# STACKING SEQUENCES OPTIMIZATION OF THE ANTI-BUCKLED LAMINATED COMPOSITES CONSIDERING VARIOUS FAILURE CRITERIA

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# ABSTRACT

## STACKING SEQUENCES OPTIMIZATION OF THE ANTI-BUCKLED LAMINATED COMPOSITES CONSIDERING VARIOUS FAILURE CRITERIA

In recent years, fiber-reinforced composite materials have been mostly used in engineering applications due to advantage of the ratio of strength to weight. Fiber-reinforced laminated composites with an optimum stacking sequences have become critical issue especially for defence and automotive industry. In this study, stacking sequences optimization of laminated composites for maximum buckling load factor has been investigated using genetic algorithm (GA). Symmetrical and balanced laminated composite plates with 48 layers graphite/epoxy are considered for optimization process. The designs of composite plates have been investigated for various in-plane loadings and aspect ratios. Fiber orientation angles are chosen as design variables. The optimum designs obtained have been controlled by Tsai-Wu and maximum stress failure criteria. Furthermore, dispersed designs for specific cases have been converted to conventional designs and the advantages and disadvantages of various designs have been examined in terms of buckling resistance. Finally, buckling behaviors of 48- and 64-layered composite plates have been studied under overloaded conditions. In design process, the increase in the reliability of the optimization has been provided independently using a variety of genetic algorithm parameters. All the results have shown that the loading conditions and dimensions of composite plates are significant in stacking sequences optimization of laminated composite materials in terms of maximum critical buckling load factor. Furthermore, it has been seen that the fiber orientation angles determine which failure modes (buckling or static failure criteria) are critical.

# ÖZET

## ÇEŞİTLİ KIRILMA KRİTERLERİ GÖZETİLEREK BURKULMAYAN TABAKALI KOMPOZİTLERİN TABAKA DİZİLİMLERİNİN OPTİMİZASYONU

Son yıllarda, fiber takviyeli kompozit malzemeler diğer malzemelere göre yüksek dayanıklılığa sahip ve hafif olmasından dolayı birçok mühendislik uygulamalarında kullanılmaktadır. Bu sebeple tabakalı kompozitlerin optimizasyonu, özellikle savunma ve otomotiv endüstrisi için kritik bir öneme sahiptir. Bu çalışmada, tabakalı kompozit plakada maksimum burkulma yük kapasitesini elde edebilmek için tabaka dizilimleri genetik algoritma optimizasyon yöntemi kullanılarak bulunmuştur. 48 tabakalı grafit /epoksi kompozit plakalar, balans ve simetrik bir yapıya sahip olup farklı en-boy oranlarında ve yükleme koşullarında burkulma davranışları incelenmiştir. Fiber yönlenme açıları da tasarım parametresi olarak düşünülüp elde edilen optimum tasarımlar Tsai-Wu ve maksimum gerilme kırılma teorileriyle kontrol edilmiştir. Ayrıca bazı yükleme koşullarında elde edilen fiber açı değerleri endüstride üretim için kullanılan fiber açı değerlerine $(90, \pm 45, 0)$ dönüştürülerek sonuçlar kompozit plakanın burkulma dayanıklılığı açısından karşılaştırılmıştır. Son olarak 48 ve 64 tabakalı kompozit plakaların yüksek yükleme ve farklı en-boy oranı esas alınarak burkulma davranışı incelenmiştir. Optimizasyon sonuçlarının güvenirliliğini artırmak için genetik algoritmanın farklı program parametreleri kullanılmıştır. Sonuçlar yükleme koşullarının ve plaka ölçülerinin tabakalı kompozitlerin maksimum burkulma yük kapasitesi açısından kritik önem taşıdığı ve fiber yönlenme açılarınında kırılmanın burkulmadan veya kırılma kriterlerinden kaynaklandığını belirlemede etkili olduğunu göstermektedir.

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# CHAPTER 1

# INTRODUCTION

## 1.1. Literature Review

In recent years, fiber-reinforced composite materials are widely used in many engineering applications including aircraft, yachts, motor vehicles, chemical process, sporting goods and military equipment because of their specific advantages as stiffness to weight and strength to weight. As the other advantage, many parameters like fiber orientations, ply thickness, stacking sequence, volume fraction of reinforcement, etc. can be altered according to designer's needs. These parameters change mechanical and thermal properties of composite materials. Micromechanics of composite material are based on volume fraction of matrix and fiber and determine tensile, compressive and shear strength. Configuration of laminates also plays an important role in determining weight minimization, cost analysis, resistance to the environment effects and buckling, etc. Therefore, design variables are too significant for optimization process.

Optimization provides engineers and researchers with a possibility that obtain the best design among the number of designs. Several methods have been developed for different fiber-reinforced composite structures like sandwich beam, laminate composite beam, column, etc (Awad, et al. 2012). The improved methods considering experimental results, numerical modeling, and design constraints are presented in their study for design optimization of composite structures. Various optimization methods are proposed and compared. Table 1.1 lists comparison of different optimization methods.

Table 1.1. Comparison of the optimization methods
(Source:Awad, et al. 2012)

| Method | Objective | Probability | Uncontrolled parameters | Free derivative | Solution cost | Optimum solution remark | Overall ranking |
|---|---|---|---|---|---|---|---|
| DSA | Single | x | x | x | Moderate | Discrete and continuous variables | Low |
| GA | Multi objective | √ | x | √ | Low in parallel optimization | Global | High |
| SA | Multi objective | √ | x | √ | Low | Multiple global optimum | Moderate |
| RBDO | Multi objective | √ | x | x | High | Convergence difficulties | Moderate |
| PSOA | Multi objective | x | x | √ | Less than GA for single objective | Global Convergence difficulties | High |
| ACO | Multi objective | √ | x | √ | Moderate | Good performance | Moderate |
| MRDO | Multi objective | √ | √ | √ | High | Enhance the design objectives | High |

The methods given in Table 1.1 are defined as: DSA: Design sensitivity analysis, GA: Genetic algorithm, SA: Simulate annealing method, RBDO: Reliability based design optimization, PSOA: Particle swarm optimization algorithm, ACO: Ant colony optimization, MRDO: Multi-objective robust design optimization.

Many researchers tried to make a better laminated composite material either by increasing static strength of composite laminates or reducing the weight for a given thickness. Especially, genetic algorithm, which is stochastic optimization method, was firstly studied to obtain the minimum thickness design of composite laminated plate by Riche and Haftka (1995). They improved selection, mutation, and permutation operators. Constraints were formed into objective function as a penalty function. In addition, Soremekun et al. (2001) solved two different composite laminate design problems; (i) maximization of the twisting displacement of a cantilever composite plate and (ii) buckling load maximization of a simply supported laminated composite plate. The results were analyzed by two types of genetic algorithm which are standard genetic algorithm (SGA) and generalized elitist genetic algorithm (GEGA). As a similar study, Liu et al. (2000) presented permutation genetic algorithms optimizing the stacking sequence of a composite laminate for maximum buckling load. Permutation genetic algorithms are mostly used in order to solve scheduling problems and also they decrease the dimensionality of the design space. For this reason, it was shown that the permutation genetic algorithm was more efficient than a standard genetic algorithm for stacking sequence optimization of a composite laminate in their study.

Buckling analyses are very critical issue for thin and large composite plates, especially in aircraft design. Therefore, researchers have studied the optimum design of composite laminates for buckling load. Erdal and Sonmez (2005) presented optimization of laminated composite plate for maximum buckling load using simulated annealing that is one of the most popular stochastic optimization techniques. Soykasap and Karakaya (2007) and Karakaya and Soykasap (2009) investigated buckling optimization of laminated composite plates using genetic algorithm and generalized pattern search algorithm. In these studies, the critical buckling load was determined for several load cases (biaxial load) and different plate aspect ratios. However, not only uniaxial and biaxial loadings but also pure shear loading and the combination of shear and biaxial loadings change the optimal solutions for maximum buckling load. Kim and Lee (2005) showed the optimal and the worst case designs for composite plates having different aspect ratios (length to width), load conditions (uniaxial, biaxial, shear loads) and number of plies. In addition, Sebaey et al. (2011) studied optimization of composite panels for the critical buckling load under strength constraints and failure index minimization including the matrix cracking and the fiber tensile failure using ant colony optimization algorithm. They also showed the benefits of dispersed laminates over conventional ones.

Optimization of the laminated composite plate generally involves strength analyses to control the feasibility of the design process. Failure criteria can be used as strength constraints to obtain optimal designs of laminated composite plates. Maximum stress, Tsai-Wu and Puck failure criteria were independently analyzed for the minimum weight and the minimum material cost of laminated composite plates by Lopez et al. (2009). They used the ply orientations, the number of layers and the layer material as the design variables. The optimization problems were solved by genetic algorithm. Narayana Naik et al. (2008) presented the minimum weight design of composite laminates using the failure mechanism based, maximum stress and Tsai-Wu failure criteria. They showed the effectiveness of the new failure mechanism based failure criterion. It involves fiber breaks, matrix cracks, fiber compressive failure and matrix crushing. Akbulut and Sonmez (2008) investigated the optimal thickness of laminated composite plates subject to in-plane loadings. Direct search simulated annealing, which is a global search algorithm, was utilized to find the optimal design. In order to check the static failure, Tsai-Wu and maximum stress criteria were used for different loading cases. Fiber orientation angles in each layer and layer thickness were taken as design variables. Akbulut and Sonmez (2011) also studied the thickness (or weight) minimization of laminated composite plates subjected to both in-plane and out-of-plane loading using new variant of the simulated annealing algorithm. As the failure mode, maximum stress and Tsai-Wu failure criteria were used to predict failure. Soden et al. (1998) analyzed various failure theories for fiber-reinforced composites. The results were compared and failure theories were identified according to method of analysis, type of analyses, thermal stresses, modes of failure, micromechanics, post-initial failure degradation models, failure criterion and computer programs.

Except genetic algorithm and simulated annealing algorithm, there are several optimization methods to obtain the optimal design for laminated composite materials. Walker and Smith (2003) improved the multi-objective optimization using finite element analysis and genetic algorithm. In this study, both the mass and deflection of laminated composite were minimized and Tsai-Wu failure criterion was chosen as a design variable. Furthermore, Aymerich and Serra (2008) studied the ant colony optimization algorithm for maximum buckling load with strength constraint. Ant colony optimization algorithm is a class of approximate heuristic search technique and it was based on the ants' ability to find the shortest paths between their nest and the food source. The results were compared with genetic algorithm and tabu search algorithm. Chang et al. (2010) proposed the maximum buckling load factor of laminate having 0, $\pm 45$, 90 angles under different loading conditions by using permutation discrete particle swarm optimization (PDPSO). The algorithm was inspired from behavior of a bird flock in searching for food. The results of PDPSO were compared with standard discrete particle swarm optimization (SDPSO), gene rank crossover (GR) and partially mapped crossover (PMX). It was shown that PDPSO was more efficient than other methods.

## 1.2. Objectives

In this study, optimal stacking sequence designs of laminated composite plates for maximum buckling load are determined using genetic algorithm (GA). Symmetric and balanced composite plates with 48 layers graphite/epoxy, which are simply supported on four sides, are analyzed under different load conditions and aspect ratios (length to width). Fiber orientation angle in each layer is taken as a design variable. Design constraints are based on Tsai-Wu and maximum stress failure criterion.

The purpose of this thesis can be listed as follows;

- To maximize the load that a composite laminate can sustain without buckling failure.
- To prevent failure of each lamina from compressive in-plane loadings by using Tsai-Wu and maximum stress failure criterion.
- Comparison of the stacking sequence designs and buckling load factors for different failure criteria.
- To determine buckling load factor under different load conditions and aspect ratios.
- Comparison of conventional and dispersed optimum stacking sequences for various loading ratios and aspect ratios.
- Determination of buckling behavior of composite plates with 48 layers and 64 layers.
- To perform combination of MATLAB *Genetic Algorithm Toolbox* and *Symbolic Math Toolbox*.
- To improve performance of genetic algorithm using other parameters like population, crossover, mutation, migration and reproduction, etc.

## 1.3. Design Approach for Composite Materials

The design of composite materials is a combined process including material selection, process specification, optimization of laminate configuration, and design of the structural components. Design objectives are based on the structural application. Specific application requirements determine one or a combination of two or more basic design objectives (Daniel and Ishai 1994):

- Design for stiffness
- Design for strength (static and fatigue)
- Design for dynamic stability
- Design for environmental stability
- Design for damage tolerance

The various design objectives, structural requirements, typical materials and applications are given in Table 1.2. In this thesis study, stiffness is considered as design objective, and therefore graphite fiber composite are chosen as design material for high buckling load and small deflection.

Table 1.2. Design Approach for Composite Materials
(Source: Daniel and Ishai 1994)

| Design Objective | Requirements | Materials | Applications |
|---|---|---|---|
| **Stiffness** | • Small Deflection<br>• High Buckling Load<br>• High flexural rigidity<br>• Low Weight | Carbon fiber composite<br>Kevlar fiber composite<br>Boron fiber composite<br>Graphite fiber composite | Aircraft control surfaces<br>Underground Vessels<br>Sporting Goods<br>Underwater Vessels<br>Thin skins in compressions |
| **Strength** | • High Load Capacity<br>• Low Weight<br>• High Inter-laminar Strength | Carbon fiber composite<br>Kevlar fiber composite<br>S-Glass fiber composite | Joints<br>Trusses<br>Pressure Vessels |
| **Dynamic Control and Stability** | • Long Fatigue Life<br>• High Resonance Frequency | Carbon, graphite fibers<br>Thermoplastic matrices | Engine Components<br>Rotor blades<br>Flywheels |
| **Dimensional Stability** | • Low coefficient of thermal and moisture expansion<br>• High Stiffness ($E_x$, $E_y$) | Carbon fiber composite<br>Kevlar fiber composite<br>Graphite fiber composite | Space Antennae<br>Satellites<br>Solar Reflectors |
| **Damage Tolerance** | • High impact resistance<br>• High fracture toughness<br>• Energy absorbent interlayers | Tough epoxy matrix<br>Thermoplastic matrices | Ballistic Armor<br>Impact Resistant Structures |

# CHAPTER 2

# COMPOSITE MATERIALS

## 2.1. Definition and Basic Characteristics

A structural composite is a kind of material system combining two or more materials on a macroscopic scale, so they have better mechanical properties than conventional materials, such as, metals. Some of these mechanical properties like strength, stiffness, fatigue and impact resistance, thermal conductivity, corrosion resistance, etc. can be improved depending on designer's needs. Composite materials are generally made from two materials: One of that is usually discontinuous, stiffer, and stronger and is called fiber (reinforcement material), the other base material that is less stiff and weaker is called matrix.

The properties of a composite material depend on the properties of the constituents (matrix and fiber), geometry and distribution of the materials. One of the most important parameters for composite materials is volume fraction and it shows fiber volume ratio of composite structure. Volume fraction can be determined by burn-out test. The distribution of reinforcing fibers determines homogeneity or heterogeneity. The more heterogeneity areas of composite structure, the higher are the possibility of failure in the weakest areas. The fiber orientation and geometry cause the isotropy or orthotropy. If the composite is isotropic, the material properties such as stiffness, strength, thermal expansion and thermal conductivity are the same in all directions. A material is anisotropic when material properties at a point vary with direction or fiber orientation (Daniel and Ishai 1994).

The materials of the composite system have different roles that depend on the type and application of composite material. For low or medium performance composite material, short fibers or particles are usually used. They provide low production cost and easy to work with. There are fewer flaws, so they have higher strength. On the other hand, the matrix is main load bearing material and it affects many mechanical properties of composite material including transverse modulus and strength, shear modulus and strength, compressive strength, interlaminar shear strength, thermal expansion coefficient, thermal resistance, and fatigue strength. For high performance composite materials, the continuous fiber reinforcement that is easy to orient and process is the most important part of the composite structure, because it has impact resistance and improved dimensional stability. The matrix material provides protection for sensitive fibers and distributes loads from one fiber to another (Kaw 2006).

## 2.2. Overview of Advantages and Drawbacks of Composite Materials

Composite materials have many advantages compared to traditional materials such as high strength, long fatigue life, low density, and high impact resistance (Table 2.1). In addition, composite materials provide us with many degrees of freedom for optimum configuration of the material (Daniel and Ishai 1994).

Table 2.1. Specific Modulus and Specific Strength of Typical Fibers, Composites, and Bulk Metals (Source: Kaw 2006)

| Material Units | Specific gravity | Young's modulus (GPa) | Ultimate strength (MPa) | Specific modulus (GPa-m³/kg) | Specific strength (MPa-m³/kg) |
|---|---|---|---|---|---|
| *System of Units: SI* | | | | | |
| Graphite fiber | 1.8 | 230.00 | 2067 | 0.1278 | 1.148 |
| Aramid fiber | 1.4 | 124.00 | 1379 | 0.08857 | 0.9850 |
| Glass fiber | 2.5 | 85.00 | 1550 | 0.0340 | 0.6200 |
| Unidirectional graphite/epoxy | 1.6 | 181.00 | 1500 | 0.1131 | 0.9377 |
| Unidirectional glass/epoxy | 1.8 | 38.60 | 1062 | 0.02144 | 0.5900 |
| Cross-ply graphite/epoxy | 1.6 | 95.98 | 373.0 | 0.06000 | 0.2331 |
| Cross-ply glass/epoxy | 1.8 | 23.58 | 88.25 | 0.01310 | 0.0490 |
| Quasi-isotropic graphite/epoxy | 1.6 | 69.64 | 276.48 | 0.04353 | 0.1728 |
| Quasi-isotropic glass/epoxy | 1.8 | 18.96 | 73.08 | 0.01053 | 0.0406 |
| Steel | 7.8 | 206.84 | 648.1 | 0.02652 | 0.08309 |
| Aluminum | 2.6 | 68.95 | 275.8 | 0.02652 | 0.1061 |

On the other hand, there are limitations for composite materials. These limitations can be listed as follows;

**Cost Effectiveness:** High cost of fabrication of composites is an important issue. Composite production still depends on skilled hand labor with limited mass production and standardization. Because of that, quality control procedures are too difficult to apply. Improvements in processing and manufacturing techniques will be lower these costs and control production parameters in the future.

**Mechanical Characterization:** The analysis of composite structure is more complex than a metal structure. Unlike metals, composite materials are not isotropic, so their mechanical properties are not the same in all directions. They have many measure parameters. For instance a single layer of a graphite/epoxy composite requires nine stiffness and strength constants for mechanical analysis. However, mechanical characterization is simple for conventional materials and they have two elastic constants and two strength parameters.

**Maintainability and Durability:** Repair of composite materials is difficult process compared with metals. Sometimes, defects and cracks in the composite structure may not be realized. Composite materials don't have a high combination of strength and fracture toughness compared to metals. On the other hand, metals show better combination of strength and fracture toughness than composites ( Figure 2.1).

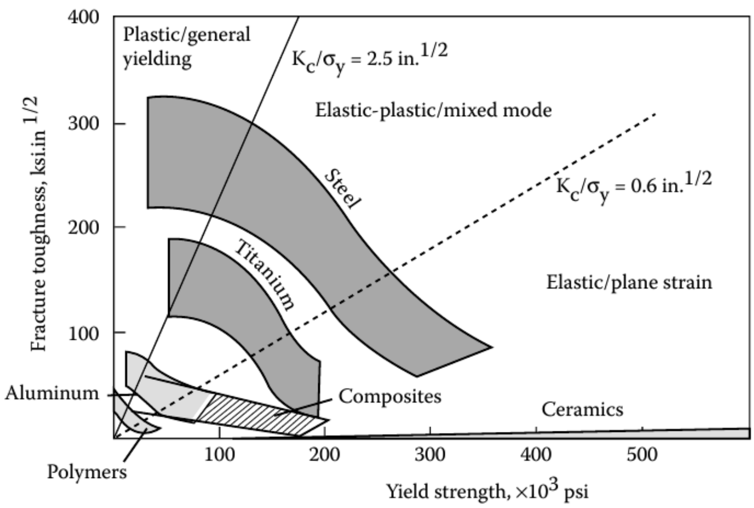



Figure 2.1. Fracture toughness as a function of yield strength for monolithic metals, ceramics, and metal–ceramic composites. (Source: Eager 1991)

Composites can't give higher performance in all the properties used for material selection. In Figure 2.2, six primary material selection parameters: strength, toughness, formability joinability, corrosion resistance, and affordability are presented. Obviously, composites show better strength than metals, but lower values for other material selection parameters.

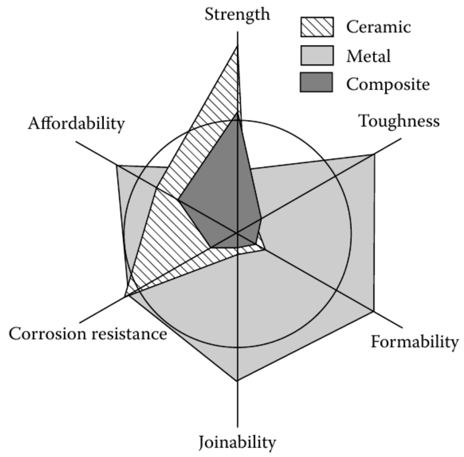



Figure 2.2. Primary material selection parameters for a hypothetical situation for metals, ceramics, and metal–ceramic composites. (Source: Eager 1991)

## 2.3. Types and Classification of Composite Materials

Composite materials are classified by the type (particulate composites), geometry (discontinuous or short-fiber composites) and orientation (continuous fiber composites) of the reinforcement material as shown in Figure 2.3.

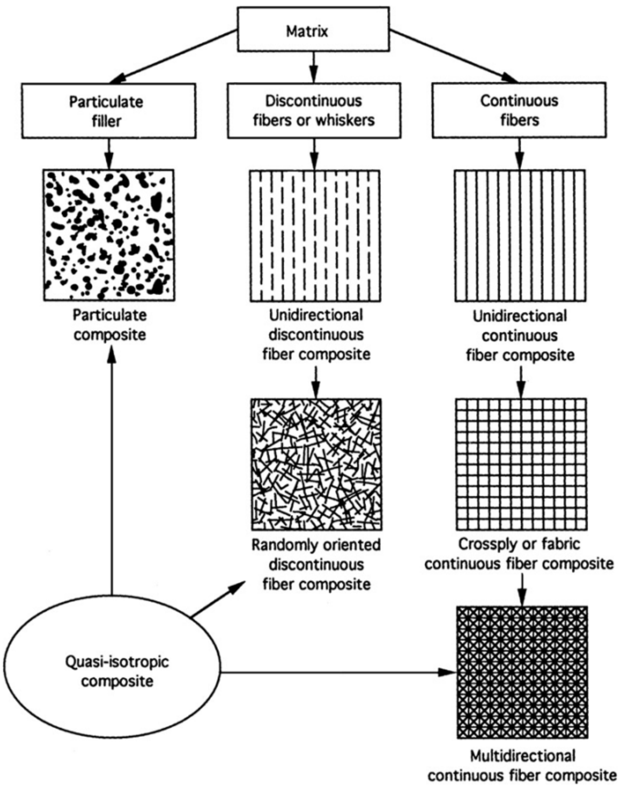



Figure 2.3. Classification of composite material systems
(Source: Daniel and Ishai 1994)

Particular composites consist of particles of different sizes and shapes immersed in the matrices such as alloys and ceramics. They are isotropic because of the randomness of particle distribution. Particular composites can be classified into four groups. These are nonmetallic particles in a nonmetalic matrix (glass feinforced with mica flakes), metallic particles in nonmetallic matrices (aluminium particles in polyurethane), metallic particles in metallic matrices (lead particles in copper alloys) and nonmetallic particles in metallic matrices (silicon carbide particles in aluminium). Particulates cause improved strength, increased operating temperature, oxidation resistance of composite materials.

Fiber composites consist of matrices reinforced by short (discontinuous) and long (continuous) fibers. Short fibers can be all oriented along one direction or randomly oriented. Continuous fibers can be all parallel (unidirectional continuous fiber composite), oriented at right angles to each other (crossply or fabric continuous fiber composite) or oriented along several directions (multidirectional continuous fiber composite). Continuous fibers affect the strength and stiffness of composite structures directly.

Fiber-reinforced composites can be classified into wide categories according to the matrix used such as polymer, metal, ceramic and carbon matrix composites (Table 2.2).

Table 2.2. Types of Composite Materials
(Source: Daniel and Ishai 1994)

| **Matrix type** | **Fiber** | **Matrix** |
|---|---|---|
| Polymer | E-glass<br>S-glass<br>Carbon (graphite)<br>Aramid (Kevlar)<br>Boron | Epoxy<br>Polyimide<br>Polyester<br>Thermoplastics<br>(PEEK, polysulfone, etc.) |
| Metal | Boron<br>Borsic<br>Carbon (graphite)<br>Silicon carbide<br>Alumina | Aluminum<br>Magnesium<br>Titanium<br>Copper |
| Ceramic | Silicon carbide<br>Alumina<br>Silicon nitride | Silicon carbide<br>Alumina<br>Glass-ceramic<br>Silicon nitride |
| Carbon | Carbon | Carbon |

The most common advanced composites are polymer matrix composites including a thermoset (e.g., epoxy, polyimide, polyester) or thermoplastic (poly-ether-ether-ketone, polysulfone) reinforced by thin diameter fibers (e.g., graphite, aramids, boron). These composites have high strength, simple manufacturing technique and low cost. Metal matrix composites consist of metals or alloys (aluminum, magnesium, titanium, copper) reinforced with boron, carbon (graphite) or ceramic fibers. The materials are widely used to provide advantages over metals such as steel and aluminum. These advantages include higher specific strength and modulus by low density metals such as aluminum and titanium, lower coefficients of thermal expansion, such as graphite. Ceramic matrix composites have ceramic matrices (silicon carbide, aluminum oxide, glass-ceramic, silicon nitride) reinforced with ceramic fibers. Advantages of ceramic matrix composites are high strength, hardness, high service temperature limits for ceramics, chemical inertness and low density. However fracture toughness is low for ceramic matrix composites. Carbon-carbon composites use carbon fibers in the carbon or graphite matrix. They have excellent properties of high strength at high temperature, low thermal expansion and low density.

Comparison of conventional matrix materials is shown in Table 2.3 and Table 2.4, each type of matrix and fiber has their advantages and drawbacks.

Table 2.3. Comparison of Conventional Matrix Materials
(Source: Daniel and Ishai 1994)

| **Property** | **Metals** | **Ceramics** | | **Polymers** |
|---|---|---|---|---|
| | | **Bulk** | **Fibers** | |
| Tensile strength | + | − | ++ | v |
| Stiffness | ++ | v | ++ | − |
| Fracture toughness | + | − | v | + |
| Impact strength | + | − | v | + |
| Fatigue endurance | + | v | + | + |
| Creep | v | v | ++ | − |
| Hardness | + | + | + | − |
| Density | − | + | + | ++ |
| Dimensional stability | + | v | + | − |
| Thermal stability | v | + | ++ | − |
| Hygroscopic sensitivity | ++ | v | + | v |
| Weatherability | v | v | v | + |
| Erosion resistance | + | + | + | − |
| Corrosion Resistance | − | v | v | + |

++, superior; +, good; –, poor; v, variable.

Table 2.4. Advantages and Disadvantages of Reinforcing Fibers
(Source: Daniel and Ishai 1994)

| Fiber | Advantages | Disadvantages |
|---|---|---|
| E-glass, S-glass | High strength<br>Low cost | Low stiffness<br>Short fatigue life<br>High temperature sensitivity |
| Aramid (Kevlar) | High tensile strength<br>Low density | Low compressive strength<br>High moisture absorption |
| Boron | High stiffness<br>High compressive strength | High cost |
| Carbon (AS4, T300, C6000) | High strength<br>High stiffness | Moderately high cost |
| Graphite (GY-70, pitch) | Very high stiffness | Low strength<br>High cost |
| Ceramic (silicon carbide, alumina) | High stiffness<br>High use temperature | Low strength<br>High cost |

In this study, graphite/epoxy composite material ( T300/5208) is considered for design materials.

## 2.4. Significance, Objectives and Applications of Composite Materials

The study of composites is an important issue of material design to obtain the optimum composition along same or different constituent materials. The basic parts for research and technology of composite materials consist of the following tasks (Daniel and Ishai 1994).

- Investigation of basic characteristics of the constituent and composite materials.
- Material optimization for given loading conditions.
- Development of effective and efficient production methods and understanding of their effect on material properties.
- Development of analytical and experimental methods in order to determine material properties, stress analysis and failure analysis.
- Assessment of durability, flaw criticality, and life prediction.

Commercial and industrial applications of fiber-reinforced polymer composites are various because many fiber-reinforced polymers have better combination of strength and modulus than traditional metallic materials. The application areas of composite materials are aircraft, space, automotive, sporting goods, marine and military. Figure 2.4 shows the relative market share of US composite shipments and it has seen that major application of composite materials is transportation. Furthermore, fiber-reinforced polymer composites are used in electronics (e.g., printed circuit boards), building construction (e.g., floor beams), furniture (e.g., chair spring s), power industry (e.g., transformer housing), oil industry (e.g., offshore oil platforms and oil sucker rods used in lifting underground oil), medical industry (e.g., bone plates for fracture fixation, implants, and prosthetics), and in many other industrial products (Kaw 2006).

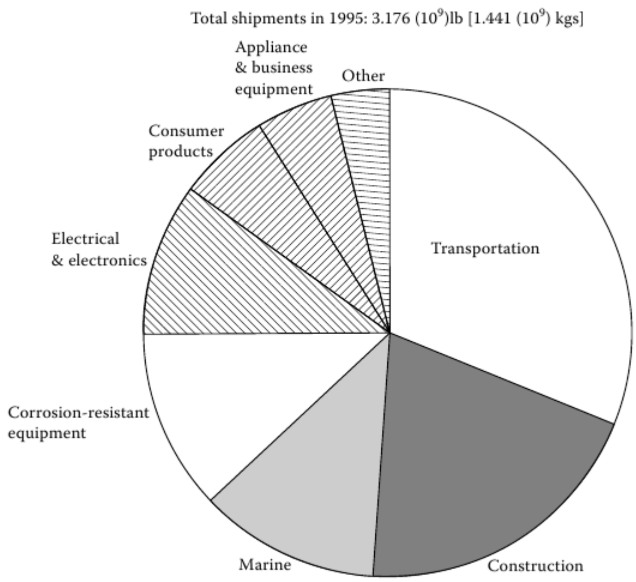



Figure 2.4. Approximate shipments of polymer-based composites
(Source: Kaw 2006)

Fiber-reinforced composites are used in the field of military and commercial aircrafts because they can be both lightweight and strength. The heavier an aircraft weighs, the more fuel it burns, so weight minimization is important for military and commercial aircrafts. Figure 2.5 shows the use of composite material in Boeing 787. With the use of carbon fibers in the 1970s, carbon fiber-reinforced composites have become the primary material in many wing, fusel age, and empennage components. The structural integrity and durability of components have increased confidence in their performance and developments of other structural aircraft components, so increasing amount of composite materials are used in military aircrafts. For example, the F-22 fighter aircraft also contains 25% by weight of carbon fiber reinforced polymers; the other major materials are titanium (39%) and aluminum (16%). The stealth aircrafts are almost all made of carbon fiber-reinforced polymers because of design features that have special coatings, reduce radar reflection and heat radiation. Furthermore, many fiber-reinforced polymers are used in military and commercial helicopters for making baggage doors, fairings, vertical fins, tail rotor spars and so on (Mallick 2007).

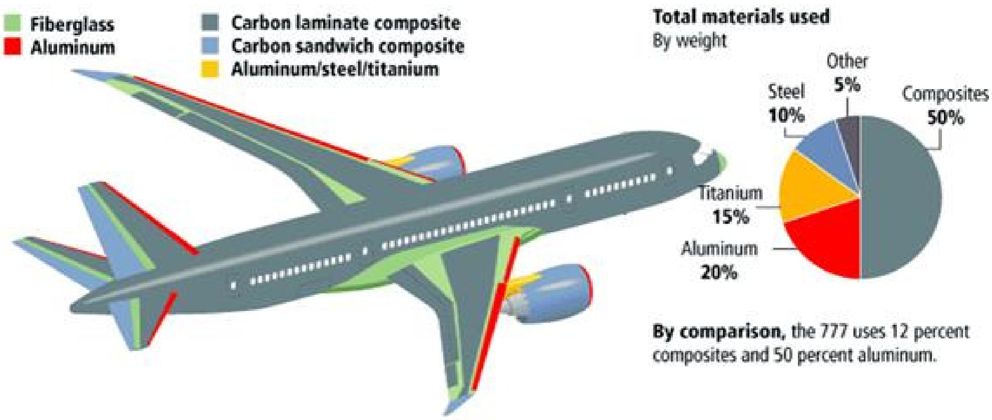



Figure 2.5. Composite Materials used in Boeing 787 body
(Source: Bintang, 2011)

Weight reduction is the primary reason in order to use fiber-reinforced composites in many space vehicles. Furthermore, composite materials are preferred due to high specific modulus and strength, and dimensional stability during large changes in temperature in space. The main applications in the structure of space shuttles are the mid-fuselage truss structure (boron fiber-reinforced aluminum tubes), payload bay door (sandwich laminate of carbon fiber-reinforced epoxy face sheets and aluminum honeycomb core), remote manipulator arm (ultrahigh-modulus carbon fiber-reinforced epoxy tube), and pressure vessels (Kevlar 49 fiber-reinforced epoxy).

The most popular application for composite materials is an automotive industry. Body components, chassis components and engine components are three main components in automotive industry. Body components such as the hood or door panels must have high stiffness and damage tolerance (dent resistance), so the composite material used for these components is E-glass fiber-reinforced sheet molding compound (SMC) composites. For the chasis components, the first structural application of fiber-reinforced composites is rear leaf spring. Unileaf E-glass fiber-reinforced epoxy springs have been used instead of multileaf steel springs with as much as 80% weight reduction. Other chassis components, such as drive shafts and wheels have been successfully tested, but they have been produced in limited quantities. The application of fiber reinforced composites in engine components has not been as successful as the other components (Mallick 2007).

# CHAPTER 3

# MECHANICS OF COMPOSITE MATERIALS

The mechanics of materials are related to stress, strain, and deformations in engineering structures subjected to mechanical and thermal loads. Many parameters and formulations such as moment, stiffness are derived from stress-strain relations. A common opinion for mechanics of conventional materials such as steel and metals is that they have homogeneous and isotropic structure. On the other hand, fiber-reinforced composites are microscopically viewed as inhomogeneous and nonisotropic (orthotropic). As a result of this, the mechanics of fiber reinforced is more complicated than that of conventional materials (Mallick 2007).

Composite materials can be analyzed in two different levels: (i) macromechanical analysis and (ii) micromechanical analysis. These terms can be defined and shown in Figure 3.1.

**Micromechanics:** Mechanical analysis of the interactions of the constituents on the microscopic level. This study deals with the state of the stress and deformation in the constituents such as matrix failure (tensile, compressive, shear), fiber failure (tensile, buckling, splitting) and interface failure (debonding).

**Macromechanics:** An analysis of the lamina level to consider the material homogeneous and to use average mechanical properties. Failure criteria may be explained by using macromechanics level.

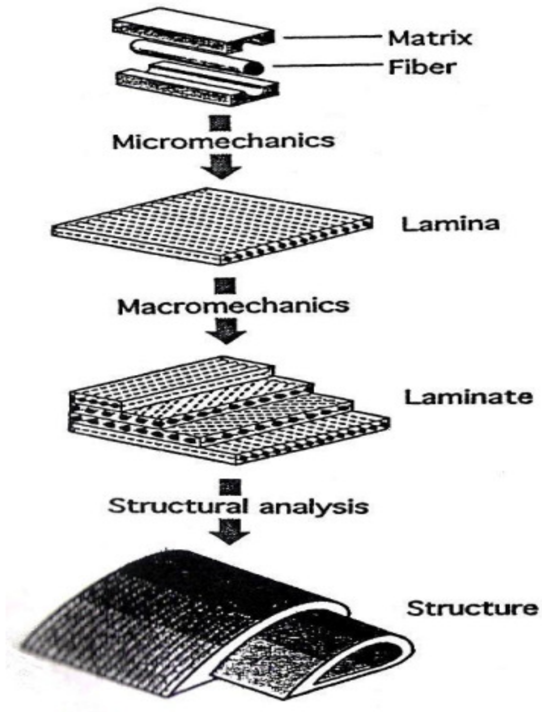



Figure 3.1. Levels of observation and types of analysis for composite materials
(Source: Daniel and Ishai 1994)

## 3.1. Macromechanical Analysis of a Laminated Composite Plate

The classical lamination theory is used to determine the mechanical behavior of the laminated composite. It is based on the following assumptions to analyze the laminated composite plate:

1- Each lamina of the laminated composite is nonhomogeneous and orthotropic.

2- The laminate is thin and edge dimensions of composite plate are much larger than its thickness. The loadings are applied in the laminate's plane and the laminate (except for their edges) are in a state of plane stress ($\sigma_z = \tau_{xz} = \tau_{yz} = 0$).

3- All displacements are small compared with the thickness of the laminate and they are continuous throughout the laminate.

4- In plane displacements in the x and y, directions are linear functions of z.

5- Transverse shear strains $\gamma_{xz}$ and $\gamma_{yz}$ are negligible.

6- Stress-strain and strain-displacement relations are linear.

7- The transverse normal strain $\varepsilon_z$ is negligible.

Cartesian coordinate system *x*, *y* and *z* explains global coordinates of the layered composite material. A layer-wise principal material coordinate system is shown as 1, 2, and 3 in Figure 3.2.

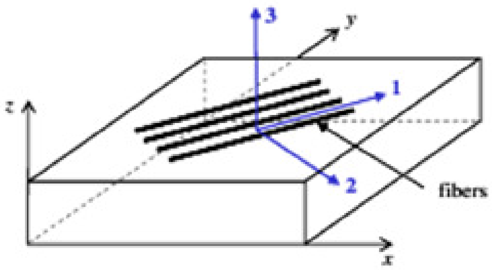



Figure 3.2. Global coordinate system (x,y,z) and material coordinate system (1,2,3). (Source: Irisarri, et al. 2009)

Mechanical in plane loadings ($N_x$, $N_y$) for thin laminated composite structure and schematic representation of a symmetric laminate for the *n*-layered structure with total thickness *h* are given in Figure 3.3.

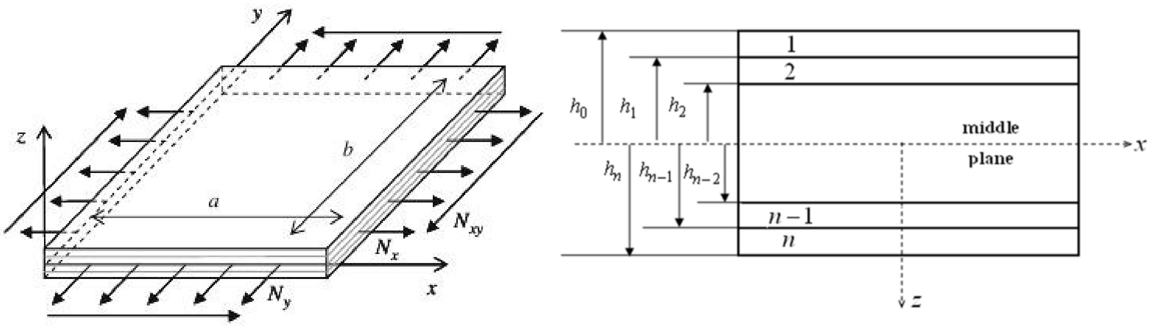



Figure 3.3. Laminate plate geometry and loading and Schematic representation of a symmetric laminate (Source: Irisarri, et al. 2009; Kaw 2006)

The strains at any point in the laminate to the reference plane can be written as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_s \end{bmatrix} = \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_s^o \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{bmatrix} \quad (3.1)$$

The in-plane stress components are related to the strain components for the *k*-th layer of a composite plate will be as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left( \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \varepsilon_{xy}^o \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right) \quad (3.2)$$

where $[\overline{Q}_{ij}]_k$ are the components of the transformed reduced stiffness matrix, $[\varepsilon^o]$ is the mid-plane strains $[\kappa]$ is curvatures. Transformation matrix [T] used in order to obtain the relation between principal axes (1, 2) and reference axes (x, y), is given by

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \qquad c = Cos\theta, \quad s = Sin\theta \quad (3.3)$$

The elements of the transformed reduced stiffness matrix $[\overline{Q}_{ij}]$ expressed in Equation 3.2 can be defined as in the following form:

$$\overline{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \quad (3.4)$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4) \quad (3.5)$$

$$\overline{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \quad (3.6)$$

$$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})sc^3 - (Q_{22} - Q_{12} - 2Q_{66})s^3c \quad (3.7)$$

$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})sc^3 \quad (3.8)$$

$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(c^4 + s^4) \quad (3.9)$$

where

$$Q_{11} = \frac{E_1}{1-\nu_{21}\nu_{12}} \tag{3.10}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{21}\nu_{12}} \tag{3.11}$$

$$Q_{22} = \frac{E_2}{1-\nu_{21}\nu_{12}} \tag{3.12}$$

$$Q_{66} = G_{12} \tag{3.13}$$

The principal stiffness terms, $Q_{ij}$, depend on elastic properties of the material along the principal directions, $E_1$, $E_2$, $G_{12}$, $\nu_{12}$, and $\nu_{21}$. The in-plane loads ($N_x$, $N_y$ and $N_{xy}$) and the moments ($M_x$, $M_y$, $M_{xy}$) in general have the following relations:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{3.14}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{3.15}$$

The matrices [A], [B] and [D] given in Equation 3.14 and 3.15 are extensional stiffness, coupling stiffness and bending laminate stiffness, respectively. These matrices can be defined as

$$A_{ij} = \sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_k - h_{k-1}) \tag{3.16}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \tag{3.17}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k^2 - h_{k-1}^2\right) \tag{3.18}$$

For the layers of a symmetric laminate with orthotropic layers, there is no coupling between in-plane loading and out of plane deformation, so the coupling stiffness matrix is $B_{ij} = 0$. The load-deformations relations are therefore reduced to

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} \tag{3.19}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{3.20}$$

The relation between the local and global stresses in each lamina can be expressed by the following transformation matrix.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} \tag{3.21}$$

Similarly, the local and global strains are written as follows:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} = [R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \tag{3.22}$$

where $[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

## 3.2. Buckling Analysis of a Laminated Composite Plate

Buckling is the most critical failure control for thin and large laminated composite plates subject to in-plane compressive loads. For the buckling analysis, we assume that the only applied loads are the in-plane compressive forces and other mechanical and thermal loads are zero (Reddy 2004). When the stress resultants $N_x$, $N_y$ and $N_{xy}$ are uniformly loaded and $w$ is the pre-buckling deformation, the equation of equilibrium in the direction normal to plate is defined as

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \qquad (3.23)$$

For simply supported plate with no shear load, $N_{xy}$ is zero. In order to simplify the equation of equilibrium, the in-plane forces are defined as follows:

$$N_x = -N_0 \qquad N_y = -kN_0 \qquad k = \frac{N_y}{N_x} \qquad (3.24)$$

The simply supported boundary conditions on all four edges of the rectangular plate (Figure 3.4) can be defined as

$$w(x,0) = 0 \quad w(x,b) = 0 \quad w(0,y) = 0 \quad w(a,y) = 0 \qquad (3.25)$$

$$M_{xx}(0,y) = 0 \;\; M_{xx}(a,y) = 0 \quad M_{yy}(x,0) = 0 \quad M_{yy}(x,b) = 0 \qquad (3.26)$$

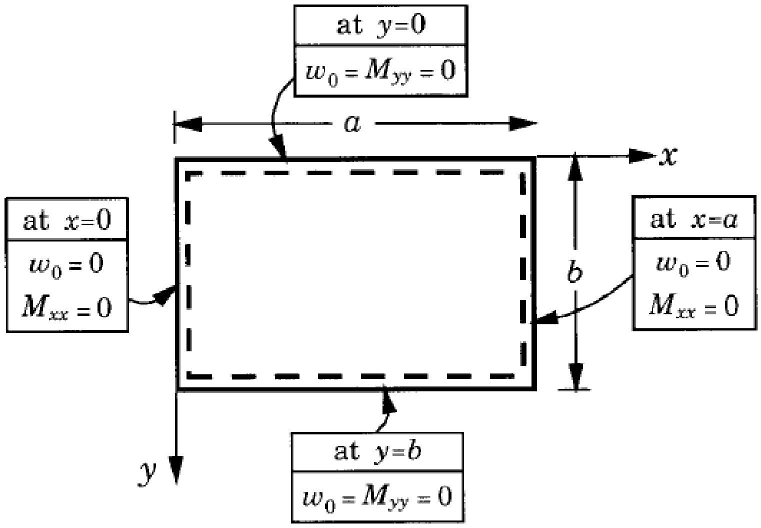



Figure 3.4. Geometry, coordinate system, and simply supported boundary conditions for a rectangular plate (Source: Reddy 2004)

As in the case of bending, Navier approach may be used for the solution considering simply supported boundary condition

$$w(x, y) = W_{mn} \sin(\alpha x)\sin(\beta y) \tag{3.27}$$

Substituting Equation 3.27 into Equation 3.23, we have obtained the following equation:

$$0 = \left\{-\left[D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right] + (\alpha^2 + k\beta^2)N_0\right\} \times W_{mn} \sin\alpha x \sin\beta y \tag{3.28}$$

For nontrivial solution $(W_{mn} \neq 0)$, the expression inside the curl brackets should be zero for every *m* and *n* half waves in x and y directions Then we obtain

$$N_0(m,n) = \frac{d_{mn}}{(\alpha^2 + k\beta^2)} \tag{3.29}$$

where $$d_{mn} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 \tag{3.30}$$

$$\alpha = \frac{m\pi}{a} \tag{3.31}$$

$$\beta = \frac{n\pi}{b} \tag{3.32}$$

where $a$ is the length of the plate, $b$ is the width of the plate. Substituting Equation 3.24 into Equation 3.29, the buckling load factor $\lambda_b$ is determined as

$$\lambda_b(m,n) = \pi^2 \left[ \frac{m^4 D_{11} + 2(D_{12} + 2D_{66})(rmn)^2 + (rn)^4 D_{22}}{(am)^2 N_x + (ran)^2 N_y} \right] \tag{3.33}$$

where $r$ is the plate aspect ratio $(a/b)$. The buckling mode is sinusoidal and if the plate is loaded as $N_x^a = \lambda_b N_x$ and $N_y^a = \lambda_b N_y$, the laminate buckles into $m$ and $n$ half waves in x and y directions, respectively. The smallest value of $\lambda_b$ over all possible combinations of $m$ and $n$ is the critical buckling load factor $\lambda_{cb}$ that determines the critical buckling loads for a specified combination of $N_x$ and $N_y$ in Equation 3.34. If $\lambda_{cb}$ is larger than 1, the laminate can sustain the applied loads $N_x$ and $N_y$ without buckling (Gurdal, et al. 1999).

$$\begin{bmatrix} N_{x_{cb}} \\ N_{y_{cb}} \end{bmatrix} = \lambda_{cb} \begin{bmatrix} N_x \\ N_y \end{bmatrix} \tag{3.34}$$

The combinations of $m$ and $n$ result in the lowest critical buckling load and, which is not easy to find. When composite plate is subjected to in-plane uniaxial loading and is simply supported for all edges, the minimum buckling load occurs at $n = 1$. The value of $m$ depends on bending stiffness matrix ($D_{ij}$) and the plate aspect ratio $(a/b)$. Therefore, it is not clear which value of $m$ will provide the lowest buckling load (Vinson 2005). In case of biaxial loading, as composite plate has low aspect ratio (such as $a/b = 3$ or smaller), or low ratios of the $D_{ij}$ (especially $D_{11}/D_{22} = 3$ or less), the critical values of $m$ and $n$ should be small (Gurdal, et al. 1999). For this reason, the smallest value of $\lambda_b$ (1, 1), $\lambda_b$(1, 2), $\lambda_b$(2, 1) and $\lambda_b$(2, 2) are considered in order to make a good prediction with respect to critical buckling load factor (Erdal and Sonmez 2005).

# CHAPTER 4

# FAILURE CRITERIA IN LAMINATED COMPOSITES

Laminated composite materials are now widely used in many applications such as aircraft, chemical, motor vehicles and military instead of conventional metals. However, laminated composite materials are more complicated than metals. They have nonhomogeneous and anisotropic structure by their nature. Therefore predicting the strength of the laminated composite material is very significant issue for applications area.

Over the last four decades, the subject of failure criteria for laminated composite materials has drawn attention to many researchers. Different types of failure criteria that have been presented for laminated composite materials show that it is still today an important research topic. Even though significant progresses have been obtained for failure criteria, it doesn't appear that there is any criterion under general load condition universally accepted by designers. In addition there is no common consent for failure criteria of laminated composite material in the industry (Figure 4.1).

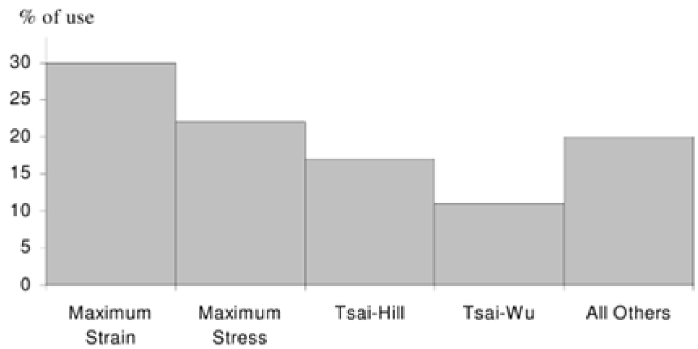



Figure 4.1. Industrial usage of composite failure criteria
(Source: Sun, et al. 1996)

Failure criteria of laminated composite materials can be classified as three groups: (i) interactive theories, (ii) non-interactive theories and (iii) failure mode-based theories.

**Interactive theories**: This group does not relate to failure modes. They contain all polynomial and tensorial criteria using mathematical expressions as a function of the material strengths. These expressions depend on the mathematical process of adjusting an expression to a curve obtained by experimental tests. This criterion may be defined in tensor notation as

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j \geq 1 \qquad (4.1)$$

The parameters $F_i$ and $F_{ij}$ are related to the lamina strengths in the principal directions. Several quadratic criteria have been presented by differing in the way in which the tensor stress components are determined. The most popular quadratic failure criteria are proposed by Tsai-Wu, Tsai-Hill, Azzi-Tsai, Hoffman and Chamis (Camanho 2002).

**Non-interactive theories:** In this theory, there are no interactions between stresses/strains acting on a lamina. This fact may causes errors in the strength predictions when multiaxial states of stress occur in a structure. The advantage of these theories is simple to use. Typical examples of non-interactive criteria are maximum stress criterion and maximum strain criterion (Kaw 2006).

**Failure mode-based theories:** These theories take into account interactions between stresses/strains acting on a lamina. Fiber and matrix failure are separately analyzed for tension and compression loading. Examples of failure mode based theories are Hashin-Rotem, Hashin and Puck failure criteria (Camanho 2002).

In this study, two of failure criteria (Maximum stress and Tsai-Wu failure theory) are examined. Detailed information about these theories is given in the following subsections.

## 4.1. Maximum Stress Failure Theory

According to the maximum stress theory, failure occurs when at least one of the stress components along one of the principal material axes exceeds the corresponding strength in that direction. The lamina is considered to be failed as

$$-\sigma_1^C < \sigma_1 < \sigma_1^T \qquad (4.2)$$

$$-\sigma_2^C < \sigma_2 < \sigma_2^T \qquad (4.3)$$

$$-\tau_{12}^{\;S} < \tau_{12} < \tau_{12}^S \qquad (4.4)$$

where

$\sigma_1$, $\sigma_2$ : Normal stresses in the 1 and 2 direction

$\sigma_1^C$ , $\sigma_1^T$ : Compressive and Tensile failure stresses in the 1 direction

$\sigma_2^C$ , $\sigma_2^T$ : Compressive and Tensile failure stresses in the 2 direction

$\tau_{12}^S$ : Shear strength

Note that all five strength parameters are treated as positive numbers, and the normal stresses are positive if tensile and negative if compressive. When shear stress is zero for a two dimensional state of stress, the failure graphic takes the form of a rectangle as shown in Figure 4.2 (Hyer 1998).

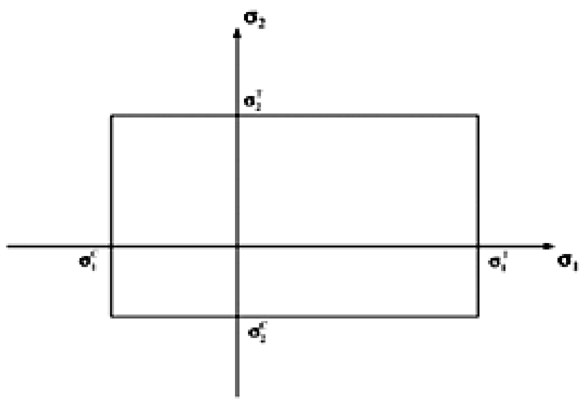



Figure 4.2. Failure envelope for the maximum stress failure theory
(Source: Daniel and Ishai 1994)

## 4.2. Tsai-Wu Failure Theory

This failure theory is based on the total strain energy failure theory of Beltrami. Tsai-Wu applied the failure theory to a lamina in plane stress. According to the theory, the lamina fails if the following condition is satisfied

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 > 1 \tag{4.5}$$

where

$$F_{11} = \frac{1}{\sigma_1^T \sigma_1^C}, \quad F_{22} = \frac{1}{\sigma_2^T \sigma_2^C}, \quad F_1 = \frac{1}{\sigma_1^T} - \frac{1}{\sigma_1^C}$$

$$F_2 = \frac{1}{\sigma_2^T} - \frac{1}{\sigma_2^C}, \quad F_{66} = \frac{1}{(\tau_{12}^F)^2}, \quad F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}}$$

$\sigma_1^T$ :tensile strength in longitudinal direction,

$\sigma_1^C$ : compressive strength in longitudinal direction,

$\sigma_2^T$ : tensile strength in transverse direction,

$\sigma_2^C$ :compressive strength in transverse direction.

$\tau_{12}^F$: shear strength

The failure criterion presents a general second order surface in the space with coordinates $\sigma_1$, $\sigma_2$ , $\tau_{12}$. For the case of no shear stress in the principal material system, the failure criterion is an ellipse in $\sigma_1$-$\sigma_2$ space (Hyer 1998). The selected criteria and the other criteria in $\sigma_1$-$\sigma_2$ stress plane are shown in Figure 4.2. Here, *X* and *Y* represent strength parameters in the fiber and transverse directions, respectively.

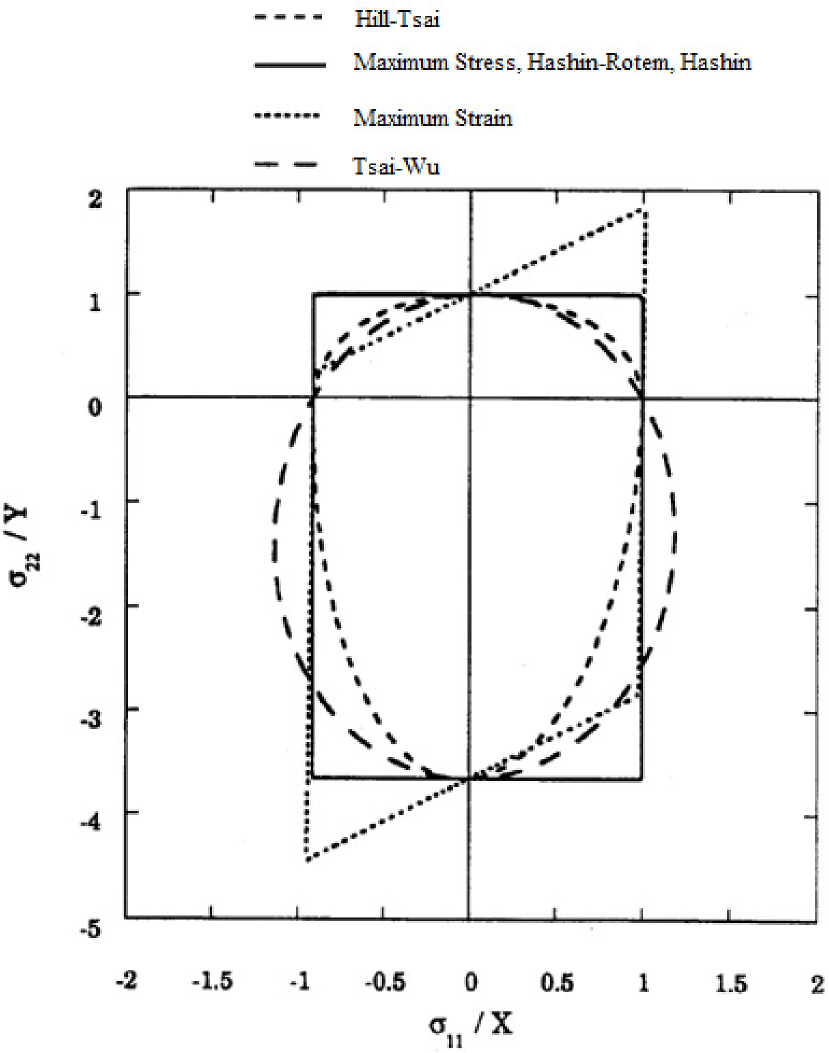



Figure 4.3. Comparison of lamina failure criteria under $\sigma_{11} - \sigma_{22}$ biaxial stress (Source: Sun, et al. 1996)

# CHAPTER 5

# OPTIMIZATION

## 5.1. General Information

Optimization is the process of maximizing or minimizing objectives to find best design by using the constraints. In nature, there are many examples where an optimum system status is sought. In metals and alloys, the atoms take positions of least energy in order to form unit cell. These unit cells describe the crystalline structure of materials. Tall trees form ribs near the base to strengthen them in bending. The honeycomb structure is one of the most compact packaging arrangements. Genetic mutation is another example of nature's optimization process. Like nature, optimization is too important issue to obtain feasible solutions for engineer and designers. For instance, a small savings in a mass-produced part will cause substantial savings for the corporation. In many industries such as aircraft, marine, automotive, weight minimization of laminated composite material can impact fuel efficiency, increased payloads or performance (Belegundu and Chandrupatla 2011).

Generally, an optimization problem has an objective function (fitness function) that determines efficiency of the design. Objective function can be classified into two groups: single objective and multi-objective. An optimization process is usually performed within some limits that determine the solution space. These limits are defined as constraint. Lastly, an optimization problem has design variables, which are parameters that are changed during the design process. Design variables can be dispersed (continuous) or discrete (limited continuous). A special case of discrete variables are integer variables. The standard formulation of optimization problem can be stated as follow:

$$\begin{aligned} &\textbf{minimize} \quad && f(x) && x \in \boldsymbol{X} \\ &\textbf{such that} \quad && h_i(x) = 0, && i = 1,......., n_e \\ & && g_j(x) \le 0, && j = 1,......., n_g \end{aligned} \tag{5.1}$$

$$x^L \leq x \leq x^U \tag{5.2}$$

where $f(x)$ is an objective function, $g(x)$ and $h(x)$ are inequality and equality constrains, respectively. Here, $x^L$ and $x^U$ define lower and upper bounds. Generally, although the objective function is minimized, for the cases of the engineering problems, it is maximized. For instance, stiffness and buckling load factor are maximized for laminated composite material. In order to convert a minimization problem into maximization problem, the sign of the objective function is changed. In other words, so as to maximize $f(x)$, we can minimize - $f(x)$ (Gurdal, et al. 1999).

Various optimization methods have been developed to obtain optimal design for laminated composite structures. The most common optimization methods used in the design of laminated composite material are design sensitivity analysis (DSA), genetic algorithm (GA), simulating annealing method (SA), reliability based design optimization (RBDO), particle swarm optimization algorithm (PSOA), ant colony optimization (ACO) and multi-objective robust design optimization (MRDO) (Awad et al., 2011). In this study, genetic algorithm is used and explained in the following section.

## 5.2. Genetic Algorithm

Many optimization problems, especially complex in engineering, can be difficult to solve using conventional optimization methods. Simulating the natural evolutionary process of nature cause stochastic optimization techniques defined as evolutionary algorithms (Walker and Smith 2003). The genetic algorithm is a stochastic optimization and evolutionary algorithm technique which is derived from biology and based on the application of Darwin's principle. Main part of genetic algorithm is an organism or individual that form from genes with chromosomes. Each gene carries coding information. At each step, genetic algorithm chooses individuals randomly from current population to be parents and uses them to create new coding for the next generation. The study of genetic algorithm depends on two main operators to produce new population: crossover and mutation (Karakaya and Soykasap 2009).

Holland (1975) developed the genetic algorithms, which replicate the mechanics of natural genetics for artificial systems derived from operations. The genetic operations involve simple, easy to program, random exchanges of numbers in strings that present the design variables. Many search algorithms move from one point to another in the design variable space, but the genetic algorithms work with a population of strings. Therefore, genetic algorithms are better than at separating the global minimum or maximum from the local minimum or maximum. For instance, the local and global optima for two dimensional space are shown in Figure 5.1.

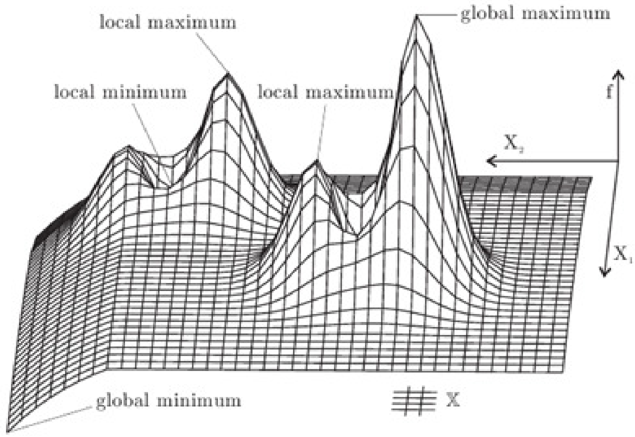



Figure 5.1. Global and local optima of $f(X_1, X_2)$ function.
(Source: Weise 2009)

Genetic algorithms have been commonly used for design of laminated composite materials. Buckling optimization, weight and cost minimization, the failure criteria optimization, the stiffness and the strength maximization have been presented by many authors. Details of genetic algorithm for composite material design are given in literature survey.

### 5.2.1. Crossover

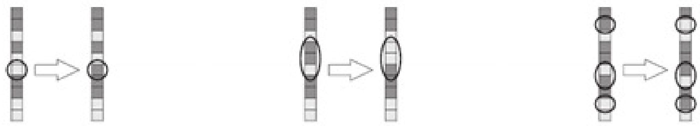
Crossover, one of the basic genetic algorithm operators, is responsible for the exchange of genetic information among the individuals of the population. A few types of crossover have been used in the composite materials, such as: one-point, two-point and uniform crossover (Figure 5.2). The effective of each type of crossover may depend on the optimization problem and also the stage of the search (Lopez, et al. 2009).

(a) (b)

Figure 5.2. Types of crossover (a) One-point crossover (b) Two-point crossover (Source: Weise 2009)

### 5.2.2. Mutation

The other basic genetic algorithm operator, the mutation, changes randomly the value of the genes of the individuals and unexplored regions of the design space can be found for valuable solution. Mutation is applied by generating a random number between 0 and 1 (Gurdal, et al. 1999). In Figure 5.3, three types of mutation are shown. The other operators and flowchart of genetic algorithm are shown in Figure 5.4.



(a) (b) (c)

Figure 5.3. Value-altering mutation of strings (a) Single-gene mutation (b) Multi-gene mutation (c) Multi-gene mutation (Source: Weise 2009)

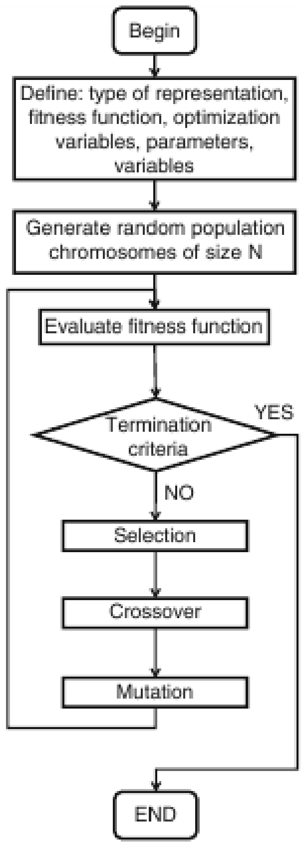



Figure 5.4. Flow chart of genetic algorithm
(Source: Cepin 2011)

## 5.3. Matlab Optimization Toolbox

MATLAB Optimization Toolbox provides methods that search for global optimum solutions that have multiple maximum or minimum points. It includes global search, multi-start, pattern search, genetic algorithm, and simulated annealing solvers. All methods have been used for design of laminated composite by many designers (Karakaya and Soykasap 2009; Apalak, et al. 2008). Using these algorithms, optimization problems where the objective or constraint function is continuous, discontinuous, stochastic, does not possess derivative can be solved.

Multi-start, genetic algorithm, and pattern search solvers can be used with Parallel Computing Toolbox to solve problems that benefit from parallel computation. By using parallel computing or by defining a custom parallel computing implementation of an optimization problem, solution time is reduced. Genetic Algorithm Solver and Multi-objective Genetic Algorithm Solver are examined in the following subsection.

### 5.3.1. Genetic Algorithm Solver

Genetic Algorithm Toolbox of MATLAB consists of two main parts. The first part is problem definition (*fitness function and number of variables*) and constraints (*linear inequalities, linear equalities, bounds and nonlinear constraint function*). The second is options (*population, fitness scaling, selection, reproduction, mutation, crossover, migration, algorithm settings, hybrid function, stopping criteria, plot functions, output function, display to command window and user function evaluation*). *Population* determines the size of population at each iteration. In order to obtain better results, large number of populations is used. *Selection* function determines the individuals to be parent for the next generation on the basis of their scaled fitness. It includes *stochastic uniform*, *remainder*, *uniform*, *roulette*, and *tournament*. *Reproduction* determines how the new generation created. *Mutation* changes randomly in the individuals using *Gaussian, uniform, adaptive feasible,* and *use constraint dependent default*. *Crossover* combines to individual so as to create new individual for the next generation by using one of the functions including *scattered*, *single point*, *two point*, *intermediate*, *heuristic*, and *arithmetic functions* (Karakaya and Soykasap 2009). A *hybrid function* is an optimization function that runs after the genetic algorithm terminates so as to improve the result of the fitness function. It includes *fminsearch, patternsearch, fminunc, fmincon*. *fminsearch* can be used for unconstraind optimization problems and *patternsearch* is utilized in case of specific bounds. *fminunc* is just used for unconstrained optimization problems and *fmincon* may be used for constrained optimization problems. Figure 5.5 shows genetic algorithm solver menu of MATLAB. In this study, the genetic algorithm options parameters are used in Table 5.1.

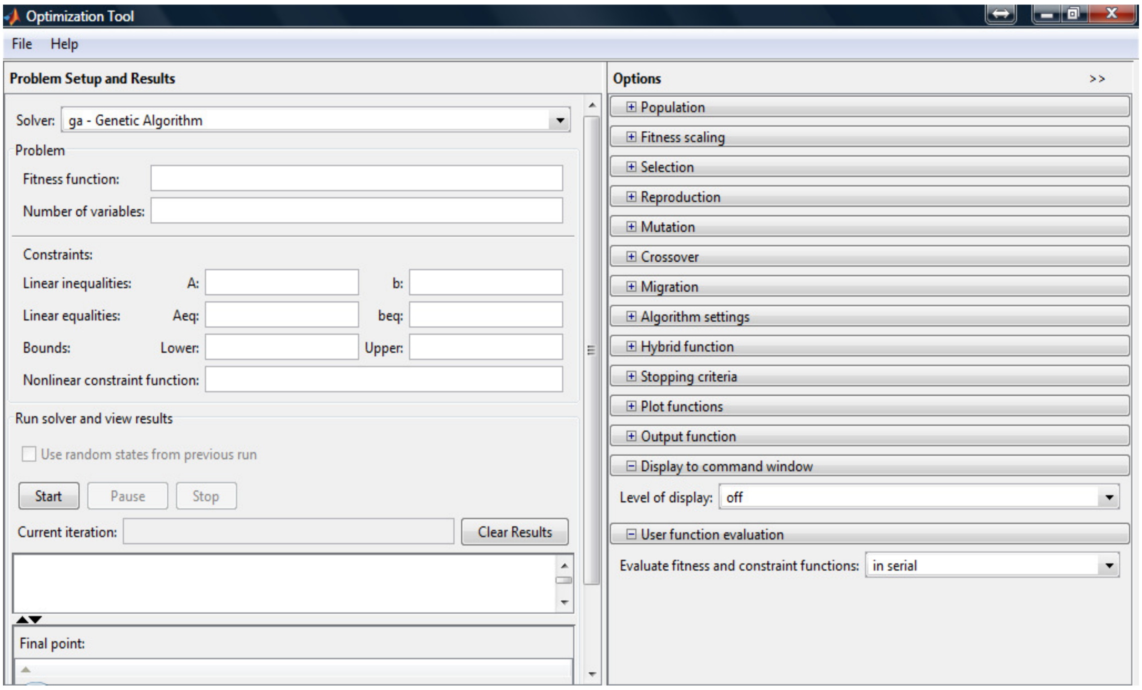



Figure 5.5. Genetic Algorithm solver menu from Matlab Optimization Toolbox

Table 5.1. Genetic Algorithm parameters for single objective function in test problems

| | |
|---|---|
| **Population Type** | Double vector |
| **Population size** | 40 |
| **Initial range** | [-90 ; 90 ] |
| **Selection function** | Roulette |
| **Crossover fraction** | 0.8 |
| **Mutation function** | Use constraint dependent default |
| **Crossover function** | Scattered |
| **Migration direction** | Both<br>Fraction=0.2, Interval=20 |
| **Initial penalty** | 10 |
| **Penalty factor** | 100 |
| **Hybrid Function** | fmincon |
| **Stopping criteria** | Generation=1000,<br>Stall generation=5000<br>Function tolerance=$10^{-6}$ |

### 5.3.2. Multi-objective Genetic Algorithm Solver

Multi-objective optimization deals with the minimization or maximization of multiple objective functions that are exposed to a set of constraints. The multi-objective genetic algorithm solver is utilized to solve multi-objective optimization problems by defining the Pareto front which is the set of evenly distributed non-dominated optimal solutions.(The Mathworks). The multi-objective genetic algorithm solver user interface is shown in Figure 5.6.

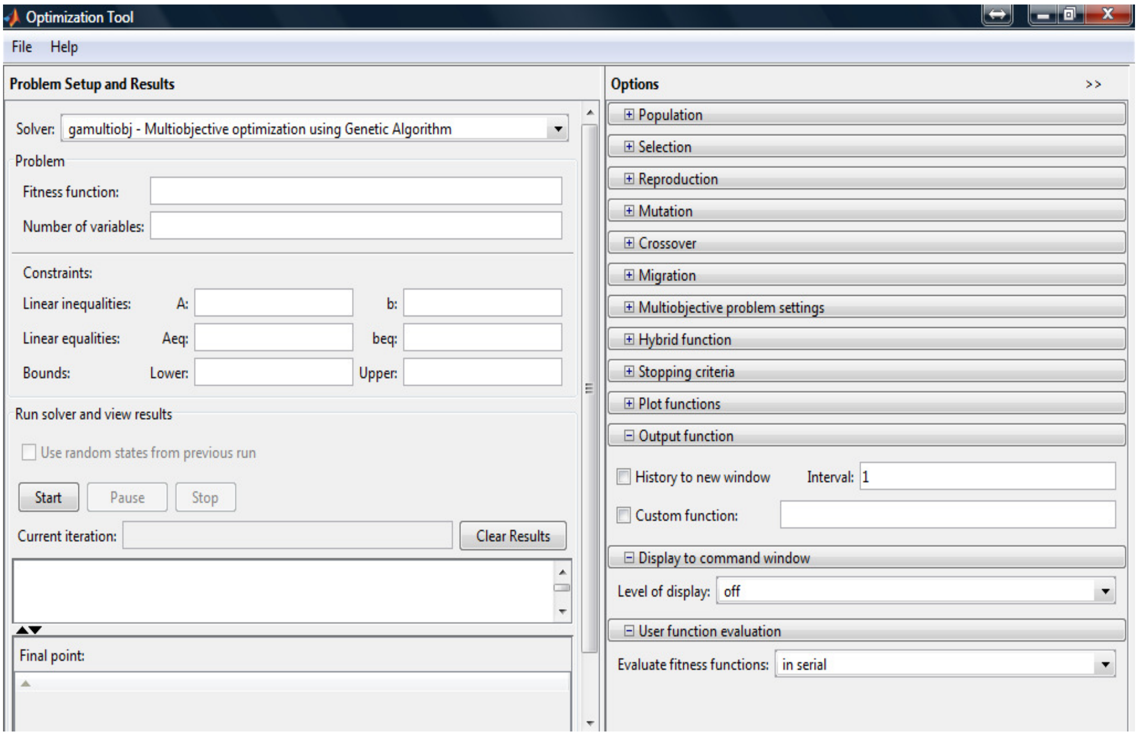



Figure 5.6. Multi-objective genetic algorithm solver menu from Matlab Optimization Toolbox

# CHAPTER 6

# RESULTS AND DISCUSSION

## 6.1. Problem Statement

The laminated composite plate under consideration is simply supported on four sides with a length of $a$ and width of $b$ (Figure 6.1). The plate is subjected to in-plane compressive loadings $N_x$ and $N_y$ in $x$ and $y$ directions, respectively.

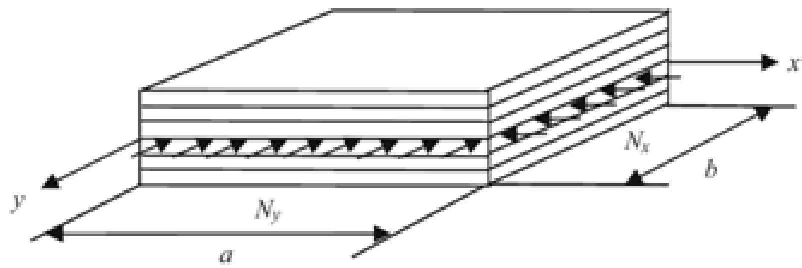



Figure 6.1. Composite plate subjected to in-plane compressive loadings
(Source: Soykasap and Karakaya 2007)

48 ply graphite/epoxy laminated composite plates are studied in this thesis. Material properties are given in Table 6.1. The original plate dimensions are length $a = 0.508$ m, width $b = 0.508$ m and a total thickness $t = 13.44$ mm (a ply thickness of $0.28$ mm). In this study, stacking sequence optimization of laminated composite materials for maximum buckling load are studied under given different loading cases ($N_x =$ 750 N/mm, 1000 N/mm, 2000 N/mm, 3000 N/mm, 5000 N/mm ) and aspect ratios ($a/b = 1$, $a/b = 2$ and $a/b = 1/2$) by considering first ply failure criteria (Maximum stress and Tsai-Wu failure criteria). During the optimization process, $N_x$ in-plane loading and $a$ the length of plate, are taken to be constant, but $N_y$ and $b$ are calculated from the load ratio ($N_x / N_y$) and the plate aspect ratio $(a/b)$, accordingly. In

design process, fiber orientation angles are taken as design variables and the allowable orientation angles are continuous ($-90 \leq \theta \leq 90$). The number of design variables is reduced from 48 to 12 by considering symmetric-balanced laminated plate. The representation of stacking sequences of 48-layered symmetric and balanced composite plate is shown as $[\pm\theta_1 / \pm\theta_2 / \pm\theta_3 / \pm\theta_4 / \pm\theta_5 / \pm\theta_6 / \pm\theta_7 / \pm\theta_8 / \pm\theta_9 / \pm\theta_{10} / \pm\theta_{11} / \pm\theta_{12}]_s$. Thus, in this thesis, the optimization problem can be stated as

Find : $\{\theta_k\}$, $\theta_k \in \{-90, 90\}$ $k = 1, \ldots, n$

Maximize : Critical buckling load factor ($\lambda_{cb}$)

Subject to : Maximum Stress and Tsai-Wu failure criteria

Table 6. 1. Elastic properties of Graphite/Epoxy (T300/5208)
(Source: Akbulut and Sonmez 2008)

| Property | Graphite/Epoxy (T300/5208) |
|---|---|
| Young's modulus, $E_1$ (GPa) | 181 |
| Young's modulus, $E_2$ (GPa) | 10.3 |
| Shear modulus, $G_{12}$ (GPa) | 7.17 |
| Poisson rate, $\upsilon_{12}$ | 0.28 |

In order to obtain optimum stacking sequences of laminated composite material, $\lambda_{cb}$, the critical buckling load factor (Equation 3.33) is maximized by using genetic algorithm. Here, the smallest value of $\lambda_b$ (1, 1), $\lambda_b$ (1, 2), $\lambda_b$ (2, 1) and $\lambda_b$ (2, 2) is taken as the critical buckling load factor ($\lambda_{cb}$). Having obtained $\lambda_{cb}$, the critical buckling loads ($N_{x,cr}$, $N_{y,cr}$) are also calculated using Equation 3.34.

A set of optimized laminated plate have been obtained and then the first ply failure of the composite plates is checked by using the maximum stress and Tsai-Wu failure criteria. The strength parameters of the laminated composite are shown in Table 6.2. In addition, conventional designs including 0, $\pm 45$, 90 orientations and dispersed designs in which fiber orientations are continuous are compared to $N_x$ = 750 N/mm, 1000 N/mm and $a/b = 2$ in order to show advantages and drawbacks of various ply orientation designs.

Lastly, the buckling behavior of 48-and 64-layered composite plates subjected to different loading conditions are investigated in terms of buckling resistance.

Table 6.2. The strength properties of the Graphite/Epoxy (T300/5208) lamina (Source: Akbulut and Sonmez 2008)

| Strength properties | Graphite/Epoxy (T300/5208) |
|---|---|
| $X_T$ (MPa) | 1500 |
| $X_C$ (MPa) | 1500 |
| $Y_T$ (MPa) | 40 |
| $Y_C$ (MPa) | 246 |
| $S_{12}$ (MPa) | 68 |

## 6.2. The Verification of Algorithms in Matlab

In this thesis, the algorithms related to buckling analysis and failure criteria of laminated composites are written in MATLAB. Before optimization process, the verification of algorithms is too significant to get optimum results. In this section, the critical buckling load factor equation (Equation 3.33) and the failure equations including maximum stress and Tsai-Wu are verified by using the literature studies.

Firstly, the critical buckling load factor is verified from the study of Karakaya and Soykasap (2009). They have used the genetic algorithm and generalized pattern search algorithm for optimum stacking sequences of a composite plate. Buckling load factor of the plate is maximized for different load cases ($N_x / N_y = 1, 2$ and 1/2) and aspect ratios ($a/b = 1, 2$ and 1/2). The obtained optimum fiber orientation angles have been converted to manufacturing values such as 90, 0, and $\pm 45$ orientations. Using specifications of the model problems, the critical buckling load factor values are calculated and it has been seen that the results are close to each other (Table 6.3).

Table 6.3. Results of different algorithms for buckling analysis

| Load cases | Stacking sequence | $\lambda_{cb}$ (the present study) | $\lambda_{cb}$ (Karakaya and Soykasap 2009) |
|---|---|---|---|
| LC1 | $[90_{10}/\pm45_2/90_2/\pm45_3/90_2/\pm45_4]_S$ | 695,660 | 695,781.3 |
| LC2 | $[\pm45_{16}]_s$ | 242,840 | 242,823.1 |
| LC3 | $[0_{10}/\pm45_2/0_2/\pm45_3/0_2/\pm45_4]_s$ | 173,920 | 173,945.3 |
| LC4 | $[90_4/\pm45_5/90_6/\pm45/90_4/\pm45_2/90_2]_s$ | 1,057,900 | 1,057,948.3 |
| LC5 | $[\pm45_{16}]_s$ | 323,790 | 323,764 |
| LC6 | $[0_{16}/\pm45/0_6/\pm45/0_2/\pm45/0_2]_s$ | 206,520 | 206,492.9 |
| LC7 | $[90_{16}/\pm45/90_6/\pm45/90_2/\pm45/90_2]$ | 413,040 | 412,985.8 |
| LC8 | $[\pm45_{16}]_s$ | 161,900 | 162,882.1 |
| LC9 | $[0_4/\pm45_5/0_6/\pm45/0_4/\pm45_2/0_2]_s$ | 132,240 | 132,243.5 |

Secondly, the algorithm for Tsai-Wu failure criteria is verified by using the study of Aydin (2011). The study focuses on the design of dimensionally stable laminated composites subjected to hygro-thermo-mechanical loading by various optimization methods. Tensile and Compression failure loads are calculated using different failure theories including Tsai-Hill, Tsai-Wu, Hoffman and Hashin-Rotem and the fiber orientation angles obtained in the optimization process. In order to verify the algorithm, compression failure loads for $[\pm27.3/\pm27.3]_S$ 8-layered E-glass/epoxy laminated composite considering Tsai-Wu failure criteria are utilized. When the limit failure loadings and material parameters are used in the algorithm, it has been seen that the failure efforts are close to one according to Equation 4.5 and shown in Table 6.4.

Table 6.4. Comparison of the failure effort values for Tsai-Wu failure criteria

| Failure effort ( $f_E$ ) for Tsai-Wu failure criteria (Aydin 2011) | Failure effort ( $f_E$ ) for Tsai-Wu (the present study) |
|---|---|
| 1.0 | 1.0864 |
| 1.0 | 1.0614 |
| 1.0 | 1.1109 |

Lastly, maximum stress failure criterion is verified by considering the study of Lopez et al. (2009). The study analyzed the minimum weight under strain, buckling and ply contiguity constraints of composite plates and also the minimum material cost of a hybrid laminate under ply failure and weight constraints. Three failure criteria are tested under in-plane loadings independently: maximum stress, Tsai-Wu and Puck failure criterion. For the verification process, $N_x$=-3000 N/mm, $N_y$=-3000 N/mm and different in-plane shear loadings are considered. As seen in Table 6.5, failure efforts are very close and even equal.

Table 6.5. Comparison of the results for maximum stress failure criteria ($N_x$=-3000 N/mm , $N_y$=-3000 N/mm)

| $N_{xy}$ (N/mm) | Stacking sequence | $f_E$ (Lopez, et al. 2009) | $f_E$ (the present study) |
|---|---|---|---|
| 100 | $[\pm 45_9]_s$ | 0.94 | 0.9366 |
| 250 | $[\pm 45_9]_s$ | 0.98 | 0.98 |
| 500 | $[\pm 45_{10}]_s$ | 0.96 | 0.955 |
| 1000 | $[\pm 45_{11}]_s$ | 1 | 0.995 |

## 6.3. Optimization Results and Discussion

In the stacking sequence optimization of the laminated composites, *Genetic Algorithm* (GA) from MATLAB Global Optimization Toolbox and *Symbolic Math Toolbox* have been used. In order to increase the reliability of GA and also find the optimum design, 50 searches are independently tested and each of them stopped after 1000 evaluations. The GA parameters used are shown in Table 5.1. Maximum stress and Tsai-Wu failure theories are used to control the feasibility of the fiber configurations generated by GA during the optimization process. To assess the results of failure theories, the failure effort ($f_E$) can be defined. According to the Tsai-Wu failure theory, the lamina does not fail if the failure effort ($f_E$: terms on the left hand side of the inequality in Equation 4.5) of the lamina is stated as

$$f_E < 1 \tag{6.1}$$

Then the maximum value of $f_{E,TW}^{k}$ is chosen as the failure effort of the laminate:

$$f_{E,TW} = \text{max of } f_{E,TW}^{k} \quad \text{for} \quad k = 1,2.......m-1, m \tag{6.2}$$

Similarly, the failure effort of the laminate can be calculated for maximum stress failure theory. Firstly, the principal stresses ($\sigma_{11}^{k}$, $\sigma_{22}^{k}$, $\tau_{12}^{k}$) in each lamina are determined. Then the failure effort for each failure mode is calculated as follows.

$$f_{E,MS}^{k} = \begin{Bmatrix} f_{E(X)}^{k} = \sigma_{11} / X_c \\ f_{E(Y)}^{k} = \sigma_{22} / Y_c \\ f_{E(S)}^{k} = |\tau_{12}| / S \end{Bmatrix} \tag{6.3}$$

Then the maximum values of $f_{E(X)}^{k}$, $f_{E(Y)}^{k}$ and $f_{E(S)}^{k}$ are chosen the failure effort of *m*th respectively.

$$f_{E(X)} = \text{max of } f_{e(X)}^{k} \quad \text{for k= 1, 2,.....,m-1, m} \tag{6.4}$$

$$f_{E(Y)} = \text{max of } f_{e(Y)}^{k} \quad \text{for k= 1, 2,.....,m-1, m} \tag{6.5}$$

$$f_{E(S)} = \text{max of } f_{e(S)}^{k} \quad \text{for k= 1, 2,.....,m-1, m} \tag{6.6}$$

It states that the lamina does not fail if $f_{E(X)}$, $f_{E(Y)}$ and $f_{E(S)}$ is smaller than 1.

The results of this study are presented in Tables (6.6-6.13). Firstly, the optimum stacking sequences for maximum buckling load factor under various in-plane loads and aspect ratios are obtained by using *Genetic Algorithm* and *Symbolic Math Toolbox.* Critical buckling loads in x and y directions are calculated. Then, the designs obtained are checked by maximum stress and Tsai-Wu failure criteria. The results can be seen from Table 6.6 to Table 6.11. In Table 6.12, the advantages of dispersed laminates over conventional designs are pointed out for some specific load cases and aspect ratios. The fiber orientation angles obtained from the optimization process are converted to manufacturing values such as 90, $\pm 45$ and 0 degrees. Furthermore, the composite plates with 48 layers and 64 layers for $N_x = 5000$ N/mm are compared in terms of buckling resistance in Table 6.13. In Figure 6.4, the investigation of genetic algorithm is shown considering different loading conditions ($N_x = 750$ N/mm, $N_x / N_y$=1, 0.5, 2, 4) and $a/b = 1$.

Table 6.6 and Table 6.7 show optimum stacking sequences for the laminate under various loading conditions ($N_x = 750$ N/mm, 1000 N/mm, 2000 N/mm, 3000 N/mm and 5000 N/mm) and for different failure criteria with $a/b = 1$. It can be understood from Table 6.6 that when in-plane loads are increased systematically, the critical buckling load factor values ($\lambda_{cb}$) decrease. As a result of this, if $\lambda_{cb}$ is smaller than 1, the plates are buckled in the x and/or y directions depending on the load ratios. Therefore, in many cases including $N_x =$ 2000 N/mm, 3000 N/mm and 5000 N/mm, buckling failure occurs. However, the cases $N_x = 1000$ N/mm, $N_x / N_y = 0.5$ and $N_x = 2000$, $N_x / N_y = 2$ are critical for buckling failure. The critical buckling load factor values for that case are close to 1 and so the buckling loads in x and y directions are close to applied loads in x and y directions, respectively. Furthermore, it can be seen from the results in Table 6.6 that all stacking sequences are formed from the combination of $\pm 45$ or $\mp 45$ fiber orientations. The results can be seen in Table 6.7 that the designs obtained in Table 6.6 are tested separately considering maximum stress and Tsai-Wu failure efforts. However, the cases which fail based on buckling analysis are not taken into consideration in Table 6.7 and also in the other tables. It can be noted that some failure efforts for Tsai-Wu failure theory have taken negative values, since in two dimensional case, Tsai-Wu failure criterion represent an ellipse and $\sigma_1$ and $\sigma_2$ principal stresses can take negative values. With regard to maximum stress theory, the failure effort ($f_E$) becomes greater when the applied loads increase. Especially, it can be seen in Table 6.7 that the failure efforts for shear stresses are more effective than the other maximum failure efforts in x and y directions. As a result, the most critical case for buckling analysis and failure criteria is to be $N_x = 2000$ N/mm and $N_x / N_y = 4$.

Table 6.6. Optimum stacking sequence for the laminate under various loading conditions (*a/b*=1)

| $N_x$ (N/mm) | $N_x/N_y$ | $\lambda_{cb}$ | $N_{x,cb}$ (N/mm) | $N_{y,cb}$ (N/mm) | Stacking Sequence |
|---|---|---|---|---|---|
| 750 | 0.5 | 1.322 | 991.185 | 1982.37 | $[\mp 45_3/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45_5]_s$ |
| | 1 | 1.982 | 1486.78 | 1486.78 | $[\pm 45/\mp 45_4/\pm 45_2/\mp 45_2/\pm 45_3]_s$ |
| | 2 | 2.643 | 1982.37 | 991.185 | $[\pm 45_2/\mp 45/\pm 45/\mp 45_2/\pm 45_2/\mp 45_2/\pm 45/\mp 45]_s$ |
| | 4 | 3.172 | 2378.84 | 594.711 | $[\pm 45/\mp 45/\pm 45_2/\mp 45/\pm 45_2/\mp 45_3/\pm 45_2]_s$ |
| 1000 | 0.5 | 0.991 | 991.185 | 1982.37 | $[\mp 45/\pm 45_3/\mp 45_4/\pm 45/\mp 45_2/\pm 45]_s$ |
| | 1 | 1.487 | 1486.78 | 1486.78 | $[\pm 45/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45_2/\pm 45/\mp 45_2/\pm 45_2]_s$ |
| | 2 | 1.982 | 1982.37 | 991.185 | $[\pm 45/\mp 45_2/\pm 45/\mp 45_3/\pm 45/\mp 45/\pm 45_2/\mp 45]_s$ |
| | 4 | 2.379 | 2378.84 | 594.711 | $[\mp 45/\pm 45_2/\mp 45_4/\pm 45_2/\mp 45_3]_s$ |
| 2000 | 0.5 | 0.496 | 991.185 | 1982.37 | $[\pm 45/\mp 45/\pm 45_4/\mp 45_2/\pm 45/\mp 45/\pm 45_2]_s$ |
| | 1 | 0.743 | 1486.78 | 1486.78 | $[\mp 45_4/\pm 45_2/\mp 45/\pm 45_2/\mp 45/\pm 45/\mp 45]_s$ |
| | 2 | 0.991 | 1982.37 | 991.185 | $[\pm 45_2/\mp 45_4/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45]_s$ |
| | 4 | 1.189 | 2378.84 | 594.711 | $[\pm 45_4/\mp 45_3/\pm 45_2/\mp 45/\pm 45/\mp 45]_s$ |
| 3000 | 0.5 | 0.330 | 991.185 | 1982.37 | $[\mp 45_3/\pm 45_2/\mp 45_2/\pm 45_2/\mp 45_3]_s$ |
| | 1 | 0.496 | 1486.78 | 1486.78 | $[\mp 45/\pm 45_2/\mp 45/\pm 45/\mp 45_2/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45]_s$ |
| | 2 | 0.661 | 1982.37 | 991.185 | $[\pm 45_2/\mp 45/\pm 45/\mp 45/\pm 45_2/\mp 45_3/\pm 45_2]_s$ |
| | 4 | 0.793 | 2378.84 | 594.711 | $[\mp 45_3/\pm 45/\mp 45/\pm 45/\mp 45_3/\pm 45/\mp 45_2]_s$ |
| 5000 | 0.5 | 0.198 | 991.185 | 1982.37 | $[\pm 45/\mp 45_4/\pm 45/\mp 45_3/\pm 45/\mp 45/\pm 45]_s$ |
| | 1 | 0.297 | 1486.78 | 1486.78 | $[\pm 45/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45_2/\pm 45/\mp 45/\pm 45_2/\mp 45]_s$ |
| | 2 | 0.396 | 1982.37 | 991.185 | $[\mp 45/\pm 45/\mp 45_2/\pm 45_3/\mp 45_2/\pm 45_3]_s$ |
| | 4 | 0.476 | 2378.84 | 594.711 | $[\pm 45/\mp 45_4/\pm 45/\mp 45/\pm 45_2/\mp 45_3]_s$ |

Table 6.7. Optimum stacking sequence for the laminate for different failure criteria ($a/b$=1)

| $N_x$ (N/mm) | $N_x/N_y$ | Stacking Sequence | Failure effort($f_E$) | | | |
|---|---|---|---|---|---|---|
| | | | TW | MS | | |
| | | | $f_{E(TW)}$ | $f_{E(X)}$ | $f_{E(Y)}$ | $f_{E(S)}$ |
| 750 | 0.5 | $[\mp 45_3/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45_5]_s$ | -0.05 | 0.10 | 0.05 | 0.41 |
| | 1 | $[\pm 45/\mp 45_4/\pm 45_2/\mp 45_2/\pm 45_3]_s$ | -0.15 | 0.07 | 0.03 | 0 |
| | 2 | $[\pm 45_2/\mp 45/\pm 45/\mp 45_2/\pm 45_2/\mp 45_2/\pm 45/\mp 45]_s$ | -0.07 | 0.05 | 0.02 | 0.21 |
| | 4 | $[\pm 45/\mp 45/\pm 45_2/\mp 45/\pm 45_2/\mp 45_3/\pm 45_2]_s$ | 0 | 0.04 | 0.02 | 0.31 |
| 1000 | 0.5 | $[\mp 45/\pm 45_3/\mp 45_4/\pm 45/\mp 45_2/\pm 45]_s$ | - | - | - | - |
| | 1 | $[\pm 45/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45_2/\pm 45/\mp 45_2/\pm 45_2]_s$ | -0.20 | 0.09 | 0.04 | 0 |
| | 2 | $[\pm 45/\mp 45_2/\pm 45/\mp 45_3/\pm 45/\mp 45/\pm 45_2/\mp 45]_s$ | -0.08 | 0.07 | 0.03 | 0.27 |
| | 4 | $[\mp 45/\pm 45_2/\mp 45_4/\pm 45_2/\mp 45_3]_s$ | 0.04 | 0.06 | 0.03 | 0.41 |
| 2000 | 0.5 | $[\pm 45/\mp 45/\pm 45_4/\mp 45_2/\pm 45/\mp 45/\pm 45_2]_s$ | - | - | - | - |
| | 1 | $[\mp 45_4/\pm 45_2/\mp 45/\pm 45_2/\mp 45/\pm 45/\mp 45]_s$ | - | - | - | - |
| | 2 | $[\pm 45_2/\mp 45_4/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45]_s$ | - | - | - | - |
| | 4 | $[\pm 45_4/\mp 45_3/\pm 45_2/\mp 45/\pm 45/\mp 45]_s$ | 0.43 | 0.12 | 0.05 | 0.82 |
| 3000 | 0.5 | $[\mp 45_3/\pm 45_2/\mp 45_2/\pm 45_2/\mp 45_3]_s$ | - | - | - | - |
| | 1 | $[\mp 45/\pm 45_2/\mp 45/\pm 45/\mp 45_2/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45]_s$ | - | - | - | - |
| | 2 | $[\pm 45_2/\mp 45/\pm 45/\mp 45/\pm 45_2/\mp 45_3/\pm 45_2]_s$ | - | - | - | - |
| | 4 | $[\mp 45_3/\pm 45/\mp 45/\pm 45/\mp 45_3/\pm 45/\mp 45_2]_s$ | - | - | - | - |
| 5000 | 0.5 | $[\pm 45/\mp 45_4/\pm 45/\mp 45_3/\pm 45/\mp 45/\pm 45]_s$ | - | - | - | - |
| | 1 | $[\pm 45/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45_2/\pm 45/\mp 45/\pm 45_2/\mp 45]_s$ | - | - | - | - |
| | 2 | $[\mp 45/\pm 45/\mp 45_2/\pm 45_3/\mp 45_2/\pm 45_3]_s$ | - | - | - | - |
| | 4 | $[\pm 45/\mp 45_4/\pm 45/\mp 45/\pm 45_2/\mp 45_3]_s$ | - | - | - | - |

The designs of different loading conditions ($N_x = 750$ N/mm, 1000 N/mm, 2000 N/mm, 3000 N/mm and 5000 N/mm) and $a/b$=2 are given in Table 6.8 and Table 6.9. As seen in Table 6.8, critical buckling load factors are greater than the previous ones (Table 6.6). Especially, when $N_x = 750$ N/mm and $N_y = 187.5$ N/mm are applied to composite plate, $\lambda_{cb}$ is 12.287. It means that the maximum loads that laminated composite sustain without buckling failure are $N_{x,cr} = 9215.35$ N/mm and $N_{y,cr} = 2303.84$ N/mm. As the results given in Table 6.6 and Table 6.8 are compared, the number of buckling failures for $a/b$=2 is less than for aspect ratio $a/b=1$. The results obtained from the failure theories are shown in Table 6.9. As $N_x = 3000$ N/mm, $N_x/N_y = 2$, $N_x/N_y = 4$ and $N_x = 5000$ N/mm $N_x/N_y = 2$, $N_x/N_y = 4$ are respectively applied to composite plate, the failure depending on maximum stress and Tsai-Wu failure criteria occurs before buckling failure mode. As the laminated composite plate is overloaded such as $N_x = 2000$ N/mm, 3000N/mm, 5000 N/mm, it has been seen that the failure efforts are close to 1 or greater than 1. However, it is not critical to control in terms of failure criteria for lower loads. Comparing the results of maximum stress failure criterion for *a/b*=1 with *a/b*=2, the failure efforts for *a/b*=2 are larger than *a/b*=1. The effective parameters of the phenomena are fiber orientation angles obtained for optimization process and therefore the principal stresses for *a/b*=2 are greater than *a/b*=1. As the failure efforts for Tsai-Wu failure criteria are investigated, they take positive values for overloaded conditions ($N_x = 3000$ N/mm, $N_x = 5000$ N/mm).

Table 6.8. Optimum stacking sequence for the laminate under various loading conditions (*a/b*=2)

| $N_x$ (N/mm) | $N_x/N_y$ | $\lambda_{cb}$ | $N_{x,cb}$ (N/mm) | $N_{y,cb}$ (N/mm) | Stacking Sequence |
|---|---|---|---|---|---|
| 750 | 0.5 | 3.436 | 2577.01 | 5154.01 | [∓78.9/±82.4/∓75.1/±82.3/±76.7/±75.6/±78/±85.4/±85.2/∓76.7/±56.6/∓55.6]$_s$ |
| | 1 | 5.947 | 4459.99 | 4459.99 | [±69.7/∓69.9/±70.7/∓70.3/∓73.5/∓72.3/∓69.6/∓68.9/±74.8/∓74.3/∓73.4/±55.5]$_s$ |
| | 2 | 9.214 | 6910.76 | 3455.38 | [∓62.1/±61.8/∓61.7/±63/∓61.3/±62.9/∓62.3/±61/±62.7/∓59.1/±56.7/±40.5]$_s$ |
| | 4 | 12.287 | 9215.35 | 2303.84 | [±53.2/∓53.1/±53.1/±53/∓53.2/±53/±52.8/∓52.9/±52.2/∓54/±49.9/∓43]$_s$ |
| 1000 | 0.5 | 2.577 | 2576.96 | 5153.92 | [±76.8/∓79.5/∓89.7/±83/±74.8/∓74.8/±77.5/∓80.1/±77.1/∓66.9/∓66/∓79.3]$_s$ |
| | 1 | 4.457 | 4456.89 | 4456.89 | [±72.1/±71.7/∓69.6/±69.3/±65.8/∓69.8/∓71/±85.3/±65.8/$90_2$/±66.8/±69.4]$_s$ |
| | 2 | 6.910 | 6910.39 | 3455.2 | [±62.2/±62.3/±61.3/±62.8/±61.6/±62.4/±61.1/±63.1/±61.2/±59.1/±52.9/±41.5]$_s$ |
| | 4 | 9.212 | 9212.27 | 2303.07 | [±52.9/∓52.8/±53.3/∓52.9/±53.1/∓52.9/±52.8/±53.3/∓51.7/∓54.8/∓68.7/±33.8]$_s$ |
| 2000 | 0.5 | 1.288 | 2576.5 | 5152.99 | [∓75.9/∓78.6/∓80/±88/∓85.2/∓77.5/∓75.1/∓75.7/±76.5/∓75.9/∓51/±74.1]$_s$ |
| | 1 | 2.228 | 4456.22 | 4456.22 | [±68.8/±72.9/∓68.6/±70.4/∓69.4/∓73.1/∓80.9/∓65/∓74/∓79.7/∓66/∓31.3]$_s$ |
| | 2 | 3.454 | 6907.22 | 3453.61 | [∓61.7/∓61.3/±65/∓62.3/∓60.4/∓64.2/∓58.6/∓60.3/∓59.1/∓62.5/∓54.6/±65.6]$_s$ |
| | 4 | 4.607 | 9214.9 | 2303.72 | [±53.1/∓52.5/±52.6/∓52.8/∓53.4/∓53.9/±54/∓53.9/±53.4/±53.1/∓50.4/±50.2]$_s$ |
| 3000 | 0.5 | 0.859 | 2576.08 | 5152.15 | [±84.7/∓77.5/∓83.6/±75.4/∓72.7/∓83.6/∓72.8/±84.7/∓75.8/∓68/±65.4/±44.5]$_s$ |
| | 1 | 1.486 | 4457.1 | 4457.1 | [∓72.6/±69.8/∓68.6/±69.1/∓74/±69.8/±66.6/∓73.9/±83.7/∓62.5/∓84.4/±40.8]$_s$ |
| | 2 | 2.303 | 6908.94 | 3454.47 | [±61.8/±62.4/±62.5/±61.8/∓$61.4_2$/±59.6/±61.9/±65.1/∓67.1/±68.3/±22.7]$_s$ |
| | 4 | 3.070 | 9210.38 | 2302.6 | [∓53.8/∓53.4/±52.9/±53.4/±52.9/∓53.2/∓51.8/∓50.9/∓52.3/∓45.1/±44.7/±75.5]$_s$ |
| 5000 | 0.5 | 0.515 | 2576.43 | 5152.86 | [±75.7/±87/±74.4/∓76.9/±86/∓82.2/$90_2$/∓77.9/±77.1/±65.7/∓69.2/∓71.6]$_s$ |
| | 1 | 0.891 | 4454.95 | 4454.95 | [∓71.8/±69.6/±68.7/±72.2/±67.3/±72.3/±80.8/∓70.8/±59.2/±85.8/±74.2/∓30]$_s$ |
| | 2 | 1.381 | 6903.16 | 3451.58 | [∓62.4/∓62/±62.7/±63.3/∓59.8/±56.1/∓65.9/±63/±57.6/±67.3/±84.6/±70.8]$_s$ |
| | 4 | 1.842 | 9208.65 | 2302.16 | [±54.2/∓52.1/∓52.9/∓53.6/∓51.7/±52.1/±53.7/∓54.2/∓50.7/±60.1/±43.8/∓28.7]$_s$ |

Table 6.9. Optimum stacking sequence for the laminate for different failure criteria ($a/b$=2)

| $N_x$ (N/mm) | $N_x/N_y$ | Stacking Sequence | Failure effort($f_E$) | | | |
|---|---|---|---|---|---|---|
| | | | TW | MS | | |
| | | | $f_{E(TW)}$ | $f_{E(X)}$ | $f_{E(Y)}$ | $f_{E(S)}$ |
| 750 | 0.5 | [∓78.9/±82.4/∓75.1/±82.3/±76.7/±75.6/±78/±85.4/±85.2/∓76.7/±56.6/∓55.6]$_s$ | -0.57 | 0.18 | 0.16 | 0.17 |
| | 1 | [±69.7/∓69.9/±70.7/∓70.3/∓73.5/∓72.3/∓69.6/∓68.9/±74.8/∓74.3/∓73.4/±55.5]$_s$ | -0.58 | 0.16 | 0.18 | 0.24 |
| | 2 | [∓62.1/±61.8/∓61.7/±63/∓61.3/±62.9/∓62.3/±61/±62.7/∓59.1/±56.7/±40.5]$_s$ | -0.23 | 0.22 | 0.12 | 0.25 |
| | 4 | [±53.2/∓53.1/±53.1/±53/∓53.2/±53/±52.8/∓52.9/±52.2/∓54/±49.9/∓43]$_s$ | -0.11 | 0.15 | 0.09 | 0.34 |
| 1000 | 0.5 | [±76.8/∓79.5/∓89.7/±83/±74.8/∓74.8/±77.5/∓80.1/±77.1/∓66.9/∓66/∓79.3]$_s$ | -0.89 | 0.18 | 0.26 | 0.22 |
| | 1 | [±72.1/±71.7/∓69.6/±69.3/±65.8/∓69.8/∓71/±85.3/±65.8/$90_2$/±66.8/±69.4]$_s$ | -0.84 | 0.11 | 0.26 | 0.26 |
| | 2 | [±62.2/±62.3/±61.3/±62.8/±61.6/±62.4/±61.1/±63.1/±61.2/±59.1/±52.9/±41.5]$_s$ | -0.28 | 0.27 | 0.16 | 0.33 |
| | 4 | [±52.9/∓52.8/±53.3/∓52.9/±53.1/∓52.9/±52.8/±53.3/∓51.7/∓54.8/∓68.7/±33.8]$_s$ | 0.01 | 0.28 | 0.13 | 0.31 |
| 2000 | 0.5 | [∓75.9/∓78.6/∓80/±88/∓85.2/∓77.5/∓75.1/∓75.7/±76.5/∓75.9/∓51/±74.1]$_s$ | -0.95 | 0.59 | 0.45 | 0.49 |
| | 1 | [±68.8/±72.9/∓68.6/±70.4/∓69.4/∓73.1/∓80.9/∓65/∓74/∓79.7/∓66/∓31.3]$_s$ | -0.31 | 0.66 | 0.31 | 0.35 |
| | 2 | [∓61.7/∓61.3/±65/∓62.3/∓60.4/∓64.2/∓58.6/∓60.3/∓59.1/∓62.5/∓54.6/±65.6]$_s$ | -0.57 | 0.28 | 0.42 | 0.79 |
| | 4 | [±53.1/∓52.5/±52.6/∓52.8/∓53.4/∓53.9/±54/∓53.9/±53.4/±53.1/∓50.4/±50.2]$_s$ | 0.06 | 0.18 | 0.26 | 0.98 |
| 3000 | 0.5 | [±84.7/∓77.5/∓83.6/±75.4/∓72.7/∓83.6/∓72.8/±84.7/∓75.8/∓68/±65.4/±44.5]$_s$ | - | - | - | - |
| | 1 | [∓72.6/±69.8/∓68.6/±69.1/∓74/±69.8/±66.6/∓73.9/±83.7/∓62.5/∓84.4/±40.8]$_s$ | -0.38 | 0.93 | 0.56 | 0.72 |
| | 2 | [±61.8/±62.4/±62.5/±61.8/∓$61.4_2$/±59.6/±61.9/±65.1/∓67.1/±68.3/±22.7]$_s$ | 0.53 | 1.01 | 0.35 | 0.54 |
| | 4 | [∓53.8/∓53.4/±52.9/±53.4/±52.9/∓53.2/∓51.8/∓50.9/∓52.3/∓45.1/±44.7/±75.5]$_s$ | 0.30 | 0.47 | 0.46 | 1.04 |
| 5000 | 0.5 | [±75.7/±87/±74.4/∓76.9/±86/∓82.2/$90_2$/∓77.9/±77.1/±65.7/∓69.2/∓71.6]$_s$ | - | - | - | - |
| | 1 | [∓71.8/±69.6/±68.7/±72.2/±67.3/±72.3/±80.8/∓70.8/±59.2/±85.8/±74.2/∓30]$_s$ | - | - | - | - |
| | 2 | [∓62.4/∓62/±62.7/±63.3/∓59.8/±56.1/∓65.9/±63/±57.6/±67.3/±84.6/±70.8]$_s$ | 4.62 | 0.70 | 1.18 | 1.73 |
| | 4 | [±54.2/∓52.1/∓52.9/∓53.6/∓51.7/±52.1/±53.7/∓54.2/∓50.7/±60.1/±43.8/∓28.7]$_s$ | 3.38 | 1.46 | 0.46 | 1.45 |

Table 6.10 and Table 6.11 show optimum stacking sequences under different loading conditions ($N_x = 750$ N/mm, 1000 N/mm, 2000 N/mm, 3000 N/mm and 5000 N/mm) and various failure criteria with regard to *a/b*=1/2. As seen in the results of buckling analysis in Table 6.10, many composite plate designs ($N_x$ =1000 N/mm $N_x / N_y$=0.5 and $N_x = 2000$ N/mm, 3000N/mm, 5000 N/mm for all load ratios) fail in terms of buckling. Comparing the previous cases ($a/b = 1$ and $a/b = 2$), critical buckling load factor values for *a/b*=1/2 are very low. Figure 6.2 also shows comparison of buckling behavior for various plate designs ($a/b = 1$, $a/b = 2$, *a/b*=1/2) and loading conditions ($N_x = 750$ N/mm and $N_x / N_y$=0.5, 1, 2 and 4). Therefore, buckling failure occurs under the applied loads in x and/or y directions. The reason for this is that width of composite plate is larger compared to length. It is interesting to note that the optimum fiber orientations for all loadings and $N_x / N_y$ = 4 are obtained as $[0_{24}]_s$. As investigated in Table 6.11, most of the designs which fail based on the buckling and therefore failure analyses are not taken into account. These designs are shown in grey color.

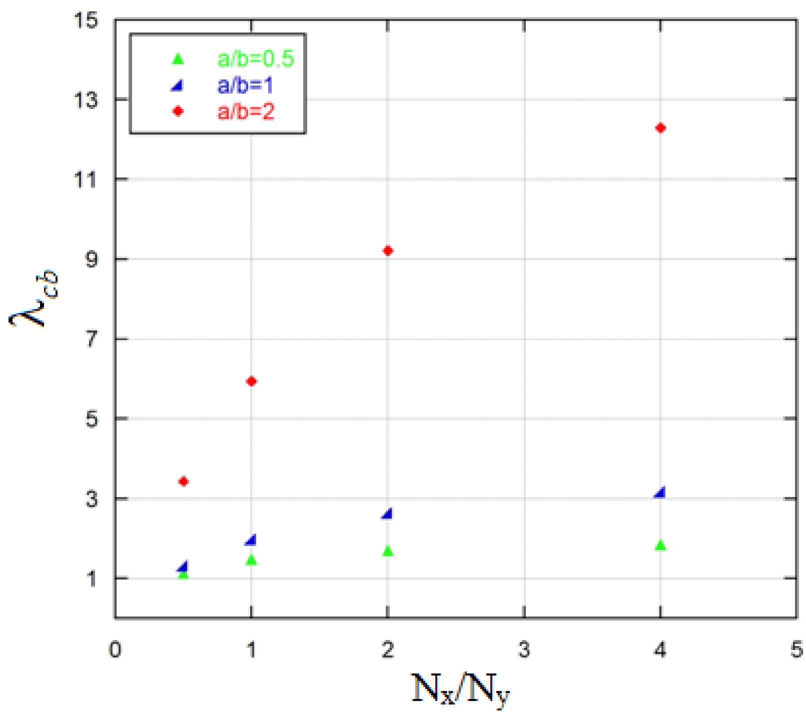



Figure 6.2. Comparison of buckling behavior of laminated composite plate for $N_x$=750 N/mm, ($N_x / N_y$=0.5, 1, 2, 4) and various aspect ratios

Table 6.10. Optimum stacking sequence for the laminate under various loading conditions (*a/b*=1/2)

| $N_x$ (N/mm) | $N_x/N_y$ | $\lambda_{cb}$ | $N_{x,cb}$ (N/mm) | $N_{y,cb}$ (N/mm) | Stacking Sequence |
|---|---|---|---|---|---|
| 750 | 0.5 | 1.150 | 862.87 | 1725.73 | $[\mp 28/\pm 24/\mp 27.8/\pm 30.8/\pm 28.9/\pm 32.9/\mp 27.9/\mp 26.3/\pm 28.6/\pm 32.3/\mp 28.8/\pm 31.8]_s$ |
| | 1 | 1.485 | 1113.64 | 1113.64 | $[\pm 20.3/\pm 22.4/\mp 21.1/\mp 19.3/\pm 16.9/\pm 20.7/\mp 10.4/\mp 9.9/\pm 11.1/\pm 6/\pm 0.9/\pm 89.9]_s$ |
| | 2 | 1.718 | 1288.81 | 644.40 | $[\pm 10.8/\mp 13.6/\pm 10.8/\pm 10.9/\pm 7.7/\pm 5.7/\mp 15/\pm 7.5/\mp 19.9/\pm 15.2/\pm 3.2/\pm 31.8]_s$ |
| | 4 | 1.855 | 1391.45 | 347.86 | $[0_{24}]_s$ |
| 1000 | 0.5 | 0.862 | 861.87 | 1723.75 | $[\pm 26.1/\pm 32.4/\mp 27.4/\pm 32.2/\pm 27.3/\mp 25.3/\mp 19.2/\mp 26.4/\mp 25/\pm 27.4/\pm 24.2/\mp 46.3]_s$ |
| | 1 | 1.114 | 1114.34 | 1114.34 | $[\pm 16/\mp 20.3/\mp 20.6/\pm 22.3/\pm 18.8/\pm 23.6/\mp 12.1/\mp 21.1/\mp 17.4/\pm 18.7/\pm 16.1/\mp 37.1]_s$ |
| | 2 | 1.289 | 1289.09 | 644.55 | $[\mp 11.7/\pm 12/\pm 10.4/\mp 13.6/\pm 11.7/\pm 7.5/\pm 10.3/\pm 10.4/\pm 6.5/\pm 8.2/\pm 6.3/\pm 7.1]_s$ |
| | 4 | 1.391 | 1391.45 | 347.86 | $[0_{24}]_s$ |
| 2000 | 0.5 | 0.431 | 861.14 | 1722.27 | $[\mp 30.4/\mp 29.3/\pm 30.1/\mp 28.3/\mp 24.7/\pm 23.7/\mp 16.5/\mp 29.2/\pm 39.8/\pm 6.7/\pm 21.1/\pm 15.9]_s$ |
| | 1 | 0.557 | 1113.06 | 1113.06 | $[\pm 22.3/\pm 21.4/\pm 16.1/\mp 17.4/\pm 12.7/\mp 20.6/\pm 7.3/\pm 24.2/\mp 28.9/\mp 32.7/\mp 23.5/\mp 31.1]_s$ |
| | 2 | 0.644 | 1288.47 | 644.23 | $[\pm 13.6/\pm 14.9/\pm 3.8/\pm 7.5/\mp 15.9/\pm 3/\mp 7.7/\pm 6.9/\pm 15.4/\pm 9.5/\pm 1.8/\pm 0.6]_s$ |
| | 4 | 0.696 | 1391.45 | 347.86 | $[0_{24}]_s$ |
| 3000 | 0.5 | 0.287 | 861.10 | 1722.20 | $[\mp 25.8/\pm 31.6/\mp 27.9/\mp 33.8/\mp 22.7/\mp 29.6/\mp 29.2/\mp 17.9/\mp 20.5/\mp 29.8/\pm 2.8/\pm 13.3]_s$ |
| | 1 | 0.371 | 1113.43 | 1113.43 | $[\pm 21.2/\pm 22.2/\mp 19.9/\mp 20.7/\pm 17.2/\pm 11.3/\pm 6.7/\pm 26.2/\pm 6.6/\mp 11.2/\pm 7.3/\mp 45.4]_s$ |
| | 2 | 0.430 | 1288.91 | 644.46 | $[\pm 9.8/\pm 10.7/\pm 7.5/\pm 12.9/\mp 14.2/\mp 14.7/\mp 15.4/\pm 11.9/\pm 6.8/\pm 1.6/\pm 11.8/\pm 19.7]_s$ |
| | 4 | 0.464 | 1391.45 | 347.86 | $[0_{24}]_s$ |
| 5000 | 0.5 | 0.173 | 862.78 | 1725.56 | $[\mp 27/\pm 30.2/\pm 25.5/\mp 26.8/\mp 28.9/\mp 29.5/\pm 25.2/\pm 32.7/\pm 35/\mp 26.1/\pm 28.9/\pm 89.1]_s$ |
| | 1 | 0.222 | 1111.99 | 1111.99 | $[\pm 18.4/\mp 19.8/\pm 25.7/\mp 13.1/\mp 20.2/\pm 21.8/\pm 8.4/\pm 9.7/\pm 34.6/\mp 9.4/\mp 11/\pm 59.7]_s$ |
| | 2 | 0.258 | 1288.39 | 644.20 | $[\pm 12/\pm 16.2/\pm 10.3/\pm 2.5/\mp 3.1/\mp 16.3/\mp 6.8/\pm 13.7/\pm 1/\mp 0.6/\pm 1.1/\pm 3.8]_s$ |
| | 4 | 0.278 | 1391.45 | 347.86 | $[0_{24}]_s$ |

Table 6.11. Optimum stacking sequence for the laminate for different failure criteria ($a/b$=1/2)

| $N_x$ (N/mm) | $N_x/N_y$ | Stacking Sequence | Failure effort($f_E$) | | | |
|---|---|---|---|---|---|---|
| | | | TW | MS | | |
| | | | $f_{E(TW)}$ | $f_{E(X)}$ | $f_{E(Y)}$ | $f_{E(S)}$ |
| 750 | 0.5 | [∓28/±24/∓27.8/±30.8/±28.9/±32.9/∓27.9/∓26.3 /±28.6/±32.3/∓28.8/±31.8]$_s$ | -0.68 | 0.15 | 0.32 | 0.58 |
| | 1 | [±20.3/±22.4/∓21.1/∓19.3/±16.9/±20.7/∓10.4/∓9.9 /±11.1/±6/±0.9/±89.9]$_s$ | -0.12 | 0.25 | 0.09 | 0.07 |
| | 2 | [±10.8/∓13.6/±10.8/±10.9/±7.7/±5.7/∓15/±7.5 /∓19.9/±15.2/±3.2/±31.8]$_s$ | -0.36 | 0.09 | 0.09 | 0.09 |
| | 4 | $[0_{24}]_s$ | -0.28 | 0.04 | 0.06 | 0 |
| 1000 | 0.5 | [±26.1/±32.4/∓27.4/±32.2/±27.3/∓25.3/∓19.2/∓26.4 /∓25/±27.4/±24.2/∓46.3]$_s$ | - | - | - | - |
| | 1 | [±16/∓20.3/∓20.6/±22.3/±18.8/±23.6/∓12.1/∓21.1 /∓17.4/±18.7/±16.1/∓37.1]$_s$ | -0.65 | 0.24 | 0.23 | 0.31 |
| | 2 | [∓11.7/±12/±10.4/∓13.6/±11.7/±7.5/±10.3/±10.4 /±6.5/±8.2/±6.3/±7.1]$_s$ | -0.61 | 0.06 | 0.14 | 0.07 |
| | 4 | $[0_{24}]_s$ | -0.36 | 0.05 | 0.08 | 0 |
| 2000 | 0.5 | [∓30.4/∓29.3/±30.1/∓28.3/∓24.7/±23.7/∓16.5/∓29.2 /±39.8/±6.7/±21.1/±15.9]$_s$ | - | - | - | - |
| | 1 | [±22.3/±21.4/±16.1/∓17.4/±12.7/∓20.6/±7.3/±24.2 /∓28.9/∓32.7/∓23.5/∓31.1]$_s$ | - | - | - | - |
| | 2 | [±13.6/±14.9/±3.8/±7.5/∓15.9/±3/∓7.7/±6.9 /±15.4/±9.5/±1.8/±0.6]$_s$ | - | - | - | - |
| | 4 | $[0_{24}]_s$ | - | - | - | - |
| 3000 | 0.5 | [∓25.8/±31.6/∓27.9/∓33.8/∓22.7/∓29.6/∓29.2/∓17.9 /∓20.5/∓29.8/±2.8/±13.3]$_s$ | - | - | - | - |
| | 1 | [±21.2/±22.2/∓19.9/∓20.7/±17.2/±11.3/±6.7/±26.2 /±6.6/∓11.2/±7.3/∓45.4]$_s$ | - | - | - | - |
| | 2 | [±9.8/±10.7/±7.5/±12.9/∓14.2/∓14.7/∓15.4/±11.9 /±6.8/±1.6/±11.8/±19.7]$_s$ | - | - | - | - |
| | 4 | $[0_{24}]_s$ | - | - | - | - |
| 5000 | 0.5 | [∓27/±30.2/±25.5/∓26.8/∓28.9/∓29.5/±25.2/±32.7 /±35/∓26.1/±28.9/±89.1]$_s$ | - | - | - | - |
| | 1 | [±18.4/∓19.8/±25.7/∓13.1/∓20.2/±21.8/±8.4/±9.7 /±34.6/∓9.4/∓11/±59.7]$_s$ | - | - | - | - |
| | 2 | [±12/±16.2/±10.3/±2.5/∓3.1/∓16.3/∓6.8/±13.7 /±1/∓0.6/±1.1/±3.8]$_s$ | - | - | - | - |
| | 4 | $[0_{24}]_s$ | - | - | - | - |

The optimum stacking sequences calculated for $a/b = 2$ and the corresponding buckling load factors are also shown in Table 6.12. The results are obtained at two different in-plane loadings; $N_x$ = 750 N/mm and $N_x$ = 1000 N/mm and four different load ratios; $N_x / N_y$ = 1, 0.5, 2, and 4. The comparison between the optimization results obtained by considering conventional and dispersed ply orientations presents that the optimum buckling load factor is higher when dispersed orientations are used. For this reason, search space for dispersed laminates is larger and it provides higher freedom to choose the ply orientations. On the other hand, the main drawback of dispersed laminated composites is too difficult to carry out considering the manufacturing process.

Table 6.13 shows the results of buckling behavior of composite plate with 48 layers and 64 layers. In order to determine and compare the results of buckling behavior, in-plane loading $N_x$ = 5000 N/mm and four different load ratios: $N_x / N_y$ = 1, 0.5, 2, and 4 and aspect ratios $a / b$ = 1, 2, 1/2 for design optimization are studied. As seen from the results in Table 6.13, two different cases for which the load ratio $N_x / N_y$ = 2 and aspect ratio $a/b = 2$, and the load ratio $N_x / N_y$ = 4 and aspect ratio $a/b = 2$ for the composite plate with 48 layers are resistant to buckling failure. On the contrary, five different cases for the composite plate with 64 layers do not fail with respect to buckling failure: $N_x / N_y$ = 1 and $a/b = 2$, $N_x / N_y$ = 0.5 and $a/b = 2$, $N_x / N_y$ = 2 and $a/b = 2$, $N_x / N_y$ = 4 and $a/b = 1, 2$, respectively. As expected, the buckling load factor of 64-layered composite plate is higher than those of 48-layered (Figure 6.3). Furthermore, the best anti-buckled design among the different aspect ratios of composite plate is obtained for the case $a/b = 2$ when comparing the optimum results in Table 6.13. On the other hand, the worst design is obtained for $a/b = 1/2$ because of the minimum buckling load factor value. For the specific cases, stacking sequences of 64-layered composite plate is similar to those of 48-layered composite plate. The optimum designs have the combination of $\pm 45$ and $\mp 45$ fiber orientations angles for $a/b = 1$ and all load ratios. Similarly, the stacking sequences are $[0_{24}]_s$ for $N_x / N_y$ = 4 and $a/b = 1/2$.

As the obtained results analyzed in Table 6.13, the design process varies according to designers' needs. Critical buckling load factor is a significant point to determine high buckling loads or weight minimization. If the durable design is desired so as to prevent high buckling loads, the critical buckling load factor should be as large as possible. Therefore, the number of layers of composite plate should be increased. However, in case weight minimization of composite plate is considered, the critical buckling load factor should be 1 or slightly higher than 1 and so the number of layers of composite plate can be decreased.

Table 6.12. Conventional (Con) and Dispersed (Dis) optimum stacking sequences for various loading ratios (*a/b*=2)

| $N_x$ (N/mm) | $N_x/N_y$ | Orientation | Stacking Sequence | $\lambda_{cb}$ |
|---|---|---|---|---|
| 750 | 0.5 | Dis. | $[\mp78.9/\pm82.4/\mp75.1/\pm82.3/\pm76.7/\pm75.6/\pm78/\pm85.4/\pm85.2/\mp76.7/\pm56.6/\mp55.6]_s$ | 3.436 |
| | | Con. | $[0_2/90_2/0_2/90_{12}/0_2/90_2/0_2]_s$ | 2.331 |
| | 1 | Dis. | $[\pm69.7/\mp69.9/\pm70.7/\mp70.3/\mp73.5/\mp72.3/\mp69.6/\mp68.9/\pm74.8/\mp74.3/\mp73.4/\pm55.5]_s$ | 5.947 |
| | | Con. | $[90_2/0_2/90_2/0_{10}/90_2/0_4/90_2]_s$ | 3.169 |
| | 2 | Dis. | $[\mp62.1/\pm61.8/\mp61.7/\pm63/\mp61.3/\pm62.9/\mp62.3/\pm61/\pm62.7/\mp59.1/\pm56.7/\pm40.5]_s$ | 9.214 |
| | | Con. | $[0_2/90_2/0_2/90_2/0_2/90_2/0_2/90_4/0_2/90_4]_s$ | 5.666 |
| | 4 | Dis. | $[\pm53.2/\mp53.1/\pm53.1/\pm53/\mp53.2/\pm53/\pm52.8/\mp52.9/\pm52.2/\mp54/\pm49.9/\mp43]_s$ | 12.287 |
| | | Con. | $[90_8/0_2/90_4/0_2/90_2/0_2/90_2/0_2]_s$ | 7.482 |
| 1000 | 0.5 | Dis. | $[\pm76.8/\mp79.5/\mp89.7/\pm83/\pm74.8/\mp74.8/\pm77.5/\mp80.1/\pm77.1/\mp66.9/\mp66/\mp79.3]_s$ | 2.577 |
| | | Con. | $[90_2/0_4/90_4/0_2/90_2/0_2/90_2/0_6]_s$ | 1.583 |
| | 1 | Dis. | $[\pm72.1/\pm71.7/\mp69.6/\pm69.3/\pm65.8/\mp69.8/\mp71/\pm85.3/\pm65.8/90_2/\pm66.8/\pm69.4]_s$ | 4.457 |
| | | Con. | $[90_4/0_2/90_4/0_4/90_{10}]_s$ | 3.507 |
| | 2 | Dis. | $[\pm62.2/\pm62.3/\pm61.3/\pm62.8/\pm61.6/\pm62.4/\pm61.1/\pm63.1/\pm61.2/\pm59.1/\pm52.9/\pm41.5]_s$ | 6.910 |
| | | Con. | $[90_{24}]_s$ | 4.676 |
| | 4 | Dis. | $[\pm52.9/\mp52.8/\pm53.3/\mp52.9/\pm53.1/\mp52.9/\pm52.8/\pm53.3/\mp51.7/\mp54.8/\mp68.7/\pm33.8]_s$ | 9.212 |
| | | Con. | $[90_2/0_2/90_2/0_2/90_2/0_2/90_4/0_6/90_2]_s$ | 5.611 |

Table 6.13. Comparison of Buckling behavior of Composite plate with 48 layers and 64 layers ($N_x$=5000 N/mm)

| Layer No | $N_x/N_y$ | a/b | Stacking Sequence | $\lambda_{cb}$ |
|---|---|---|---|---|
| 48 | 0.5 | 1/2 | $[\mp 27/\pm 30.2/\pm 25.5/\mp 26.8/\mp 28.9/\mp 29.5/\pm 25.2/\pm 32.7/\pm 35/\mp 26.1/\pm 28.9/\pm 89.1]_s$ | 0.173 |
| | | 1 | $[\pm 45/\mp 45_4/\pm 45/\mp 45_3/\pm 45/\mp 45/\pm 45]_s$ | 0.198 |
| | | 2 | $[\pm 75.7/\pm 87/\pm 74.4/\mp 76.9/\pm 86/\mp 82.2/90_2/\mp 77.9/\pm 77.1/\pm 65.7/\mp 69.2/\mp 71.6]_s$ | 0.515 |
| | 1 | 1/2 | $[\pm 18.4/\mp 19.8/\pm 25.7/\mp 13.1/\mp 20.2/\pm 21.8/\pm 8.4/\pm 9.7/\pm 34.6/\mp 9.4/\mp 11/\pm 59.7]_s$ | 0.222 |
| | | 1 | $[\pm 45/\mp 45/\pm 45/\mp 45/\pm 45/\mp 45_2/\pm 45/\mp 45/\pm 45_2/\mp 45]_s$ | 0.297 |
| | | 2 | $[\mp 71.8/\pm 69.6/\pm 68.7/\pm 72.2/\pm 67.3/\pm 72.3/\pm 80.8/\mp 70.8/\pm 59.2/\pm 85.8/\pm 74.2/\mp 30]_s$ | 0.891 |
| | 2 | 1/2 | $[\pm 12/\pm 16.2/\pm 10.3/\pm 2.5/\mp 3.1/\mp 16.3/\mp 6.8/\pm 13.7/\pm 1/\mp 0.6/\pm 1.1/\pm 3.8]_s$ | 0.258 |
| | | 1 | $[\mp 45/\pm 45/\mp 45_2/\pm 45_3/\mp 45_2/\pm 45_3]_s$ | 0.396 |
| | | 2 | $[\mp 62.4/\mp 62/\pm 62.7/\pm 63.3/\mp 59.8/\pm 56.1/\mp 65.9/\pm 63/\pm 57.6/\pm 67.3/\pm 84.6/\pm 70.8]_s$ | 1.381 |
| | 4 | 1/2 | $[0_{24}]_s$ | 0.278 |
| | | 1 | $[\pm 45/\mp 45_4/\pm 45/\mp 45/\pm 45_2/\mp 45_3]_s$ | 0.476 |
| | | 2 | $[\pm 54.2/\mp 52.1/\mp 52.9/\mp 53.6/\mp 51.7/\pm 52.1/\pm 53.7/\mp 54.2/\mp 50.7/\pm 60.1/\pm 43.8/\mp 28.7]_s$ | 1.842 |
| 64 | 0.5 | 1/2 | $[\mp 28.2/\mp 22.8/\mp 28.3/\pm 28.8/\pm 25.3/\pm 30/\pm 33.9/\mp 33.5/\mp 29.3/\pm 28.6/\pm 25.9/\pm 24.9/\pm 30.4/\pm 36.8/\pm 25.4/\pm 61.4]_s$ | 0.409 |
| | | 1 | $[\mp 45_4/\pm 45_8/\mp 45/\pm 45/\mp 45_2]_s$ | 0.470 |
| | | 2 | $[\mp 80.6/\mp 79.4/\mp 87/\mp 74.5/\pm 85.1/\mp 74.3/\pm 81.6/\pm 77/\mp 73.1/\pm 83.8/\mp 69.6/\pm 77.4/\pm 60.4/\pm 69.2/\pm 70.7/\pm 64.9]_s$ | 1.221 |
| | 1 | 1/2 | $[\mp 18.6/\mp 21.2/\pm 16.7/\mp 18.1/\pm 25.9/\pm 14/\mp 22.3/\pm 15.2/\pm 18.7/\pm 8.8/\pm 23.1/\pm 38.1/\pm 12.2/\pm 6.6/\pm 60.8/\pm 10.4]_s$ | 0.527 |
| | | 1 | $[\mp 45/\pm 45_2/\mp 45/\pm 45_3/\mp 45_2/\pm 45/\mp 45/\pm 45/\mp 45/\pm 45_2/\mp 45]_s$ | 0.705 |
| | | 2 | $[\pm 68.4/\pm 71.4/\pm 75.4/\mp 68/\pm 79.1/\mp 72.1/\pm 66.4/\mp 67.7/\pm 89.8/\mp 66.8/\pm 59/\mp 56.2/\mp 66.3/\pm 49/\mp 62.6/\pm 84.5]_s$ | 2.109 |
| | 2 | 1/2 | $[\mp 15.7/\pm 9.6/\pm 10.7/\pm 9.5/\pm 8.8/\pm 7.8/\pm 6/\pm 22.1/\pm 6.9/\pm 6.4/\pm 12.7/\pm 7/\pm 2.3/\pm 3.9/\pm 4.7/\mp 22]_s$ | 0.611 |
| | | 1 | $[\mp 45/\pm 45/\mp 45_2/\pm 45_2/\mp 45_2/\pm 45_2/\mp 45_4/\pm 45_2]_s$ | 0.940 |
| | | 2 | $[\pm 61.7/\mp 60.6/\mp 63/\pm 63.9/\mp 62.6/\pm 62.6/\mp 60.2/\mp 59.2/\mp 60.5/\pm 69.3/\mp 60/\mp 58.2/\mp 63/\pm 52/\pm 61.8/\pm 72.5]_s$ | 3.272 |
| | 4 | 1/2 | $[0_{32}]_s$ | 0.660 |
| | | 1 | $[\mp 45_2/\pm 45_2/\mp 45/\pm 45/\mp 45_5/\pm 45_2/\mp 45_2/\pm 45]_s$ | 1.128 |
| | | 2 | $[\pm 53.2/\pm 52.9/\pm 53/\mp 53.5/\pm 53.1/\mp 52.7/\pm 53.4/\pm 53.1/\mp 52.3/\mp 54.1/\pm 52.4/\mp 53.3/\mp 49.9/\mp 54.4/\mp 32.2/\pm 18.1]_s$ | 4.367 |

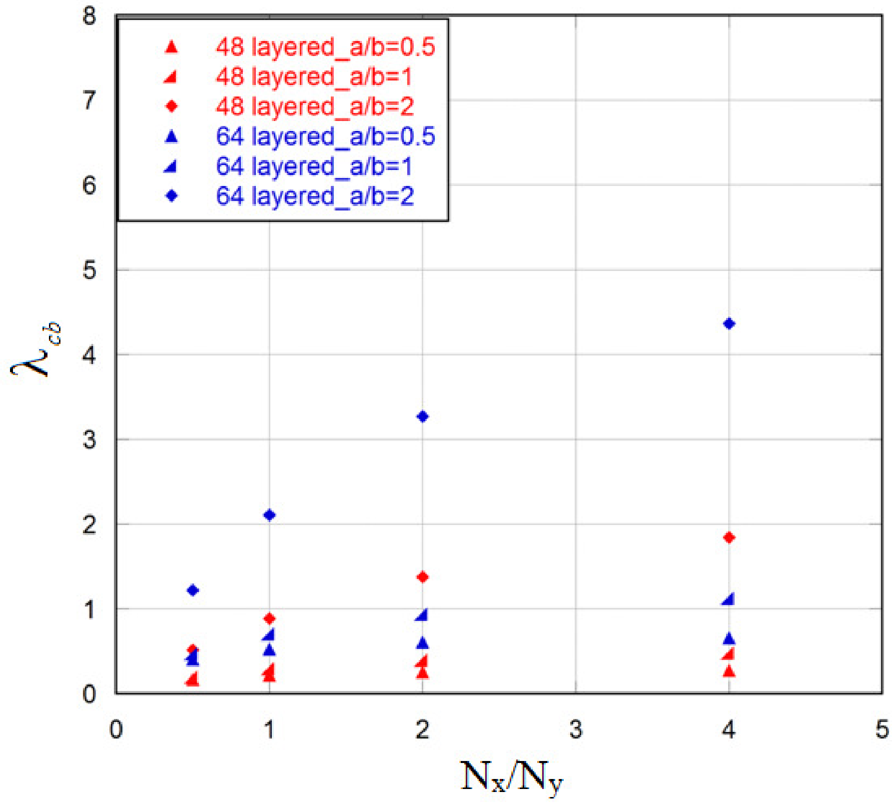
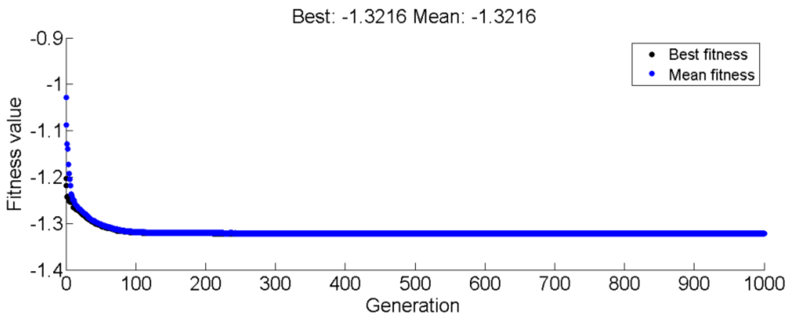
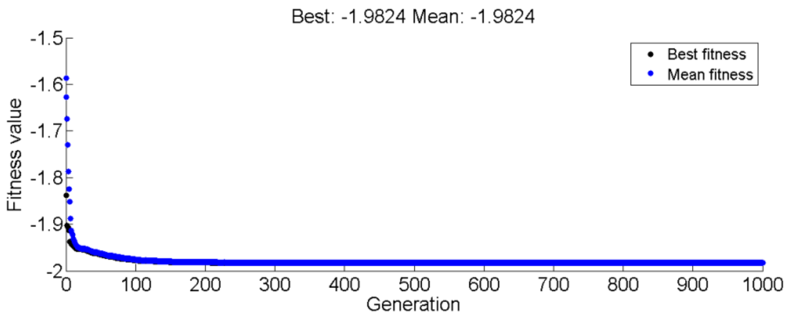



Figure 6.3. Comparison of buckling behavior of 48-and 64-layered composite plates under $N_x = 5000$ N/mm, ($N_x / N_y$=0.5, 1, 2, 4) and various aspect ratios

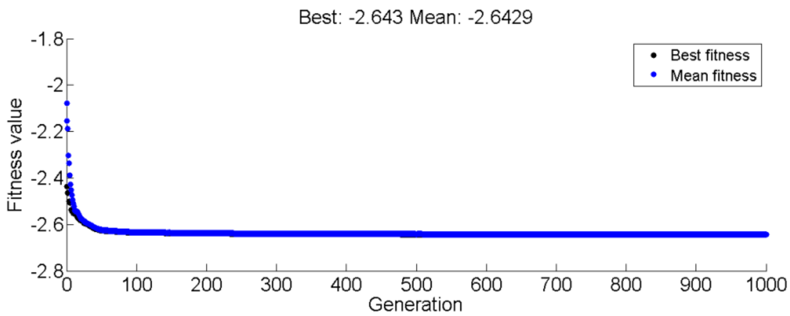
In Figure 6.4, the performance of genetic algorithm is shown by considering various load conditions ($N_x = 750$ N/mm and $N_x / N_y$=0.5, 1, 2, 4) and the aspect ratio ($a/b=1$). The best fitness value is defined as objective function (critical buckling load factor equation), and mean fitness value is the average of the fitness values in each generation. The number of generation determines in which the algorithm stops. In this study, the number of generation is 1000 for each case. The best fitness value is close to mean fitness value and the optimum solutions are obtained after approximately 600 function evaluations.



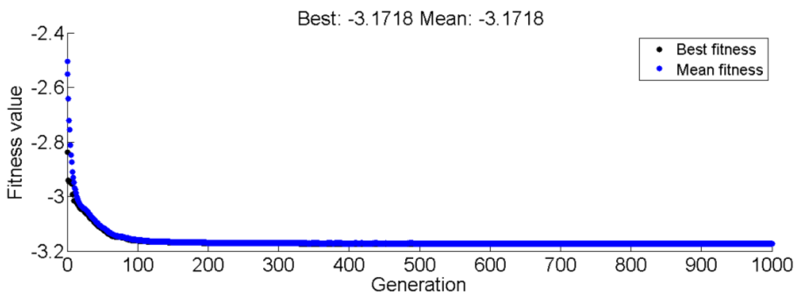

(a)




(b)




(c)




(d)

Figure 6.4. GA generation steps for model problems (a) $N_x = 750$ N/mm, $N_x / N_y = 0.5$, (b) $N_x = 750$ N/mm, $N_x / N_y = 1$, (c) $N_x = 750$ N/mm, $N_x / N_y = 2$, (d) $N_x = 750$ N/mm, $N_x / N_y = 4$

# CHAPTER 7

# CONCLUSION

In this thesis, the stacking sequences optimization of the 48-layered graphite/epoxy composite materials subjected to in-plane compressive loadings have been presented considering various failure criteria. Genetic algorithm is considered for optimization process and buckling load factor is taken as the objective function. The designs for maximized critical buckling load factor have also been checked based on maximum stress and Tsai-Wu failure criteria.

In order to assess the results of optimization process and the behavior of buckling of the laminated composites, the cases for various loading ratios and aspect ratios have been studied. To increase the reliability of optimization process, genetic algorithm is run 50 times and stopped after 1000 evaluations. Furthermore, by using plot function options in genetic algorithm toolbox, it has been shown that the best fitness value and the averages of fitness value are close to each other after the specific evaluations (e.g. 600 evaluations).

In this study, as the results have been investigated in terms of buckling resistance, the aspect ratio having the smallest critical buckling load factor value for the cases having the same $N_x$ is $1/2$ and hence the resistance to buckling becomes low. On the other hand, the most resistant design to buckling failure is to be the cases with $a/b = 2$.

In general, when the in-plane loadings have increased systematically, the critical buckling load factor value decreases. However, buckling loads in x and y directions are close to each other for the same load and aspect ratios. Furthermore, it can be noted that the higher the $N_x / N_y$ load ratio is, the higher the relative increase in buckling load is factor for the same in-plane loading $N_x$. It is noted that optimum fiber orientation angles for some specific cases are similar. For the case $N_x / N_y$=4 and $a/b = 1/2$, all of the sequences have been obtained as $[0_{24}]_s$. The optimum fiber orientation angles are formed as the combination of $\pm 45$ and $\mp 45$ for all load ratios and $a/b = 1$.

The designs obtained for maximum critical buckling load factor in the optimization process have been controlled by various failure criteria: Tsai-Wu and maximum stress. The cases which buckling failure occurs have not been taken into consideration for the failure analysis. As the results have been examined regarding the failure theories, failure effort values are based on the stacking sequences and in-plane loadings. When the composite plates are overloaded ($N_x = 3000$ N/mm, $N_x / N_y = 2$, $N_x / N_y = 4$ and $N_x = 5000$ N/mm $N_x / N_y = 2$, $N_x / N_y = 4$), static failure (maximum stress and/or Tsai-Wu) occurs before buckling failure. Therefore, not only buckling analysis but also static failure analysis for composite plates should be done. However, it is not a critical point to check the failure criteria for relatively low in-plane loadings.

Comparison of buckling behavior of dispersed and conventional laminates have been performed by regarding different loading conditions ($N_x = 750$ N/mm, $N_x / N_y =$ 0.5, 1, 2, 4 and $N_x$=1000 N/mm, $N_x / N_y =$ 0.5, 1, 2, 4) and the aspect ratio ($a/b = 2$). The higher critical buckling load factor has been obtained when dispersed orientations are used since the search space is larger for dispersed orientations. On the contrary, it is easy to manufacture conventional laminates.

Finally, the buckling behavior of the composite plates with 48 and 64 layers have been compared by considering different loading conditions ($N_x = 5000$ N/mm, $N_x / N_y$=0.5, 1, 2, 4) and aspect ratios ($a/b =$ 1, 2, 1/2). As expected, critical buckling load factor of 64-layered composite plate is higher than 48-layered. For this reason, the composite plate with 64 layers is more resistant to buckling.

It can also be concluded that all the results showed that critical buckling load factor is an important parameter to determine buckling resistance and weight minimization. The critical buckling load factor should be as high as possible in order to increase buckling resistance. Thus, the number of layers must be increased or geometry (length to width) of the composite plate should be changed. On the other hand, if the weight minimization of composite plate has been taken into consideration, the critical buckling load factor should be equal to one or slightly higher than one. Therefore the number of layers of composite plate can be decreased according to critical buckling load factor.

# REFERENCES

Akbulut, Mustafa and Sonmez, Fazil O., 2008, Optimum design of composite laminates for minimum thickness. *Computers and Structures*. 86(21-22): p. 1974-1982.

Akbulut, Mustafa and Sonmez, Fazil O., 2011, Design optimization of laminated composites using a new variant of simulated annealing. *Computers and Structures*. 89(17-18): p. 1712-1724.

Awad, Ziad K., Aravinthan, Thiru, Zhuge, Yan and Gonzalez, Felipe, 2012, A review of optimization techniques used in the design of fibre composite structures for civil engineering applications. *Materials & Design*. 33(0): p. 534-544.

Aydin, Levent., 2011, Design of dimensionally-stable laminated composites subjected to hygro-thermo-mechanical loading by stochastic optimization methods, *PhD İzmir Institute of Technology,* Izmir, TURKEY.

Aymerich, F. and Serra, M., 2008, Optimization of laminate stacking sequence for maximum buckling load using the ant colony optimization (ACO) metaheuristic. *Composites Part A: Applied Science and Manufacturing*. 39(2): p. 262-272.

Belegundu, Ashok D. and Chandrupatla, Tirupathi R., 2011, Optimization Concepts and Applications in Engineering, Cambridge University Press.

Bintang, Boeing 787/7E7, *www.bintang.site11.com/Boeing_787/Boeing787_files/Specifications.html*

Camanho, Pedro Ponces, 2002, Failure Criteria for Fibre-Reinforced Polymer Composites, Faculdade de Engenharia da Universidade do Porto.

Cepin, Marko, 2011, Assessment of Power System Reliability Methods and Applications, Springer.

Chang, Nan, Wang, Wei, Yang, Wei and Wang, Jian, 2010, Ply stacking sequence optimization of composite laminate by permutation discrete particle swarm optimization. *Structural and Multidisciplinary Optimization*. 41(2): p. 179-187.

Daniel, Isaac M. and Ishai, Ori, 1994, Engineering Mechanics of Composite Materials, Oxford University Press.

Eager, T. W., 1991, Whither advanced materials, ASM International.

Erdal, Ozgur and Sonmez, Fazil O., 2005, Optimum design of composite laminates for maximum buckling load capacity using simulated annealing. *Composite Structures*. 71(1): p. 45-52.

Gurdal, Z., Haftka, R. T. and Hajela, P., 1999, Design and optimization of laminated composite material, John Wiley & Sons, Inc.

Hyer, M. H., 1998, Stress Analysis of Fiber-Reinforced Composite Materials McGraw-Hill.

Irisarri, François-Xavier, Bassir, David Hicham, Carrere, Nicolas and Maire, Jean-François, 2009, Multiobjective stacking sequence optimization for laminated composite structures. *Composites Science and Technology*. 69(7-8): p. 983-990.

Karakaya, Şükrü and Soykasap, Ömer, 2009, Buckling optimization of laminated composite plates using genetic algorithm and generalized pattern search algorithm. *Structural and Multidisciplinary Optimization*. 39(5): p. 477-486.

Kaw, A. K., 2006, Mechanics of Composite Materials, CRC Press Taylor & Francis Group.

Kemal Apalak, M., Yildirim, Mustafa and Ekici, Recep, 2008, Layer optimisation for maximum fundamental frequency of laminated composite plates for different edge conditions. *Composites Science and Technology*. 68(2): p. 537-550.

Kim, C W and Lee, J S, 2005, Optimal Design of Laminated Composite Plates for Maximum Buckling Load Using Genetic Algorithm. *Mechanical Engineering Science*. 219: p. Part C.

Liu, Boyang, T. Haftka, Raphael, A. Akgün, Mehmet and Todoroki, Akira, 2000, Permutation genetic algorithm for stacking sequence design of composite laminates. *Computer Methods in Applied Mechanics and Engineering*. 186(2-4): p. 357-372.

Lopez, R. H., Luersen, M. A. and Cursi, E. S., 2009, Optimization of laminated composites considering different failure criteria. *Composites Part B: Engineering*. 40(8): p. 731-740.

Mallick, P. K., 2007, Fiber-reinforced composites materials, manufacturing, and design, Taylor & Francis Group, LLC.

Narayana Naik, G., Gopalakrishnan, S. and Ganguli, Ranjan, 2008, Design optimization of composites using genetic algorithms and failure mechanism based failure criterion. *Composite Structures*. 83(4): p. 354-367.

Reddy, J. N., 2004, Mechanics of Laminated Composite Plates and Shells, CRC Press.

Riche, R. Le and Haftka, R. T., 1995, Improved genetic algorithm for minimum thickness composite laminate design. *Composite Engineering*. 5(2): p. 143-161.

Sebaey, T. A., Lopes, C. S., Blanco, N. and Costa, J., 2011, Ant Colony Optimization for dispersed laminated composite panels under biaxial loading. *Composite Structures*. 94(1): p. 31-36.

Soden, P. D., Hinton, M. J. and Kaddour, A. S., 1998, A comparison of the predictive capabilities of current failure theories for composite laminates. *Composites Science and Technology*. 58(7): p. 1225-1254.

Soremekun, G., Gürdal, Z., Haftka, R. T. and Watson, L. T., 2001, Composite laminate design optimization by genetic algorithm with generalized elitist selection. *Computers and Structures*. 79(2): p. 131-143.

Soykasap, Ömer and Karakaya, Şükrü, 2007, Structural optimization of laminated composite plates for maximum buckling load capacity using genetic algorithm. *Key Engineering Materials* 348-349 p. 725-728.

Sun, C.T., Quinn, B.J., Tao, J., Oplinger, D.W. and Hughes, William J., 1996, Comparative Evaluation of Failure Analysis Methods for Composite Laminates National Technical Information Service, Springfield.

The Mathworks, Inc, Global Optimization Toolbox 3.1.

Vinson, J. R., 2005, Plate and Panel Structures of Isotropic, Composite and Piezoelectric Materials, Including Sandwich Construction, Springer.

Walker, M. and Smith, R. E., 2003, A technique for the multiobjective optimisation of laminated composite structures using genetic algorithms and finite element analysis. *Composite Structures*. 62(1): p. 123-128.

# APPENDIX A

# MATLAB COMPUTER PROGRAM

In this section, the computer program calculating the buckling load factor of the laminated composites, failure theories for fiber-reinforced composite materials and GA codes generated by Global Optimization Toolbox are given. After obtaining the expressions for buckling analysis, they can be used in optimization toolbox. Then the obtained results are controlled by the failure theories.

```
clear all;
close all;
clc;
format short
theta_half = [sym('th(1)') -sym('th(1)') sym('th(2)') -sym('th(2)') sym('th(3)') -
sym('th(3)') sym('th(4)') -sym('th(4)') sym('th(5)') -sym('th(5)') sym('th(6)') -sym('th(6)')
sym('th(7)') -sym('th(7)') sym('th(8)') -sym('th(8)') sym('th(9)') -sym('th(9)')
sym('th(10)') -sym('th(10)') sym('th(11)') -sym('th(11)') sym('th(12)') -sym('th(12)') ];%-
-> Half fiber orientation
theta = [theta_half fliplr(theta_half)];
Nplies = length(theta)
E1 = 181;   %[GPa] % Elastic Modulus
E2 = 10.3;    %[GPa] % Elastic Modulus
G12 = 7.17;    %[GPa] % Shear Modulus
NU12 = 0.28;   %%%%%%% Poisson ratio %%%%%%%%
NU21 = (NU12*E2)/E1;
Q11 = E1/(1 - NU12*NU21);
Q12 = (NU21*E1)/(1 - NU12*NU21);
Q22 = E2/(1 - NU12*NU21);
Q66 = G12;
Q = [ Q11 Q12 0; Q12 Q22 0; 0 0 Q66];
t = Nplies * h_ply ;
for i = 1:(Nplies+1);
 h(i) = -(t/2-((i-1)*(t/Nplies)));
end
D=0;
for i=1:Nplies
a=theta(1,i);
m=cos((a*pi)/180);
n=sin((a*pi)/180);
T = [ m^2 n^2 2*m*n; n^2 m^2 -2*m*n; -m*n m*n (m^2 - n^2)];
Qbar = inv(T) * Q * (inv(T))' ;
D = D + 1/3 * Qbar * (h(1,i+1)^3 - h(1,i)^3);
end
```

```
D;
 %%%% In-Plane Loads%%%%
 Nx = 1000000 %[N/m]edge:b
 Ny = 1000000 %[N/m] edge:a
 Nxy = 0;  %[N/m]
 a=0.508; %[m](length)
 b=0.254; %[m] (width)
 r=a/b
 m=1;n=1;
Nfbl = (pi^2)*(D(1,1)*(m^4) + 2*(D(1,2) + 2*D(3,3))*((r*m*n)^2) +
D(2,2)*((r*n)^4)); %failure buckling load
 Nal = ((a*m)^2)*Nx + ((r*a*n)^2)*Ny; %applied load
 Lamda_buckling11 = Nfbl/Nal
 m=1;n=2;
 Nfbl = (pi^2)*(D(1,1)*(m^4) + 2*(D(1,2) + 2*D(3,3))*((r*m*n)^2) +
D(2,2)*((r*n)^4)); %failure buckling load
 Nal = ((a*m)^2)*Nx + ((r*a*n)^2)*Ny; %applied load
 Lamda_buckling12 = Nfbl/Nal
 m=2;n=1;
 Nfbl = (pi^2)*(D(1,1)*(m^4) + 2*(D(1,2) + 2*D(3,3))*((r*m*n)^2) +
D(2,2)*((r*n)^4)); %failure buckling load
 Nal = ((a*m)^2)*Nx + ((r*a*n)^2)*Ny; %applied load
 Lamda_buckling21 = Nfbl/Nal
 m=2;n=2;
 Nfbl = (pi^2)*(D(1,1)*(m^4) + 2*(D(1,2) + 2*D(3,3))*((r*m*n)^2) +
D(2,2)*((r*n)^4)); %failure buckling load
 Nal = ((a*m)^2)*Nx + ((r*a*n)^2)*Ny; %applied load
 Lamda_buckling22 = Nfbl/Nal
 fid=fopen('bucklingloadfactor.m','w');
 int=('function y = bucklingloadfactor(th)');
 fprintf(fid, '%s\n',int);
 Lamda_buckling11=char(Lamda_buckling11);
 Lamda_buckling12=char(Lamda_buckling12);
 Lamda_buckling21=char(Lamda_buckling21);
 Lamda_buckling22=char(Lamda_buckling22);
 fprintf(fid,'%s','y=-min([');
 fprintf(fid,'%s%f',Lamda_buckling11);
 fprintf(fid,'%s\n',',...');
 fprintf(fid,'%s%f',Lamda_buckling12);
 fprintf(fid,'%s\n',',...');
 fprintf(fid,'%s%f',Lamda_buckling21);
 fprintf(fid,'%s\n',',...');
 fprintf(fid,'%s%f',Lamda_buckling22);
 fprintf(fid,'%s',']);');
 fclose(fid);
 display 'bitti!'
 %%%% Tsai-Wu and Max Stress  failure analyses %%%%%%%%
clear all;
close all;
clc;
```

```
format short
th=optimresults.x;
theta_half = [th(1,1) -th(1,1) th(1,2) -th(1,2) th(1,3) -th(1,3) th(1,4) -th(1,4) th(1,5) -
th(1,5) th(1,6) -th(1,6) th(1,7) -th(1,7) th(1,8) -th(1,8) th(1,9) -th(1,9) th(1,10) -th(1,10)
th(1,11) -th(1,11) th(1,12) -th(1,12)]
theta = [theta_half fliplr(theta_half)];
Nplies = length(theta)
h_ply = 0.28 %--> [mm]
E1 = 181000; %[MPa]
E2 = 10300; %[MPa]
G12 = 7170; %[MPa]
NU12 = 0.28;
NU21 = (NU12*E2)/E1;
Q11 = E1/(1 - NU12*NU21);
Q12 = (NU21*E1)/(1 - NU12*NU21);
Q22 = E2/(1 - NU12*NU21);
Q66 = G12;
Q = [ Q11 Q12 0; Q12 Q22 0; 0 0 Q66];
 t = Nplies * h_ply ;
for i = 1:(Nplies+1);
 h(i) = -(t/2-((i-1)*(t/Nplies)));
end
 A=0;B=0;D=0;
  for i=1:Nplies
  a=theta(1,i);
  m=cos((a*pi)/180);
  n=sin((a*pi)/180);
  T = [ m^2 n^2 2*m*n; n^2 m^2 -2*m*n; -m*n m*n (m^2 - n^2)];
  eval(['Qbar'  num2str(i) '=inv(T) * Q *T;'])
    Qbar=eval(['Qbar' int2str(i)]);
  A = A + Qbar * (h(1,i+1) - h(1,i));
  B = B + 1/2 * Qbar * (h(1,i+1)^2 - h(1,i)^2);
  D = D + 1/3 * Qbar * (h(1,i+1)^3 - h(1,i)^3);
 end
  A;
 B;
 D;
 Nx =-3000 %[N/mm]
 Ny =-1500 %[N/mm]
 Nxy = 0; %[N/mm]
 N = [Nx;Ny;Nxy]
a11=A(2,2)*A(3,3)-(A(3,2)*A(2,3));
a12=A(1,3)*A(3,2)-(A(3,3)*A(1,2));
a13=A(1,2)*A(2,3)-(A(2,2)*A(1,3));
a21=A(2,3)*A(3,1)-(A(3,3)*A(2,1));
a22=A(1,1)*A(3,3)-(A(3,1)*A(1,3));
a23=A(1,3)*A(2,1)-(A(2,3)*A(1,1));
a31=A(2,1)*A(3,2)-(A(3,1)*A(2,2));
a32=A(1,2)*A(3,1)-(A(3,2)*A(1,1));
a33=A(1,1)*A(2,2)-(A(2,1)*A(1,2));
```

```
aa=[a11,a12,a13;a21,a22,a23;a31,a32,a33];
detA=A(1,1)*A(2,2)*A(3,3) + A(1,2)*A(2,3)*A(3,1) + A(1,3)*A(2,1)*A(3,2) -
A(1,3)*A(2,2)*A(3,1) - A(1,2)*A(2,1)*A(3,3) - A(1,1)*A(2,3)*A(3,2);
Ainv=(1/detA)*aa;
 for i=1:Nplies
    eps = Ainv*N;
    Qbar=eval(['Qbar' int2str(i)]);
    eval(['sigma' num2str(i) '=Qbar*eps;']);
 end
 %%%%%% Principal Material System Stresses %%%%%%
 clear m n
 for i=1:Nplies
    a=theta(1,i);
    m=cos((a*pi)/180);
    n=sin((a*pi)/180);
    sigma=eval(['sigma' int2str(i)]);
    eval(['sigmaf1_'
num2str(i)'=m^2*sigma(1,1)+n^2*sigma(2,1)+2*m*n*sigma(3,1);'])
     eval(['sigmaf2_' num2str(i) '=n^2*sigma(1,1)+m^2*sigma(2,1)-
2*m*n*sigma(3,1);'])
    eval(['T12_' num2str(i) '=-m*n*sigma(1,1)+m*n*sigma(2,1)+(m^2-
n^2)*sigma(3,1);'])
   end
Xt = 1500;
Xc = 1500;
Yt = 40;
Yc = 246;
S12 = 68;
%%%%%%%%% Strength parameters %%%%%%%%
F1 = (1/Xt) - (1/Xc);
F2 = (1/Yt) - (1/Yc);
F11 = 1/(Xt*Xc);
F22 = 1/(Yt*Yc);
F12 = -0.5*(F11*F22)^0.5;
F21 = 1/S12^2;
 %%%%%%% Tsai - Wu theory %%%%%%%%%%%%
display 'Tsai-Wu failure theory'
 for i=1:Nplies
    sigmaf1_=eval(['sigmaf1_' int2str(i)]);
    sigmaf2_=eval(['sigmaf2_' int2str(i)]);
    T12_=eval(['T12_' int2str(i)]);
    eval(['TWfunction' num2str(i)
'=F1*sigmaf1_+F2*sigmaf2_+F11*sigmaf1_^2+2*F12*sigmaf1_*sigmaf2_+F22*sigm
af2_^2+F21*T12_^2;'])
    TWfunction(1,i)=eval(['TWfunction' int2str(i)]);
 end
 TWfunction;
 display 'Max TW function value'
 [failurevalue,laminate_no]=max(TWfunction)
 display 'Min TW function value'
```

```
[failurevalue,laminate_no]=min(TWfunction)
%%%%%%%% Max Stress Theory %%%%%%%%%%%%%%%%%%%
display 'Max Stress failure theory'
for i=1:Nplies
    a=theta(1,i);
    m=cos((a*pi)/180);
    n=sin((a*pi)/180);
    sigma=eval(['sigma' int2str(i)]);
    eval(['sigmaf1_' num2str(i) '=-
(m^2*sigma(1,1)+n^2*sigma(2,1)+2*m*n*sigma(3,1));'])  %%%%%%%%%% Xc
failure control
    eval(['sigmaf2_' num2str(i) '=-(n^2*sigma(1,1)+m^2*sigma(2,1)-
2*m*n*sigma(3,1));'])  %%%%%%%%%% Yc failure control
    eval(['T12_' num2str(i) '=abs(-m*n*sigma(1,1)+m*n*sigma(2,1)+(m^2-
n^2)*sigma(3,1));'])%%%%%%%%%% S12 failure control
    sigmaf1_(1,i)=eval(['sigmaf1_' int2str(i)]);
    sigmaf2_(1,i)=eval(['sigmaf2_' int2str(i)]);
    T12_(1,i)=eval(['T12_' int2str(i)]);
end
sigmaf1_;
sigmaf2_;
T12_;
display 'Xc strength_value=1500 MPa'
display 'Max MS_sigmaf1_ value'
[failurevalue,laminate_no]=max(sigmaf1_)
display 'Min MS_sigmaf1_value'
[failurevalue,laminate_no]=min(sigmaf1_);
display 'Ycstrength_value=246 MPa'
display 'Max MS_sigmaf2_value'
[failurevalue,laminate_no]=max(sigmaf2_)
display 'Min MS_sigmaf2_value'
[failurevalue,laminate_no]=min(sigmaf2_);
display 'S12strength_value=68 MPa'
display 'Max MS_T12_value'
[failurevalue,laminate_no]=max(T12_)
display 'Min MS_T12_value'
[failurevalue,laminate_no]=min(T12_);
```