DEVELOPMENT OF UNIVARIATE CONTROL CHARTS FOR NON-NORMAL DATA

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ABSTRACT

DEVELOPMENT OF UNIVARIATE CONTROL CHARTS FOR NON-NORMAL DATA

In this study, a new control chart methodology was developed to address statistical process monitoring issue associated with non-normally distributed process variables. The new method (NM) was compared aginst the classical Shewhart control chart (OM) using synthetic datasets from normal and non-normal distributions as well as over an industrial example. The NM involved taking the difference between the specified probability density estimate and non-parametric density estimate of the variable of interest to calculate an error value. Both OM and NM were found to work well for normally distributed data when process is in-control and out-of control situation. Both methods could be returned back to normal operation upon feeding in-control data.

In case of non-normally distributed data, the OM failed significantly to detect small shifts in mean and standard deviation, however the NM maintained its performance to detect such changes.

In the application to an industrial case (data were obtained from a local cement manufacturer about a 90 micrometer sieve fraction of the final milled cement product), the NM methodology outperformed the OM by recognizing the change in the mean and variance of the measured parameter. The data were tested for its distribution and were found to be non-normally distributed. Violations beyond the control limits in the new developed technique were easily observed. The NM was found to successfully operate without the necessity to apply run rules.

ÖZET

NORMAL DAĞILIMA SAHİP OLMAYAN DEĞİŞKENLER İÇİN KONTROL GRAFİKLERİNİN GELİŞTİRİLMESİ

Bu çalışmada, normal olarak dağılmayan proses değişkenleri ile ilintili istatistiksel proses gözleme amacına cevap verecek yeni bir kontrol grafiği metodolojisi geliştirilmiştir. Rastgele normal, normal olmayan dağılımlara sahip olan ve sanayi veri kümelerine Shewhart kontrol grafiklerinin (EY) uygulanması ve bunların yeni yöntem (YY) ile kıyaslanması yapılmıştır. YY bir hata değeri hesaplamak için, ilgili değişkenin parametrik olmayan yoğunluk tahmini değerinin ve belirlenen ihtimal yoğunluğu tahmininin arasındaki farkın bulunmasını kapsar. Normal dağılmış verinin kontrol altında ve kontrol dışındaki durumları için EY ve YY yöntemlerinin her ikisinin de iyi çalıştığı gözlendi. Kontrol sınırları içinde veri beslenince her iki yöntemin de normal operasyona döndürülebildiği gösterildi.

Normal dağılmayan veri durumunda ise, EY ortalama ve standard sapmadaki küçük değişiklikleri yakalamada başarısız olurken YY bu değişiklikleri yakalamada başarılı oldu.

YY'in çimento üreticisinden temin edilen ve 90 mikron elek üstü öğütülmüş çimento miktarını (DACK 90) içeren sanayi verilerine uygulanması durumunda EY'e göre ölçülen parametrenin ortalaması ve varyansındaki değişiklikleri tanımada daha başarılı olduğu tespit edildi. Verinin dağılımı sınandı ve normal olmayan şekilde dağıldığı tespit edildi. YY'ta kontrol sınırları dışında kalan ihlaller kolaylıkla gözlendi. YY'un, çalışma kurallarına gerek duymadan, başarıyla uygulanabildiği görüldü.

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GLOSSARY OF ABBREVIATIONS

OM	Old Method or the Shewhart Control Chart Method
NM	New Method That is Developed in This Thesis
NDD	Normally Distributed Data
NNDD	Non-normally Distributed Data
NND-ID	Non-normally Distributed Industrial Data
ATS	Average Time to Signal
ARL	Average Run Length
NNDNOD	Non-normally Distributed Normal Operation Data
NDNOD	Normally Distributed Normal Operation Data
EWMA	Exponentially Weighted Moving Average
CUSUM	Cumulative Sum
CC	Control Chart
UCL	Upper Control Limit
LCL	Lower Control Limit
SPC	Statistical Process Control
UWL	Upper Warning Limit
LWL	Lower Warning Limit
VSI	Variable Sampling Interval
ID	Industrial Data
CPCT	Cumulative Percent Coarser Than
ND	Normally Distributed
RF	Relative Frequency
KSDENSITY	Kernel Smoothing Density Estimation
NORMPDF	Normal Probability Density Function
NPDF	Normal Probability Density Function
CHI2PDF	Chi-square Probability Density Function
NORMRND	Random Matrices from Normal Distribution
KDE	Kernel Density Estimators
NORMCDF	Normal Cumulative Distribution Function
CHI2CDF	Non-normal Cumulative Distribution Function
STD	Standard Deviation

CHAPTER 1

INTRODUCTION

The main work of SQC (Statistical Quality Control) is to control the central tendency and variability of some processes. A common monitoring tool is to construct control charts "(Dou and Ping 2002)". A control chart (CC) is a statistical scheme (usually allowing graphical implementation) devised for the purpose of checking and then monitoring the statistical stability of a process.

An efficient CC must continue sampling as long as the process is in-control and must give an out-of control signal to stop sampling as quickly as possible when the process becomes out-of-control "(Bakir 2004)". The major function of control charting is to detect the occurrence of assignable causes so that the necessary corrective action may be taken before a large quantity of non-conforming product is manufactured "(Chou et al. 2001)". The most widely used method to control the central tendency of a process is Shewhart-X chart "(Shewhart 1931)" which includes a centerline and two control limit lines.

There are two other possible alternatives to the Shewhart control charts in the construction of the central location control charts. One is the CUSUM (Cumulative Sum) chart and the other is the EWMA (Exponentially Weighted Moving Average) chart. Both of these concentrate on improving the performance of control charts in detecting small shifts by using historical data "(Dou and Ping 2002)".

Various control chart techniques have been developed and widely applied in process control. Duncan's cost model "(Duncan 1956)" includes the cost of an out-of-control condition, the cost of false alarms, the cost of searching for an assignable cause, and the cost of sampling, inspection, evaluation, and plotting. In addition to the economic design of control charts, another approach to designing a control chart is called statistical design "(Chou et al. 2001)".

In real industrial applications, the process populations are frequently not normally distributed. Burr discussed the effects of non-normality and provided appropriate control constants for different non-normal populations. All the distributions considered belong to the Burr family "(Burr 1967)", which is of the following form:

$$F(y)=\begin{pmatrix} 1-(1+y^c)^{-q}, & y \geq 0, \\ 0, & y < 0, \end{pmatrix} \quad (1)$$

.

"(Yourstone and Zimmer 1992)" used the Burr distribution to represent various non-normal distributions and consequently to statistically design the control limits of an X control chart. "(Chou and Cheng 1997)" extended the model presented by Yourstone and Zimmer to design the control limits of the ranges control chart under non-normality. Also, "(Tsai 1990)" employed the Burr distribution to design the probabilistic tolerance for a subsystem. "(Rahim 1985)" proposed an economic model of the x chart for non-normal data by transforming the standardized normal random variate to non-normal variates.

If the non-normal process distribution really should be non-normal, it is necessary to use new knowledge to manage and improve the process. One software package can even adjust the control limits and the center line of the control chart so that control charts for non-normal data are statistically equivalent to Shewhart control charts for normal data.

The aim of this study is to design the control chart for non-normal data. Firstly the error chart was developed by taking difference between the probability density function (pdf) and non-parametric density estimation (ksdensity) for the normal distribution. Upper control limit (UCL) and lower control limit (LCL) of the process were determined. Then this procedure was applied for chi-square which is one of the non-normal distributions and applied for industrial data.

The second chapter presents the principles of statistical process control and the work done in the literature on control charts. The data and the method used in this study is given in chapter three. Fourth chapter presents the results of the control charts developed in this study and their comparison to the Shewhart method. Finally, conclusions are given in the fifth chapter.

CHAPTER 2

LITERATURE SURVEY

2.1. Statistical Process Control

It is impossible to inspect or test quality into a product; the product must be built right the first time. This implies that the manufacturing process must be stable and that all individuals involved with the process (including operators, engineers, quality assurance personel, and management) must continuously seek to improve process performance and reduce variability in key parameters. On-line statistical process control (SPC) is a primary tool for achieving this objective.

If a product is to meet or exceed customer expectations, generally it should be produced by a process that is stable or repetable. More precisely, the process must be capable of operating with little variability around the target or nominal dimensions of the product's quality characteristics. SPC is a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability.

SPC can be applied to *any* process. Its seven major tools are:

1. Histogram or stem-and-leaf plot
2. Check sheet
3. Pareto chart
4. Cause-and-effect diagram
5. Defect concentration diagram
6. Scatter diagram
7. Control chart

Although these tools, often called "the magnificent seven", are an important part of SPC, they comprise only its technical aspects. SPC builds an environment in which all individuals in an organization seek continous improvement in quality and productivity. This environment is best developed when management becomes involved in the process. Once this environment is established, routine application of the magnificent seven becomes part of the usual manner of doing business, and organization is well on its way to achieving its quality improvement objectives.

Control charts are the simplest type of on-line statistical process control procedure.

2.2. Control Charts

Control charts are useful for tracking process statistics over time and detecting the presence of special causes. A special cause results in variation that can be detected and controlled. Examples of special causes include supplier, shift, or day of the week differences. Common cause variation, on the other hand, is variation that is inherent in the process. A process is in control when only common causes - not special causes - affect the process output.

Variable control charts, described here, plot statistics from measurement data, such as length or pressure. Attributes control charts plot count data, such as the number of defects or defective units. For instance, products may be compared against a standard and classified as either being defective or not. Products may also be classified by their number of defects.

A process statistic, such as a subgroup mean, individual observation, or weighted statistic, is plotted versus sample number or time. As with variables control charts, a process statistic, such as the number of defects, is plotted versus sample number or time for attributes control charts.A “center line” is drawn at the average of the statistic being plotted for the time being charted. Two other lines - the upper and lower control limits - are drawn, by default, 3σ above and below the center line (Figure 2.1.)

A process is in control when most of the points fall within the bounds of the control limits, and the points do not display any nonrandom pattern.

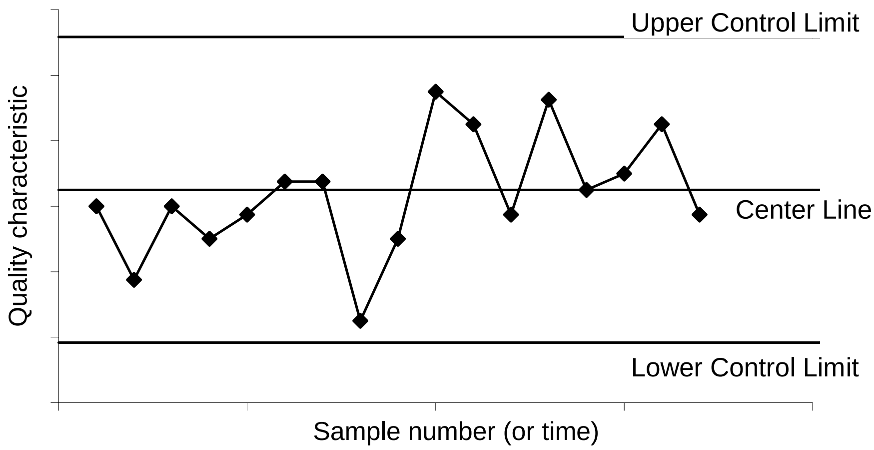

Figure 2.1. Structure of a control chart

The number of sampling instances before the CC signals is called the run length, which we denote by L. The efficiency of a CC depends on the distribution of the run length L. The most common and simplest efficiency criterion is to consider the average run length (ARL), which is the expected value of the run length distribution. It is desirable that the ARL of a CC be large if the process is in-control and be small if the process is out-of-control "(Chou et al. 2004)". The false alarm rate is the probability that the CC gives an out-of-control signal when in fact the process is in-control. Most control charts are distribution-based procedures in the sense that the observations made on the process output are assumed to follow a specified probability CCs. "(Amin et al. 1995)" found a pronounced difference in the values of the in-control ARL of the Shewhart X-bar chart under various distributions. Assuming a known standard deviation and a sample size of n=10, they found the exact values of the in-control ARLs of the traditional (one-sided) $3\sigma\bar{x}$ Shewhart X-bar to be: 1068.7 under a uniform distribution, 740.8 under a normal distribution, 441.9 under a double exponential distribution, and 11.7 under a Cauchy distribution. This implies that for heavy-tailed underlying distributions, false alarms will occur much more frequently than expected when the process is operating in-control. For example, when the process has a Cauchy distribution, the in-control ARL will only be 11.7, which entails almost 63 times as many false alarms as the anticipated ARL value of 740.8 associated with the traditional $3\sigma\bar{x}$ Shewhart control limits "(Bakir 2004)".

Traditionally, when the issue on designing control chart is discussed, one usually assumes the measurements in each sampled subgroup (or say population) are normally distributed; therefore, the sample mean $\overline{X}$ is also normally distributed "(Chou et al. 2004)".

2.2.1. Control Charts for Normal Data

In this section control charts for normal data are explained. First Shewhart Control Charts are discussed followed by some introductory information about the ARL and ATS.

2.2.1.1. Shewhart Control Chart

Since 1924 when Dr. Shewhart presented the first control chart, various control chart techniques have been developed and widely applied as a primary tool in statistical process control. A survey by "(Saniga and Shirland 1977)" indicated that the control chart for averages (or the X chart) dominates the use of any other control chart technique if quality is measured on a continuous scale "(Chou 2001)".

When the $\overline{X}$ control chart is used to monitor a process, three parameters should be determined: the sample size, the sampling interval between successive samples, and the control limits of the chart which are *UCL*(Upper Control Limit) and *LCL* (Lower Control Limit) "(Chou 2000)":

$$UCL = \overline{X} + K\sigma\overline{X} \tag{2}$$

$$CL = \overline{X} \tag{3}$$

$$LCL = \overline{X} - K\sigma\overline{X} \tag{4}$$

where

$$\overline{X} = \frac{\overline{X}_1 + \overline{X}_2 + ... + \overline{X}_m}{m} \tag{5}$$

is the grand average, K is the control constant and $\sigma\overline{X}$ is the standard deviation of sample mean and m is the number of samples taken. There are tables available for values of K based on the sample sizes and process requirements "(Montgomery 2005)". Shewhart also suggested 3-sigma control limits as action limits and sample sizes of four or five, leaving the interval between successive subgroups to be determined by the practitioner "(Chou 2001)".

In process control using Shewhart Control Charts, common practice is to observe for data points that lie outside of the control limits during normal operation. When such data are collected there are criteria that are developed to determine whether the process is out of control. These are known as run-rules. Western Electric Handbook presents a comprehensive discussion of the issue "(Western Electric Handbook 1956)".

2.2.1.2. Calculation of ARL and ATS

In-control average run length(ARL), out-of-control ARL and average time to signal were evaluated for α=0.01. These values were computed by equation 6, 7 and 8 "(Montgomery 2005)".

$$\text{In-control } ARL = ARL_0 = \frac{1}{\alpha} \tag{6}$$

$$\text{Out-of-control } ARL = ARL_1 = \frac{1}{1-\beta} \tag{7}$$

$$ATS = ARL*h \tag{8}$$

$$\beta = \Phi\left[\frac{UCL-(\mu_0+k\sigma)}{\sigma/\sqrt{n}}\right] - \Phi\left[\frac{LCL-(\mu_0+k\sigma)}{\sigma/\sqrt{n}}\right] \tag{9}$$

where α is the probability of making type I error, β is the probability of making type II error, h is time, Φ is the cumulative distribution function and μ_o is the mean in the in-control case.

2.2.1.3. Statistically Designed Control Charts

Statistically designed control charts are those in which control limits (which determine the Type I error probability, α) and power are preselected. These then determine the sample size and, if the average time to signal is specified, the sampling interval "(Woodall,1985)". "(Saniga 1989)" incorporated the concept of statistical considerations into the economic design of the control charts and then presented the `economic statistical design' of the joint X and R charts for normal data

2.2.2. Control Charts for Non-Normal Data

The normality assumption may not be tenable every time. For example, the distributions of measurements from chemical processes, semiconductor processes, or cutting tool wear process are often skewed "(Chang and Bai, 2001)". If the measurements are really normally distributed, the statistic X, which is the sample characteristic of interest, is also normally distributed. If the measurements are asymmetrically distributed, the statistic X will be approximately normally distributed only when the sample size n is sufficiently large (based on the central limit theorem) and when its. Unfortunately, when a control chart is applied to monitor the process, the sample size n is always not sufficiently large due to the sampling cost. Therefore, if the measurements are not normally distributed, the traditional way for designing the control chart may reduce the ability that a control chart detects the assignable causes "(Chou 2001)".

The non-normal behavior of measurements may imply that the traditional design approach is improper for the operation of control charts.

2.2.2.1. Burr Distribution

The Burr cumulative distribution function "(Burr, 1942)" is

$$F(y) = 1 - \frac{1}{(1+y^c)^q}, \qquad \text{for} \geq 0, \tag{10}$$

where c and q are greater than 1. “(Burr 1967)” applied his distribution to study the effect of non-normality on the constants of X and R control charts. “(Burr 1942)” tabulated the expected value, standard deviation, skewness coefficient, and kurtosis coefficient of the Burr distribution for various combinations of c and q. These tables allow the users to make a standardized transformation between a Burr variate (say, Y) and another random variate (X)

Denote UCL, LCL, UWL, and LWL as the upper control limit, lower control limit, upper warning limit, and lower warning limit, respectively. Expressed mathematically, we obtain

$$UCL = \mu_0 + k\sigma/\sqrt{n}, \qquad LCL = \mu_0 - k\sigma/\sqrt{n}, \tag{11}$$

$$UWL = \mu_0 + w\sigma/\sqrt{n}, \qquad LWL = \mu_0 - w\sigma/\sqrt{n}, \tag{12}$$

where μ_0 is the process mean when the process is in control. In this article, to simplify the model, we assume that when the process is out of control, the process mean shifts to $\mu_1 = \mu_0 + \delta\sigma$, but the process standard deviation remains unchanged. The Burr random variate Y can be transformed to the sample statistic X by the standardized procedure as follows:

$$\frac{Y - M}{S} = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \tag{13}$$

That is, the scale and origin of the fitted Y values are changed to those of the X values, and from Equation above, when the process is in-control, we obtain,

$$\overline{X} = \mu_0 + (Y - M)\frac{\sigma/\sqrt{n}}{S} \tag{14}$$

When the process is out-of-control, X is assumed to follow a Burr distribution with mean $\mu_0 + \delta\sigma$ and standard deviation $\sigma/\sqrt{n}$ “(Chou 2004)”.

2.2.2.2. The Variable Sampling Interval (VSI) $\overline{X}$ control chart

Assume that the distribution of measurements from a process is non-normally distributed, and has the mean μ and standard deviation σ. When an $\overline{X}$ control chart (with centerline μ_0; the upper control limit $\mu_0 + k_1(\sigma/\sqrt{n})$ and the lower control limit $\mu_0 - k_1'(\sigma/\sqrt{n})$ where k_1 and k_1' are not necessarily equal) is used for monitoring the process, a sample of size n is taken and calculated its sample mean at each sampling point to judge whether or not the process remains in-control state. If the sample mean $\overline{X}_i$ plotted on the control $\overline{X}$ chart goes beyond the control limits, then a signal will be given to inform the operator to search for the assignable cause. Otherwise, the process is considered being in-control, and the next sample is continually taken at next sampling point

In this situation, the control chart operates with fixed sampling interval (say h_1) regardless of $\overline{X}_i$ which is said to be the FSI control chart.

For VSI control charts, if the sample mean falls inside the control limits, the monitored process is also considered stable as FSI chart. However, with a difference to the FSI charts, the next sampling interval will be a function of this sample mean. That is, the next sampling rate depends on the current sample mean.

Assume the VSI $\overline{X}$ chart uses a unite number of sampling interval lengths, say h_1, h_2, … h_m; where $h_1<h_2<\ldots<h_m$; and $m\geq 2$. The choice of a sampling interval can be made by a function h(x) when the value of $\overline{X}_i$ is measured. Burr distribution can be implemented for the economic design of the VSI chart in monitoring non-normal process data.

CHAPTER 3

THE PROPOSED METHOD AND THE MODEL

The methodology used in this study for the development of the new method (NM) is briefly presented in this section. First of all different data sets were created or collected from the industry to test the effectiveness of the OM and the NM. Matlab v.7 software was used for computations. A number of built-in functions were employed in Matlab environment for computations. These functions are explained in section 3.3 and a complete list of program codes is given in Appendix D.

3.1. The Data

Different types of data were used in this study. Two random selections of 10000 data points were generated in Matlab, the first being normally distributed while the second was non-normally distributed (chi-square distribution). In addition, industrial data about the fineness of Portland cement were collected from a local cement plant (Çimentaş A.Ş.).

3.1.1. Normally Distributed Random Data

Matlab v.7 commercial software was used to create the randomly selected normally distributed data set. 10000 randomly selected data were produced with an average of 20 and a standard deviation of 1 (Appendix A). Batches with as much as 1000 data were taken from this 10000 data stock. The first 1000 data (Figure 3.1) were named “normally distributed normal operation data (NDNOD)” and were used for calculation of the control limits.

3.1.2. Non-Normally Distributed Random Data

Matlab v.7 commercial software was again used to create the randomly selected non-normally distributed data set. 10000 randomly selected data points with a chisquare non-normal distribution were created with 5 degrees of freedom (Appendix B). Batches with as much as 1000 data were taken from this 10000 data stock. The first 1000 data (Figure 3.2) were named “non-normally distributed normal operation data (NNDNOD)” and were used for calculation of the control limits.

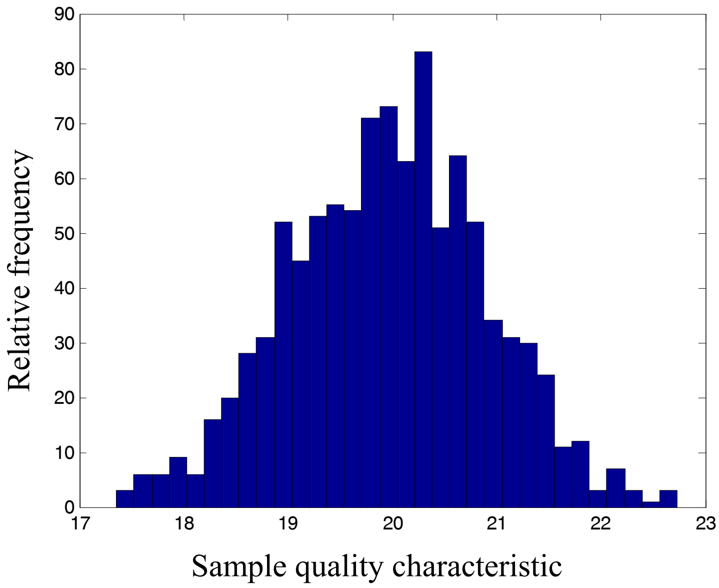

Figure 3.1. Histogram of the normally distributed data used in this study (NDNOD).

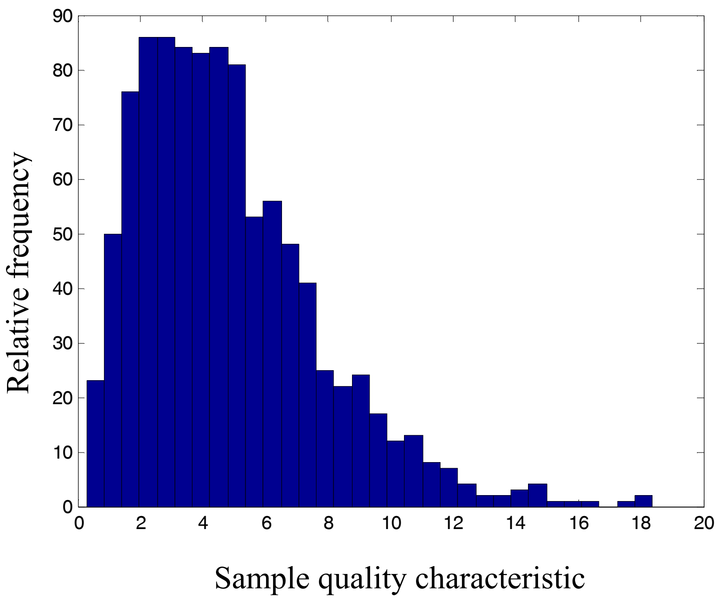

Figure 3.2. Histogram of the non- normally distributed (chi-square) data used in this study (NNDNOD)

3.1.3. Industrial Data (ID)

Portland cement manufacture process starts with a rotary furnace step in which a semi-product of clinker is produced. The semi-product is fast-cooled to preserve the cement phases and milled in tube mills with heavy iron ball media. The tube mill is actually a tumbling ball mill. The product of the mill is a finely ground powder whose particle size and surface area are closely monitored as a process control tool. One of the most important parameters is the CPCT 90 micrometers (Cumulative Percent Coarser Than) which is the oversized weight percentage above a 90 micrometer sieve. This data is collected once in every hour. For effective cement hydration reaction this percentage must be controlled within predetermined limits. The percentage of milled product larger than (cumulative percent larger than: CPCT) the 90 micrometer sieve was measured and recorded (DACK 90) in the cement plant as a process control parameter.

There were a total of 1179 data points that were collected from the plant (Figure 3.3 and Appendix C). Average CPCT 90 micrometer value was 0.5614 with a standard deviation of 0.3262. The data was partitioned into two sets based on the observation that the process was out-of control after the 951st data point. Therefore the first part with 950 data points was called the ID for normal operation. The remaining 229 data points were for the out-of-control situation.

Histogram for industrial data is presented in Figure 3.4 for normal operation. In order to identify the type of the reference distribution with which the industrial data could be associated, a number of tests were done. As a result of the first visual inspection the data could be identified non-normal. Therefore, a chisquare type of distribution was attempted first. For this purpose the normplot function of Matlab was employed and the resulting graph is shown in Figure 3.5. When the crosses are located on the diagonal red line the distribution is normal. The more the crosses deviate away from the diagonal red line, the less becomes the normality. As can be seen in Figure 3.5. the ID used in this study was not normally distributed.

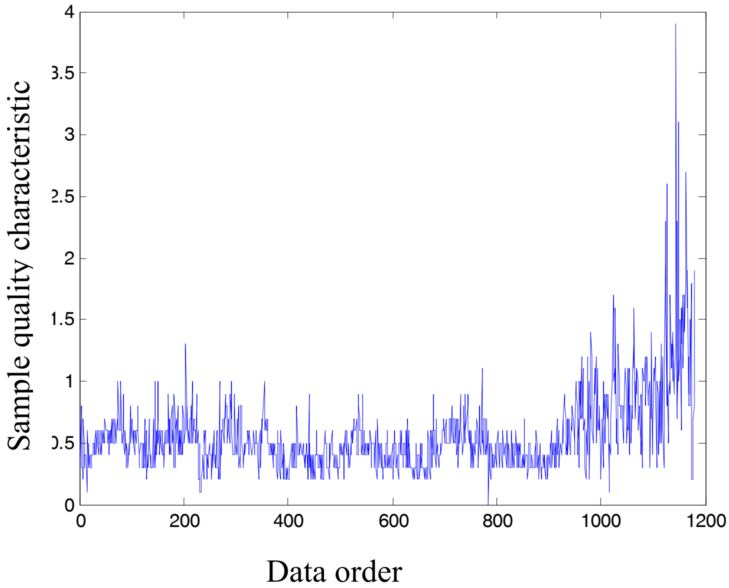

Figure 3.3. Complete industrial data used in this study.

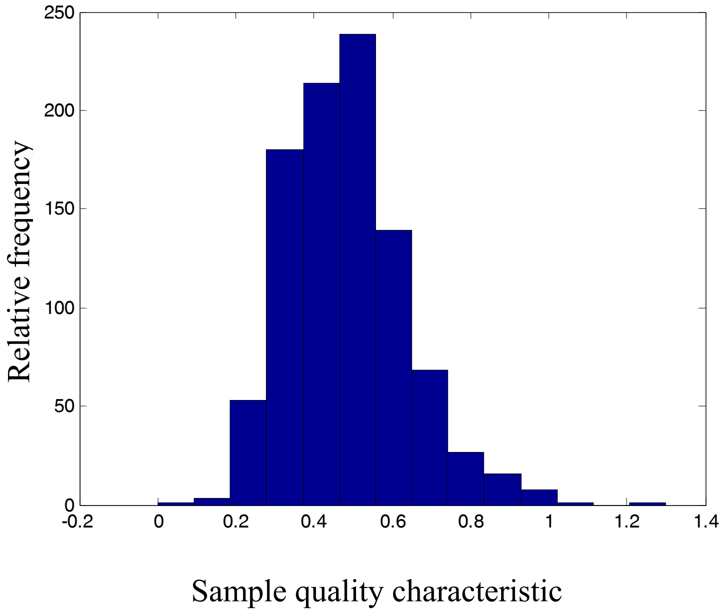

Figure 3.4. Histogram for normal operation part (950 data points) of industrial data.

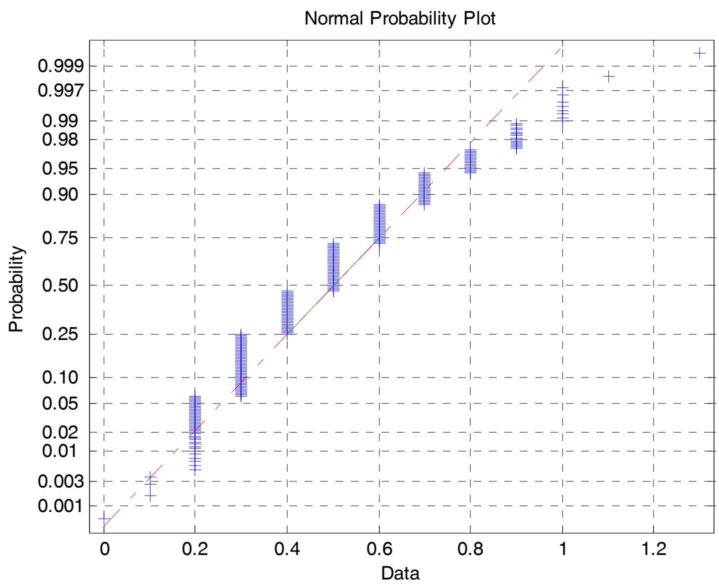

Figure 3.5. Check for normality of the ND part (950 data points) of the ID.

3.2. Proposed Methodology

SPC is usually carried out by plotting the control charts for normal operation data in order to check for violations that are outside the control limits. In this study, two separate control charts were produced from the normal operation data which was regarded as ideal, in-control situation. The first was the OM which utilized the well known Shewhart Control Chart approach while the second was developed in this study and was named the new method (NM).

3.2.1. The Shewhart Method (OM)

The theory of Shewhart control charts is given in section 2.2.1. Normal operation data (NDNOD) were analyzed to determine its standard deviation, upper and lower control limits via equations 2, 3, 4 and 5. Using these values the control chart was created. Different levels of shifts were imposed on the data to observe the control chart performance when the process gets out of control. In such a case operator intervention is required. Shifts as much as 0.1, 0.25, 0.5 and 1σ were made to the mean. In addition, the standard deviation was increased by 1.1, 1.25, 1.5 and 2. Finally, tests were conducted to make sure the process could be returned to normal operation. Same procedure was repeated for NNDNOD.

3.2.2. The New Method (NM)

Shewhart charts (OM) are well known and effectively used for normally distributed data as a process control tool. However, they are known to be deficient in the monitoring of non-normally distributed data. Therefore, a new method is proposed in this thesis to create a control chart for non-normally distributed data. The method works by first computing a ksdensity function using the built-in function of Matlab (ksdensity). Secondly either a normpdf or a chi2pdf function is computed. The mathematical basis for the two functions is presented in section 3.3. When the data was normally distributed the error was calculated by taking the difference between ksdensity and normpdf via equation 16. For non-normally distributed data, however, the difference between ksdensity and chi2pdf was used for error calculation (Eq. 17). The control chart developed in the new method contained the error as a process control measure in contrast to the original data itself that was used in the OM.

Figures 3.6. and 3.7 show a typical pdf and ksdensity estimation by using the Matlab statistics toolbox. The ksdensity points distributed around pdf plot illustrate the behavior of the process. The difference between the points of ksdensity (*ks*) and pdf(*pdf*) gives the error(*E*). Normpdf (*npdf*) used for normal distribution and chi2pdf (*cpdf*) used for chi-square distribution.

$$En = ksn - npdf \qquad (16)$$

$$Ec = ksc - cpdf \qquad (17)$$

Lower and upper control limits were then computed following the same procedure with the OM. However, the variable in this case was the error, not the original data. Control limits in the NM could be created based on the assumption that the error was normally distributed.

The effects of shifts to the data were also investigated for non-normally distributed data. The program code was written in such a way that gradually out-of-control data was added to the normal operation data and the effect of the appending process was observed on the NM error chart. Such error charts were plotted using batches of data with 500 data points.

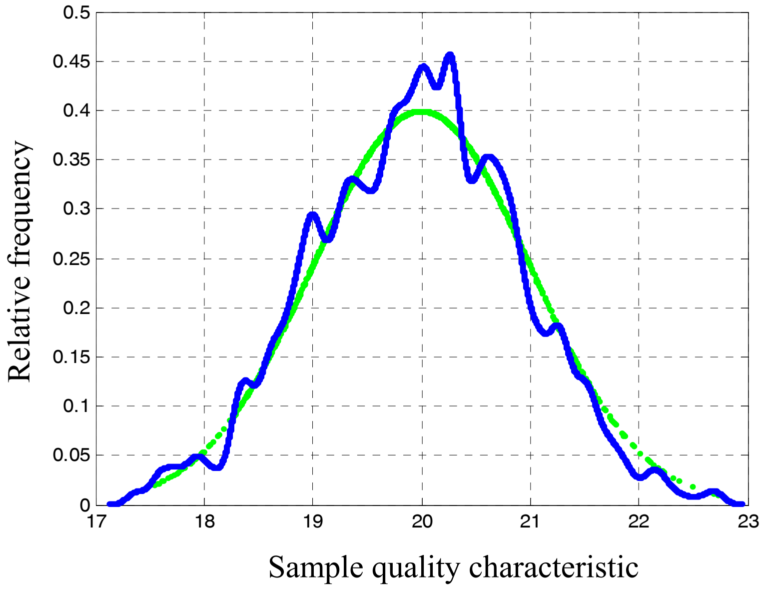

Figure 3.6. Plots of ksdensity and normal probability density function (normpdf). Wavy line indicates the ksdensity estimate while the other smooth line shows pdf.

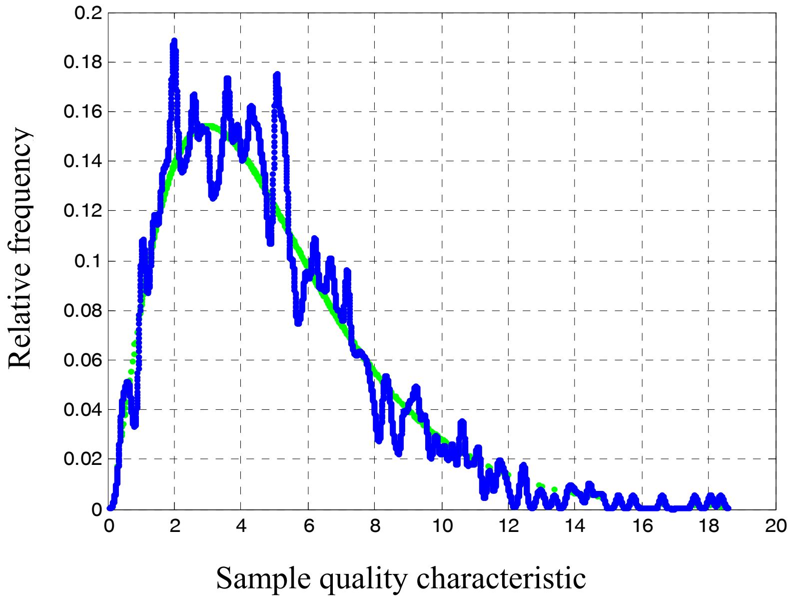

Figure 3.7. Plots of ksdensity and chi-square probability density function (chi2pdf). Wavy line indicates the ksdensity estimate while the other smooth line shows pdf.

3.3. Functions Used in Matlab

In this study, Matlab software was widely used for creation of control charts and for other computations. Some of the important built-in functions are explained below to help the reader.

3.3.1. Random Matrices from Normal Distribution (Normrnd)

The following commands illustrate how to call the Random matrices from normal distribution

$$R=normrnd\ (\mu,\sigma) \qquad (18)$$

Returns a matrix of random numbers chosen from the normal distribution with parameters μ and σ. The size of R is the common size of μ and σ if both are matrices. If either parameter is a scalar, the size of R is the size of the other parameter. Alternatively, $R = normrnd\ (\mu,\sigma, M, N)$ returns an M by N matrix.

3.3.2. Normpdf and Chi2pdf

Probability distribution function (pdf) command in Matlab returns the ordinate of the normal distribution at a given x value. The pdf command is used when x is known and the corresponding y value, which is the relative frequency, on the curve is desired (Figure 3.8).

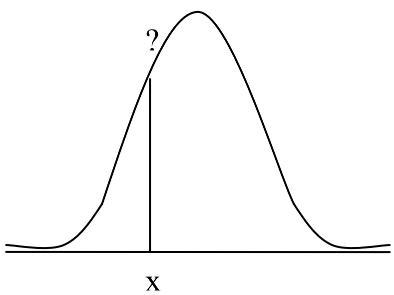

Figure 3.8. Visualization of the probability density function

A pdf is not a single function. Rather a pdf is a family of functions characterized by one or more parameters.

The pdf function call has the same general format for every distribution in the Statistics Toolbox of Matlab. The following commands illustrate how to call the pdf for the normal distribution.

$$Y = normpdf\,(x,\mu,\sigma) \tag{19}$$

This computes the normal pdf at each of the values in X using the corresponding parameters in mu and sigma. X, μ, and σ can be vectors, matrices, or multidimensional arrays that all have the same size. A scalar input is expanded to a constant array with the same dimensions as the other inputs. The parameters in σ must be positive.

The normal pdf is (Abramowitz and Stegun 1964).

$$y = f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{20}$$

3.3.3. Kernel Density Estimators (Ksdensity)

A data sample can de described by estimating its density in a nonparametric way. The ksdensity function does this by using a kernel smoothing function and an associated bandwidth to estimate the density.

There are several methods for choosing the interval width. These nonparametric estimators result in figures which are smoother than histograms, allowing easy recognition of characteristics such as outliers, skewness, and multimodality. Most of these methods have been employed to describe young and size distribution of each sample was analyzed by means of Kernel Density. Estimators (KDE), a statistical method first proposed by Silverman (1986) and defined as:

$$M_o(z) = \int \hat{f}^2 - \frac{2}{n}\sum_i \hat{f}_{-i}(X_i) \tag{21}$$

where ,

$\hat{f}^2$ = density estimation of the variable x

n = number of observations

z = bandwidth

Xi = length of the *i-th* fish specimen

[F, Xi]=ksdensity(X) computes a probability density estimate of the sample in the vector *X*.

Ksdensity evaluates the density estimate at 100 points covering the range of the data. *F* is the vector of density values and *Xi* is the set of 100 points. The estimate is based on a normal kernel function, using a window parameter (bandwidth) that is a function of the number of points in *X*.

F=ksdensity(X, Xi) specifies the vector *Xi* of values where the density estimate is to be evaluated. *[F, Xi,U]=ksdensity(...)* also returns the bandwidth of the kernel smoothing window.

[...]=ksdensity(...,'PARAM1',val1,'PARAM2',val2,...) specifies parameter name/value pairs to control the density estimation. Valid parameters are the following:

kernel : The type of kernel smoother to use, chosen from among 'normal' (default), 'box', 'triangle', and 'epanechnikov'.

Npoints : The number of equally-spaced points in *XI*.

Width : The bandwidth of the kernel smoothing window. The default is optimal for estimating normal densities, a smaller value to reveal features such as multiple modes can be chosen (Bowman and Azzalini 1997). In this thesis, this value was taken as 0.075 for the randomly selected data (NDD and NNDD) and as 0.05 for the industrial data (ID). The second value (0.05) was smaller because the numerical value of the industrial data (range from 0 to 1.4) was smaller.

3.3.4. Normal Cumulative Distribution Function (normcdf) and Chi-square Cumulative Distribution Function (chi2cdf)

Cdf computes a chosen cumulative distribution function. *P = cdf (NAME,X,A)* returns the named cumulative distribution function, which uses parameter A, at the values in *X*. Similarly for *P = cdf (NAME,X,A,B,C)*. The name can be: 'beta' or 'Beta', 'bino' or 'Binomial', 'chi2' or 'Chisquare', 'exp' or 'Exponential', 'ev' or 'Extreme Value', 'f' or 'F', 'gam' or 'Gamma', 'geo' or 'Geometric','hyge' or 'Hypergeometric', 'logn' or 'Lognormal', 'nbin' or 'Negative Binomial', 'ncf' or 'Noncentral F','nct' or 'Noncentral t', 'ncx2' or 'Noncentral Chi-square', 'norm' or 'Normal', 'poiss' or 'Poisson', 'rayl' or 'Rayleigh', 't' or 'T', 'unif' or 'Uniform', 'unid' or 'Discrete Uniform', 'wbl' or 'Weibull'.

CDF calls many specialized routines that do the calculations. *P = cdf('name',X,A1,A2,A3)* returns a matrix of probabilities, where name is a string containing the name of the distribution, *X* is a matrix of values, and *A, A2,* and *A3* are matrices of distribution parameters. Depending on the distribution, some of these parameters may not be necessary. Vector or matrix inputs for *X, A1, A2*, and *A3* must have the same size, which is also the size of *P*. A scalar input for *X, A1, A2, or A3* is expanded to a constant matrix with the same dimensions as the other inputs. *cdf* is a utility routine allowing you to access all the cdfs in the Statistics Toolbox by using the name of the distribution as a parameter.

3.3.4.1. Normal Cumulative Distribution Function (Normcdf)

$P = normcdf(X, \mu,\sigma)$

$[P, PLO, PUP] = normcdf(X, \mu, \sigma, PCOV, alpha)$

$normcdf(X, \mu,\sigma)$ computes the normal cdf at each of the values in *X* using the corresponding parameters in μ and σ. X, μ, and σ can be vectors, matrices, or multidimensional arrays that all have the same size. A scalar input is expanded to a constant array with the same dimensions as the other inputs. The parameters in σ must be positive. $[P, PLO, PUP] = normcdf(X,\mu, \sigma, PCOV,\alpha)$ produces confidence bounds for *P* when the input parameters μ and σ are estimates. PCOV is the covariance matrix of the estimated parameters. α specifies $100(1 -\alpha)\%$ confidence bounds. The default value of α is 0.05. PLO and PUP are arrays of the same size as P containing the lower and upper confidence bounds. The function normdf computes confidence bounds for P using a normal approximation to the distribution of the estimate and then transforming those bounds to the scale of the output P. The computed bounds give approximately the desired confidence level when you estimateμ, σ, and PCOV from large samples, but in smaller samples other methods of computing the confidence bounds might be more accurate. The normal cdf is he result, p, is the probability that a single observation from a normal distribution with parameters μ and will fall in the interval *(- x]*. The standard normal distribution has $\mu = 0$ *and* $\sigma = 1$

3.3.4.2. Chi- square Cumulative Distribution Function (chi2cdf)

P = chi2cdf(X,V) computes the chi2cdf at each of the values in *X* using the corresponding parameters in *V*. *X* and *V* can be vectors, matrices, or multidimensional arrays that have the same size. A scalar input is expanded to a constant array with the same dimensions as the other input. The degrees of freedom parameters in *V* must be positive integers, and the values in *X* must lie on the interval [0 1]. The chi2cdf for a given value *x* and ν degrees-of- freedom is where is the Gamma function. The result, *p,* is the probability that a single observation from a chi2 distribution with ν degrees of freedom will fall in the interval *[0 x]*. The chi2 density function with ν degrees-of-freedom is the same as the gamma density function with parameters ν /2 and 2.

CHAPTER 4

RESULTS AND DISCUSSION

In this chapter the results of OM and NM are given and compared for normally and non-normally distributed data as well as for industrial data.

4.1. Results of OM and NM for Normally Distributed Data

Normally distributed random data that was described in section 3.1.1 was used to plot the control charts. Two different techniques were used to create these charts: Shewhart control chart called the old method in this study (OM) and the new method (NM) that was developed in this thesis.

Upper and lower control limits of the OM charts were computed using the procedure described in section 2.2.1. The resulting Shewhart control charts (OM) are shown in Figure 4.1. As can be seen from the figure, the data is obviously normally distributed and there are very few data that are outside of the control limits.

Additional batches of randomly selected data of 500 data points were taken from the initial large random data set of 10000 members. These 500 data sets were shifted by addition of 0.1, 0.25, 0.5 and 1 to the mean. The purpose for imposing shifts to the data was to find out how the resulting control charts would be affected. Of course the mean increased and this was observed on the control chart (Figures 4.2-4.5). As a result of the increase in the mean the total number of violations was observed to increase in the UCL (upper control limit) area of the graph. This result was expected and the OM procedure was confirmed.

Table 4.1 shows the different scenarios that were tested in this study. Three different types of data were tested in the in-control and out of control situation for both the OM and the NM.

Table 4.1. List of the figures which show the progression of error charts in this study.

		Type of Data		
		NDD	NNDD	ID
In-Control	OM	Figure 4.1	Figure 4.22	Figure 4.35
	NM	Figure 4.11	Figure 4.28	Figure 4.38
Out-of-control	OM	Figures 4.2-4.9	Figures 4.23-4.26	Figure 4.36
	NM	Figures 4.12-4.19	Figures 4.29-4.32	Figure 4.39
Back to in-control	OM	Figure 4.10	Figure 4.27	
	NM	Figures 4.20-4.21	Figures 4.33-4.34	

4.1.1. Application of OM for α = 0.01

In this section the control charts for an α value of 0.01 are given (Figures 4.1-4.9). When the α value increases the L coefficient in UCL or LCL calculation is decreased and hence the control limits are narrower. The progression of the increase in the number of violations (out-of-the control limits) with an increase in the amount of shift of the mean (Figures 4.2-4.5) and the standard deviation (Figure 4.6-4.9) can be clearly observed. In Figure 4.10 the capability to return back to normal operation is tested and demonstrated. This was achieved by appending the already shifted data with in-control data.

The lower and upper control limits are calculated for two different α values. This was done to check how much a narrower control interval could lead to an increase in the number of violations.

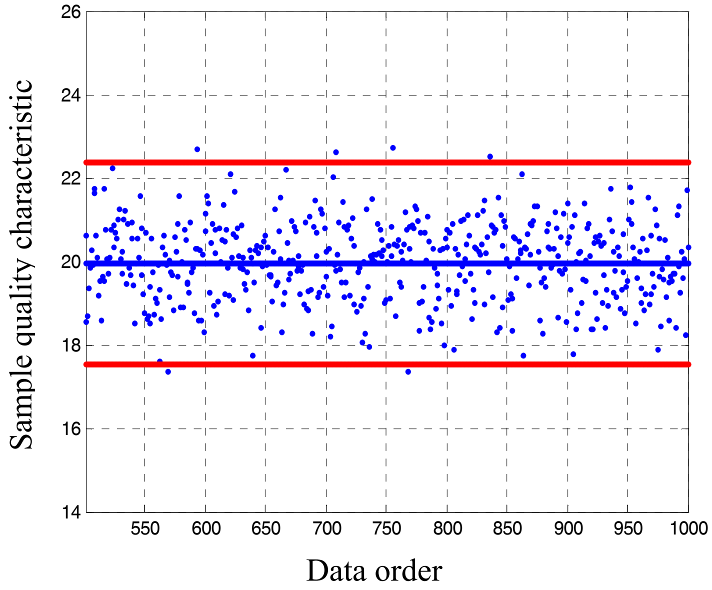

Figure 4.1. Shewhart control chart for normally distributed random data.

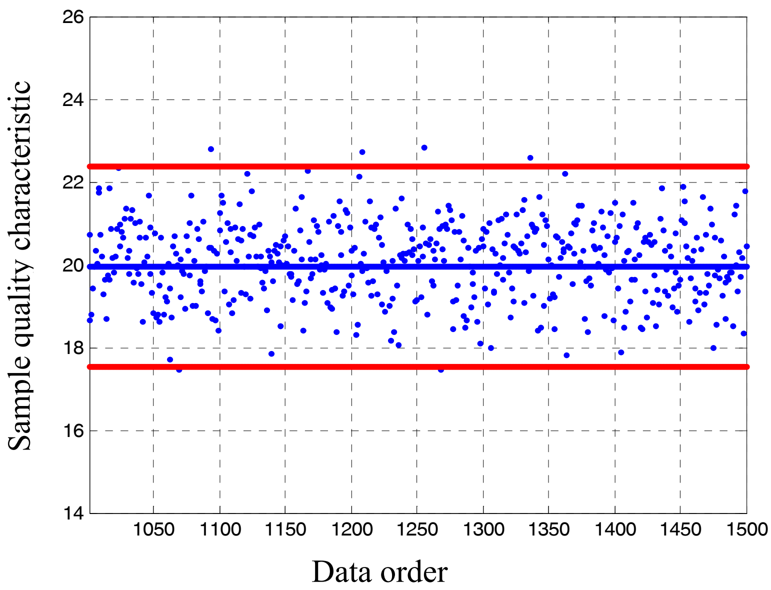

Figure 4.2. Shewhart control chart for normally distributed random data after its mean is shifted by 0.1 unit.

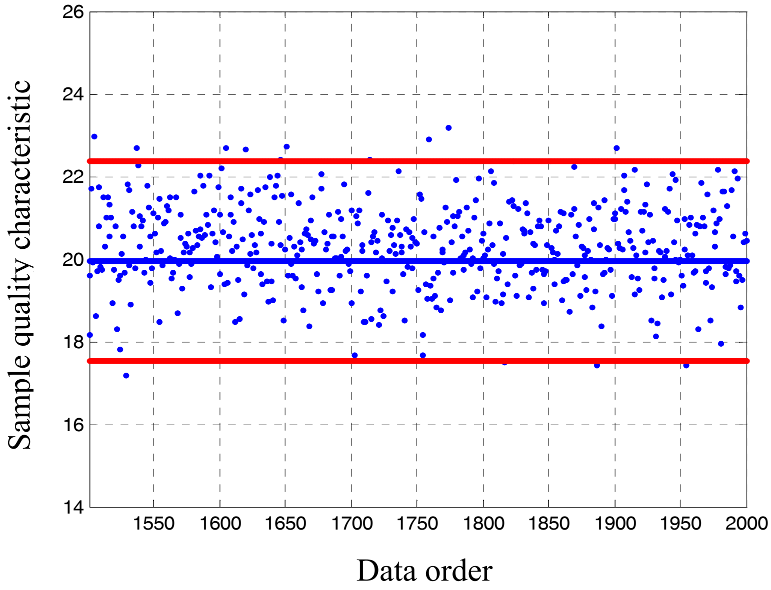

Figure 4.3. Shewhart control chart for normally distributed random data after its mean is shifted by 0.25 unit.

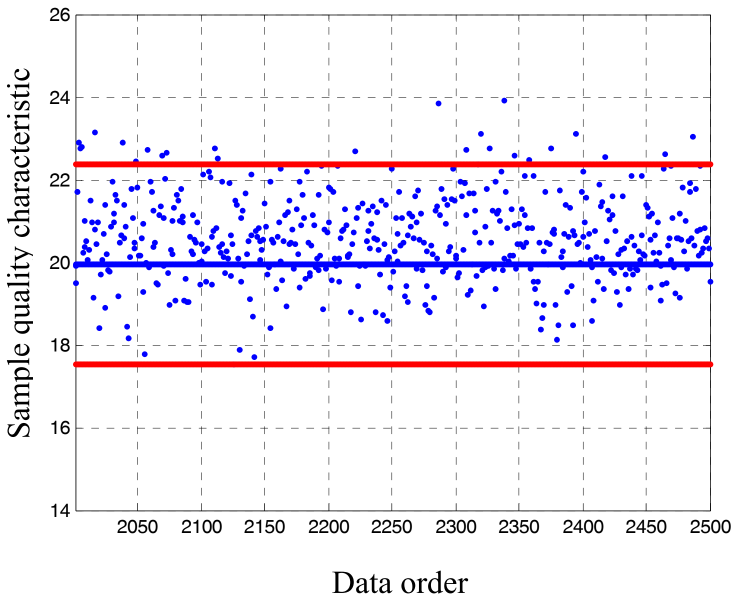

Figure 4.4. Shewhart control chart for normally distributed random data after its mean is shifted by 0.5 unit.

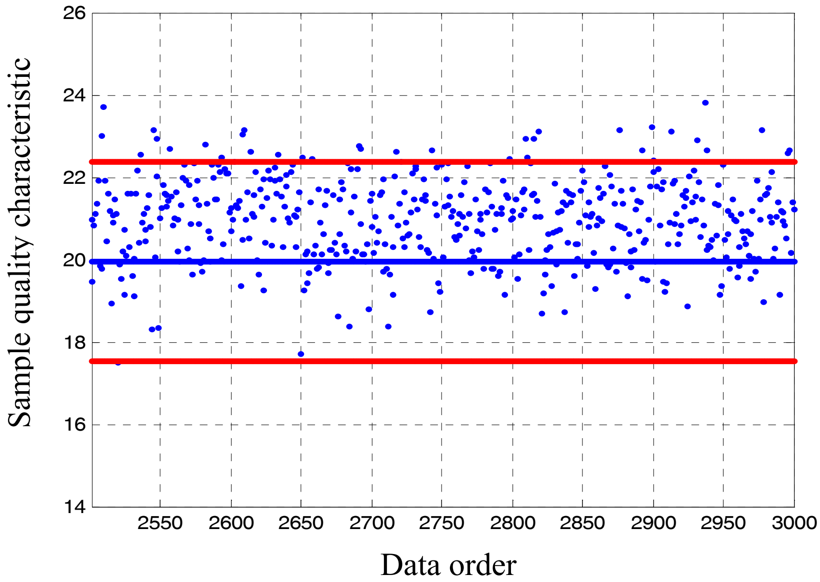

Figure 4.5. Shewhart control chart for normally distributed random data after its mean is shifted by 1 unit.

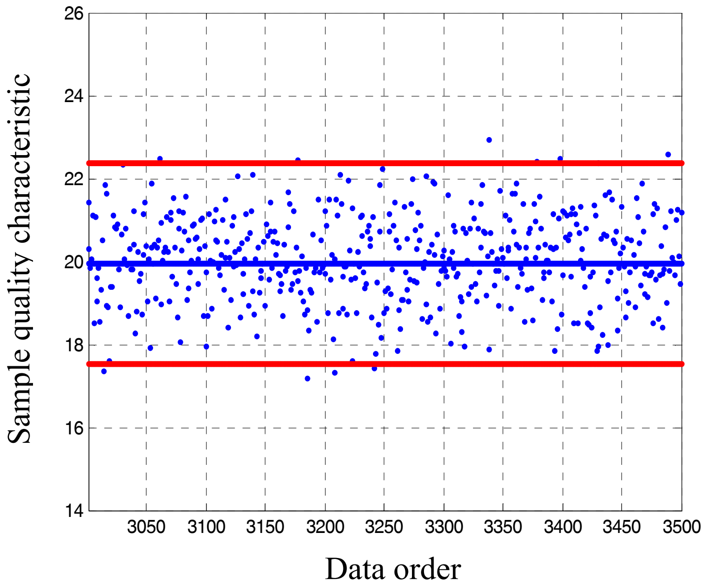

Figure 4.6. Shewhart control chart for normally distributed random data after its std is increased by 0.1 unit. The new standard deviation is 1.1.

Figure 4.7. Shewhart control chart for normally distributed random data after its std is increased by 0.25 unit. The new standard deviation is 1.25.

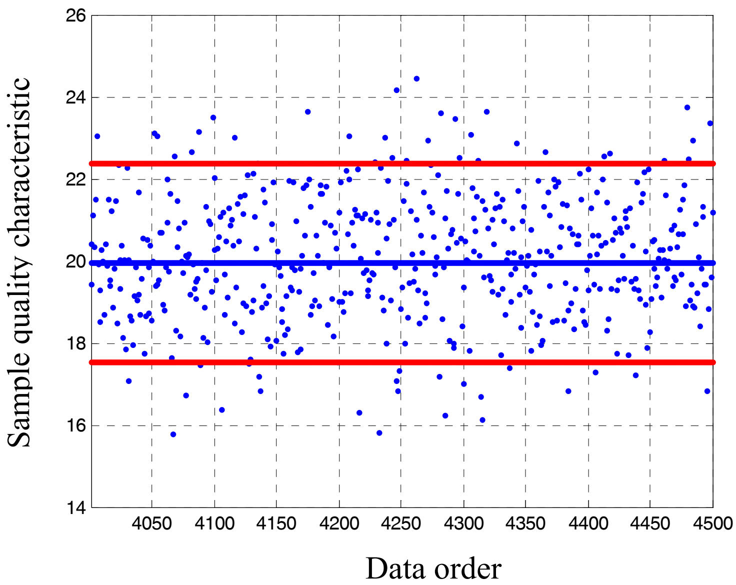

Figure 4.8. Shewhart control chart for normally distributed random data after its std is increased by 0.5 unit. The new standard deviation is 1.5.

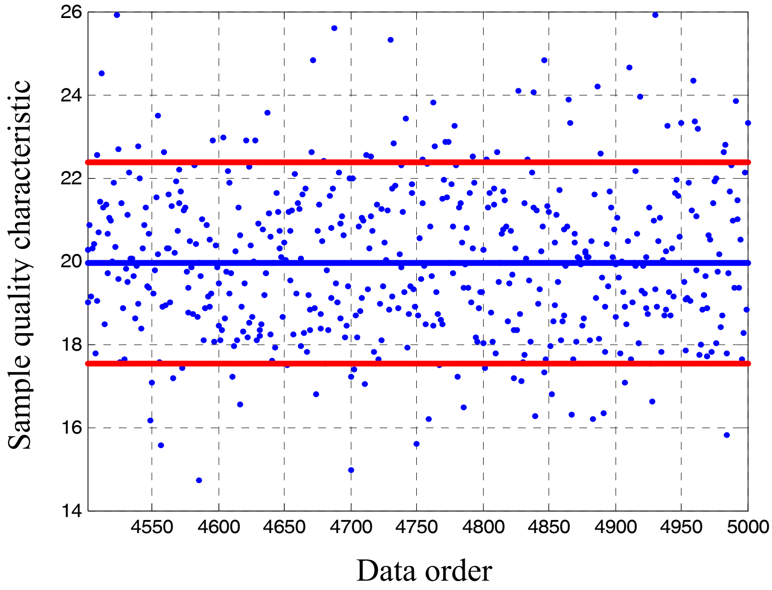

Figure 4.9. Shewhart control chart for normally distributed random data after its mean is shifted by 1 unit. The new standard deviation is 2.

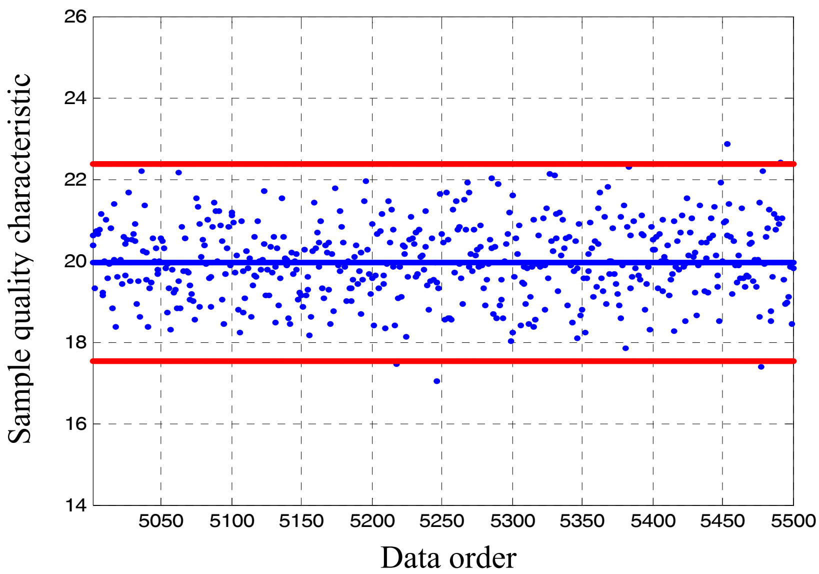

Figure 4.10. Shewhart control chart for normally distributed random data after the data is modified to return it back to normal operation.

4.1.2. Application of NM for α = 0.01

In this section the results of the NM are given on control charts that were developed with control limits calculated for an α value of 0.01. In Figure 4.11 the error chart for normally distributed data (NDD) is presented for the in-control situation upon application of the new method (NM). The same data (NDD) was purposefully shifted to observe its response on the error charts (Figures 4.12-4.19). In Figures 4.20 and 4.21 the capability to return back to normal operation is tested and demonstrated. This was achieved by appending the already shifted data with in-control data.

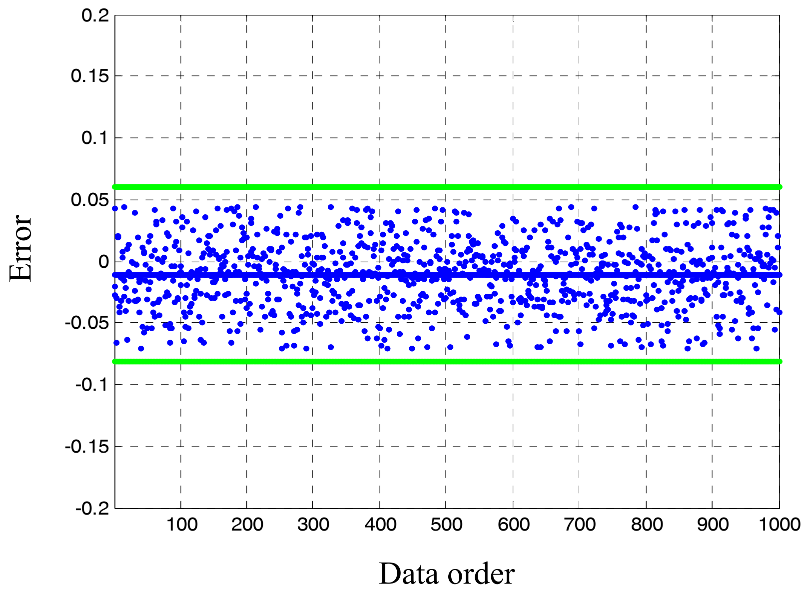

Figure 4.11. Error control chart for NM for normally distributed random data when the operation is in-control.

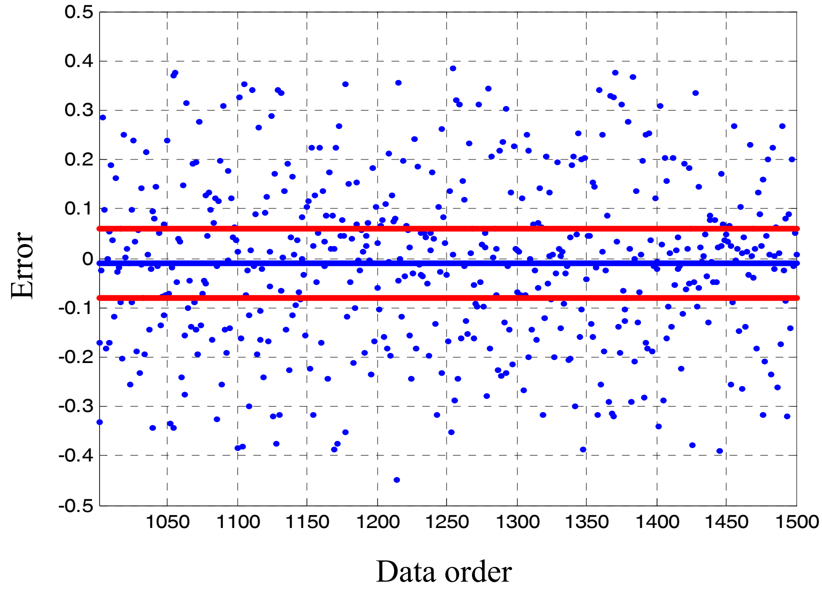

Figure 4.12. Error chart for normally distributed data after its mean is shifted by 0.1 unit.

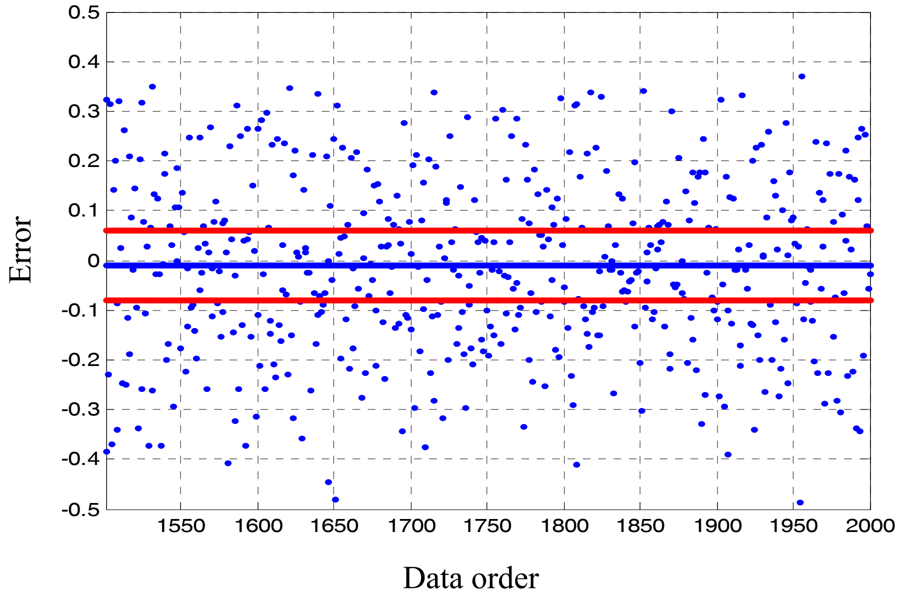

Figure 4.13. Error chart for normally distributed data after its mean is shifted by 0.25 unit.

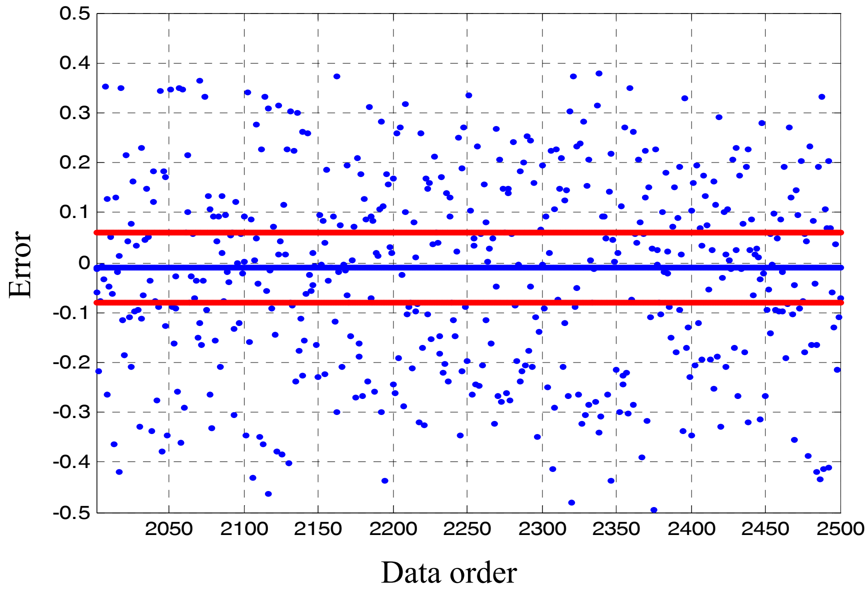

Figure 4.14. Error chart for normally distributed data after its mean is shifted by 0.5 unit.

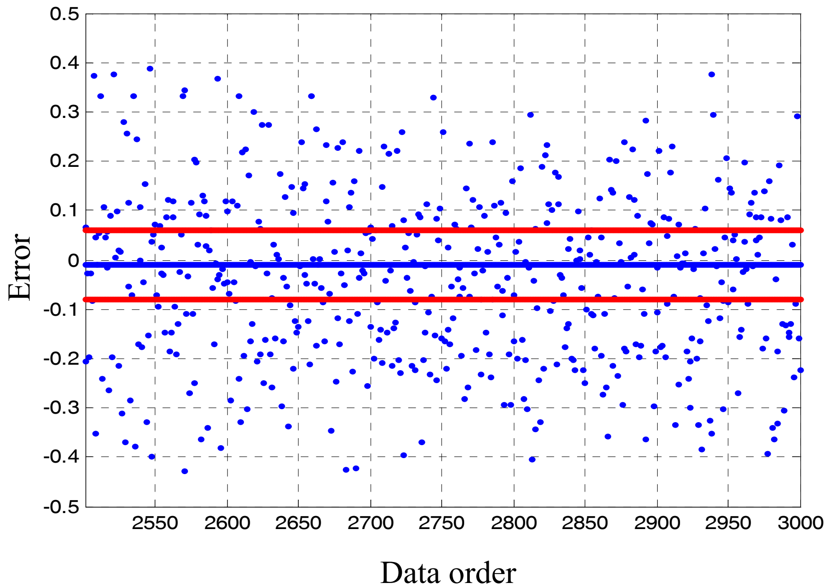

Figure 4.15. Error chart for normally distributed data after its mean is shifted by 1 unit.

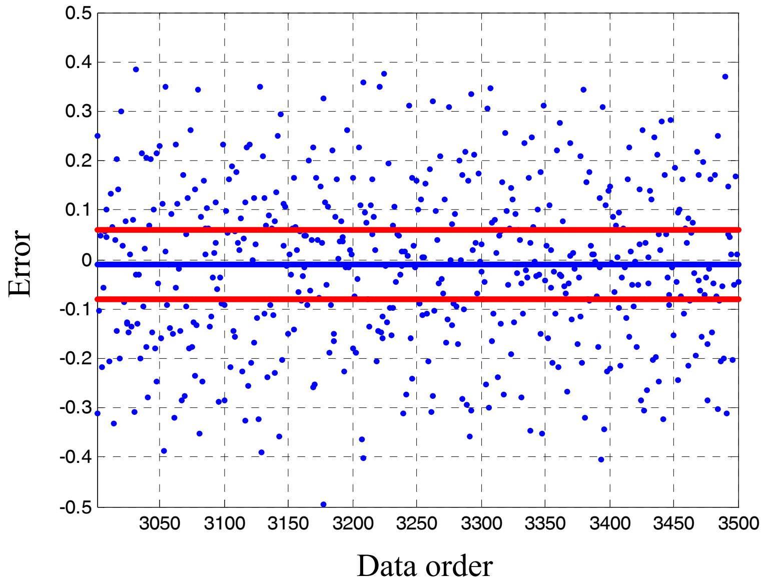

Figure 4.16. Error chart for normally distributed data after its std is increased by 0.1 unit. The new standard deviation is 1.1.

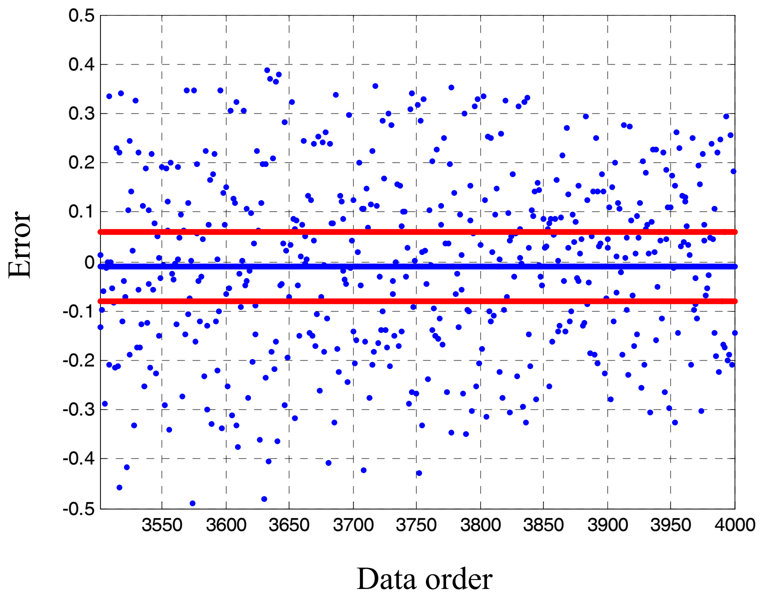

Figure 4.17. Error chart for normally distributed data after its std is increased by 0.25 unit. The new standard deviation is 1.25.

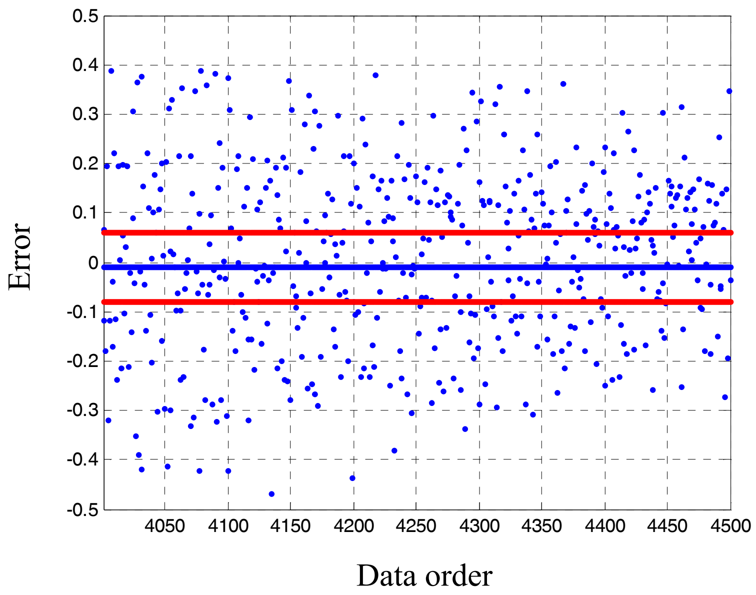

Figure 4.18. Error chart for normally distributed data after its std is increased by 0.5 unit. The new standard deviation is 1.5.

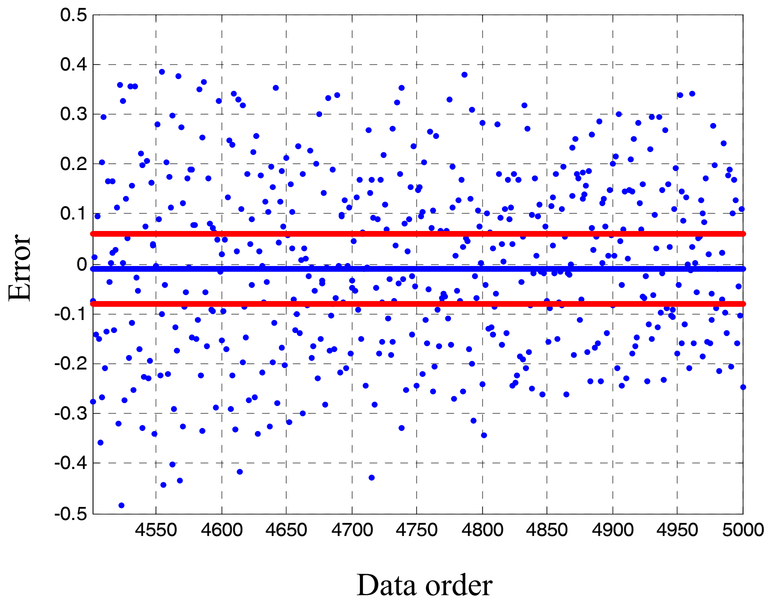

Figure 4.19. Error chart for normally distributed data after its std is increased by 1. unit.The new standard deviation is 2.

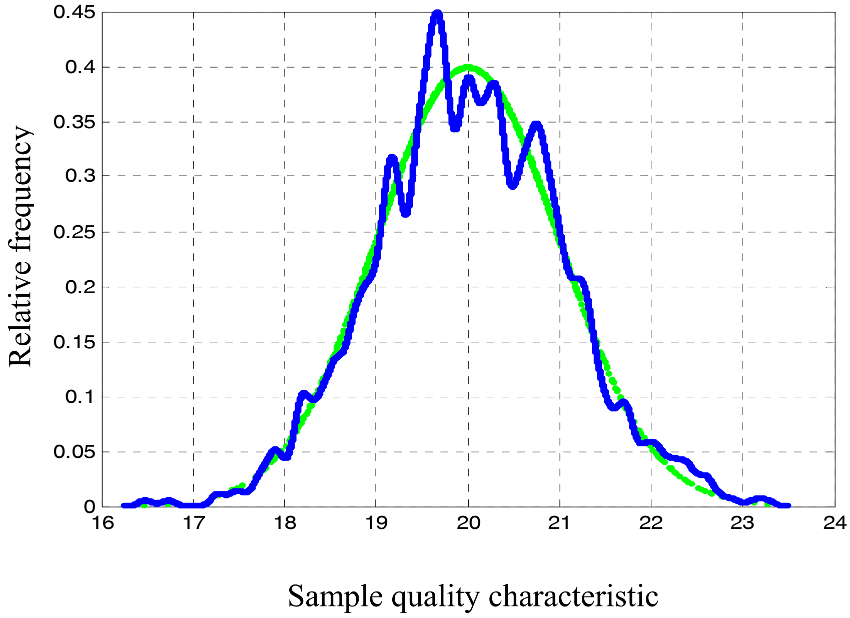

Figure 4.20. Plots of ksdensity and normal probability density function (normpdf) after the data is modified to return it back to normal operation. Wavy line indicates the ksdensity estimate while the other smooth line shows pdf.

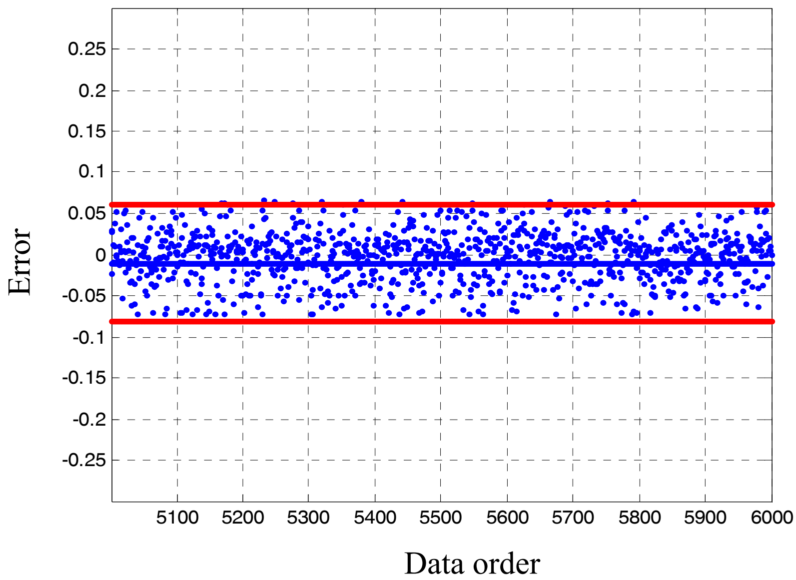

Figure 4.21. Error control chart for normally distributed random data after the data is modified to return it back to normal operation.

4.2. Results of OM and NM for Non-normally Distributed Data

Non-normally distributed random data that was described in section 3.1.2 was used to plot the control charts. OM and the NM were used for this non-normally distributed data (NNDD) as well.

The Shewhart control chart (OM) is shown in Figure 4.22. As can be seen from the figure, the data is obviously non-normally distributed and there are many data points that are outside of the control limits. These violations are unevenly located. There is no violation below the LCL when all violations are located above the UCL. This confirmed that the Shewhart control charts cannot be used for NNDD. Therefore, a new method is necessary for monitoring such data.

It should be noted that from this point on the OM is not used because it was identified to be unfit for NNDD. The NM was, however, further tested by giving shifts to the randomly selected NNDD.

Additional batches of randomly selected data of 500 data points were taken from the initial large random data set of 10000 members. These 500 data sets were shifted by 0.1, 0.25, 0.5 and 1. The purpose for imposing shifts to the data was to find out how the resulting control charts would be affected.

4.2.1. Application of the OM for $\alpha = 0.01$

In this section the control charts for an α value of 0.01 are given (Figures 4.22-4.26). As a result of the increase in the mean the total number of violations was not found to change significantly (Figures 4.23-4.26). In Figure 4.27 the capability to return back to normal operation is tested and marginally achieved. This was done by appending the already shifted data with in-control data. The resulting plot, however, contained a higher number of out-of-control data points leaving the impression that OM cannot be used for NNDD.

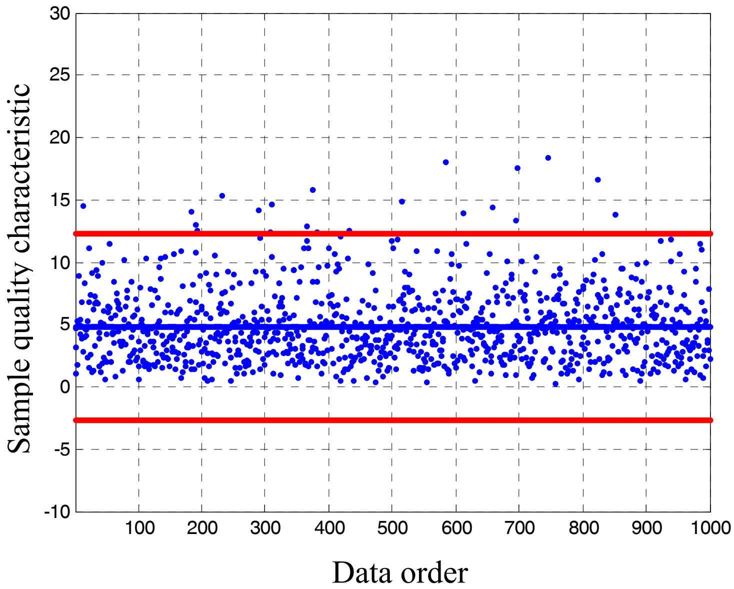

Figure 4.22. Shewhart control chart for non-normally distributed random data for in-control situation.

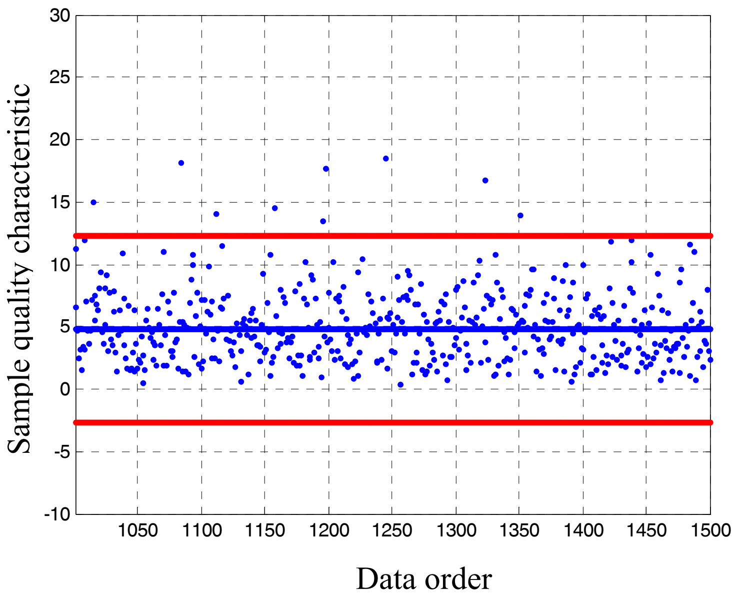

Figure 4.23. Shewhart control chart for non-normally distributed random data after it is shifted by 0.1 unit.

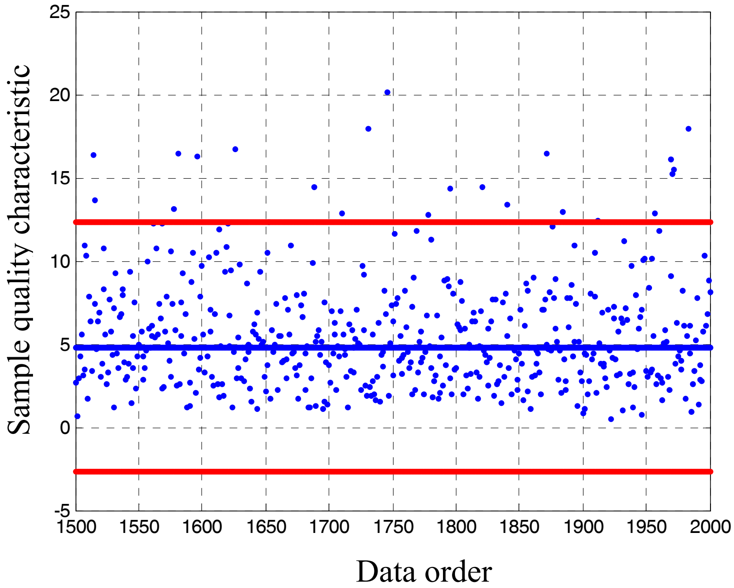

Figure 4.24. Shewhart control chart for non-normally distributed random data after it is shifted by 0.25 unit.

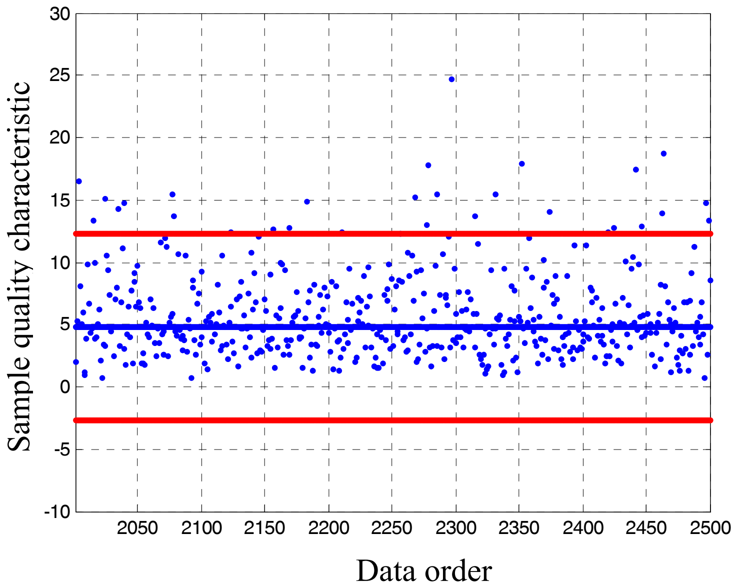

Figure 4.25. Shewhart control chart for non-normally distributed random data after it is shifted by 0.5 unit.

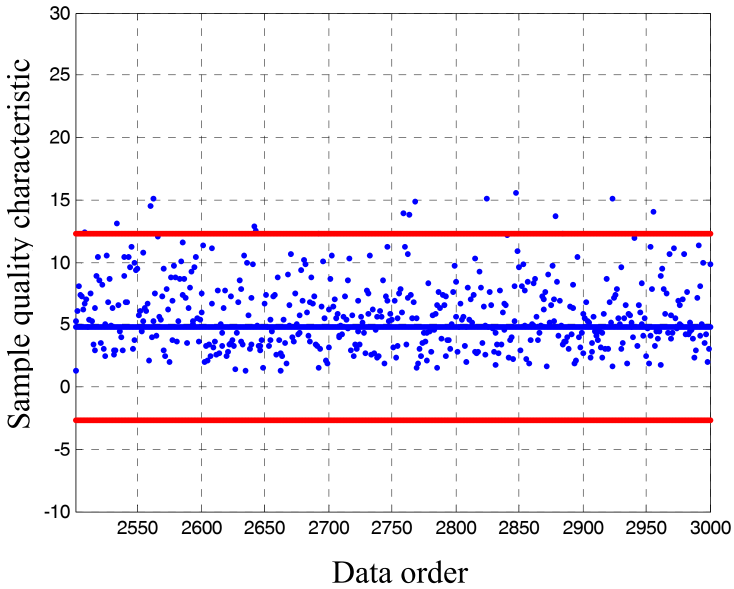

Figure 4.26. Shewhart control chart for non-normally distributed random data after it is shifted by 1 unit.

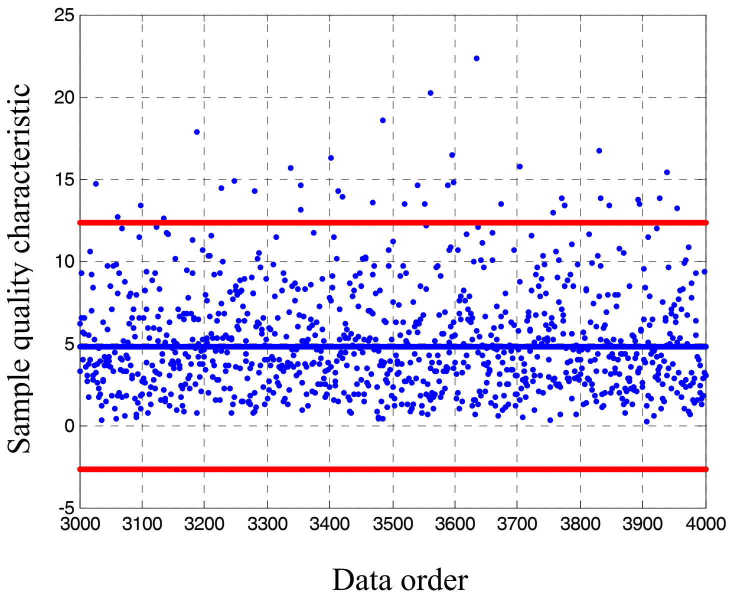

Figure 4.27. Shewhart control chart for non-normally distributed random data after the data is modified to return it back to normal operation.

4.2.2. Application of the NM for α = 0.01

In this section the results of the NM are given on control charts that were developed with control limits calculated for an α value of 0.01. In Figure 4.28 the error chart for non-normally distributed data (NNDD) is presented for the in-control situation upon application of the new method (NM). The same data (NDD) was purposefully shifted to observe its response on the error charts (Figures 4.29-4.32). The number of violations significantly increased even with the smallest amount of shift (0.01) and remained roughly the same for higher amounts of shifts. In Figures 4.33 and 4.34 the capability to return back to normal operation is tested and demonstrated. This was achieved by appending the already shifted data with in-control data.

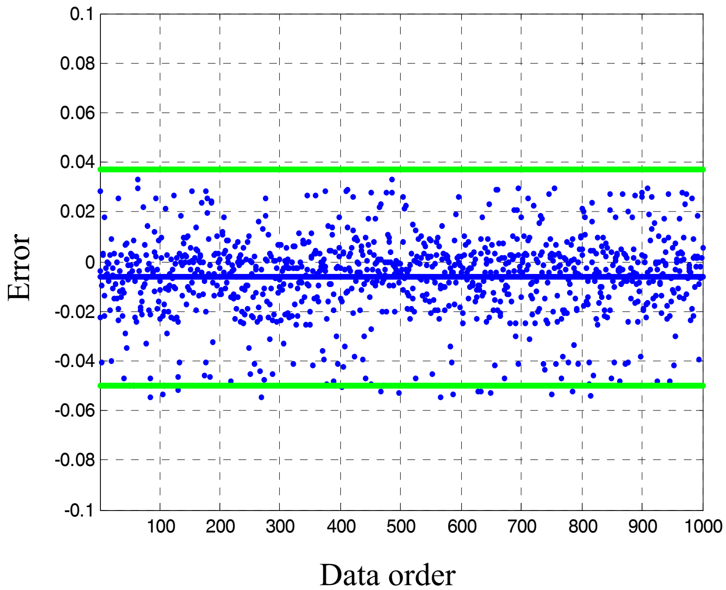

Figure 4.28. Error chart for chi-square distributed data (NNDD) for in-control situation.

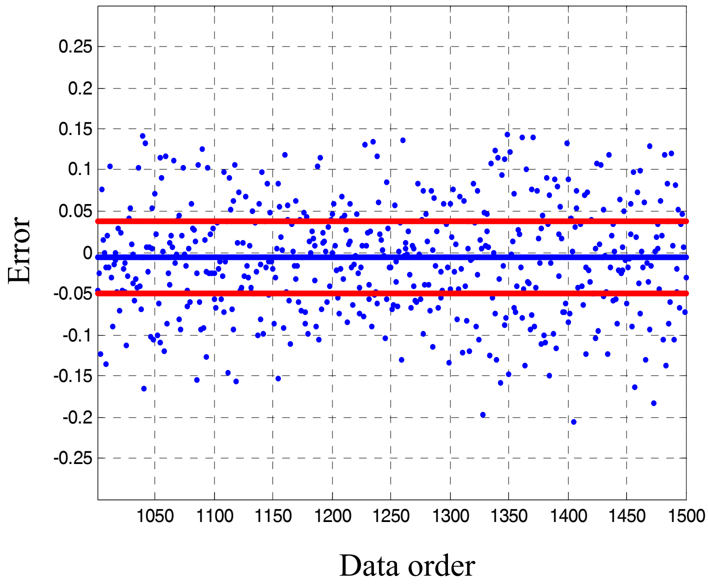

Figure 4.29. Error chart for chi-square distributed data (NNDD) after it is shifted by 0.1 unit.

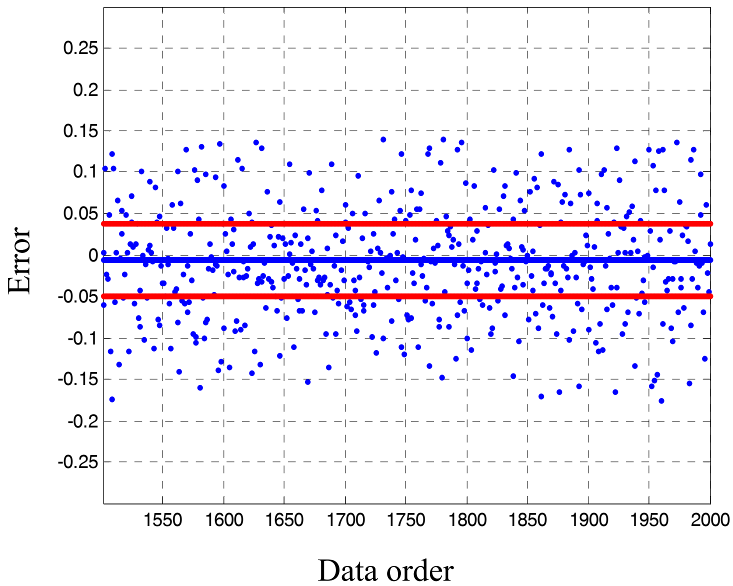

Figure 4.30. Error chart for chi-square distributed data (NNDD) after it is shifted by 0.25 unit.

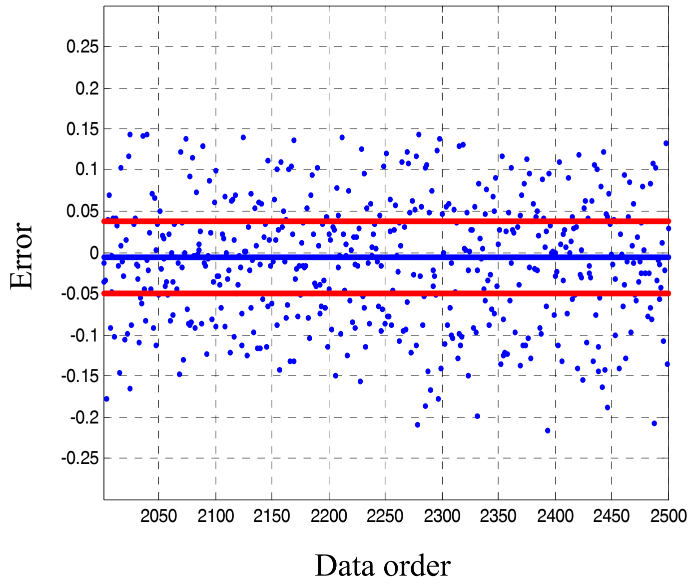

Figure 4.31. Error chart for chi-square distributed data (NNDD) after it is shifted by 0.5 unit.

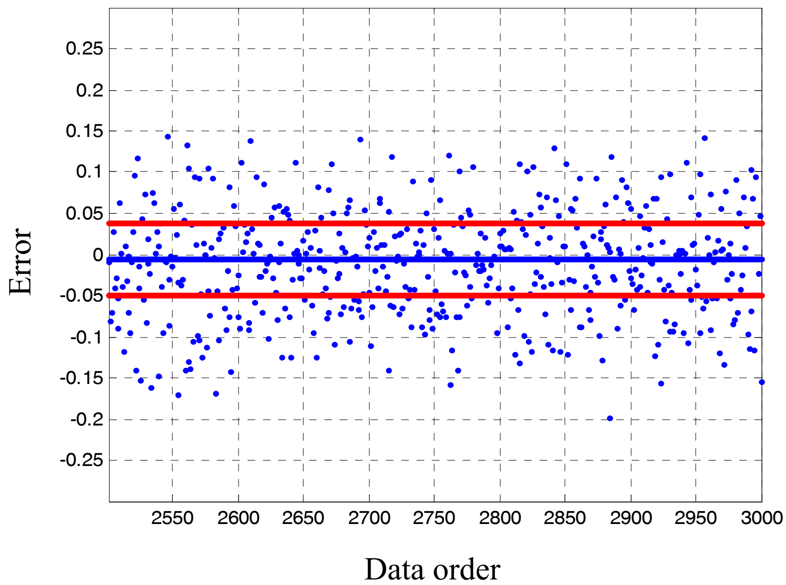

Figure 4.32. Error chart for chi-square distributed data (NNDD) after it is shifted by 1 unit.

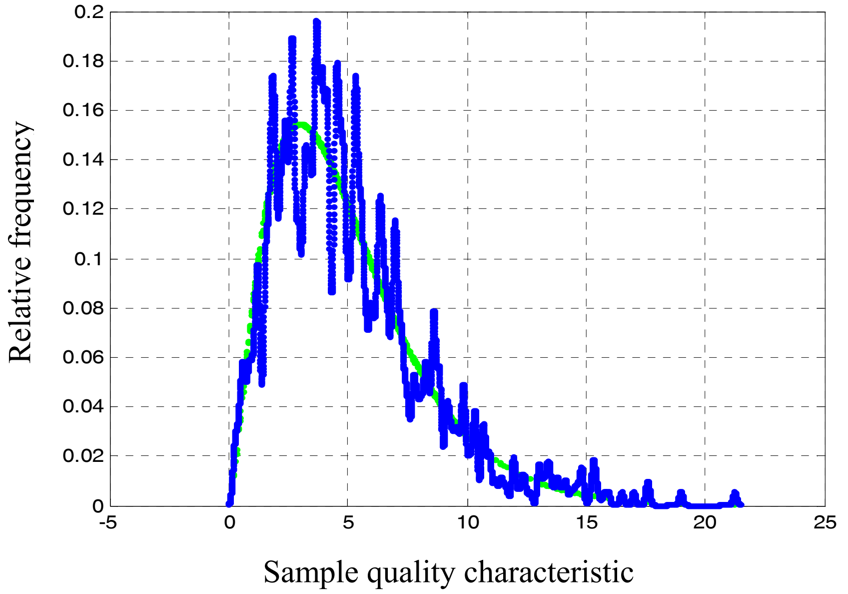

Figure 4.33. Plots of ksdensity and chi-square probability density function (chi2pdf) after the process is returned back to in-control situation. Wavy line indicates the ksdensity estimate while the other smooth line shows pdf.

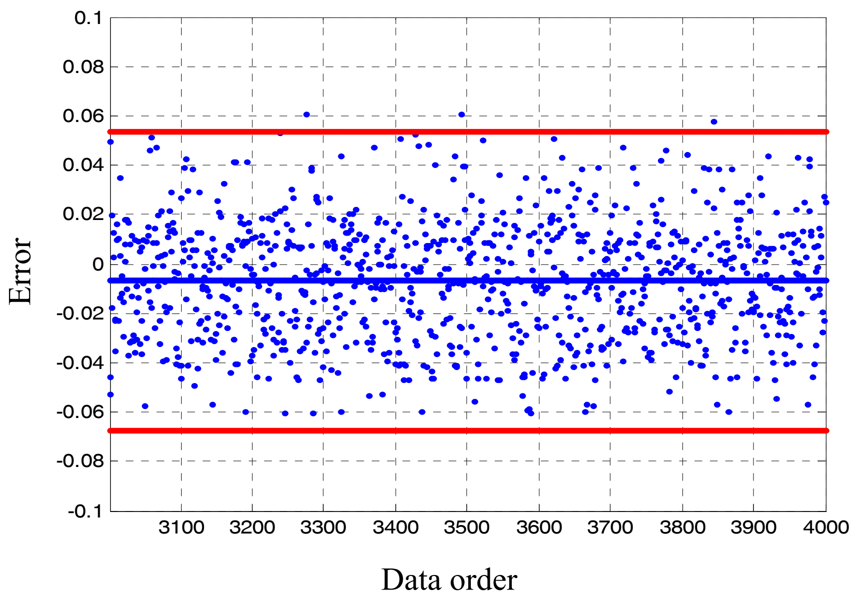

Figure 4.34. Error chart for chi-square distributed data (NNDD) after the process is returned back to in-control situation.

4.3. Results of OM and NM for Industrial Data

Non-normally distributed industrial data that was described in section 3.1.3 was used to plot the control charts. OM and the NM were used for this non-normally distributed industrial data (NND-ID) as well. The histogram for this data for in-control situation is shown in Figure 3.4. The NND-ID used in this study looked non-normal and skewed to the right. The same result was clear from Figure 3.5 where the normal probability plot of the NND-ID is given. The diagonal red line on the figure shows the conformity to normal distribution and any deviation from this line is an indication of non-normality.

4.3.1. Application of OM for $\alpha = 0.01$

The Shewhart control chart (OM) is shown in Figure 4.35. As can be seen from the figure, the data is obviously non-normally distributed and there are several data points that are outside of the control limits. These violations are unevenly distributed. There is only one violation below the LCL when all other violations are located above the UCL. This confirmed that the Shewhart control charts cannot be used for NND-ID. Therefore, a new method is necessary for monitoring such industrial data.

OM was applied to the out of control part of the industrial data and the results are given in Figure 4.36. The OM appears to successfully identify the out of control situation. But this was observed only above the UCL line, there was no violation below the LCL line.

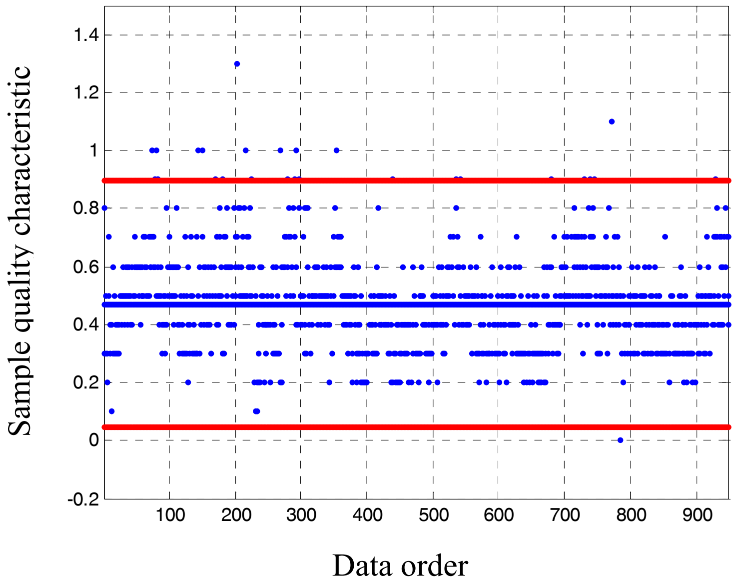

Figure 4.35. Shewhart control chart for industrial data for normal operation

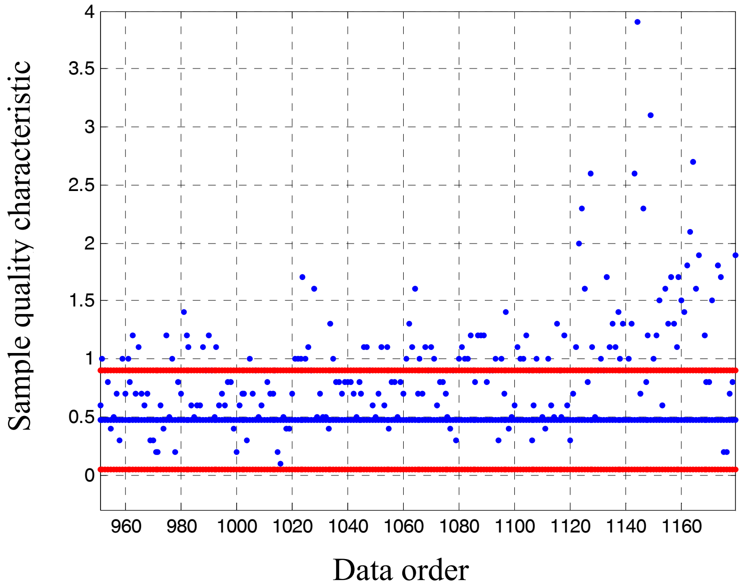

Figure 4.36. Shewhart control chart for industrial data for out-of-control situation.

4.3.2. Application of the NM for α = 0.01

In this section the results of the NM are given on control charts that were developed with control limits calculated for an α value of 0.01. In Figure 4.37 the plot for the two functions (ksdensity and pdf) used in this study is given. As can be seen from Figure 4.37, the two functions closely followed each other for the in-control situation. In Figure 4.38, the error chart for the non-normally distributed industrial data (NND-ID) is presented for the in-control situation upon application of the new method (NM). There were no violations in this error chart.

When the NND-ID was analyzed by NM in the out-of-control situation the error chart as shown in Figure 4.39 was obtained. In Figure 4.39 it was clear that the new method was successful in capturing the violations. Another significant achievement of the NM was that the violations occurred equally above the UCL and below the LCL. This could not be possible with the OM because of the nature of the data that cannot contain any negative values. The percentage of coarse fraction of cement remaining above a certain sieve cannot be negative.

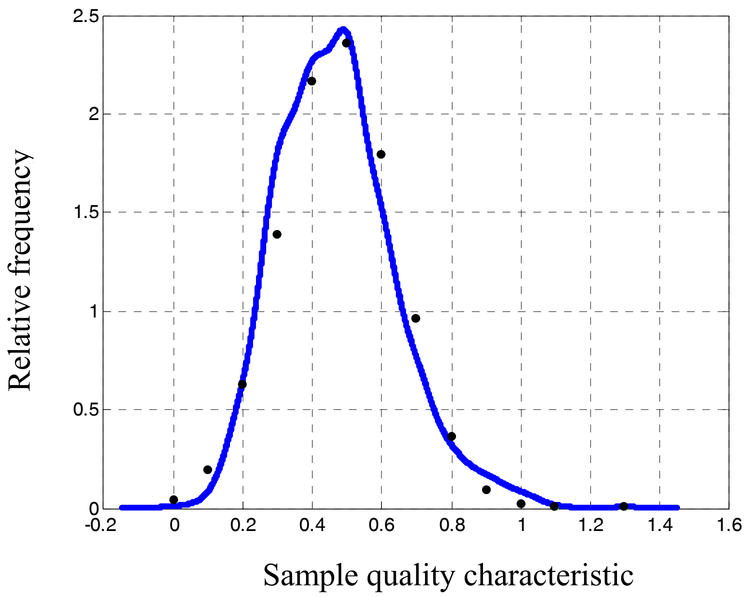

Figure 4.37. Plots of ksdensity and normpdf for industrial data for the in-control situation Dark continuous line shows ksdensity values while the dots show pdf values.

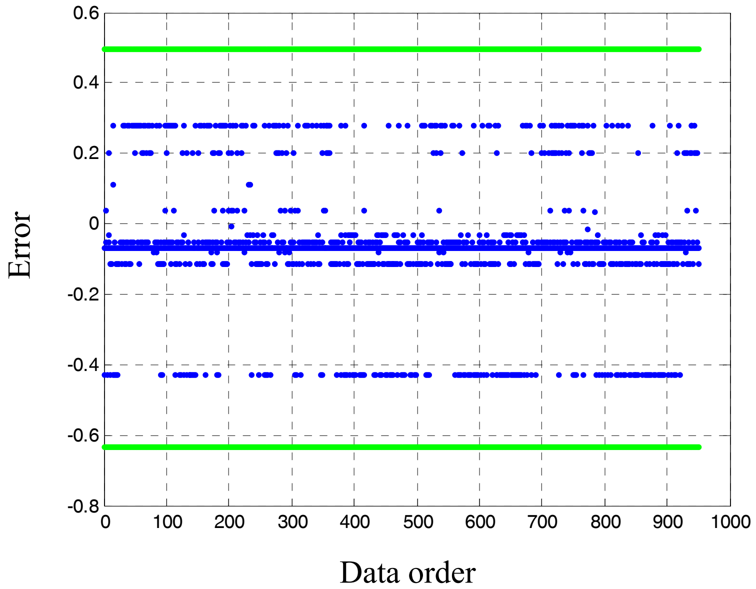

Figure 4.38. Error chart for industrial data for normal operation

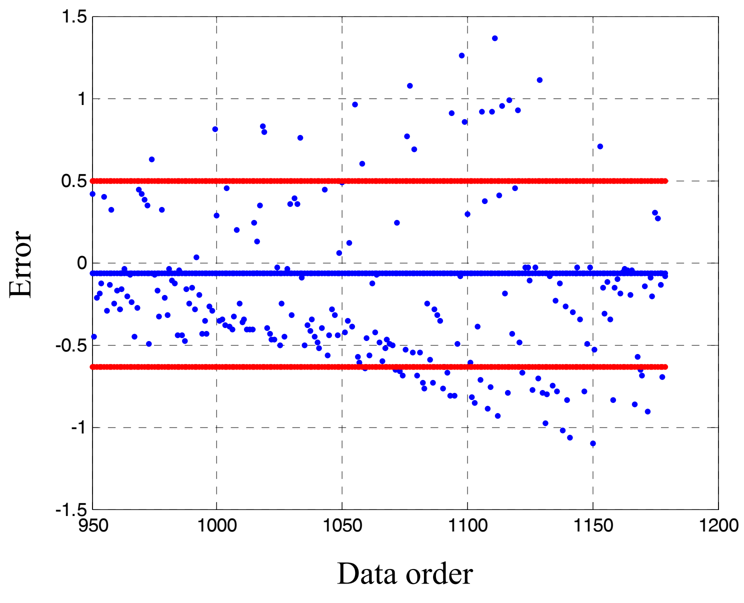

Figure 4.39. Error chart for industrial data for out-of control situation

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE STUDY

For normally distributed data (NDD) both methods (OM and NM) worked well for in-control situation. The OM contained a total of six false alarms while the NM had none. This is a burden on the OM because run rules must be created to determine whether the alarm signals are a part of an out-of-control situation or just assignable causes (Montgomery 2005). The NM removes this necessity and hence is more practical. The effects of shifts to mimic an out-of-control situation showed a gradual increase in the number of violations in both methods. Upon feeding in-control data both methods could be returned back to in-control status from out-of-control case.

Non-normally distributed data NNDD were used to test performance of both OM and NM. For in-control case the OM was found to be unsuccessfull by causing a large number of violations. When changes to mean and standard deviation introduced, the OM was found to be insensitive and unable to capture such conditions. On the other hand, the NM was found to be successful in both the in-control and out-of-control cases. Small amounts of shifts (i.e. less than 0.01) could be quickly captured by the NM. The NM could also be easily returned back to normal operation by feeding in-control data.

ID were first tested for its underlying distribution and it was identified as an NNDD. Then both OM and NM were applied for the in-control case and several false violations were observed in the OM while the NM worked well without violations. When the out-of-control data were analyzed by OM and NM, the OM was found to provide violations that were all located above the UCL. This one-sided biased result was expected because of the type of the distribution. The NM, however, was very well in that regard. The violations occurred equally balanced above the UCL and below the LCL. The NM was successfully implemented and the task outlined in the aim of the study part (section 2.1.3) was achieved.

The approach of this study was mainly aimed at developing a control chart for non-normal data, not on the economic aspects of such NND processes.

More NND-ID must be collected to further check and confirm the utility of the NM. The author is optimistic that this would yield good results for better process control. The effects of varying smoothing parameters, shift ratios and sample sizes can be tested. Care must be taken to collect ID with higher measurement precision. New method's performance can be compared against other methods based on ARL and ATS criteria.

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APPENDICES

APPENDIX A

Normally Distributed Data							
19.5674	20.7143	17.8293	19.7041	20.5690	20.6565	19.4535	17.9457
18.3344	21.6236	19.9408	18.5249	19.1783	18.8322	19.1532	20.1326
20.1253	19.3082	18.9894	19.7660	19.7344	19.5394	19.7537	21.5929
20.2877	20.8580	20.6145	20.1184	18.8122	19.7376	20.6630	21.0184
18.8535	21.2540	20.5077	20.3148	17.7977	18.7868	19.1458	18.4196
21.1909	18.4063	21.6924	21.4435	20.9863	18.6806	18.7987	19.9213
21.1892	18.5590	20.5913	19.6490	19.4814	20.9312	19.8801	19.3183
19.9624	20.5711	19.3564	20.6232	20.3274	20.0112	19.9347	18.9754
20.3273	19.6001	20.3803	20.7990	20.2341	19.3549	20.4853	18.7656
20.1746	20.6900	18.9909	20.9409	20.0215	20.8057	19.4045	20.2888
19.8133	20.8156	19.9805	19.0079	18.9961	20.2316	19.8503	19.5707
20.7258	20.7119	19.9518	20.2120	19.0529	19.0102	19.5652	20.0558
19.4117	21.2902	20.0000	20.2379	19.6256	21.3396	19.9207	19.6321
22.1832	20.6686	19.6821	18.9922	18.8141	20.2895	21.5352	19.5350
19.8636	21.1908	21.0950	19.2580	18.9441	21.4789	19.3935	20.3710
20.1139	18.7975	18.1260	21.0823	21.4725	21.1380	18.6526	20.7283
21.0668	19.9802	20.4282	19.8685	20.0557	19.3159	20.4694	22.1122
20.0593	19.8433	20.8956	20.3899	18.7827	18.7081	19.0964	18.6427
19.9044	18.3959	20.7310	20.0880	19.9588	19.9271	20.0359	18.9774
19.1677	20.2573	20.5779	19.3645	18.8717	19.6694	19.3725	21.0378
20.2944	18.9435	20.0403	19.4404	18.6507	19.1564	20.5354	19.6102
18.6638	21.4151	20.6771	20.4437	19.7389	20.4978	20.5529	18.6187
19.0781	19.1949	20.5689	19.0501	20.9535	19.0224	19.5532	21.0821
20.3155	20.5287	19.7444	20.7812	21.2781	21.4885	19.7963	20.1286
21.5532	20.2193	19.6225	19.2879	19.4522	20.1068	20.3271	19.2438
20.7079	19.8868	18.8929	19.9887	20.2608	21.8482	19.3270	19.9109
21.9574	20.3792	20.4855	19.9992	19.9868	19.7249	19.8507	17.9911
20.5045	20.9442	19.9950	19.7506	19.4197	22.2126	17.5510	21.0839
21.8645	17.8796	19.7238	20.3966	22.1363	21.5085	20.4733	19.0188
19.6602	19.3553	21.2765	19.7360	19.7424	18.0549	20.1169	19.3115
18.8602	19.2957	21.8634	18.3360	18.5905	18.3195	19.4089	21.3395
19.7889	18.9819	19.4774	18.9710	21.7701	19.4265	19.3453	19.0908
21.1902	19.8179	20.1034	20.2431	20.3255	19.8142	18.9193	19.5871
18.8838	21.5210	19.1924	18.7434	18.8810	20.0089	19.9523	19.4938
20.6353	19.9616	20.6804	19.6528	20.6204	20.8369	20.3793	21.6197

19.3986	21.2274	17.6354	19.0586	21.2698	19.2777	19.6696	20.0809
20.5512	19.3038	20.9901	18.8254	19.1040	19.2785	19.5001	18.9189
18.9002	20.0075	20.2189	18.9789	20.1352	19.7988	19.9640	18.8755
20.0860	19.2171	20.2617	19.5983	19.8610	19.9795	19.8252	21.7357
17.9954	20.5869	21.2134	20.1737	18.8366	20.2789	19.0427	21.9375
19.5069	19.7488	19.7253	19.8839	21.1837	21.0583	21.2925	21.6351
20.4620	20.4801	19.8669	21.0641	19.9846	20.6217	20.4409	18.7441
19.6790	20.6682	18.7295	19.7546	20.5362	18.2494	21.2809	19.7865
21.2366	19.9217	18.3364	18.4825	19.2836	20.6973	19.5023	19.8011
19.3687	20.8892	19.2964	20.0097	19.3444	20.8115	18.8813	20.3075
17.6748	22.3093	20.2809	20.0714	20.3144	20.6363	20.8076	19.4277
18.7684	20.5246	19.4588	20.3165	20.9131	21.3101	20.0412	19.7334
21.0556	19.9882	18.6665	19.8986	17.3650	20.0281	19.1237	19.7345
19.5124	20.8564	18.6546	20.4998	20.0559	22.3726	20.2293	20.7017
21.8625	20.2685	21.4819	19.1095	20.9443	19.2663	21.7513	19.4573
21.1069	20.6250	20.0327	20.1391	17.5760	19.9355	20.7532	20.9122
18.7724	18.9527	21.8705	19.7639	19.7762	18.5560	20.0650	19.8279
19.3301	21.5357	18.7910	19.9245	20.0581	20.6123	19.7072	19.6640
21.3409	20.4344	19.2174	19.6414	19.5754	18.6765	20.0828	20.5415
20.3881	18.0829	19.2327	17.9224	19.7971	19.3384	20.7662	20.9321
20.3931	20.4699	19.8928	19.8565	18.4869	19.8539	22.2368	19.4297
18.2927	21.2744	19.0229	21.3933	18.8736	20.2481	20.3269	18.5014
20.2279	20.6385	19.0360	20.6518	19.1850	19.9234	20.8633	19.9497
20.6856	21.3808	17.6208	19.6229	20.3666	21.7382	20.6794	20.5530
19.3632	21.3198	19.1618	19.3386	19.4139	21.6220	20.5548	20.0835
18.9974	19.0906	20.2573	20.2490	21.5374	20.6264	21.0016	21.5775
19.8144	17.6944	19.8162	19.6165	20.1401	20.0918	21.2594	19.6692
18.9460	21.7887	19.8324	19.4715	18.1372	19.1924	20.0442	20.7952
19.9285	20.3908	19.8830	20.0554	19.5458	19.5387	19.6859	19.2152
20.2792	20.0203	20.1685	21.2538	19.3479	18.5940	20.2267	18.7369
21.3733	19.5940	19.4988	17.4800	20.1033	19.6255	20.9967	20.6667
20.1798	18.4651	19.2949	20.5849	19.7794	19.5291	21.2159	21.5783
19.4580	20.2214	20.5082	18.9919	19.7210	17.5854	19.6724	18.8918
21.6342	18.6255	19.5791	18.6074	19.7226	19.3057	18.8418	19.9741
20.8252	19.1607	20.2291	18.6994	18.7063	18.6086	20.5801	18.8894
20.2308	19.7914	19.0405	19.3950	19.1116	20.3296	20.2398	20.7508
20.6716	20.7559	19.8540	18.5114	19.0135	20.5985	19.6491	20.5002
19.4919	20.5080	20.1319	20.2801	19.0172	19.0559	19.9869	20.3543
19.4408	20.3757	20.7445	20.5585	19.9284	20.1472	20.8921	19.4827
19.2466	19.0470	20.0755	20.3253	20.9778	21.4419	20.1733	19.8396
20.9258	20.7782	19.4734	19.6649	20.0183	20.6723	20.9232	18.9164
19.7515	19.9937	19.3145	19.6776	20.8180	20.1387	19.8214	18.0458
19.8502	20.5245	19.7316	19.6176	20.7023	19.1405	19.4783	19.0905

18.7416	21.3643	18.8117	19.0466	19.7687	19.2477	21.4320	19.9944
20.3126	20.4820	20.2486	20.2336	19.8863	21.2296	19.1299	18.2765
22.6903	19.2129	20.1025	21.2352	20.1279	21.1508	20.8075	21.2631
20.2897	20.7520	19.9590	19.4215	19.2006	19.3920	19.4894	19.3996
18.5772	19.8331	17.7524	19.4985	19.7614	20.8062	20.7435	17.9361
20.2468	19.1838	19.4892	20.7229	19.9105	20.2171	20.8479	20.1109
18.5642	22.0941	20.2492	20.0395	18.9767	19.6265	19.1701	21.4876
20.1486	20.0802	20.3692	21.5413	20.9375	19.1680	20.5330	20.0530
18.3069	19.0627	20.1792	18.2989	18.8683	20.2869	21.0328	20.1620
20.7192	20.6357	19.9627	18.9663	19.2893	18.1811	18.9480	19.9731
21.1418	21.6820	18.3967	19.2363	18.8305	18.4269	20.3621	20.1736
21.5519	20.5936	20.3394	22.1764	21.0654	22.0157	19.9632	20.8822
21.3836	20.7902	19.8689	20.4316	19.3196	19.9280	18.7724	20.1823
19.2419	20.1053	20.4852	19.5562	18.2742	22.6289	19.7249	20.7553
20.4427	19.8414	20.5988	20.0300	20.8132	19.7567	20.2137	20.2349
20.9111	20.8709	19.9140	19.6843	22.7316	20.4995	19.5995	19.4022
18.9259	19.8052	19.1053	20.4194	20.4111	19.4892	20.0649	20.0208
20.2018	21.3242	20.8121	21.1911	18.6931	20.7712	18.2420	20.8261
20.7629	19.8735	20.1095	20.8584	20.3838	17.3558	21.6867	19.9919
18.7118	19.2628	19.2245	20.2710	21.3059	18.9477	20.6256	20.1098
18.3455	19.3081	20.7748	21.2315	21.3171	20.2854	20.3274	18.8670
19.0096	20.6802	20.2450	19.0740	20.2280	19.8206	18.3534	19.3169
20.6852	18.9275	20.1650	19.8881	18.5704	18.5329	20.4287	19.7221
19.0251	20.8998	20.4062	19.1970	19.8503	21.3953	19.2628	20.6548
19.3933	17.8769	21.2160	18.3350	19.4950	20.4408	20.5649	18.7516
20.6868	20.2847	21.4484	19.0986	18.2709	20.5654	18.6158	19.4025
20.0200	19.2667	18.9749	20.5883	19.5825	19.3064	20.4603	19.5182
21.0638	19.2266	20.2054	20.5542	19.3850	20.8339	20.6294	20.9834
18.6590	20.1518	20.5889	19.5848	20.7208	17.7626	20.3798	21.7621
20.4795	19.6632	19.7360	20.0618	20.3394	21.0976	18.9867	21.4274
18.3660	20.9708	22.4953	20.4574	20.8828	19.9984	19.6528	20.9118
18.5573	19.8928	20.8559	20.1990	20.2842	18.3854	20.4419	20.3268
20.2938	21.0135	19.1490	20.2576	19.8545	18.7713	18.4098	20.0696
19.8596	19.5247	20.8119	22.0807	19.9104	20.2074	19.2986	18.5002
18.8697	20.0689	20.7002	17.7228	20.2892	20.2209	18.9224	19.5818
19.7075	20.3986	20.7599	20.3390	21.1648	18.9939	21.0022	19.9790
19.4175	21.1163	18.2871	20.2899	20.8057	19.5469	21.7295	20.2284
19.1037	20.6205	21.5370	20.6623	18.6444	21.3995	20.7090	18.9918
20.2486	19.7123	18.3902	19.4191	20.1209	19.5380	19.2521	19.3354
18.5103	18.6282	21.1095	20.8878	19.7778	20.0327	20.2289	20.5582
20.3135	19.3141	18.8903	20.1719	20.5717	20.7988	19.7765	18.8115
17.9749	20.3317	20.3855	20.8488	19.6999	20.8968	19.1467	20.8585
20.5290	19.0023	20.9652	20.9638	21.1343	20.1379	20.3456	17.8947

20.3435	20.2914	20.8183	21.3219	21.5350	18.3809	19.3978	18.5861
20.7582	19.7933	19.5744	20.4938	19.1291	20.0798	19.4784	19.6157
20.5536	21.1071	20.0370	19.9357	19.2024	19.6865	21.2591	
19.5421	19.6391	19.7085	19.6988	18.4114	21.0943		

APPENDIX B

Non-Normally Distributed Data							
3.1806	4.2753	1.9871	8.4224	3.5193	2.0197	2.7189	0.5458
1.0469	3.8881	6.4056	0.8265	1.2798	4.9880	2.8859	7.9339
4.7129	1.1187	6.0065	5.7200	3.6803	5.0720	5.7750	2.9788
5.2366	5.1358	11.4692	7.5410	4.6915	1.9921	2.0953	5.3703
1.7586	1.9079	6.3175	6.8611	5.3277	2.4953	7.0639	5.0595
8.8698	9.9771	2.7014	6.2668	10.1234	8.3642	6.2334	4.3968
8.8615	2.3691	5.5522	4.4531	3.3800	3.9576	2.3365	1.9988
4.2235	6.0837	1.9897	6.6479	6.4392	5.5854	3.5975	2.1003
5.3700	5.0116	4.2761	6.2332	7.1370	4.5976	1.6928	5.9248
4.8681	2.1465	4.1929	3.6235	7.7353	2.2940	6.5930	2.8060
3.8066	0.5696	4.3335	3.3144	5.2427	5.9701	2.2737	3.9075
6.8404	4.1614	3.4633	1.6532	10.3081	10.3584	1.6716	3.1753
2.8219	3.3219	1.4513	7.6935	8.6212	2.9154	3.9899	4.1040
14.4616	1.6958	3.6092	4.3665	2.6152	2.2880	6.1669	10.6059
3.9441	1.9089	7.7899	2.6980	1.5342	8.1630	1.8350	2.7819
4.6775	10.2743	4.7232	7.1645	4.1222	3.2841	4.5914	4.5001
8.2936	4.4995	6.5675	5.0516	1.9628	1.4063	0.7056	1.8230
4.5102	1.6468	1.7244	2.0238	1.6206	5.3302	3.0378	5.9255
4.0578	4.2131	3.1139	10.9179	5.2404	10.7029	5.8409	4.3186
2.3158	2.7360	3.7609	8.0761	3.1884	6.7692	3.4554	3.5700
5.2591	6.1083	0.6628	1.1472	4.4997	13.0274	9.0882	9.2820
1.4699	6.1733	4.7354	4.1059	3.3381	5.9948	2.7279	2.9698
6.7948	5.2139	4.3002	3.6016	3.1036	12.4662	0.4599	7.3749
11.0856	2.9274	4.3309	1.0487	5.5197	3.4080	1.6248	5.1728
2.5992	1.4737	3.6397	1.9551	6.8504	1.7695	8.2432	6.4459
7.3818	8.3205	5.6088	5.0891	13.9997	3.7410	4.0085	1.9235
9.1726	2.5569	1.8670	1.5869	1.4399	8.8666	5.5484	10.6087
1.1311	1.0299	4.1943	3.3895	1.9662	6.4855	7.7497	5.7422
1.3249	2.8547	5.5488	2.1108	3.0075	2.7930	0.6087	0.7851
6.2416	3.8090	3.4317	1.7137	11.0646	1.6310	2.6990	5.8692
3.2595	4.3597	3.0220	1.9687	4.5759	2.6453	2.5730	9.2717
6.6986	7.2937	4.2283	3.2552	1.8663	9.6010	1.9740	6.4981
7.2053	2.5358	3.8389	4.8650	1.7943	5.5792	3.8191	9.8018
6.7852	2.5375	2.0820	4.0004	11.7145	5.5965	10.5305	9.4959
9.3496	3.7676	9.3609	8.2816	12.9055	1.0000	4.2212	2.1696
6.6147	4.2734	5.7653	3.6503	11.1491	5.0393	9.0443	0.4729
8.8694	5.2073	9.3040	1.2252	1.5878	6.6814	2.5899	12.0203
1.6698	8.2552	3.0270	4.3621	3.7346	2.7160	4.3555	5.5886

4.6450	6.4332	1.8038	4.5469	3.7737	2.0012	2.4130	4.3934
2.3640	0.9528	7.1724	5.3336	5.3031	3.8095	6.3011	3.2447
6.6610	6.7275	4.4558	5.9776	2.8574	1.9121	3.6351	1.2034
0.4344	7.1882	2.4666	9.2900	2.0454	4.1261	5.9061	5.0183
7.9506	6.4896	4.0762	2.9124	3.1466	5.2083	6.6130	1.4158
5.0102	9.4475	0.7018	5.1474	8.3633	9.7637	4.1068	2.4442
5.1502	5.3694	8.3716	4.2947	15.7433	4.8846	7.5134	4.0253
8.9772	2.6387	2.0391	2.8397	5.0440	2.9255	15.3066	2.0464
3.5740	3.9084	2.6061	14.1556	3.5949	11.1443	6.0686	2.0699
3.9531	5.8813	9.5937	3.6183	6.7446	7.2450	4.2987	0.4252
1.5660	4.6868	2.1699	1.3674	3.0506	5.0488	7.6157	2.3045
1.0492	2.8157	3.2280	11.9123	12.4540	6.6265	4.3147	5.1359
2.5746	2.6776	3.6057	5.3641	8.4768	3.0031	6.0680	3.8144
5.0435	3.7630	11.0769	6.4282	3.4176	2.5938	9.7182	3.8584
2.0780	1.2309	6.4515	9.2494	6.1309	1.3923	5.9130	3.9980
3.9175	1.7913	4.6093	2.1945	7.6974	5.3780	2.4047	4.8485
6.9155	2.3496	2.3641	4.7436	2.8620	6.3450	6.9456	3.0190
2.2049	5.5047	3.1121	3.9368	1.2494	4.7812	3.8604	2.5714
4.7557	2.8268	1.3722	1.7315	4.1868	4.0415	2.3472	6.0081
3.6747	4.7589	3.3216	8.8357	6.1738	4.4164	13.8836	3.2085
4.1150	0.8375	3.0896	4.2881	4.5838	2.2316	4.5736	3.3023
3.3611	3.1290	11.8040	6.1114	10.8340	3.5978	2.1183	2.0890
0.6433	2.6832	6.9505	2.5479	3.4307	3.4387	6.4873	5.0580
3.9243	4.6447	4.5275	2.6757	7.1210	1.7397	11.4107	9.0819
9.8656	3.7157	3.5273	5.3262	2.4092	6.2751	6.3264	2.8436
6.5494	3.5627	4.5818	4.6555	1.5770	5.0782	7.1005	3.0182
3.3152	2.5125	7.0029	12.3693	6.6071	3.3803	4.6506	6.8287
2.6632	4.1461	14.8173	3.5729	1.3906	7.5259	3.8831	4.4507
5.1083	1.3209	5.3687	14.6556	1.5215	10.8383	7.4361	10.6387
3.2995	6.3975	7.4041	10.4642	2.7851	1.8212	3.7849	1.0069
2.9563	1.4882	6.6569	2.6204	1.2623	4.2574	4.5593	1.9469
4.4984	2.6629	6.1803	4.4646	6.1944	1.8171	2.9605	2.4515
9.1715	3.9173	8.0015	3.4728	3.5670	6.9409	2.6124	14.4172
6.2932	5.1055	1.7217	5.0356	1.5316	5.9789	3.5903	5.7322
1.9916	4.1116	8.2875	7.9796	2.2088	2.9820	1.6919	3.1538
7.7501	11.7288	2.6231	8.9890	2.0296	2.8867	5.1071	4.4222
0.3977	1.7881	0.9797	2.9240	4.1259	2.4723	4.6420	3.4689
3.7072	7.1719	7.1953	7.6118	0.4034	7.6702	4.2137	7.8965
4.5065	2.9972	10.1149	3.8461	7.8655	3.6421	0.5130	4.3874
3.1996	6.9114	6.6290	2.8009	4.0252	3.9070	2.9969	7.2149
2.2635	7.3394	4.7545	4.3948	8.0541	1.5841	5.1093	6.7473
2.4746	2.3206	0.7507	5.6891	3.0792	5.3204	5.5136	3.6875
9.0547	6.0994	2.1695	8.8707	4.5395	18.0582	4.8826	4.0072

8.6806	8.1406	4.3169	7.0233	5.6159	5.2433	4.2245	4.7210
2.7785	1.9155	0.9822	0.2798	8.5205	1.3493	1.1196	2.3803
7.1662	5.4892	9.2167	5.2291	6.4286	5.1012	5.4112	3.6682
5.0045	4.2259	2.7952	7.2486	3.5404	1.3319	3.9586	4.0753
3.3243	1.6309	4.6681	4.3095	1.4195	4.7856	5.9244	1.9651
2.3164	3.5729	10.3538	7.3836	2.6116	1.0158	6.3458	7.7208
5.2339	3.8780	4.4912	7.0379	5.3850	6.8141	4.0850	1.7826
0.8814	1.8621	4.8279	9.4270	2.0098	8.6386	5.3632	2.5597
13.3880	6.6600	4.2547	9.0638	5.2491	10.6959	3.4197	7.3732
4.1249	1.8807	4.8647	7.8122	8.4778	9.8162	3.4517	2.2799
17.5944	7.5586	7.4837	1.0596	5.0952	2.4628	1.9618	7.1898
3.6558	0.6065	4.8925	2.0227	4.8374	5.7715	4.9666	6.7390
4.8637	5.2267	6.9589	6.6798	5.6426	7.6070	6.3084	6.9776
7.6590	2.5132	6.0076	2.0504	8.9893	1.8781	3.6016	0.9938
3.8287	2.4320	4.7333	2.7814	10.1489	4.9549	16.6123	10.6159
2.9718	4.7959	5.2113	6.6860	7.8413	6.9895	2.1389	1.1119
10.0637	3.4154	2.0361	4.3926	7.2163	1.5397	4.0121	8.4891
2.2434	6.5900	2.1059	8.2801	4.4433	2.0898	2.3731	1.8186
6.3064	2.8384	4.2950	1.4630	5.0399	7.0516	1.0476	5.5701
6.1780	7.5074	5.4624	5.9038	1.3401	5.4111	2.1845	3.8265
3.2224	4.8593	2.1970	1.0834	3.9074	7.4866	7.1645	1.2904
4.5178	7.3433	7.1908	1.3227	3.0103	5.2251	1.4422	9.8758
5.8243	7.8349	4.6639	5.2570	0.9760	3.9188	4.6991	5.7650
4.9460	9.5062	18.3754	3.9331	3.2169	4.0747	3.7116	6.2200
5.1366	4.1467	5.6597	1.7848	2.7633	5.2416	6.2438	7.2809
13.7985	9.4820	1.5123	3.5279	6.8204	8.7466	3.5084	2.3024
5.4100	1.1008	5.5643	2.8347	2.6173	3.2150	8.6037	3.7477
5.2440	1.0687	5.9764	2.1939	3.5657	4.2717	2.9720	6.9614
0.5203	5.7219	2.9968	5.1072	6.5609	5.0412	1.3615	5.5363
8.4345	2.5052	5.0624	1.2609	1.5992	1.9914	3.2976	1.4567
4.3286	6.2183	9.1987	5.3234	2.8015	2.6564	3.1202	10.3236
1.1062	1.4023	1.8721	6.0846	3.0640	6.1930	3.5307	4.4304
1.6292	5.8345	8.0041	5.4252	7.9208	1.6916	3.5057	12.5015
4.9730	6.4628	11.6792	6.9706	11.8663	2.4278	1.1373	1.6597
5.0169	5.5505	6.7738	2.5988	10.0402	5.1812	8.4191	2.4135
1.9951	1.9824	2.4834	7.3838	7.6098	10.6049	9.5174	4.9934
3.1317	3.3894	5.0426	0.6208	5.3684	1.9151	3.9716	3.2579
9.8968	5.7688	3.7081	3.3552	4.5415	6.4481	2.5053	4.5273
3.1107	1.1353	2.2755	6.1758	1.2479	2.3838	4.1439	0.9449
4.4304	2.5790	5.4325	1.1766	3.4311	3.4743	2.9004	11.4372
7.1359	10.9490	4.6646	3.7530	3.7986	2.7913	5.2721	5.3704
2.4921	0.6540	1.7804	3.1974	3.5044	9.1972	1.6325	6.8013
4.8730	7.8051	4.7521	5.9556	9.5943	3.6478	4.5726	1.4540
6.0804	2.9452	2.1851	2.2419	7.5460			

APPENDIX C

Industrial Data															
0.3	0.4	0.4	0.3	0.6	0.5	0.6	0.2	0.6	0.5	0.5	0.5	0.3	0.4	0.3	0.2
0.3	0.6	0.5	0.3	0.6	0.8	0.6	0.5	0.6	0.5	0.6	0.4	0.3	0.2	0.3	0.4
0.8	0.5	0.6	0.5	0.5	0.4	0.5	0.5	0.5	0.5	0.8	0.3	0.6	0.4	0.3	0.6
0.5	0.5	0.4	0.3	0.4	0.6	0.8	0.3	0.8	0.5	0.8	0.2	0.8	0.2	0.2	0.6
0.3	0.6	0.3	0.7	0.6	0.4	0.9	0.6	0.9	0.6	1.0	0.3	0.5	0.4	0.5	0.5
0.3	0.4	0.5	0.5	0.4	0.6	0.5	0.4	0.7	0.6	0.6	0.6	0.5	0.3	0.3	0.4
0.2	0.5	0.4	0.3	0.4	0.4	0.6	0.3	1.0	0.5	0.7	0.4	0.4	0.2	0.5	0.6
0.7	0.4	0.4	0.6	0.5	0.6	0.6	0.4	0.6	0.4	0.6	0.3	0.4	0.3	0.5	0.4
0.5	0.6	0.3	0.2	0.3	0.5	0.2	0.3	0.4	0.6	0.7	0.4	0.5	0.4	0.6	0.4
0.3	0.5	0.5	0.4	0.6	0.5	0.2	0.6	0.4	0.5	0.6	0.2	0.4	0.4	0.5	0.3
0.4	0.5	0.4	0.4	0.5	0.4	0.5	0.5	0.4	0.5	0.6	0.5	0.5	0.6	0.4	0.5
0.4	0.6	0.8	0.3	0.5	0.8	0.1	0.3	0.9	0.5	0.7	0.3	0.5	0.4	0.4	0.3
0.4	0.5	0.6	0.7	0.5	0.7	0.1	0.3	0.8	0.4	0.5	0.2	0.4	0.4	0.4	0.3
0.1	0.7	0.5	0.5	0.6	0.5	0.2	0.3	0.5	0.4	0.5	0.2	0.4	0.3	0.3	0.5
0.3	0.6	0.7	0.5	0.6	0.6	0.2	0.2	0.4	0.5	0.4	0.2	0.4	0.3	0.4	0.4
0.6	0.6	0.6	0.3	0.5	0.7	0.3	1.0	0.5	0.5	0.4	0.3	0.5	0.4	0.3	0.6
0.3	0.6	0.6	0.4	0.9	1.3	0.4	0.2	0.4	0.6	0.5	0.2	0.4	0.4	0.3	0.5
0.5	0.6	0.5	0.3	0.4	0.8	0.2	0.6	0.7	0.5	0.5	0.3	0.4	0.3	0.3	0.6
0.3	0.6	0.6	0.3	0.7	0.6	0.6	0.4	0.5	0.4	0.4	0.2	0.3	0.4	0.4	0.5
0.4	0.5	0.5	0.3	0.5	0.8	0.5	0.6	0.8	0.5	0.4	0.2	0.4	0.2	0.2	0.7
0.4	0.6	0.4	0.3	0.4	0.5	0.6	0.6	0.3	0.5	0.5	0.3	0.3	0.4	0.3	0.7
0.3	0.4	0.6	0.7	0.7	0.5	0.5	0.7	0.4	0.6	0.3	0.3	0.5	0.4	0.3	0.6
0.3	0.5	0.5	0.3	0.8	0.6	0.4	0.6	0.3	0.5	0.5	0.4	0.5	0.2	0.3	0.6
0.4	0.6	0.5	0.5	0.5	0.5	0.4	0.5	0.8	0.5	0.4	0.4	0.2	0.2	0.4	0.6
0.4	0.5	0.6	1.0	0.6	0.7	0.2	0.7	0.8	0.4	0.5	0.4	0.4	0.3	0.5	0.7
0.5	0.7	0.6	0.3	0.5	0.7	0.4	0.6	0.6	0.2	0.5	0.5	0.2	0.4	0.4	0.5
0.5	0.5	0.8	0.6	0.3	0.8	0.3	0.9	0.5	0.4	0.4	0.3	0.9	0.6	0.4	0.4
0.5	0.6	0.6	0.4	0.9	0.5	0.4	0.7	0.4	0.5	0.4	0.4	0.3	0.5	0.5	0.4
0.4	0.7	0.4	0.4	0.7	1.0	0.5	0.8	0.3	0.6	0.3	0.5	0.5	0.5	0.4	0.9
0.5	0.6	0.3	0.7	0.3	0.5	0.4	0.8	0.3	0.3	0.2	0.3	0.2	0.4	0.4	0.8
0.6	0.5	0.4	1.0	0.6	0.5	0.4	0.5	0.5	0.3	0.6	0.5	0.3	0.3	0.5	0.7
0.5	0.5	0.5	0.5	0.7	0.7	0.4	0.7	0.4	0.7	0.5	0.4	0.3	0.7	0.8	0.4
0.4	0.5	0.5	0.6	0.6	0.7	0.4	0.7	0.5	0.3	0.6	0.4	0.3	0.4	0.7	1.2
0.6	0.7	0.5	0.4	0.3	0.4	0.5	0.7	1.1	0.5	0.3	0.2	0.3	0.6	0.7	0.5
0.5	0.5	0.4	0.3	0.2	0.3	0.5	0.8	0.5	0.6	0.5	0.3	0.4	0.5	0.7	1.0
0.6	0.6	0.3	0.4	0.3	0.7	0.8	0.9	0.6	0.6	0.4	0.5	0.3	0.3	0.5	0.2
0.6	0.7	0.2	0.3	0.5	0.4	0.7	0.6	0.4	0.5	0.3	0.5	0.4	0.5	0.4	0.8

0.6	0.6	0.3	0.5	0.3	0.5	0.6	0.5	0.7	0.3	0.5	0.5	0.3	0.5	0.6	0.7
0.4	1.0	0.7	0.5	0.4	0.4	0.5	0.6	0.7	0.3	0.4	0.5	0.2	0.5	1.0	1.4
0.5	0.7	0.4	0.6	0.5	0.4	0.7	0.6	0.5	0.5	0.4	0.3	0.3	0.4	0.9	1.2
0.9	0.7	0.5	0.3	0.3	0.3	0.7	0.4	0.4	0.5	0.6	0.3	0.4	0.4	0.8	1.1
0.4	0.6	0.3	0.4	0.3	0.3	0.6	0.3	0.5	0.5	0.4	0.3	0.4	0.4	0.4	0.6
0.4	0.6	0.5	0.4	0.3	0.4	0.5	0.6	0.7	0.4	0.3	0.4	0.2	0.7	0.5	0.5
0.6	0.9	0.4	0.3	0.2	0.4	0.7	0.3	0.4	0.3	0.5	0.3	0.4	0.5	0.7	0.6
0.4	1.0	0.3	0.4	0.2	0.5	0.6	0.5	0.6	0.6	0.4	0.3	0.4	0.7	0.3	0.6
0.5	0.5	0.5	0.3	0.3	0.4	0.4	0.3	0.5	0.4	0.3	0.3	0.4	0.9	1.0	1.1
0.5	0.5	0.5	0.3	0.2	0.5	0.5	0.5	0.0	0.4	0.5	0.3	0.3	0.8	0.7	0.9
0.4	0.6	0.2	0.3	0.3	0.6	0.7	0.5	0.5	0.4	0.4	0.4	0.5	0.7	1.0	1.2
0.4	0.9	0.3	0.4	0.3	0.6	0.3	0.4	0.3	0.5	0.5	0.5	0.4	0.4	0.8	0.9
0.5	0.5	0.3	0.5	0.3	0.6	0.6	0.6	0.3	0.5	0.3	0.3	0.6	0.5	1.2	0.5
0.4	0.7	0.3	0.2	0.2	0.7	0.5	0.5	0.2	0.4	0.3	0.6	0.3	0.4	0.7	1.1
0.5	0.7	0.7	0.3	0.6	0.5	0.9	0.5	0.5	0.3	0.3	0.5	0.5	0.7	1.1	0.6
0.4	0.3	0.5	0.4	0.5	0.5	0.7	0.5	0.4	0.5	0.4	0.4	0.4	0.4	0.7	0.7
0.5	1.0	0.5	0.6	0.3	0.6	0.6	0.7	0.3	0.6	0.3	0.3	0.5	0.6	0.6	0.6
0.6	0.7	0.4	0.5	0.2	0.7	0.5	0.6	0.4	0.3	0.4	0.4	0.5	0.7	0.7	0.8
0.6	0.9	1.3	0.5	0.5	0.5	0.5	0.3	0.4	0.5	0.3	0.3	0.3	0.6	0.3	0.8
0.5	0.5	1.0	0.4	0.6	0.5	0.4	0.6	0.3	0.4	0.7	0.2	0.4	0.4	0.3	0.4
0.3	0.6	0.8	0.3	0.4	0.5	0.5	0.8	0.3	0.4	0.3	0.5	0.5	0.5	0.2	0.2
0.5	0.9	0.8	0.4	0.3	0.5	0.8	0.5	0.4	0.3	0.4	0.4	0.3	0.7	0.2	0.6
0.5	0.8	0.7	0.4	0.6	0.5	0.7	0.4	0.5	0.3	0.4	0.2	0.5	0.6	0.6	0.6
0.5	0.7	0.8	0.3	0.4	0.5	0.5	0.4	0.4	0.4	0.4	0.3	0.8	0.6	2.0	1.6
0.5	0.7	0.8	0.2	0.9	0.7	0.9	1.3	0.8	1.5	1.0	1.0	0.7	0.3	2.3	1.3
0.5	0.9	0.8	0.5	0.6	0.7	0.6	2.6	1.5	1.4	0.9	1.2	0.5	0.5	1.6	1.7
0.3	0.2	0.7	0.5	0.6	0.8	0.5	3.9	0.9	1.8	1.7	0.9	0.4	0.3	0.8	1.3
0.6	0.1	0.5	0.6	0.5	1.2	1.3	0.7	1.8	2.1	1.1	0.3	0.9	0.3	2.6	1.1
0.5	0.5	0.8	0.4	0.8	1.2	0.9	2.3	1.7	2.7	1.3	0.6	0.3	0.3	1.1	1.7
0.2	0.4	0.7	0.6	0.7	1.2	0.5	0.8	0.2	1.6	1.1	1.0	1.0	0.3	0.5	0.5
0.2	0.4	1.1	0.5	1.0	0.8	1.2	1.2	0.2	1.9	1.4	0.9	1.1	0.3	0.9	0.6
0.3	0.7	1.1	0.4	1.3	0.9	0.6	3.1	0.7	0.9	1.0	0.5	1.0	0.2	0.6	1.1
0.3	1.0	0.9	0.4	1.1	0.9	0.3	1.0	0.8	1.2	1.3	0.4	1.0	0.3	0.8	1.0
0.5	1.0	0.6	0.3	1.6	1.0	0.7	1.2	1.9	0.8	0.9	1.0	1.2	0.3	0.3	0.4
0.3	1.0	0.5	0.5	0.7	0.3	1.1	1.5	0.8	0.7	1.0	0.6	0.4	1.1	0.6	0.9
0.2	1.7	0.7	0.5	1.0	1.0	1.6	1.1	0.8	0.3	0.9	0.6	0.4	1.1	1.4	0.9
0.4	1.0	1.1	0.4	0.7	0.7	0.5	1.0	1.1	0.4	0.4					

APPENDIX D

D.1. Matlab Codes For OM

D.1.1. For Normally Distributed Data

```
x0=normrnd(20,1,10000,1);
x=x0(1:500,1);
XT=[]
XT=x(1:500,1);
m=mean(XT)
s=std(XT)
ucl=m+2.57*s
lcl=m-2.57*s
m =

  20.0512
s =

   0.9889
ucl =

  22.5926
lcl =

  17.5097
plot(1:500,XT,'.');grid;hold on
plot(1:500,ones(1,500)*m,'.b',1:500,ones(1,500)*ucl,'.r',1:500,ones(1,500)*lcl,'.r');hold
off
axis([1 500 14 26])
```

01 MEAN SHIFT

```
XT1=[]
xt1=[]
 XT1=x0(501:1000,1);
xt1=XT+0.1;
plot(501:1000,xt1,'.');grid;hold on
plot(501:1000,ones(1,500)*m,'.b',501:1000,ones(1,500)*ucl,'.r',501:1000,ones(1,500)*l
cl,'.r');hold off
axis([501 1000 14 26])
```

025 MEAN SHIFT

```
XT2=[]
xt2=[]
 XT2=x0(1001:1500,1);
xt2=XT2+0.25;
```

```
plot(1001:1500,xt2,'.');grid;hold on
plot(1001:1500,ones(1,500)*m,'.b',1001:1500,ones(1,500)*ucl,'.r',1001:1500,ones(1,50
0)*lcl,'.r');hold off
axis([1001 1500 14 26])
```

0.5 MEAN SHIFT

```
XT3=[]
xt3=[]
 XT3=x0(1501:2000,1);
xt3=XT3+0.5;
plot(1501:2000,xt3,'.');grid;hold on
plot(1501:2000,ones(1,500)*m,'.b',1501:2000,ones(1,500)*ucl,'.r',1501:2000,ones(1,50
0)*lcl,'.r');hold off
axis([1501 2000 14 26])
```

1 MEAN SHIFT

```
XT4=[]
xt4=[]
 XT4=x0(2001:2500,1);
xt4=XT4+1;
plot(2001:2500,xt4,'.');grid;hold on
plot(2001:2500,ones(1,500)*m,'.b',2001:2500,ones(1,500)*ucl,'.r',2001:2500,ones(1,50
0)*lcl,'.r');hold off
axis([2001 2500 14 26])
```

1.1 STD SHIFT

```
XST1=[]
XST1=normrnd(20,1.10,500,1);
plot(2501:3000,XST1,'.');grid;hold on
plot(2501:3000,ones(1,500)*m,'.b',2501:3000,ones(1,500)*ucl,'.r',2501:3000,ones(1,50
0)*lcl,'.r');hold off
axis([2501 3000 14 26])
```

1.25 STD SHIFT

```
XST2=[]
XST2=normrnd(20,1.25,500,1);
plot(3001:3500,XST2,'.');grid;hold on
plot(3001:3500,ones(1,500)*m,'.b',3001:3500,ones(1,500)*ucl,'.r',3001:3500,ones(1,50
0)*lcl,'.r');hold off
axis([3001 3500 14 26]
```

1.5 STD SHIFT

```
XST3=[]
XST3=normrnd(20,1.50,500,1);
plot(3501:4000,XST3,'.');grid;hold on
```

```
plot(3501:4000,ones(1,500)*m,'.b',3501:4000,ones(1,500)*ucl,'.r',3501:4000,ones(1,50
0)*lcl,'.r');hold off
axis([3501 4000 14 26])
```

2 STD SHIFT

```
XST4=[]
XST4=normrnd(20,2,500,1);
plot(4001:4500,XST4,'.');grid;hold on
plot(4001:4500,ones(1,500)*m,'.b',4001:4500,ones(1,500)*ucl,'.r',4001:4500,ones(1,50
0)*lcl,'.r');hold off
```

BACK TO NORMAL OPERATION

```
XT5=[]
XT5=normrnd(20,1,500,1);
plot(4501:5000,XT5,'.');grid;hold on
plot(4501:5000,ones(1,500)*m,'.b',4501:5000,ones(1,500)*ucl,'.r',4501:5000,ones(1,50
0)*lcl,'.r');hold off
axis([4501 5000 14 26])
```

D.1.2. For Non-normally Distributed Data

```
x0=chi2rnd(5,10000,1);
x=x0(1:1000,1);
XT=[]
XT=x(1:1000,1);
m=mean(XT)
s=std(XT)
ucl=m+2.57*s
lcl=m-2.57*s
m =
    4.8199
s =

    2.9079
ucl =

   12.2931
lcl =

   -2.6534
plot(1:1000,XT,'.');grid;hold on
plot(1:1000,ones(1,1000)*m,'.b',1:1000,ones(1,1000)*ucl,'.r',1:1000,ones(1,1000)*lcl,'.r
');hold off
axis([1 1000 -10 30])
```

01 MEAN SHIFT

```
XT1=[]
xt1=[]
 XT1=x0(1001:1500,1);
xt1=XT(501:1000,1)+0.1;
plot(1001:1500,xt1,'.');grid;hold on
plot(1001:1500,ones(1,500)*m,'.b',1001:1500,ones(1,500)*ucl,'.r',1001:1500,ones(1,50
0)*lcl,'.r');hold off
axis([1001 1500 -10 30])
```

025 MEAN SHIFT

```
XT2=[]
xt2=[]
 XT2=x0(1501:2000,1);
xt2=XT2+0.25;
plot(1501:2000,xt2,'.');grid;hold on
plot(1501:2000,ones(1,500)*m,'.b',1501:2000,ones(1,500)*ucl,'.r',1501:2000,ones(1,50
0)*lcl,'.r');hold off
axis([1501 200 -10 30]
```

0.5 MEAN SHIFT

```
XT3=[]
xt3=[]
 XT3=x0(2001:2500,1);
xt3=XT3+0.5;
plot(2001:2500,xt3,'.');grid;hold on
plot(2001:2500,ones(1,500)*m,'.b',2001:2500,ones(1,500)*ucl,'.r',2001:2500,ones(1,50
0)*lcl,'.r');hold off
axis([2001 2500 -10 30])
```

1 MEAN SHIFT

```
XT4=[]
xt4=[]
 XT4=x0(2501:3000,1);
xt4=XT4+1;
plot(2501:3000,xt4,'.');grid;hold on
plot(2501:3000,ones(1,500)*m,'.b',2501:3000,ones(1,500)*ucl,'.r',2501:3000,ones(1,50
0)*lcl,'.r');hold off
axis([2501 3000 -10 30])
```

1.1 STD SHIFT

```
XST1=[]
XST1=normrnd(20,1.10,500,1);
plot(2501:3000,XST1,'.');grid;hold on
```

```
plot(2501:3000,ones(1,500)*m,'.b',2501:3000,ones(1,500)*ucl,'.r',2501:3000,ones(1,50
0)*lcl,'.r');hold off
axis([2501 3000 14 26])
```

1.25 STD SHIFT

```
XST2=[]
XST2=normrnd(20,1.25,500,1);
plot(3001:3500,XST2,'.');grid;hold on
plot(3001:3500,ones(1,500)*m,'.b',3001:3500,ones(1,500)*ucl,'.r',3001:3500,ones(1,50
0)*lcl,'.r');hold off
axis([3001 3500 14 26])
```

1.5 STD SHIFT

```
XST3=[]
XST3=normrnd(20,1.50,500,1);
plot(3501:4000,XST3,'.');grid;hold on
plot(3501:4000,ones(1,500)*m,'.b',3501:4000,ones(1,500)*ucl,'.r',3501:4000,ones(1,50
0)*lcl,'.r');hold off
axis([3501 4000 14 26])
```

2 STD SHIFT

```
XST4=[]
XST4=normrnd(20,2,500,1);
plot(4001:4500,XST4,'.');grid;hold on
plot(4001:4500,ones(1,500)*m,'.b',4001:4500,ones(1,500)*ucl,'.r',4001:4500,ones(1,50
0)*lcl,'.r');hold off
```

BACK TO NORMAL OPERATION

```
XT5=[]
XT5=normrnd(20,1,500,1);
plot(4501:5000,XT5,'.');grid;hold on
plot(4501:5000,ones(1,500)*m,'.b',4501:5000,ones(1,500)*ucl,'.r',4501:5000,ones(1,50
0)*lcl,'.r');hold off
axis([4501 5000 14 26])
```

D.1.3. For Industrial Data

```
XT=[]
XT=x(1:950,1);
hist(XT)
normplot(XT)
m=mean(XT)
s=std(XT)
ucl=m+2.57*s
lcl=m-2.57*s
```

```
m =

   0.4709
s =

   0.1660
ucl =

   0.8975
lcl =

   0.0444
plot(1:950,XT,'.');grid;hold on
plot(1:950,ones(1,950)*m,'.b',1:950,ones(1,950)*ucl,'.r',1:950,ones(1,950)*lcl,'.r');hold
off
axis([1 950 -0.2 1.5])

XT1=[]
XT1=x(951:1179,1);
plot(951:1179,XT1,'.');grid;hold on
plot(951:1179,ones(1,229)*m,'.b',951:1179,ones(1,229)*ucl,'.r',951:1179,ones(1,229)*l
cl,'.r');hold off
axis([951 1179 -0.3 4])
```

D.2. Matlab Codes For NM

D.2.1. For Normally Distributed Data

```
x0=normrnd(20,1,10000,1);
x=x0(1:1000,1);
[F2,xi2]=ksdensity(x,'npoints',5000,'width',0.075);
K2=[];
for i=1:1000
 [d,id]=min(abs(x(i)-xi2));
 K2(i,1)=F2(id);
end
y=normpdf(x,20,1);
plot(x,y,'g.',xi2,F2,'b.');grid
>> E0=[];
E0=y-K2;
m=mean(E0)
s=std(E0)
ucl=m+2.57*s
lcl=m-2.57*s
 m =
   -0.0107
s =
```

```
   0.0276
ucl =
   0.0603
lcl =

  -0.0817
figure(2);plot(1:1000,E0,'.');grid;hold on
>> figure(3);plot(1:1000,E0,'.');grid;hold on
plot(ones(1,1000)*m,('.b'))
plot(ones(1,1000)*ucl,('.g'))
plot(ones(1,1000)*lcl,('.g'))
axis([1 1000 -0.3 0.3])
```

01 MEAN SHIFT

```
XT=[]
xt1=[]
 XT=x0(1001:1500,1);
xt1=XT+0.1;
ET=[];
zt=[];
for j=1:500
zt=[x((j+500):1000);xt1(1:j)];
 [FZ,xiZ]=ksdensity(zt,'npoints',5000,'width',0.075);
Kt=[];
for i=1:500
 [d,id]=min(abs(zt(i)-xiZ));
 Kt(i,1)=FZ(id);
end
%yt=normpdf(zt(j),20,1);
yt=normpdf(xt1(j),20,1);
ET(j,1)=yt-Kt(500);
end
figure(2);plot(1001:1500,ET,'.');grid;hold on
plot(1001:1500,ones(1,500)*m,'.b',1001:1500,ones(1,500)*ucl,'.r',1001:1500,ones(1,50
0)*lcl,'.r')
axis([1001 1500 -0.50 0.50])
```

025 MEAN SHIFT

```
XT1=[]
xt2=[]
 XT1=x0(1501:2000,1);
xt2=XT1+0.25;
ET1=[];
zt1=[];
for j=1:500
zt1=[x((j+500):1000);xt2(1:j)];
 [FZ1,xiZ1]=ksdensity(zt1,'npoints',5000,'width',0.075);
```

```
Kt1=[];
for i=1:500
 [d,id]=min(abs(zt1(i)-xiZ1));
 Kt1(i,1)=FZ1(id);
end
%yt=normpdf(zt(j),20,1);
yt1=normpdf(xt2(j),20,1);
ET1(j,1)=yt1-Kt1(500);
end
figure(2);plot(1501:2000,ET1,'.');grid;hold on
plot(1501:2000,ones(1,500)*m,'.b',1501:2000,ones(1,500)*ucl,'.r',1501:2000,ones(1,50
0)*lcl,'.r')
axis([1501 2000 -0.50 0.50])
```

0.5 MEAN SHIFT

```
XT2=[]
xt3=[]
 XT2=x0(2001:2500,1);
xt3=XT2+0.5;
ET2=[];
zt2=[];
for j=1:500
zt2=[x((j+500):1000);xt3(1:j)];
 [FZ2,xiZ2]=ksdensity(zt2,'npoints',5000,'width',0.075);
Kt2=[];
for i=1:500
 [d,id]=min(abs(zt2(i)-xiZ2));
 Kt2(i,1)=FZ2(id);
end
%yt=normpdf(zt(j),20,1);
yt2=normpdf(xt3(j),20,1);
ET2(j,1)=yt2-Kt2(500);
end
figure(2);plot(2001:2500,ET2,'.');grid;hold on
plot(2001:2500,ones(1,500)*m,'.b',2001:2500,ones(1,500)*ucl,'.r',2001:2500,ones(1,50
0)*lcl,'.r')
axis([2001 2500 -0.50 0.50])
```

1 MEAN SHIFT

```
XT3=[]
xt4=[]
 XT3=x0(2501:3000,1);
xt4=XT3+1;
ET3=[];
zt3=[];
for j=1:500
zt3=[x((j+500):1000);xt4(1:j)];
 [FZ3,xiZ3]=ksdensity(zt3,'npoints',5000,'width',0.075);
```

```
Kt3=[];
for i=1:500
 [d,id]=min(abs(zt3(i)-xiZ3));
 Kt3(i,1)=FZ3(id);
end
%yt=normpdf(zt(j),20,1);
yt3=normpdf(xt4(j),20,1);
ET3(j,1)=yt3-Kt3(500);
end
figure(2);plot(2501:3000,ET3,'.');grid;hold on
plot(2501:3000,ones(1,500)*m,'.b',2501:3000,ones(1,500)*ucl,'.r',2501:3000,ones(1,50
0)*lcl,'.r')
axis([2501 3000 -0.50 0.50])
```

1.1 STD SHIFT

```
XT4=[]
XT4=normrnd(20,1.1,500,1);
ET4=[];
zt4=[];
for j=1:500
zt4=[x((j+500):1000);XT4(1:j)];
 [FZ4,xiZ4]=ksdensity(zt4,'npoints',5000,'width',0.075);
Kt4=[];
for i=1:500
 [d,id]=min(abs(zt4(i)-xiZ4));
 Kt4(i,1)=FZ4(id);
end
%yt=normpdf(zt(j),20,1);
yt4=normpdf(XT4(j),20,1);
ET4(j,1)=yt4-Kt4(500);
end
plot(3001:3500,ET4,'.');grid;hold on
figure(2);plot(3001:3500,ET4,'.');grid;hold on
plot(3001:3500,ones(1,500)*m,'.b',3001:3500,ones(1,500)*ucl,'.r',3001:3500,ones(1,50
0)*lcl,'.r')
axis([3001 3500 -0.50 0.50])
```

1.25 STD SHIFT

```
XT5=[]
XT5=normrnd(20,1.25,500,1);
ET5=[];
zt5=[];
for j=1:500
zt5=[x((j+500):1000);XT5(1:j)];
 [FZ5,xiZ5]=ksdensity(zt5,'npoints',5000,'width',0.075);
Kt5=[];

for i=1:500
```

```
 [d,id]=min(abs(zt5(i)-xiZ5));
 Kt5(i,1)=FZ5(id);
end
%yt=normpdf(zt(j),20,1);
yt5=normpdf(XT5(j),20,1);
ET5(j,1)=yt5-Kt5(500);
end
figure(2);plot(3501:4000,ET5,'.');grid;hold on
plot(3501:4000,ones(1,500)*m,'.b',3501:4000,ones(1,500)*ucl,'.r',3501:4000,ones(1,50
0)*lcl,'.r')
axis([3501 4000 -0.50 0.50])
```

1.5 STD SHIFT

```
XT6=[]
XT6=normrnd(20,1.5,500,1);
ET6=[];
zt6=[];
for j=1:500
zt6=[x((j+500):1000);XT6(1:j)];
 [FZ6,xiZ6]=ksdensity(zt6,'npoints',5000,'width',0.075);
Kt6=[];
for i=1:500
 [d,id]=min(abs(zt6(i)-xiZ6));
 Kt6(i,1)=FZ6(id);
end
%yt=normpdf(zt(j),20,1);
yt6=normpdf(XT6(j),20,1);
ET6(j,1)=yt6-Kt6(500);
end
figure(2);plot(4001:4500,ET6,'.');grid;hold on
plot(4001:4500,ones(1,500)*m,'.b',4001:4500,ones(1,500)*ucl,'.r',4001:4500,ones(1,50
0)*lcl,'.r')
axis([4001 4500 -0.50 0.50])
```

2 STD SHIFT

```
XT7=[]
XT7=normrnd(20,2,500,1);
ET7=[];
zt7=[];
for j=1:500
zt7=[x((j+500):1000);XT7(1:j)];
 [FZ7,xiZ7]=ksdensity(zt7,'npoints',5000,'width',0.075);
Kt7=[];
for i=1:500
 [d,id]=min(abs(zt7(i)-xiZ7));
 Kt7(i,1)=FZ7(id);
end
%yt=normpdf(zt(j),20,1);
```

```
yt7=normpdf(XT7(j),20,1);
ET7(j,1)=yt7-Kt7(500);
end
figure(2);plot(4501:5000,ET7,'.');grid;hold on
plot(4501:5000,ones(1,500)*m,'.b',4501:5000,ones(1,500)*ucl,'.r',4501:5000,ones(1,50
0)*lcl,'.r')
axis([4501 5000 -0.50 0.50])
```

BACK TO NORMAL OPERATION

```
X8=[]
X8=normrnd(20,1,1000,1);
E8=[];
 [F8,xi8]=ksdensity(X8,'npoints',5000,'width',0.075);
K8=[];
for i=1:1000
 [d,id]=min(abs(X8(i)-xi8));
 K8(i,1)=F8(id);
end
y8=normpdf(X8,20,1);
plot(X8,y8,'g.',xi8,F8,'b.');grid
>> E8=[];
E8=y8-K8;
figure(2);plot(5001:6000,E8,'.');grid;hold on
plot(5001:6000,ones(1,1000)*m,'.b',5001:6000,ones(1,1000)*ucl,'.r',5001:6000,ones(1,
1000)*lcl,'.r')
axis([5001 6000 -0.30 0.30])
```

D.2.2. For Non-normally Distributed Data

```
x0=chi2rnd(5,10000,1);
x=x0(1:1000,1),
[F2,xi2]=ksdensity(x,'npoints',5000,'width',0.075);
K2=[];
for i=1:1000
 [d,id]=min(abs(x(i)-xi2));
 K2(i,1)=F2(id);
end
y=chi2pdf(x,5);
plot(x,y,'g.',xi2,F2,'b.');grid
E0=[];
E0=y-K2;
figure(2);plot(1:1000,E0,'.');grid;hold on
>> figure(3);plot(1:1000,E0,'.');grid;hold on
m=mean(E0)
s=std(E0)
ucl=m+2.57*s
lcl=m-2.57*s
```

```
 plot(ones(1,1000)*m,('.b'))
plot(ones(1,1000)*ucl,('.g'))
plot(ones(1,1000)*lcl,('.g'))
axis([1 1000 -0.1 0.1])
m =

   -0.0064
s =

    0.0169
ucl =

    0.0370
lcl =

   -0.0499
```

01 MEAN SHIFT

```
XT=[]
xt1=[]
XT=x0(1001:1500,1);
xt1=XT+0.1;
ET=[];
zt=[];
for j=1:500
zt=[x((j+500):1000);xt1(1:j)];
 [FZ,xiZ]=ksdensity(zt,'npoints',5000,'width',0.075);
Kt=[];
for i=1:500
 [d,id]=min(abs(zt(i)-xiZ));
 Kt(i,1)=FZ(id);
end
%yt=chi2pdf(zt(j),10);
yt=chi2pdf(xt1(j),5);
ET(j,1)=yt-Kt(500);
end
plot(1001:1500,ET,'.');grid;hold on
plot(1001:1500,ones(1,500)*m,'.b',1001:1500,ones(1,500)*ucl,'.r',1001:1500,ones(1,50
0)*lcl,'.r')
axis([1001 1500 -0.3 0.3])
```

025 MEAN SHIFT

```
XT1=[]
xt2=[]
XT1=x0(1501:2000,1);
xt2=XT1+0.25;
ET1=[];
zt1=[];
```

```
for j=1:500
zt1=[x((j+500):1000);xt2(1:j)];
 [FZ1,xiZ1]=ksdensity(zt1,'npoints',5000,'width',0.075);
Kt1=[];
for i=1:500
 [d,id]=min(abs(zt1(i)-xiZ1));
 Kt1(i,1)=FZ1(id);
end
%yt=chi2pdf(zt(j),10);
yt1=chi2pdf(xt2(j),5);
ET1(j,1)=yt1-Kt1(500);
end
plot(1501:2000,ET1,'.');grid;hold on
plot(1501:2000,ones(1,500)*m,'.b',1501:2000,ones(1,500)*ucl,'.r',1501:2000,ones(1,50
0)*lcl,'.r')
axis([1501 2000 -0.3 0.3])
```

0.5 MEAN SHIFT

```
XT2=[]
xt3=[]
XT2=x0(2001:2500,1);
xt3=XT2+0.5;
ET2=[];
zt2=[];
for j=1:500
zt2=[x((j+500):1000);xt3(1:j)];
 [FZ2,xiZ2]=ksdensity(zt2,'npoints',5000,'width',0.075);
Kt2=[];
for i=1:500
 [d,id]=min(abs(zt2(i)-xiZ2));
 Kt2(i,1)=FZ2(id);
end
%yt=chi2pdf(zt(j),10);
yt2=chi2pdf(xt3(j),5);
ET2(j,1)=yt2-Kt2(500);
end
plot(2001:2500,ET2,'.');grid;hold on
plot(2001:2500,ones(1,500)*m,'.b',2001:2500,ones(1,500)*ucl,'.r',2001:2500,ones(1,50
0)*lcl,'.r')
axis([2001 2500 -0.3 0.3])
```

1 MEAN SHIFT

```
XT3=[]
xt4=[]
 XT3=x0(2501:3000,1);
xt4=XT3+1;
ET3=[];
zt3=[];
```

```
for j=1:500
zt3=[x((j+500):1000);xt4(1:j)];
 [FZ3,xiZ3]=ksdensity(zt3,'npoints',5000,'width',0.075);
Kt3=[];
for i=1:500
 [d,id]=min(abs(zt3(i)-xiZ3));
 Kt3(i,1)=FZ3(id);
end
%yt=normpdf(zt(j),20,1);
yt3=chi2pdf(xt4(j),5);
ET3(j,1)=yt3-Kt3(500);
end
plot(2501:3000,ET3,'.');grid;hold on
figure(2);plot(2501:3000,ET3,'.');grid;hold on
plot(2501:3000,ones(1,500)*m,'.b',2501:3000,ones(1,500)*ucl,'.r',2501:3000,ones(1,50
0)*lcl,'.r')
axis([2501 3000 -0.3 0.3])
```

BACK TO NORMAL OPERATION

```
x0=chi2rnd(5,10000,1);
xS=x0(3001:4000,1),
[FS,xiS]=ksdensity(xS,'npoints',5000,'width',0.075);
KS=[];
for i=1:1000
 [d,id]=min(abs(xS(i)-xiS));
 KS(i,1)=FS(id);
end
yS=chi2pdf(xS,5);
plot(xS,yS,'g.',xiS,FS,'b.');grid
ES=[];
ES=yS-KS;
m=mean(ES)
s=std(ES)
ucl=m+2.57*s
lcl=m-2.57*s
plot(3001:4000,ES,'.');grid;hold on
plot(3001:4000,ones(1,1000)*m,'.b',3001:4000,ones(1,1000)*ucl,'.r',3001:4000,ones(1,
1000)*lcl,'.r')
axis([3001 4000 -0.1 0.1])
```

D.2.3. For Industrial Data

```
xt=x(1:950);
 normplot(xt)
 [F2,xi2]=ksdensity(xt,'npoints',5000,'width',0.05,'kernel','normal');
 figure(1);y2=normpdf(xt,0.4741,0.1674);
 plot(xi2,F2,'b.',xt,y2,'g.');grid
```

```
K2=[];
for i=1:950
 [d,id]=min(abs(xt(i)-xi2));
 K2(i,1)=F2(id);
end
E0=[];
E0=y2-K2;
m=mean(E0)
s=std(E0)
ucl=m+2.57*s
lcl=m-2.57*s
m =
   -0.0674
s =

    0.2198
ucl =

    0.4976
lcl =

   -0.6324
 figure(3);plot(1:950,E0,'.');grid;hold on
plot(ones(1,950)*m,('.b'))
plot(ones(1,950)*ucl,('.g'))
plot(ones(1,950)*lcl,('.g'))

 XT=[]
XT=x(950:1179,1);
ET=[];
zt=[];
for j=1:230
zt=[x((j+1):230);XT(1:j)];
[FZ,xiZ]=ksdensity(zt,'npoints',5000,'width',0.05,'kernel','normal');
Kt=[];
for i=1:230
 [d,id]=min(abs(zt(i)-xiZ));
 Kt(i,1)=FZ(id);
end
yt=normpdf(XT(j),0.4741,0.1674);
ET(j,1)=yt-Kt(230);
end
plot(950:1179,ET,'.');grid;hold on
plot(950:1179,ones(1,230)*m,'.b',950:1179,ones(1,230)*ucl,'.r',950:1179,ones(1,230)*l
cl,'.r')
```