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Range Identification for Nonlinear Parameterizable Paracatadioptric Systems

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***Abstract*—In this paper, a new range identification technique for a calibrated paracatadioptric system mounted on a moving platform is developed to recover the range information and the three-dimensional (3D) Euclidean coordinates of a static object feature. The position of the moving platform is assumed to be measurable. To identify the unknown range, first a function of the projected pixel coordinates is related to the unknown 3D Euclidean coordinates of an object feature. This function is nonlinearly parameterized (*i.e.*, the unknown parameters appear nonlinearly in the parameterized model). An adaptive estimator based on a min-max algorithm is then designed to estimate the unknown 3D Euclidean coordinates of an object feature relative to a fixed reference frame which facilitates the identification of range. A Lyapunov-type stability analysis is used to show that the developed estimator provides an estimation of the unknown parameters within a desired precision.**

***Index Terms*—Nonlinear parameterization, range identification, paracatadioptric systems, Lyapunov methods, vision-based estimation, min-max algorithm.**

I. Introduction

The problem of range identification, where the estimation of the unknown time-varying distance of the object from the camera along its optical axis, has received noteworthy attention over the last several years due to its significance in several applications such as autonomous vehicle navigation, aerial tracking, path planning, surveillance, etc. These applications require either the range or the 3D Euclidean coordinates of features of a moving or a static object to be recovered from their two-dimensional (2D) image sequence. The range estimation is usually done by mounting a camera on a moving vehicle such as a mobile robot or an unmanned aerial vehicle (UAV) which captures images of the static objects or features. However, the use of conventional (perspective) cameras pose restrictions for some applications because of their limited field-of-view (FOV).

One efficient way to enhance the FOV is to use mirrors (spherical, elliptical, hyperboloid, or paraboloid) in conjunction with conventional cameras, commonly known as catadioptric systems [1]. However, the use of curved mirrors reduce the resolution and distort the images to a large extent. As stated in [2], the distorted image mapping can be dealt with by using computer vision techniques, but the nonlinearity which is introduced in the transformation makes it difficult to recover 3D coordinates of the object features. Catadioptric systems that have a single effective viewpoint are known as central catadioptric systems, and are desirable because they allow for distortion-free reconstruction of panoramic images [3]. A paracatadioptric system is a special case of central catadioptric systems which employs a paraboloid mirror along with an orthographic lens. These systems are advantageous due to the fact that the paraboloid constant of the mirror along with its physical size do not need to be determined during the calibration. Furthermore, mirror alignment requirements are relaxed which means that the mirror can be arbitrarily translated enabling the camera to zoom in on a part of the paraboloid mirror for higher resolution; however, with a reduced FOV [1].

In the past, many researchers have proposed various range identification techniques for perspective vision systems. Some of which have utilized the extended Kalman filter (EKF) [4], [5], [6]. However, EKF involves linearization of the nonlinear vision model and requires *a priori* knowledge of the noise distribution. To overcome the shortcomings of the linear model, many researchers focused on utilizing nonlinear system analysis and estimation tools to develop nonlinear observers to identify the range when the motion parameters were known [7], [8], [9], [10], [11], [12]. More recently, in [13], measurement of camera position was utilized to develop an adaptive estimator to recover the structure which was extended in [14] to recover the range.

Although, there have been several reports on range identification for perspective vision systems, very few results have been shown for range identification for catadioptric systems. In [15], Ma *et al.* proposed a range identification technique for paracatadioptric system based on a sequence of linear approximation-based observers. In [16], Gupta *et al.* designed a nonlinear observer to asymptotically identify the range for a paracatadioptric system. However, both of these reports assumed the focal point of the paraboloid mirror to be at its vertex. This assumption was recently relaxed in [2]. In the current work, we also base our development on a more practical approach that the focal length of the paraboloid mirror is not at its vertex. In [3], an omnidirectional light projector was embedded in a paracatadioptric system, and the range

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was calculated by triangulation. In [2], Hu *et al.* developed a nonlinear estimator similar to [9] to identify the range for paracatadioptric systems where the motion parameters were assumed to be known, and it assumed that the object must translate in at least one direction.

In this paper, we present a method to identify the range of a static object using a moving paracatadioptric system whose position is measurable. For many applications, position measurements are considerably less noisy than velocity measurements; hence, we are motivated to develop an estimator based on position measurements. The estimator is designed by first developing a geometric model along with a paracatadioptric projection model that relates an object feature with the paracatadioptric system mounted on a moving mechanical system. The novelty of this work lies in the compensation for nonlinear parameterization of the model which relates the projected pixel coordinates to the Euclidean coordinates of the object feature. It should be noted that contrary to [13], where the unknown terms appear linearly in the parameterized model for a perspective vision system, in the current work the unknown parameters appear nonlinearly in the model for a paracatadioptric system. This fact makes it difficult to use a standard adaptive estimator or a gradient based estimator [17]. The estimator presented in this paper which facilitates range identification to the desired precision is based on a min-max optimization algorithm. We show that the developed estimator identifies the range and the 3D coordinates of the object feature upon the satisfaction of a Nonlinear Persistent Excitation (NLPE) condition. The contributions of this paper are that: i) the developed estimator utilizes position measurements instead of velocity measurements, ii) is continuous, and iii) provides estimation of unknown parameters within a desired precision.

II. Model Development

A. Geometric Model

For the development of a geometric relationship between a moving paracatadioptric system and a stationary object, an orthogonal coordinate frame, denoted by $\mathcal{M}$, which is centered at the focal point of the moving paraboloid mirror whose Z-axis is aligned with the optical axis of the camera, is defined (see Fig. 1). As shown in Fig. 1, an inertial coordinate frame, denoted by $\mathcal{W}$, and an orthogonal coordinate frame, denoted by $\mathcal{B}$, are defined. $\mathcal{F}$ denotes a static feature on a stationary object. Let the unknown 3D Euclidean coordinates of the object feature be denoted as the constant $\theta \in \mathbb{R}^3$ relative to the world frame $\mathcal{W}$ and $m(t) \in \mathbb{R}^3$ relative to $\mathcal{M}$ be defined as follows

$$m \triangleq [x \;\; y \;\; z]^T. \tag{1}$$

To relate the coordinate systems, let $R_b(t) \in SO(3)$ and $x_b(t) \in \mathbb{R}^3$ denote the measurable rotation matrix and the translation vector, respectively, from $\mathcal{B}$ to $\mathcal{W}$ expressed in $\mathcal{W}$. Let $R_m \in SO(3)$ and $x_m \in \mathbb{R}^3$ be the known rotation matrix and the translation vector, respectively, from $\mathcal{M}$ to $\mathcal{B}$ expressed in $\mathcal{B}$.

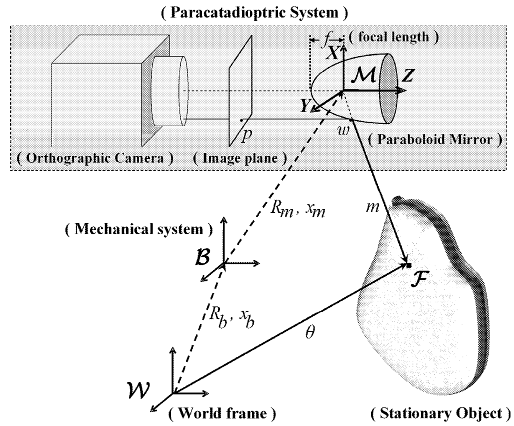

Fig. 1. Geometric relationships between the stationary object, mechanical system, and the paracatadioptric system.

B. Paracatadioptric System Projection Model

In a paracatadioptric system, a Euclidean point is projected onto a paraboloid mirror and is then reflected to an orthographic camera (see Fig. 1); thus, to facilitate the subsequent development, and to relate the geometric model to the vision system, let the projection of the object feature on the surface of the paraboloid mirror with its focus at the origin be denoted by $w(t) \in \mathbb{R}^3$ relative to $\mathcal{M}$ and defined as follows

$$w \triangleq [u \;\; v \;\; q]^T. \tag{2}$$

The projection $w(t)$ can be expressed as follows [18]

$$w = \frac{2f}{\lambda} m = \frac{2f}{\lambda} [x \;\; y \;\; z]^T \tag{3}$$

where $f \in \mathbb{R}$ is the known focal length of the mirror and $\lambda(t) \in \mathbb{R}$ is the unknown nonlinear signal defined as follows

$$\lambda \triangleq -z + \sqrt{x^2 + y^2 + z^2}. \tag{4}$$

It is worthwhile to mention that the use of a paracatadioptric system results in an orthographic projection from the paraboloid mirror to the image plane. In other words, the reflected rays are parallel to the optical axis; thus, the distance from the mirror to the image plane is irrelevant. After utilizing (2) and (3), the projection can be expressed as follows

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{2f}{\lambda} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{5}$$

However, when measured from a CCD chip as in any practical case, $[u, \, v]^T$ is transformed as follows [19]

$$p \triangleq \begin{bmatrix} u' \\ v' \end{bmatrix} = K \begin{bmatrix} u \\ v \end{bmatrix} + C \tag{6}$$

where $p(t) \in \mathbb{R}^2$ are the measured pixel coordinates on the image plane, $K \in \mathbb{R}^{2\times 2}$ and $C \in \mathbb{R}^2$ are defined as follows

$$K \triangleq \begin{bmatrix} a_1 & a_2 \\ 0 & a_1^{-1} \end{bmatrix} \qquad C \triangleq \begin{bmatrix} c_x \\ c_y \end{bmatrix} \tag{7}$$

where $a_1^2, a_2 \in \mathbb{R}$ are the aspect ratio and the skew factor, respectively, and C is the image center. Since a central catadioptric camera can be calibrated using a single image of three lines [19], [20], we assume the camera to be calibrated. It is clear from (6) that the coordinates in the mirror frame, $u(t)$ and $v(t)$, can be obtained from the measured pixel coordinates as follows

$$[u \;\; v]^T = K^{-1}(p - C). \tag{8}$$

Also, since the paraboloid mirror is rotationally symmetric, $q(t) \in \mathbb{R}$ can be computed from $u(t)$ and $v(t)$ as follows [2]

$$q = \frac{u^2 + v^2}{4f} - f. \tag{9}$$

Assumption 1: It is assumed that the object feature does not intersect the optical axis *i.e.*, $x(t), y(t) \neq 0$ simultaneously and thus, $\lambda(t) \neq 0$.

III. Nonlinear Parameterization of the Model

In this section, the parameterization of the nonlinear function $q(t)$ is presented after relating it to the unknown 3D Euclidean coordinates of the object feature. From Fig. 1, $m(t)$ can be written as follows [13]

$$m = R_m^T \left[R_b^T (\theta - x_b) - x_m \right]. \tag{10}$$

After utilizing (1), the 3D coordinates of the object feature relative to $\mathcal{M}$ can be expressed as follows

$$x = R_{m1}^T \left[R_b^T (\theta - x_b) - x_m \right] \tag{11}$$
$$y = R_{m2}^T \left[R_b^T (\theta - x_b) - x_m \right] \tag{12}$$
$$z = R_{m3}^T \left[R_b^T (\theta - x_b) - x_m \right] \tag{13}$$

where $R_{mi}^T \in \mathbb{R}^{1\times 3}$ is the i^{th} row of R_m^T, and $z(t)$ is the range of the object feature. After substituting (5) into the nonlinear model given in (9), $q(t)$ is nonlinearly parameterized (NLP) as follows

$$q(\theta, \Pi) = \frac{\left(\frac{2f}{\lambda} x\right)^2 + \left(\frac{2f}{\lambda} y\right)^2}{4f} - f \tag{14}$$

where $\Pi(t) \in \mathbb{R}^{q\times r}$ contains the combinations of known and measurable quantities (*i.e.*, f, R_m, x_m, $R_b(t)$, and $x_b(t)$).

Assumption 2: The unknown parameter vector θ is assumed to belong to a known hypercube $\Theta \subset \mathbb{R}^3$. In other words, the 3D coordinates of the object feature relative to $\mathcal{W}$ are assumed to lie within their known minimum and maximum values.

Assumption 3: For any $\Pi(t)$, the function $q(t)$ is either concave or convex on a simplex[1] Θ_s in $\mathbb{R}^3$ such that $\Theta_s \supset \Theta$ (see Fig. 2).

Assumption 4: The function $\Pi(t)$ is bounded, continuous function of its arguments, and is Lipschitz in t such that

$$\|\Pi(t_1) - \Pi(t_2)\| \leq L_1 |t_1 - t_2| \qquad \forall t_1, t_2 \in \mathbb{R}^+$$

where $L_1 \in \mathbb{R}^+$ is the Lipschitz constant.

[1]A simplex in $\mathbb{R}^n$ is a convex polyhedron having exactly $n+1$ vertices.

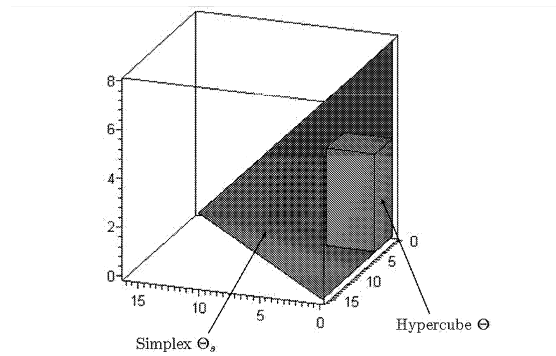

Fig. 2. Simplex Θ_s, and hypercube Θ.

Assumption 5: $q(\theta_0, \Pi)$ is Lipschitz with respect to its arguments such that

$$|q(\theta_0 + \Delta\theta_0, \Pi + \Delta\Pi) - q(\theta_0, \Pi)| \leq L_2(\|\Delta\Pi\| + \|\Delta\theta_0\|)$$

where $L_2 \in \mathbb{R}^+$ is the Lipschitz constant, $\Delta\Pi = \Pi(t_1) - \Pi(t_2)$, and $\Delta\theta_0 = \theta_0(t_1) - \theta_0(t_2)$.

Definition 1: A function $H(\varsigma)$ is said to be convex on Θ if it satisfies the following inequality

$$H(\sigma\varsigma_1 + (1-\sigma)\varsigma_2) \leq \sigma H(\varsigma_1) + (1-\sigma) H(\varsigma_2) \quad \forall \varsigma_1, \varsigma_2 \in \Theta \tag{15}$$

and concave if it satisfies the following inequality

$$H(\sigma\varsigma_1 + (1-\sigma)\varsigma_2) \geq \sigma H(\varsigma_1) + (1-\sigma) H(\varsigma_2) \quad \forall \varsigma_1, \varsigma_2 \in \Theta \tag{16}$$

where $0 \leq \sigma \leq 1$.

Remark 1: It should be noted that the Assumptions 2 and 3 essentially characterize the nature of the nonlinear parameterization, and the convexity or concavity of the function $q(\cdot)$ is required in a region Θ_s which is larger than the hypercube Θ. Also, since $\Pi(t)$ contains the combinations of known constants (*i.e.*, f, R_m, and x_m), and continuous measurable signals $R_b(t)$ and $x_b(t)$, Assumption 4 is valid. Assumption 5 is valid owing to the fact that the developed estimator is continuous as shown later in the paper.

Remark 2: The hypercube Θ can be found using the minimum and the maximum values of θ. The vertices of the simplex Θ_s, denoted by $\theta_{s1}, \theta_{s2}, \theta_{s3}, \theta_{s4} \in \mathbb{R}^3$, can be found by first inscribing Θ in a 3-dimensional sphere and then inscribing this sphere inside a 4-dimensional polyhedron [17], [21].

It should be noted that in (11)-(13), θ (*i.e*, the constant 3D coordinates of the object feature relative to $\mathcal{W}$) is the only unknown vector, and if we estimate this we can obtain an estimation of the 3D coordinates of the object feature relative to $\mathcal{M}$ as follows

$$\hat{x} = R_{m1}^T[R_b^T(\hat{\theta} - x_b) - x_m] \quad (17)$$
$$\hat{y} = R_{m2}^T[R_b^T(\hat{\theta} - x_b) - x_m] \quad (18)$$
$$\hat{z} = R_{m3}^T[R_b^T(\hat{\theta} - x_b) - x_m] \quad (19)$$

where $\hat{x}(t), \hat{y}(t) \in \mathbb{R}$ are the estimates of $x(t)$ and $y(t)$, respectively, $\hat{z}(t) \in \mathbb{R}$ is the estimate of the corresponding range $z(t)$, and $\hat{\theta}(t) \in \mathbb{R}^3$ is the estimate of θ.

IV. Range Estimation

In this section, an estimator for the unknown constant parameter vector θ which appears nonlinearly in the model given in (14) is presented. There are very few researchers who have addressed adaptive control or estimation for NLP systems [17], [22], [23], [24]. Parameter convergence in NLP systems was addressed in [25]. As pointed out in [17], the gradient algorithm employed in [22], [23], [24] are not only inadequate but can also lead to instability for general NLP systems. In this work, we design an adaptive estimator that facilitates the identification of range within a desired precision based on the min-max algorithm developed in [17]. The maximization is that of a tuning function over all the possible values of the nonlinear parameters, and the minimization is over all the possible sensitivity functions that can be used in the adaptive law. The sensitivity function which differs from the gradient depending upon the sign of a tuning error is incorporated in the adaptive law. The stability analysis ensures that the use of the tuning function along with the adaptive law has globally bounded error signals, and upon the satisfaction of an NLPE condition similar to [25], the parameter estimation follows; hence, the identification of range.

A. Estimator Design

To facilitate the estimator design, the estimate of (14) is defined as follows

$$\hat{q} \triangleq \frac{\left(\frac{2f}{\hat{\lambda}}\hat{x}\right)^2 + \left(\frac{2f}{\hat{\lambda}}\hat{y}\right)^2}{4f} - f \quad (20)$$

where $\hat{q}$ denotes $q(\hat{\theta}) \in \mathbb{R}$, $\hat{\lambda}(t) \in \mathbb{R}$ is the estimate of $\lambda(t)$, and is defined as follows

$$\hat{\lambda} \triangleq -\hat{z} + \sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2}. \quad (21)$$

To further facilitate the development, we define a filter signal $q_f(t) \in \mathbb{R}$ as follows

$$\dot{q}_f \triangleq -\alpha q_f + q; \qquad q_f(0) \triangleq 0 \quad (22)$$

where $\alpha \in \mathbb{R}^+$. The estimate of (22) is written as follows

$$\dot{\hat{q}}_f = -\alpha\left(\hat{q}_f - \varepsilon \text{sat}(r)\right) + \hat{q} - a^* \text{sat}(r) \quad (23)$$

where $\hat{q}_f(t) \in \mathbb{R}$, and $\dot{\hat{q}}_f(t)$ are the estimates of $q_f(t)$, and $\dot{q}_f(t)$, respectively, $\varepsilon \in \mathbb{R}^+$ is the desired precision, $a^*(t)$ is the tuning function obtained from the subsequently presented min-max optimization problem, and $r(t) \in \mathbb{R}$ is defined as follows

$$r \triangleq \frac{\tilde{q}_f}{\varepsilon} \quad (24)$$

where the filter error $\tilde{q}_f(t) \in \mathbb{R}$ is defined as follows

$$\tilde{q}_f \triangleq \hat{q}_f - q_f. \quad (25)$$

Also, in (23), $\text{sat}(r)$ is a saturation function given as follows

$$\text{sat}(r) = \begin{cases} +1 & \text{if } r \geq 1 \\ r & \text{if } |r| < 1 \\ -1 & \text{if } r \leq -1. \end{cases} \quad (26)$$

To proceed with the development, we define a tuning error $\tilde{q}_{f\varepsilon}(t) \in \mathbb{R}$ as follows

$$\tilde{q}_{f\varepsilon} \triangleq \tilde{q}_f - \varepsilon \text{sat}(r). \quad (27)$$

After taking the time derivative of (25), the following expression can be written

$$\dot{\tilde{q}}_f = -\alpha \tilde{q}_{f\varepsilon} + \hat{q} - q - a^* \text{sat}(r) \quad (28)$$

where (22), (23), and (27) were utilized.

Remark 3: It should be noted that the inclusion of the tuning error $\tilde{q}_{f\varepsilon}(t)$ provides the following expressions

$$\tilde{q}_{f\varepsilon} = 0 \qquad \text{when } |\tilde{q}_f| \leq \varepsilon$$
$$\dot{\tilde{q}}_{f\varepsilon} = \dot{\tilde{q}}_f \qquad \text{when } |\tilde{q}_f| > \varepsilon.$$

This remark is utilized later in the stability analysis.

Based on the stability analysis an estimator $\dot{\hat{\theta}}(t) \in \mathbb{R}^3$ is designed with a projection strategy which facilitates the estimation of θ as follows

$$\dot{\hat{\theta}} = \text{Proj}\{-\tilde{q}_{f\varepsilon}\phi^*\}. \quad (29)$$

The projection strategy $\text{Proj}\{\cdot\}$ in (29) ensures that $\hat{\theta}(t)$ always belongs to the hypercube Θ. The strategy is as follows

$$\hat{\theta}_j = \begin{cases} \hat{\theta}_j & \text{if } \hat{\theta}_j \in [\theta_{j,\min}, \theta_{j,\max}] \\ \theta_{j,\min} & \text{if } \hat{\theta}_j < \theta_{j,\min} \\ \theta_{j,\max} & \text{if } \hat{\theta}_j > \theta_{j,\max} \end{cases} \quad (30)$$

where the subscript j denotes the j^{th} element of the corresponding vector $\forall j = 1, 2, 3,\ \theta_{j,\min}, \theta_{j,\max} \in \mathbb{R}$ are the minimum and maximum values of the j^{th} component of θ, respectively, and $\phi^*(t) \in \mathbb{R}^3$ is the sensitivity function.

Similar to [17], the solutions for $\phi^*(t)$ and $a^*(t)$ are obtained from a min-max optimization problem of the following form

$$a^* = \min_{\phi \in \mathbb{R}^3} \max_{\theta \in \Theta_s} J(\phi, \theta) \quad (31)$$

$$\phi^* = \arg\min_{\phi \in \mathbb{R}^3} \max_{\theta \in \Theta_s} J(\phi, \theta) \quad (32)$$

where the performance index $J(\cdot) \in \mathbb{R}$ is given by the following expression

$$J(\cdot) = \text{sat}(r)\left[\hat{q} - q - \tilde{\theta}^T\phi\right] \quad (33)$$

where $\tilde{\theta}(t) \in \mathbb{R}^3$ is the parameter estimation error defined as follows

$$\tilde{\theta} \triangleq \hat{\theta} - \theta. \quad (34)$$

The solutions of (31) and (32) are given as follows[2]

$a)$ when $\tilde{q}_f < 0$

$$a^* = \begin{cases} 0 & \text{if } q \text{ is concave on } \Theta_s \\ A_1 & \text{if } q \text{ is convex on } \Theta_s \end{cases} \quad (35)$$

$$\phi^* = \begin{cases} \nabla q(\hat{\theta}) & \text{if } q \text{ is concave on } \Theta_s \\ A_2 & \text{if } q \text{ is convex on } \Theta_s \end{cases} \quad (36)$$

$b)$ when $\tilde{q}_f \geq 0$

$$a^* = \begin{cases} A_1 & \text{if } q \text{ is concave on } \Theta_s \\ 0 & \text{if } q \text{ is convex on } \Theta_s \end{cases} \quad (37)$$

$$\phi^* = \begin{cases} A_2 & \text{if } q \text{ is concave on } \Theta_s \\ \nabla q(\hat{\theta}) & \text{if } q \text{ is convex on } \Theta_s \end{cases}. \quad (38)$$

In (35)-(38), $A(t) \in \mathbb{R}^4$ is given as follows

$$A = [A_1 \; A_2]^T = G^{-1} b \quad (39)$$

where $A_1(t) \in \mathbb{R}$, and $A_2(t) \in \mathbb{R}^3$, $G(t) \in \mathbb{R}^{4\times4}$ is given as follows

$$G = \begin{bmatrix} -1 & \beta(\hat{\theta} - \theta_{s1})^T \\ -1 & \beta(\hat{\theta} - \theta_{s2})^T \\ -1 & \beta(\hat{\theta} - \theta_{s3})^T \\ -1 & \beta(\hat{\theta} - \theta_{s4})^T \end{bmatrix} \quad (40)$$

and $b(t) \in \mathbb{R}^4$ is given as follows

$$b = \begin{bmatrix} \beta\,(\hat{q} - q_{s1}) \\ \beta\,(\hat{q} - q_{s2}) \\ \beta\,(\hat{q} - q_{s3}) \\ \beta\,(\hat{q} - q_{s4}) \end{bmatrix} \quad (41)$$

where $\beta(\Pi) \in \mathbb{R}$ is defined as follows

$$\beta = \begin{cases} 1 & \text{if } q \text{ is convex on } \Theta_s \\ -1 & \text{if } q \text{ is concave on } \Theta_s. \end{cases} \quad (42)$$

In (41), $g_{sh} \triangleq q(\theta_{sh}, \Pi)\, \forall h = 1, 2, 3, 4$. As mentioned earlier in Remark 2, θ_{sh} are the vertices of the simplex Θ_s. In (36) and (38), $\nabla q(\hat{\theta}) \in \mathbb{R}^3$ is the gradient function given as follows

$$\nabla q(\hat{\theta}) = (\partial q/\partial\theta)\,|_{\theta=\hat{\theta}}. \quad (43)$$

It is evident that the estimate of the constant 3D coordinates of the object feature relative to the world frame (*i.e.*, $\hat{\theta}(t)$) can be used to obtain the estimates of all its 3D coordinates relative to the vision system including the range (*i.e.*, $\hat{z}(t)$) from (17)-(19).

Remark 4: It should be noted that the inclusion of the tuning error $\tilde{q}_{f\varepsilon}(t)$ with the saturation function $\text{sat}(r)$ ensures that the estimator is continuous even if a discontinuous solution of the min-max algorithm is obtained (see [17] for more detailed description).

[2]The reader is referred to [17] for the proof of the solutions.

Remark 5: It should be noted that $\hat{\theta}(t)$ is bounded because of the projection strategy in (30); thus, $\phi^*(t)$ can be upper bounded as follows

$$\|\phi^*(t)\| \leq L_\phi \qquad \forall t \geq t_0 \quad (44)$$

where $L_\phi \in \mathbb{R}^+$.

B. Stability Analysis

Theorem 1: The adaptive update law given in (29) along with the solutions of $a^*(t)$ and $\phi^*(t)$ given in (35)-(38) ensures that $\tilde{q}_{f\varepsilon}(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$; hence, the stability of the estimator, and the global boundedness of the overall adaptive system are ensured.

Proof: See [26]. ■

Theorem 2: The developed estimation technique ensures that $\|\tilde{\theta}(t)\| \leq \sqrt{\gamma}$ as $t \to \infty$ provided the following NLPE condition holds

$$\beta(\Pi(t_2))\left(q(\hat{\theta}(t_1), \Pi(t_2)) - q(\theta, \Pi(t_2))\right) \geq \varepsilon_u \left\|\hat{\theta}(t_1) - \theta\right\| \quad (45)$$

where

$$\gamma = \frac{8\varepsilon c_1}{\varepsilon_u^2} \quad ; \qquad c_1 = 4L_1L_2 + 2L_2L_\phi + L_\phi^2, \quad (46)$$

$t_2 \in [t_1, t_1 + T_0]$, $t_1 > t_0$, and $T_0, \varepsilon_u \in \mathbb{R}^+$.

Proof: See [26]. ■

Remark 6: From the definition of γ in (46), it follows that γ can be made smaller by choosing smaller ε. As the desired precision $\varepsilon \to 0$, then $\gamma \to 0$; thus, the parameter estimation error $\|\tilde{\theta}(t)\| \to 0$.

V. Conclusion

A novel technique for range indentification and 3D Euclidean coordinates of a static object feature with a calibrated paracatadioptric system mounted on a moving platform with measurable position was presented. An adaptive estimator for nonlinearly parameterized function of projected pixel coordinates was presented which facilitated the range estimation along with the estimation of 3D Euclidean coordinates of an object feature. A Lyapunov-type stability analysis was presented to prove that the proposed estimator is stable, and ensures global boundedness of the error signals. Further, the parameter estimation error signals were shown to be bounded by a desired precision upon satisfaction of an NLPE condition. The developed estimator can be used for range identification for the applications with paracatadioptric systems where position measurements are readily avaliable. Future work will focus on extending the work for all catadioptric systems with practical implementation.

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