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A note on the (G'/G)-expansion method again

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ABSTRACT

We report an observation on two recent analytic methods; the (G'/G)-expansion method and the simplest equation method.

Let us consider a nonlinear ordinary differential equation in the form

$$P(u, u', u'', u''', \ldots) = 0, \tag{1}$$

where $u = u(z)$ is an unknown function, P is a polynomial in its arguments. According to the (G'/G)-expansion method [1], to solve Eq. (1), we assume its solutions $u(z)$ in the finite series form

$$u(z) = \sum_{i=0}^{N} a_i \left(\frac{G'(z)}{G(z)}\right)^i, \quad a_i = \text{const.}, \quad a_N \neq 0, \tag{2}$$

where $G(z)$ is the solution of the auxiliary linear ordinary differential equation

$$G''(z) + \lambda G'(z) + \mu G(z) = 0, \tag{3}$$

where λ and μ are real constants.

Theorem. *Let $F(z)$ and $G(z)$ be two functions such that $G(z) = F(z)\exp(-\lambda z/2)$. Then, $G(z)$ satisfies Eq. (3) if and only if $F(z)$ satisfies the differential equation*

$$F''(z) + \alpha F(z) = 0, \quad \alpha = (4\mu - \lambda^2)/4. \tag{4}$$

Moreover, the fraction $G'(z)/G(z)$ can be expressed in terms of the fraction $F'(z)/F(z)$, namely,

$$\frac{G'(z)}{G(z)} = \frac{F'(z)}{F(z)} - \frac{\lambda}{2}. \tag{5}$$

Corollary. *The ansatz (2) can be reduced to the special case*

$$u(z) = \sum_{i=0}^{N} b_i \left(\frac{F'(z)}{F(z)}\right)^i, \quad b_i = \text{const.}, \quad b_N \neq 0, \tag{6}$$

where $F(z)$ satisfies the auxiliary Eq. (4).

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Obviously, this theorem reduces the study of Eq. (3) to the special case $G''(z) + \mu G(z) = 0$. To be more specific, solving this special equation is tantamount to solving Eq. (3). Hence, the constant λ in the (G'/G)-expansion method can be taken as zero for simplicity. For application purposes, the ansatz (6) is much more practical than the ansatz (2) in the sense that it minimizes the number of parameters by combining the constants λ and μ.

On the other hand, to solve Eq. (1) by means of the simplest equation method [2,3], we assume its solutions $u(z)$ in the form

$$u(z) = A_0 + A_1 Y(z) + \cdots + A_n (Y(z))^N + B_1 \left(\frac{Y'(z)}{Y(z)}\right) + \cdots + B_n \left(\frac{Y'(z)}{Y(z)}\right)^N, \tag{7}$$

where $Y(z)$ is the general solution of the simplest equation and A_i, B_i are arbitrary constants. As a special example, one can consider the following simplest equation

$$Y'(z) + (Y(z))^2 + \lambda Y(z) + \mu = 0, \tag{8}$$

where λ and μ are real constants.

Now, let us take $Y(z) = G'(z)/G(z)$ in both (7) and (8). Then Eq. (8) turns into Eq. (3). Since

$$\frac{Y'(z)}{Y(z)} = -\lambda - \frac{G'(z)}{G(z)} - \mu \left(\frac{G'(z)}{G(z)}\right)^{-1}, \tag{9}$$

the ansatz (7) becomes

$$u(z) = A_0 + A_1 \frac{G'(z)}{G(z)} + \cdots + A_n \left(\frac{G'(z)}{G(z)}\right)^N + B_1 \left(-\lambda - \frac{G'(z)}{G(z)} - \mu \left(\frac{G'(z)}{G(z)}\right)^{-1}\right) + \cdots + B_n \left(-\lambda - \frac{G'(z)}{G(z)} - \mu \left(\frac{G'(z)}{G(z)}\right)^{-1}\right)^N, \tag{10}$$

which can be rewritten as

$$u(z) = \sum_{i=-N}^{N} c_i \left(\frac{G'(z)}{G(z)}\right)^i, \quad c_i = \text{const}. \tag{11}$$

The ansatz (11) together with Eq. (3) is known as the extended (or improved) (G'/G)-expansion method.

Recently, Kudryashov [4] demonstrated that (G'/G)-expansion method is equivalent to the tanh-method. We think that, for a new equation or previously unstudied problem, one should not receive a criticism for using one of the equivalent methods. It should be considered as a personal choice. Besides, it is a well-known fact that the simplest equation method was introduced to the research community earlier than the (G'/G)-expansion method. The discussion made above convinced us that the later one is a specific form of the former one.

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