state of non-correlation but the performance order of the multiple antenna methods remained the same.

Considering this result and the results for non-correlated state, it can be said that the proposed scheme is more resistant to correlated channels rather than SLS and correlation channels as their performance degraded more than the proposed scheme

As seen in Figure 4.21, for non-correlated state, there were no difference of gain between Proposed and SLS and Collaboration methods. On the other hand when the correlation is considered, there's a gain difference of $0.4 - 0.5dB$ occurs.

In Figure 4.22, the relation between detection probability and antenna number for both uncorrelated and correlated channels is given. As seen in this figure, proposed scheme performs better than SLS and Collaborative OR techniques as the correlation between the channel increases.

In the following step, we considered the correlated antennas for the systems employing OFDM modulation. In the first figure the its effect is given in terms of missing

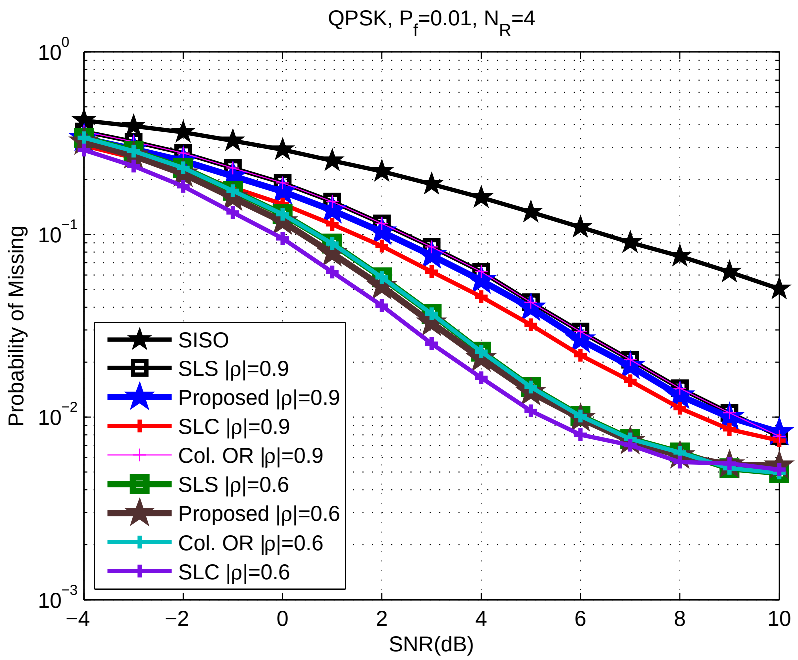

Figure 4.20. P_m vs. SNR for QPSK correlated SIMO systems

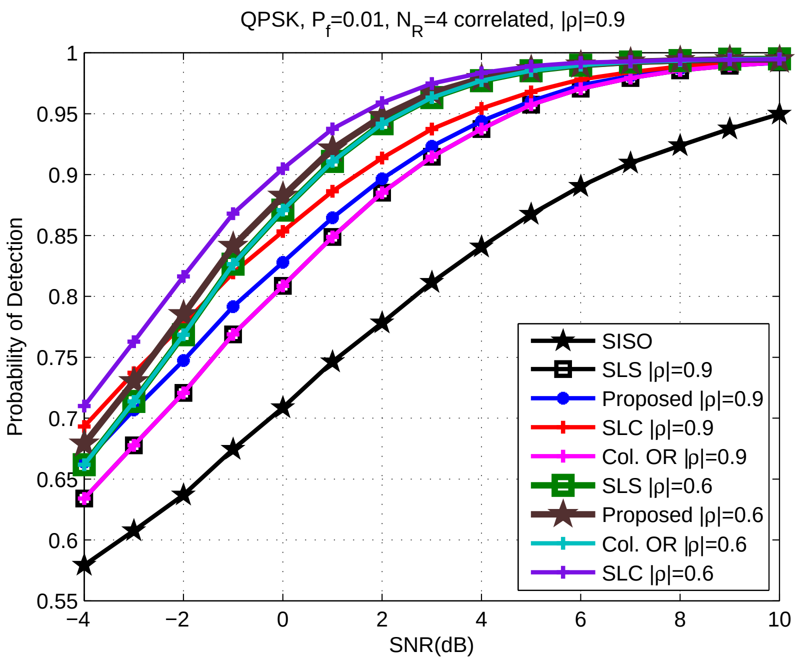

Figure 4.21. P_d vs. SNR for QPSK correlated SIMO systems

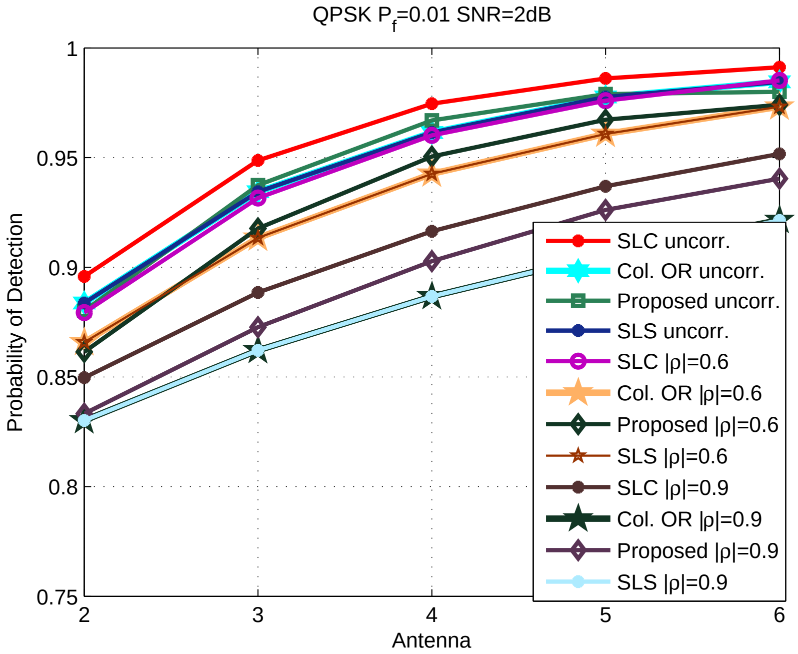

Figure 4.22. P_d vs. SNR for QPSK correlated SIMO systems

probability vs. SNR. Likewise in other modulation schemes, the detection performances of all multiple antenna schemes degrades with respect to the correlation coefficient. Increasing correlation coefficient lead to the worse performance.

On the other hand, it is seen in the Figure 4.23 that Proposed scheme provides gain as much as other methods in the case of correlated antennas.

In the Figure 4.24, the detection performance of multiple antenna schemes with respect to SNR for the case of correlated antennas is given. Likewise the previous figure, it indicates that proposed scheme catches the performance of SLC.

To sum up the simulation results, it is shown that proposed scheme provides satisfying gain under both non-correlated and correlated cases. It's performance is very close to SLC and better than SLS and Collaboration schemes for some cases. EGC provides gain more than other methods but considering the fact that it requires perfect knowledge on the wireless channel, it can not be sensible to implement in spectrum sensing schemes.

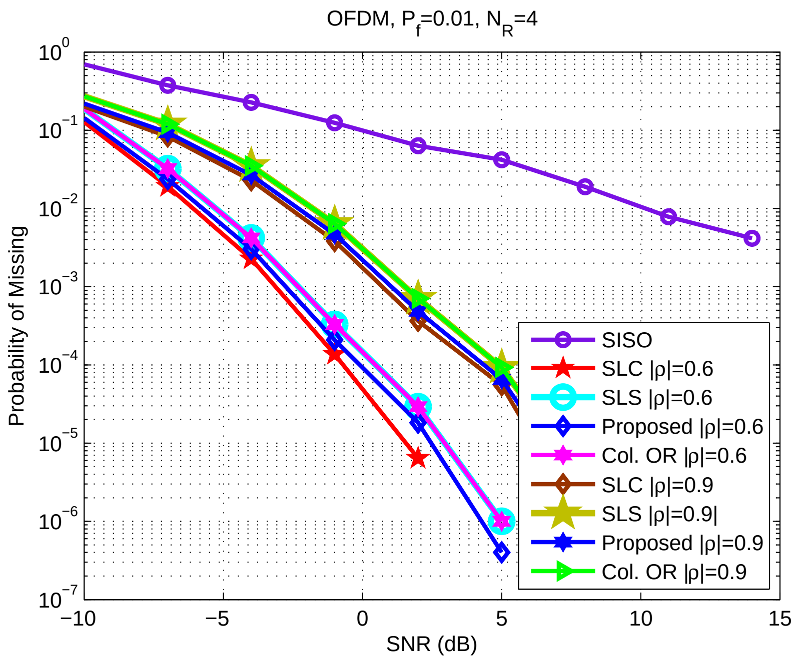

Figure 4.23. P_m vs. SNR for OFDM correlated SIMO systems

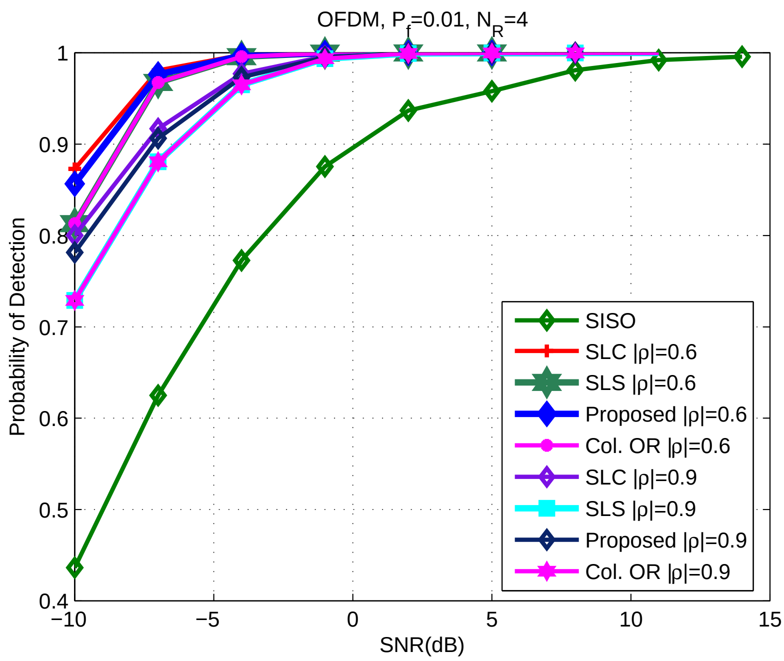

Figure 4.24. P_d vs. SNR for OFDM correlated SIMO systems

4.3. Multiple Antenna Methods Based on Cyclostationary Feature Detection

We see in literature that there is an increasing interest on multiple antenna techniques for the systems based on cyclostationary feature detection in order to improve the performance of primary user detection. In (Sadeghi and Azmi 2008), the authors propose to use multiple cyclostationary detectors and combine the statistics from each detector. This techniques provide a gain but on the other hand, it requires a detector dedicated to each antenna. This refers to the increase of the implementation costs. Even the fact whether a single cyclostationary detectors are affordable or not is still on debate, so we think that this technique can not be implemented.

The authors of (Rajarshi and Krusheel 2008) propose ”Maximum Ratio Combining” for cyclostationary feature detection. As it requires perfect channel knowledge and a changing threshold according with channel, we did not consider it to implement in our system.

Therefore we propose a technique which does not require several detectors and channel state information.

4.3.1. The Proposed Technique

The previous multiple antenna techniques for cyclostationary feature detection requires multiple cyclostationary feature detection which leads more implementation costs as cyclostationary feature detectors are more complex to implement. Therefore we need to reduce the number of cyclostationary feature detector numbers used in spectrum sensing systems.

We propose a multiple antenna technique which requires only one cyclostationary feature detection. On the other hand we need several detectors to compute the energy of each branch in this algorithm as seen in Figure 4.25.

According to this algorithm, we employ energy detectors for each antenna as follows:

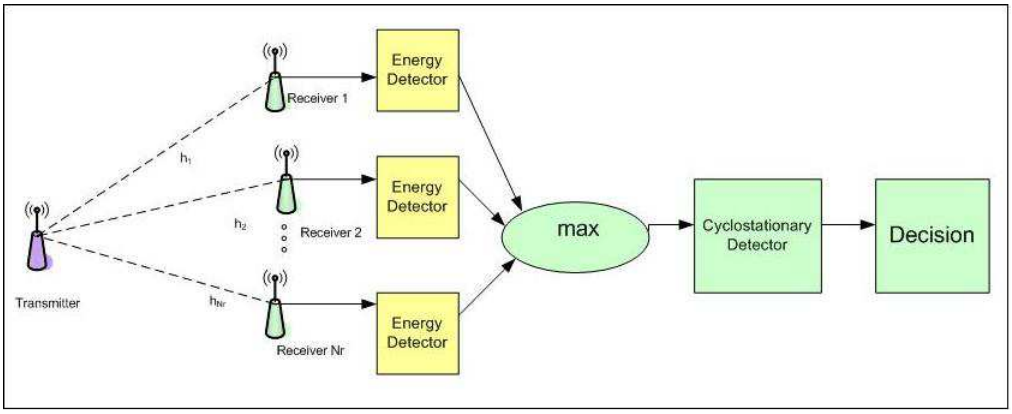

Figure 4.25. Proposed Scheme

$$\Xi_j = \sum_{k=1}^{N} |x_j(k)|^2 \tag{4.78}$$

Then we choose the branch having the maximum energy and then apply it to the cyclostationary feature detector;

$$j_{max} = \max_j \Xi_j \tag{4.79}$$

After applying j_{max} branch to the cyclostationary feature detector, we compare the statistics with a threshold and decide whether the band is idle or not.

$$I_{j_{max}}(\alpha) \lessgtr \lambda \tag{4.80}$$

For a system having four antennas in the receive side, we'll need four energy detectors and 1 cyclostationary feature detectors which costs less than other methods requiring four cyclostationary feature detectors.

4.4. Simulation Results for Multiple Antenna Methods Based on Cyclostationary Feature Detection

In this section the simulation results are given for the proposed multiple antenna technique in spectrum sensing systems based on cyclostationary feature detection. In this simulation, we evaluated the simulations for BPSK modulated systems under $P_f = 0.01$ and the state of both uncorrelated and correlated antennas. BPSK signal is assumed to have 125 MHz carrier frequency, 20 MHz bandwidth and 4GHz sampling frequency. We analyse the systems for both 2 and 4 antennas in the receiver side.

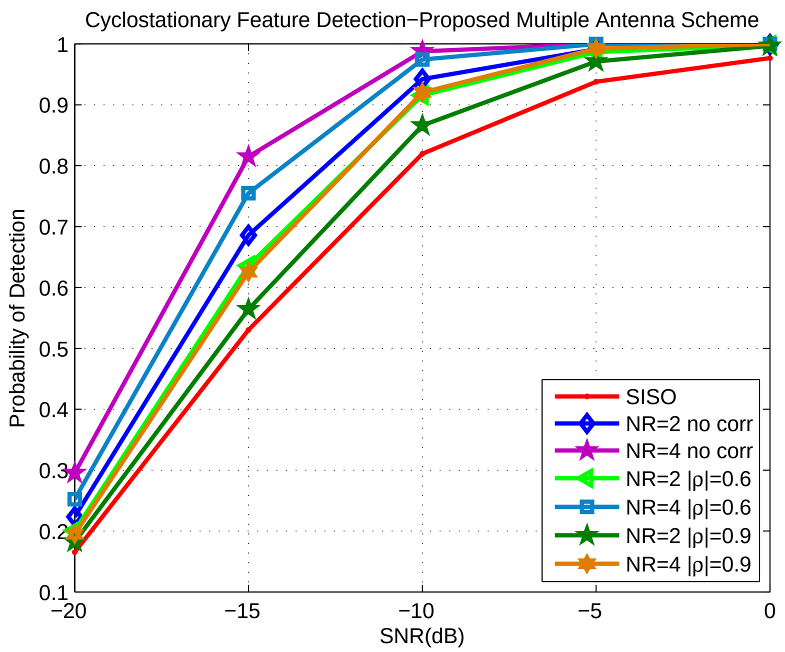

Figure 4.26. P_d vs. SNR for $P_f = 0.01$-BPSK systems

Figure 4.26 indictes that proposed technique provides a gain with respect to antenna numbers. Using 2 antennas in the receiver, the proposed scheme can detect the primary user with a probability of 90 percent approximately at -8dB while it can achieve the same detection performance approximately at -12dB with 4 antennas in the receiver.

It is also seen in this figure that the correlation between the antennas degrades the performance of the system, on the other hand it still provides somewhat gain for 2 antennas with strong correlation of $\rho = 0.9$.

Figure 4.27 refers to the performance of the proposed scheme in terms of Missing Probability. Using multiple antennas we achieved lower missing probability that refers to lower probability of interference to primary user caused by secondary user.

We can not see the gain that proposed technique provide for strongly correlated $\rho = 0.9$ two antennas in Figure 4.26 as clearly as in Figure 4.27. Using 4 antennas, the proposed scheme offers a significant gain even with strongly correlated channels.

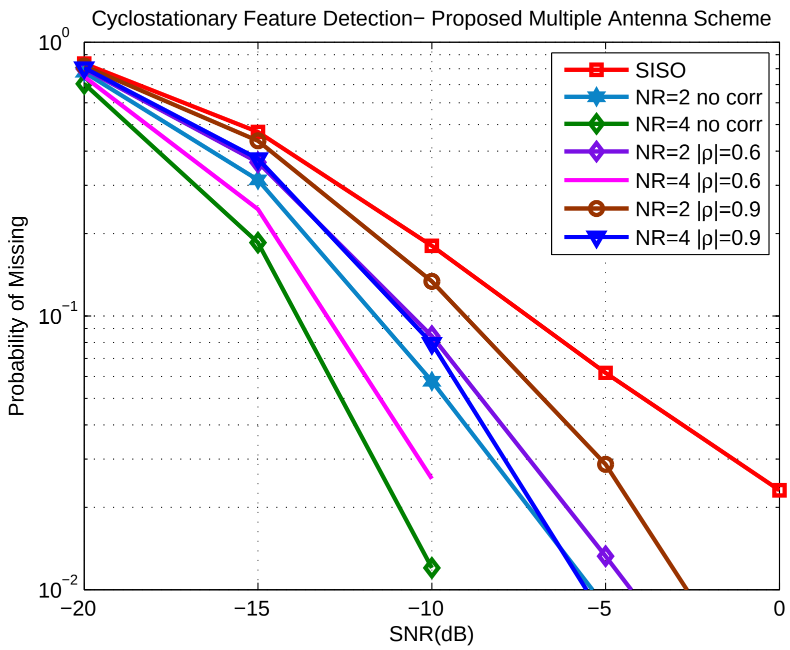

Figure 4.27. P_m vs. SNR for $P_f = 0.01$-BPSK systems

To sum up then we improved the performance of the spectrum sensing system based on cyclostationary feature detection with the proposed multiple antenna scheme with a lower implementation cost compared to the other multiple antenna techniques proposed in literature.

CHAPTER 5

CONCLUSIONS

In this thesis, we have focused on spectrum sensing techniques for Cognitive Radio systems with Multiple Antennas. We have aimed to propose multiple antenna techniques in order to increase the performance of the spectrum sensing systems in literature and reduce the implementation costs.

We have studied some spectrum sensing techniques proposed in literature and obtained simulation performances of Energy Detection and Cyclostationary Feature Detection methods. We have studied three different ways of energy detection implementation; time domain, periodogram and Welch's periodogram; and showed that they provide the same performance. We have also evaluated the simulations for different symbol lengths and showed that longer symbol length refers to better detection probability. However it increases the sensing time. We have evaluated the simulations for BPSK, QPSK and OFDM modulation schemes and given the results in terms of Detection Probability, Missing Probabiliy vs. SNR. We have also showed how detection probability changes with respect to False Alarm Probability. In the next step we have implemented Cyclostationary Feature Detection technique for SISO-BPSK systems. We have showed the cyclic frequencies giving the maximum values of spectral correlation function of BPSK signals and the fact that noise is not a cyclostationary process. Then we have evaluated the system for both AWGN and Rayleigh Fading Channel and given the results in terms of Detection Probability and Missing Probability. Then we have compared Cyclostationary Feature Detection with Energy Detection for the same signal under same conditions and show that Cyclostationary Feature Detection performs better than Energy Detection for lower SNRs and there's an approximately 5dB difference between two techniques. However the implementation of cyclostationary feature detection is much more complex.

We have studied some multiple antenna techniques in literature in order to improve the performances of spectrum sensing techniques. We have studied SC, MRC, EGC, SLC, SLS, Collaborative HD techniques and evaluated the simulations in order to examine their performances. We have given the results showing that EGC offers the highest gain but on the other hand it requires perfect channel information. SLC provides considerably high gain and it does not require any prior channel knowledge but on the other hand it requires several detectors which increase the implementation costs.

Finally we have proposed some multiple antenna techniques for both Energy Detection and Cyclostationary Feature Detection based systems. For Energy Detection we have proposed to select signal at sample level and apply it to only one energy detector. We have computed how variance changes and normalize the data with these coefficients given for 2,3,4,5 and 6 antennas in the receiver side. We have simulated this technique for BPSK, QPSK and OFDM modulation schemes and showed that it provides gain approximately as much as SLC while it does not require several detectors and any prior knowledge on the channel. Then we have proposed a technique with several energy detectors and a cyclostationary feature detection. We have evaluated the simulations and showed that it provides a high gain and threshold remains the same with SISO systems.

As a future work, MIMO spectrum sensing techniques with cooperation among multiple cognitive radios can be implemented.

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APPENDIX A

CHI-SQUARE DISTRIBUTION

A direct relation exists between a chi-square-distributed random variable and a gaussian random variable. The chi-square random variable is in a certain form a transformation of the gaussian random variable. If we have W as a gaussian variable and we take the relation $\Xi = W^2$ then Ξ has a chi-square distribution with one degree of freedom.

If we define the random variable Ξ as

$$\Xi = W^2 \tag{A.1}$$

Then the pdf of Ξ in terms of the pdf of W can be expressed as;

$$\begin{aligned} F_\Xi(\xi) &= P(\Xi \leq \xi) = P(W^2 \leq \xi) \\ &= P(|W| \leq \sqrt{\xi}) \end{aligned} \tag{A.2}$$

Hence;

$$F_\Xi(\xi) = F_W(\sqrt{\xi}) - F_W(-\sqrt{\xi}) \tag{A.3}$$

Differentiating this equation with respect to ξ, we obtain the pdf of Ξ in terms of the pdf of W in the form;

$$f_\Xi(\xi) = \frac{f_W[\sqrt{\xi}]}{2\sqrt{\xi}} + \frac{f_W[-\sqrt{\xi}]}{2\sqrt{\xi}} \tag{A.4}$$

Using this pdf, we can now derive the pdf of the chi-square distribution with one degree of freedom as we know the pdf of w is zero mean gaussian variable.

$$f_{\Xi}(\xi) = \frac{1}{\sqrt{2\pi\xi}\sigma_w} e^{-\xi/(2\sigma_w^2)} \text{with } y \geq 0 \tag{A.5}$$

The characteristic function of Ξ can be expressed as;

$$\begin{aligned} \Psi_{\Xi}(jw) &= \int_{-\infty}^{\infty} e^{jw\xi} f_{\Xi}(\xi) d\xi \\ &= \frac{1}{(1 - j2w\sigma_w^2)^{1/2}} \end{aligned} \tag{A.6}$$

If we define our random variable Ξ as

$$\Xi = \sum_{t=1}^{N} W_t^2 \tag{A.7}$$

with the W_t, $t = 1, 2, ...N$ being statistically independent and identically distributed gaussian random variables with zero mean and variance σ_w^2. Thus we obtain the characteristic function of Y as

$$\Psi_{\Xi}(jw) = \frac{1}{(1 - j2w\sigma_w^2)^{N/2}} \tag{A.8}$$

Taking the inverse transform of this, we get the pdf of Ξ as

$$f_{\Xi}(\xi) = \frac{1}{\sigma_w^N 2^{N/2} \Gamma(\frac{1}{2}N)} \xi^{(N/2)-1} e^{-\xi/2\sigma_w^2}, \text{for } \xi \geq 0 \tag{A.9}$$

APPENDIX B

THE DERIVATIONS FOR THE PROPOSED TECHNIQUE

Here we derive the variances of the proposed maximum selection for 3,4,5 and 6 antennas.

For 3 Antennas;

We first derive the statistics for 3 Antennas at the receiver side.

$$j_{max}(n) = arg\max(\max(|x_1(n)|, |x_2(n)|), |x_3(n)|) \tag{B.1}$$

$$\begin{aligned} x_{j_{max}(n)}(n) = \max(\max(|x_1(n)|, |x_2(n)|), x_3(n)) \\ \cup \quad \min(\max(|x_1(n)|, |x_2(n)|), x_3(n)) \end{aligned} \tag{B.2}$$

Let denote $A = arg\max(|x_1|, |x_2|)$.;

$$\begin{aligned} E\left(\max\left(X_A, X_3\right)\right) &= \mu_A \Phi(\frac{\mu_3 - \mu_A}{\theta}) + \mu_3 \Phi(\frac{\mu_A - \mu_3}{\theta}) + \theta\phi(\frac{\mu_3 - \mu_A}{\theta}) \\ &= -\theta\phi(\frac{\mu_A - \mu_3}{\theta}) \end{aligned} \tag{B.3}$$

We know that $\mu_A = 0, \sigma_A^2 = 1.3634\sigma_w^2, \mu_3 = 0, \sigma_3^2 = \sigma_w^2$. Hence;

$$E(\max(\max(|X_1|, |X_2|), X_3)) = \sqrt{\sigma_w^2 + 1.3634\sigma_w^2}\frac{1}{\sqrt{2\pi}} = 0.6133\sigma_w \tag{B.4}$$

$$E\left(\left(\max\left(\max\left(|X_1|, |X_2|\right), X_3\right)\right)^2\right) = \left(\sigma_A^2 + \mu_A^2\right)\Phi\left(\frac{\mu_A - \mu_3}{\theta}\right) \ldots \tag{B.5}$$

$$\begin{aligned} + \left(\sigma_3^2 + \mu_3^2\right)\Phi\left(\frac{\mu_3 - \mu_A}{\theta}\right) - \left(\mu_A + \mu_3\right)\theta\phi\left(\frac{\mu_A - \mu_3}{\theta}\right) \\ = (1.3634)\sigma_w^2.(1/2) + (1/2)\sigma_w^2 = 1.1817\sigma_w^2 \end{aligned} \tag{B.6}$$

$$var\left(\max\left(\max\left(|X_1|,|X_2|\right),X_3\right)\right)= E\left(\left(\max\left(\max(X_1,X_2),X_3\right)\right)^2\right) \tag{B.7}$$
$$-\left(E\left(\max\left(\max(X_1,X_2(n)\right),X_3\right)\right)^2$$
$$= 1.1817\sigma_w^2 - (0.6133\sigma_w)^2 = 0.8056\sigma_w^2$$

$$\begin{aligned} E(\min(\max(|X_1|,|X_2|)),X_3) &= -1.5373.\frac{1}{\sqrt{2\pi}} \\ &= -0.6133\sigma_w \\ E((\min(\max(|X_1|,|X_2|)),X_3)^2) &= 1/2\sigma_w^2 + 0.6817\sigma_w^2 \\ &= 1.1817\sigma_w^2 \\ var(min(max(|X_1|,|X_2|)),X_3) &= 1.1817\sigma_w^2 - \left(0.6133\sigma_w\right)^2 \\ &= 0.8056\sigma_w^2 \end{aligned} \tag{B.8}$$

$$E\left(\max\left(\max\left(|X_1|,|X_2|\right),|X_3|\right)\right)= .... \tag{B.9}$$
$$= E\left(\max\left(\max\left(|X_1|,|X_2|\right),X_3\right)\right)+E\left(\min\left(\max\left(|X_1|,|X_2|\right),X_3\right)\right)$$
$$= 0.6133\sigma_w - 0.6133\sigma_w = 0$$

$$var\left(\max\left(\max\left(|X_1|,|X_2|\right)\right),|X_3|\right)= ... \tag{B.10}$$
$$= var\left(\max\left(\max\left(|X_1|,|X_2|\right),X_3\right)\right)+var\left(\min\left(\max\left(|X_1|,|X_2|\right),X_3\right)\right)$$
$$= 0.8056\sigma_w^2 + 0.8056\sigma_w^2 = 1.6112\sigma_w^2$$

For 4 antennas

After the computations for 3 antennas, we derive the statistics for 4 antennas at the receiver side.

$$x_{j_{max(n)}}(n) = \max\left(\max\left(\max\left(|x_1(n)|,|x_2(n)|\right)\right),|x_3(n)|\right),|x_4(n)|\right) \tag{B.11}$$
$$= \underbrace{\max(\max(\max(|x_1|,|x_2|)),|x_3|),x_4)}_{P} \quad \cup \quad \underbrace{\min(\max(\max(|x_1|,|x_2|)),|x_3|),x_4)}_{Q}$$

Here we need to find $\max(\max(|x_1(n)|, |x_2(n)|, |x_3(n)|, |x_4(n)|))$. In order to find it, let's denote $A = arg\max(|x_1(n)|, |x_2(n)|, |x_3(n))$. Hence;

$$j_{max}(n) = arg\max(|x_A(n)|, |x_4(n)|) \tag{B.12}$$

$$j_{max}(n) = arg\max(x_A(n), x_4(n)) \quad \cup \quad arg\min(x_A(n), x_4(n)) \quad \text{for all regions}$$

We previously computed that $\mu_A = 0$, $\sigma_A^2 = 1.6112\sigma_w^2$ and know that $\mu_4 = 0$ and $\sigma_4^2 = \sigma_w^2$.

Therefore we can calculate the mean and variance for $\max(X_A, X_4)$

$$E(max(X_A, X_4)) = \theta\phi(0) \tag{B.13}$$

We can compute θ as follows

$$\begin{aligned} \theta &= \sqrt{\sigma_A^2 + \sigma_4^2 - 2\rho\sigma_A\sigma_4} \\ &= \sqrt{1.6112\sigma_w^2 + \sigma_w^2 - 0} \\ &= 1.6159\sigma_w \end{aligned} \tag{B.14}$$

and we know $\phi(0) = 1/\sqrt{(}2\pi)$. Therefore

$$\begin{aligned} E(max(X_A, X_4)) &= 1.6159\sigma_w/\sqrt{2\pi}) \\ &= 0.6447\sigma_w \end{aligned} \tag{B.15}$$

To find the variance, we compute $E((\max(x_A(n), x_4(n))^2)$ as follows;

$$E\left(\left(\max\left(X_A, X_4\right)\right)^2\right)=\left(\sigma_A^2+\mu_A^2\right)\Phi\left(\frac{\mu_A-\mu_4}{\theta}\right)+... \tag{B.17}$$

$$...+\left(\sigma_4^2+\mu_4^2\right)\Phi\left(\frac{\mu_A-\mu_4}{\theta}\right)+\left(\mu_A+\mu_4\right)\theta\phi\left(\frac{\mu_A-\mu_4}{\theta}\right)$$

$$=(1.6112\sigma_w^2.(1/2)+\sigma_w^2.(1/2)=1.3056\sigma_w^2 \quad .$$

Then the variance is ;

$$var\left(\max\left(X_A, X_4\right)\right)=1.3056\sigma_w^2-(0.6447\sigma_w)^2 \tag{B.18}$$

$$=0.8901\sigma_w^2$$

Now we need to find the statistics of $\min\left(X_A, X_4\right)$

$$E\left(\min\left(X_A, X_4\right)\right) = -1.6159\sigma_w/\sqrt{2\pi}) \tag{B.19}$$

$$= -0.6447\sigma_w$$

$$E\left(\min\left(X_A, X_4\right)^2\right)=\left(\sigma_A^2+\mu_A^2\right)\Phi\left(\frac{\mu_A-\mu_4}{\theta}\right)+... \tag{B.21}$$

$$...+\left(\sigma_4^2+\mu_4^2\right)\Phi\left(\frac{\mu_A-\mu_4}{\theta}\right)-\left(\mu_A+\mu_4\right)\theta\phi\left(\frac{\mu_A-\mu_4}{\theta}\right)$$

$$=(1.6112\sigma_w^2.(1/2)+\sigma_w^2.(1/2)=1.3056\sigma_w^2 \quad .$$

Then the variance is ;

$$var\left(\min\left(X_A, X_4\right)\right)=1.3056\sigma_w^2-(-0.6447\sigma_w)^2 \tag{B.22}$$

$$=0.8901\sigma_w^2$$

Hence the mean and variance for 4 Antennas is;

$$E\left(\max\left(|X_A, X_4|\right)\right)=0 \tag{B.23}$$

$$var\left(\max\left(|X_A, X_4|\right)\right)=0.8901\sigma_w^2+0.8901\sigma_w^2=1.7802\sigma_w^2$$

For 5 Antennas

Likewise in the previous steps, we derive the statistics for 5 Antennas at the receiver. Here we need to find $\max(\max(|x_1(n)|, |x_2(n)|, |x_3(n)|, |x_4(n)|), |x_5(n)|)$. In order to find it, let's denote $A = arg\max(|x_1(n)|, |x_2(n)|, |x_3(n)|, |x_4(n)|)$. Hence;

$$j_{max}(n) = arg\max(|x_A(n)|, |x_5(n)|) \tag{B.24}$$

$$j_{max}(n) = arg\max(x_A(n), x_5(n)) \quad \cup \quad arg\min(x_A(n), x_5(n))\text{for all regions}$$

We previously computed that $\mu_A = 0$, $\sigma_A^2 = 1.7802\sigma_w^2$ and know that $\mu_5 = 0$ and $\sigma_5^2 = \sigma_w^2$.

Therefore we can calculate the mean and variance for $\max(X_A, X_5)$

$$E(max(X_A, X_5)) = \theta\phi(0) \tag{B.25}$$

We can compute θ as follows

$$\begin{aligned} \theta &= \sqrt{\sigma_A^2 + \sigma_5^2 - 2\rho\sigma_A\sigma_5} \\ &= \sqrt{1.7802\sigma_w^2 + \sigma_w^2 - 0} \\ &= 1.667393\sigma_w \end{aligned} \tag{B.26}$$

and we know $\phi(0) = 1/\sqrt{2\pi}$. Therefore;

$$\begin{aligned} E(max(X_A, X_5)) &= 1.667383\sigma_w/\sqrt{2\pi} \\ &= 0.6651933\sigma_w \end{aligned} \tag{B.27}$$

To find the variance, we compute $E(\max(X_A, X_5)^2)$ as follows;

$$E\left(\left(\max\left(X_A, X_5\right)\right)^2\right) = \left(\sigma_A^2 + \mu_A^2\right)\Phi\left(\frac{\mu_A - \mu_5}{\theta}\right) + ... \tag{B.29}$$

$$... + \left(\sigma_5^2 + \mu_5^2\right)\Phi\left(\frac{\mu_A - \mu_5}{\theta}\right) + \left(\mu_A + \mu_5\right)\theta\phi\left(\frac{\mu_A - \mu_5}{\theta}\right)$$

$$= (1.7802\sigma_w^2.(1/2) + \sigma_w^2.(1/2) = 1.3901\sigma_w^2 \quad .$$

Then the variance is ;

$$var\left(\max\left(X_A, X_5\right)\right) = 1.3901\sigma_w^2 - (0.6651933\sigma_w)^2 \tag{B.30}$$

$$= 0.9476\sigma_w^2$$

Now we need to find the statistics of $\min\left(x_A(n), x_5(n)\right)$

$$E\left(\min\left(X_A, X_5\right)\right) = -1.667383\sigma_w/\sqrt{2\pi} \tag{B.31}$$

$$= -0.6651933\sigma_w$$

$$E\left(\left(\min\left(X_A, X_5\right)\right)^2\right) = \left(\sigma_A^2 + \mu_A^2\right)\Phi\left(\frac{\mu_A - \mu_5}{\theta}\right) + ... \tag{B.32}$$

$$... + \left(\sigma_5^2 + \mu_5^2\right)\Phi\left(\frac{\mu_A - \mu_5}{\theta}\right) - \left(\mu_A + \mu_5\right)\theta\phi\left(\frac{\mu_A - \mu_5}{\theta}\right)$$

$$= (1.7802\sigma_w^2.(1/2) + \sigma_w^2.(1/2) = 1.3901\sigma_w^2 \quad .$$

Then the variance is ;

$$var\left(\min\left(X_A, X_5\right)\right) = 1.3901\sigma_w^2 - (0.6651933\sigma_w)^2 \tag{B.33}$$

$$= 0.9476\sigma_w^2$$

Hence the mean and variance for 5 Antennas is;

$$E\left(\max\left(|X_A, X_5|\right)= 0\right. \tag{B.34}$$
$$var\left(\max\left(|X_A, X_5|\right)= 0.9476\sigma_w^2 + 0.9476\sigma_w^2 = 1.8952\sigma_w^2\right.$$

For 6 Antennas;

Finally we compute the statistics for 6 antennas at the receiver side. Here we need to find :

$\max\left(\max\left(\max\left(|x_1(n)|, |x_2(n)|, |x_3(n)|, |x_4(n)|\right), |x_5(n)|\right)|x_6(n)|\right)$. In order to find it, let's denote $A = \max\left(|x_1(n)|, |x_2(n)|, |x_3(n)|, |x_4(n)|, |x_5(n)|\right)$. Hence;

$$j_{max}(n) = arg\max\left(|x_A(n)|, |x_6(n)|\right) \tag{B.35}$$
$$j_{max}(n) = arg\max\left(x_A(n), x_6(n)\right) \quad \cup \quad arg\min\left(x_A(n), x_6(n)\right)\text{for all regions}$$

We previously computed that $\mu_A = 0$, $\sigma_A^2 = 1.8952\sigma_w^2$ and know that $\mu_6 = 0$ and $\sigma_6^2 = \sigma_w^2$. Therefore we can calculate the mean and variance for $\max\left(x_A(n), x_6(n)\right)$

$$E\left(max\left(X_A, X_6\right)\right)= \theta\phi(0) \tag{B.36}$$

We can compute θ as follows

$$\begin{aligned}
\theta &= \sqrt{\sigma_A^2 + \sigma_6^2 - 2\rho\sigma_A\sigma_5} && \text{(B.37)}\\
&= \sqrt{1.8952\sigma_w^2 + \sigma_w^2 - 0} && \text{(B.38)}\\
&= 1.7015\sigma_w && \text{(B.39)}
\end{aligned}$$

and we know $\phi(0) = 1/\sqrt{2\pi}$. Therefore

$$\begin{aligned}
E\left(max\left(X_A, X_6\right)\right) &= 1.7015\sigma_w/\sqrt(2\pi)) && \text{(B.40)}\\
&= 0.678\sigma_w && \text{(B.41)}
\end{aligned}$$

To find the variance, we compute $E\left(\max\left(X_A, X_6\right)^2\right)$ as follows;

$$E\left(\left(\max\left(X_A, X_6\right)\right)^2\right) = \left(\sigma_A^2 + \mu_A^2\right)\Phi\left(\frac{\mu_A - \mu_6}{\theta}\right) + ... \tag{B.42}$$
$$... + \left(\sigma_6^2 + \mu_6^2\right)\Phi\left(\frac{\mu_A - \mu_6}{\theta}\right) + \left(\mu_A + \mu_6\right)\theta\phi\left(\frac{\mu_A - \mu_6}{\theta}\right)$$
$$= (1.8952\sigma_w^2.(1/2) + \sigma_w^2.(1/2) = 1.4476\sigma_w^2 \quad .$$

Then the variance is ;

$$var\left(\max\left(X_A, X_5\right) = 1.4476\sigma_w^2 - (0.678\sigma_w)^2 \tag{B.43}$$
$$= 0.9879\sigma_w^2$$

Now we need to find the statistics of $\min\left(X_A, X_6\right)$

$$E\left(\min\left(X_A, X_6\right)\right) = \theta\phi(0) \tag{B.44}$$

$$E\left(\min\left(X_A, X_6\right)\right) = -1.7015\sigma_w/\sqrt{2\pi} \tag{B.45}$$
$$= -0.678\sigma_w \tag{B.46}$$

To find the variance, we compute $E\left(\min\left(X_A, X_6\right)^2\right)$ as follows;

$$E\left(\left(\min\left(X_A, X_6\right)\right)^2\right) = \left(\sigma_A^2 + \mu_A^2\right)\Phi\left(\frac{\mu_A - \mu_6}{\theta}\right) + ... \tag{B.48}$$
$$... + \left(\sigma_6^2 + \mu_6^2\right)\Phi\left(\frac{\mu_A - \mu_6}{\theta}\right) - \left(\mu_A + \mu_6\right)\theta\phi\left(\frac{\mu_A - \mu_6}{\theta}\right)$$
$$= (1.8952\sigma_w^2.(1/2) + \sigma_w^2.(1/2) = 1.4476\sigma_w^2 \quad .$$

Then the variance is ;

$$var\left(\max\left(X_A, X_5\right)\right) = 1.4476\sigma_w^2 - (0.678\sigma_w)^2 \tag{B.49}$$
$$= 0.9879\sigma_w^2$$

Hence the mean and variance for 6 Antennas is;

$$E\left(\max\left(|X_A, X_5|\right)\right) = 0 \tag{B.50}$$
$$var\left(\max\left(|X_A, X_5|\right)\right) = 0.9868\sigma_w^2 + 0.9868\sigma_w^2 = 1.9758\sigma_w^2$$