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Generalized modified gravity in large extra dimensions

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Abstract

We discuss effective interactions among brane matter induced by modifications of higher-dimensional Einstein gravity through the replacement of Einstein–Hilbert term with a generic function $f(\mathcal{R}, \mathcal{R}_{AB}\mathcal{R}^{AB}, \mathcal{R}_{ABCD}\mathcal{R}^{ABCD})$ of the curvature tensors. We determine gravi-particle spectrum of the theory, and perform a comparative analysis of its predictions with those of the Einstein gravity within Arkani-Hamed–Dvali–Dimopoulos (ADD) setup. We find that this general higher-curvature quantum gravity theory contributes to scatterings among both massive and massless brane matter (in contrast to much simpler generalization of the Einstein gravity, $f(\mathcal{R})$, which influences only the massive matter), and therefore, can be probed via various scattering processes at present and future colliders and directly confronted with the ADD expectations. In addition to collision processes which proceed with tree-level gravi-particle exchange, effective interactions among brane matter are found to exhibit a strong sensitivity to higher-curvature gravity via the gravi-particle loops. Furthermore, particle collisions with missing energy in their final states are found to be sensitive to additional gravi-particles not found in Einstein gravity. In general, road to a correct description of quantum gravity above Fermi energies depends crucially on if collider and other search methods end up with a negative or positive answer for the presence of higher-curvature gravitational interactions.

1. Introduction

The extra spatial dimensions (large [1], warped [2] or hyperbolic [3]) have proven direct relevance in solving the gauge hierarchy problem within quantum gravitational framework. In particular, large extra dimensions induce Newton's constant in four dimensions from TeV scale Einstein gravity thanks to large volume of the extra space. The basic setup of this scenario i.e., the ADD scenario [1] is based on Einstein gravity

$$S_{\text{ADD}} = \int d^D x \sqrt{-g}\left\{-\frac{1}{2}\bar{M}_D^{D-2}\mathcal{R}(g_{AB}) + \mathcal{L}_{\text{matter}}(g_{AB}, \psi)\right\} \tag{1}$$

in $D = 4 + \delta$ dimensions. Here ψ collectively denotes the matter fields, and $\bar{M}_D$ is the fundamental scale of higher-dimensional gravity. The D-dimensional metric can be expanded about the flat background $g_{AB} = \eta_{AB} + 2\bar{M}_D^{1-D/2}h_{AB}$ where $\eta_{AB} = \text{diag}(1, -1, -1, \ldots, -1)$ and h_{AB} are perturbations. This expansion is admissible as long as surface tension of the brane does not exceed $\bar{M}_D$. The Newton's law of attraction on the brane still holds since $\bar{M}_{\text{Pl}} = \sqrt{V_\delta}\bar{M}_D^{1+\delta/2}$ which equals $(2\pi R)^{1/2}\bar{M}_D^{1+\delta/2}$ when δ extra dimensions are compactified on a torus of radius R. Obviously, larger the R closer the $\bar{M}_D$ to the electroweak scale [1]. Upon compactification, the higher-dimensional graviton gives rise to a tower of graviton states on the brane, and they participate in various scattering processes whose collider signatures have been discussed in detail in seminal papers [4,5].

One notes, however, that given general covariance alone, there is no symmetry reason to guarantee that the action density in (1) is unique. Indeed, general covariance does not forbid the action density therein to be generalized to a generic function

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of curvature invariants. In fact, such modifications of Einstein gravity have already been proposed and utilized for purposes of improving the renormalizability of the theory [6,7] and for explaining recent acceleration of the universe [8]. Of course, once we depart from the minimal Einstein–Hilbert regime there is no rule whatsoever which can limit numbers and types of the invariants. Our approach here is to consider only those invariants which are of lowest mass dimension and are quadratic contractions of the curvature tensors: $\mathcal{R}_{AB}\mathcal{R}^{AB}$ and $\mathcal{R}_{ABCD}\mathcal{R}^{ABCD}$ in spite of fact that we do not have any symmetry reason for not considering the higher-derivative ones $\Box\mathcal{R}$, $\nabla_A\mathcal{R}\nabla^A\mathcal{R}$, $\nabla_C\mathcal{R}_{AB}\nabla^C\mathcal{R}^{AB}$, etc. Hence, we generalize Einstein–Hilbert term to a generic function $f(\mathcal{R}, \mathcal{R}_{AB}\mathcal{R}^{AB}, \mathcal{R}_{ABCD}\mathcal{R}^{ABCD})$ of the curvature invariants, and examine induced anomalous interactions among the brane matter.

The simplest generalization of (1) would be to consider a generic function $f(\mathcal{R})$ of the curvature scalar. This possibility has been analyzed in detail in the recent work [9], and it has been found that $f(\mathcal{R})$ gravity effects are particularly pronounced and become distinguishable from those of the Einstein gravity in high-energy processes involving massive brane matter, i.e., heavy fermions, weak bosons and the Higgs boson (for recent work on Lovelock gravity see [10]). The reason is that $f(\mathcal{R})$ theory is equivalent to Einstein gravity plus an independent scalar field theory, and it is the propagation of this additional scalar that causes observable differences from the ADD predictions [9].

Here it is worth emphasizing that considering $f(\mathcal{R}, \mathcal{R}_{AB}\mathcal{R}^{AB}, \mathcal{R}_{ABCD}\mathcal{R}^{ABCD})$ theory instead of $f(\mathcal{R})$ gravity is not a straightforward generalization. The reason is that the former is a four-derivative theory, and it is generically endowed with a $J = 2$ ghost [7]. The presence of such negative-norm states constitutes the main difference between the two, and goal of the present work is to examine their signatures in high-energy processes in a comparative fashion. Such ghosty states are obviously dangerous in four dimensions especially at large distances [8]; however, in a higher-dimensional setting, it is the experiment (at the LHC or ILC) which will eventually establish presence or absence of such states whereby providing a deeper understanding of yet-to-be found quantum theory of gravity.

In Section 2 we derive graviton propagator, identify the gravi-particles which couple to brane matter with special emphasis on virtual graviton exchange. In Section 3 we study a number of higher-dimensional operators which are sensitive to modified gravity effects, and list down their collider and other phenomenological signatures. In Section 4 we conclude.

2. Virtual gravi-particle exchange

The modification of the Einstein gravity we consider is parameterized by

$$S = \int d^D x \sqrt{-g}\left\{-\frac{1}{2}\bar{M}_D^{D-2} f(\mathcal{R}, P, Q) + \mathcal{L}_{\text{matter}}(g_{AB}, \psi)\right\}, \tag{2}$$

where couplings to matter fields ψ are identical to those in (1). Here P and Q, as in [8], stand, respectively, for the quadratic contractions of the Ricci and Riemann tensors:

$$P = \mathcal{R}_{AB}\mathcal{R}^{AB}, \qquad Q = \mathcal{R}_{ABCD}\mathcal{R}^{ABCD} \tag{3}$$

which contain four derivatives. The metric field obeys

$$\begin{aligned}&[\nabla_A\nabla_B - g_{AB}\Box - \mathcal{R}_{AB}]f_R + \left[2\nabla_A\nabla^C\mathcal{R}_{CB} - \Box\mathcal{R}_{AB} - g_{AB}\nabla^C\nabla^D\mathcal{R}_{CD} - 2\mathcal{R}_{CA}\mathcal{R}^C_B\right]f_P\\ &\quad + \left[4\nabla^C\nabla^D\mathcal{R}_{CBAD} - 2\mathcal{R}_{CDEA}\mathcal{R}^{CDE}_B\right]f_Q + \frac{1}{2}f g_{AB} = \frac{\mathcal{T}_{AB}}{\bar{M}_D^{D-2}},\end{aligned} \tag{4}$$

where $f_R \equiv \partial f/\partial R$, $f_P \equiv \partial f/\partial P$, $f_Q \equiv \partial f/\partial Q$, and

$$\mathcal{T}_{AB} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\text{matter}})}{\delta g^{AB}} = \delta^\delta(\vec{y})\delta^\mu_A\delta^\nu_B T_{\mu\nu}(z) \tag{5}$$

is the stress tensor of the brane matter where y_i and z_μ stand, respectively, for coordinates in extra space and on the brane. The second equality here reflects the fact that entire energy and momentum are localized on the brane. Clearly, energy–momentum flow has to be conserved $\nabla^A\mathcal{T}_{AB} = 0$, and this is guaranteed to happen provided that $\nabla^\mu T_{\mu\nu} = 0$.

Obviously, the equations of motion (4) reduce to Einstein equations when $f(\mathcal{R}, P, Q) = \mathcal{R}$. In general, for analyzing dynamics of small oscillations about a background geometry, $g_{AB} = g^0_{AB}$ with curvature scalar $\mathcal{R}_0$, $f(\mathcal{R}, P, Q)$ must be regular at $\mathcal{R} = \mathcal{R}_0$. In particular, as suggested by (4), $f(\mathcal{R}, P, Q)$ must be regular at the origin and $f(0, 0, 0)$ must vanish (i.e., bulk cosmological constant must vanish) for $f(\mathcal{R}, P, Q)$ to admit a flat background geometry.

For determining how higher curvature gravity (2) influences interactions among the brane matter, it is necessary to determine the propagating modes which couple to the matter stress tensor. This requires expansion of the action density about the flat background up to the desired order in h_{AB}. The zeroth order term obviously vanishes. The terms first order in h_{AB} vanish by the equations of motion (4). The quadratic part, on the other hand, turns out to be

$$S_h = \int d^D x\left[\frac{1}{2}h_{AB}(x)\mathcal{O}^{ABCD}(x)h_{CD}(x) - \frac{1}{\bar{M}_D^{(D-2)/2}}h_{AB}(x)\mathcal{T}(x)^{AB}\right] \tag{6}$$

such that propagator of $h_{AB}(x)$, defined via the relation

$$\mathcal{O}_{ABCD}(x)\mathcal{D}^{CDEF}(x,x') = \frac{1}{2}\delta^D(x-x')\big(\delta_A^E\delta_B^F + \delta_B^E\delta_A^F\big), \tag{7}$$

takes the form

$$\begin{aligned}-i\mathcal{D}^{ABCD}\big(p^2\big) = {} & d_1\big(p^2\big)\eta^{AB}\eta^{CD} + d_2\big(p^2\big)\big(\eta^{AC}\eta^{BD}+\eta^{AD}\eta^{BC}\big) + d_3\big(p^2\big)\big(p^A p^B\eta^{CD} + \eta^{AB}p^C p^D\big) \\ & + d_4\big(p^2\big)\big(\eta^{BC}p^A p^D + \eta^{AD}p^B p^C + \eta^{AC}p^B p^D + \eta^{BD}p^A p^C\big) + d_5\big(p^2\big)p^A p^B p^C p^D,\end{aligned} \tag{8}$$

where the form factors $d_{1,\ldots,5}(p^2)$ depend on the underlying theory of gravitation. In ADD setup, based on Einstein gravity, they are given by $d_1(p^2) = -1/(D-2)p^2$, $d_2(p^2) = 1/2p^2$, $d_4(p^2) = (\xi-1)/2p^4$, $d_3(p^2) = d_5(p^2) = 0$. In $f(\mathcal{R})$ gravity none of them vanishes and their explicit expressions can be found in [9]. In the framework of modified gravity discussed here, they obtain nontrivial structures, too. For graviton-mediated interactions among brane-localized matter with conserved energy–momentum, only $d_1(p^2)$ and $d_2(p^2)$ are relevant, and they are given by

$$\begin{aligned}d_1\big(p^2\big) = {} & -\frac{1}{(D-2)f_R(0)p^2} + \frac{1}{(D-1)f_R(0)(p^2-m_1^2)} - \frac{m_0^2}{f_R(0)(\xi(D-1)(m_0^2+m_1^2)-(D-2)m_0^2)(p^2-m_1^2)} \\ & + \frac{\xi^2(m_0^2+m_1^2)}{(D-2)f_R(0)(\xi(D-1)(m_0^2+m_1^2)-(D-2)m_0^2)}\left(\frac{a}{p^2-m_+^2} + \frac{1-a}{p^2-m_-^2}\right), \\ d_2\big(p^2\big) = {} & \frac{1}{2f_R(0)p^2} - \frac{1}{2f_R(0)(p^2-m_1^2)},\end{aligned} \tag{9}$$

where we introduced

$$\begin{aligned}m_\pm^2 = {} & \frac{(2\xi-1)m_1^2}{4\xi((D-1)m_1^2+m_0^2)}\Bigg\{(D-3)m_0^2-(D-1)m_1^2 \\ & \pm\sqrt{4(D-2)\left(1+\frac{1}{(2\xi-1)^2}\right)m_0^2\big(m_0^2+(D-1)m_1^2\big) + \big((D-3)m_0^2-(D-1)m_1^2\big)^2}\Bigg\}, \\ a = {} & \frac{1}{m_+^2-m_-^2}\left\{m_+^2 - \frac{m_1^2((-D+2\xi+1)m_0^2+(D-1)m_1^2(2\xi-1))}{2\xi(m_0^2+(D-1)m_1^2)}\right\}\end{aligned} \tag{10}$$

with the fundamental mass scales

$$m_0^2 = -\frac{4f_R(0)}{f_P(0)-8f_{RR}(0)}, \qquad m_1^2 = -\frac{4f_R(0)}{f_P(0)+4f_Q(0)} \tag{11}$$

parameterizing the nonminimal nature of the gravity theory considered. The remaining form factors $d_{3,4,5}(p^2)$ can be obtained from (7) straightforwardly. The parameter ξ appearing in the formulae above arises from the gauge fixing term given in [4,9]. The de Donder gauge, $\xi = 1$, is frequently employed in quantum gravity.

Having determined the propagator, it is timely to analyze gravi-particles in the system and their propagation characteristics. The propagating modes and their properties are determined by the pole structures of $d_1(p^2)$ and $d_2(p^2)$ (and by the remaining form factors $d_{3,4,5}(p^2)$ when the longitudinal polarizations are taken into account). The pole at $p^2 = 0$ corresponds to a massless $J = 2$ excitation in D dimensions as is clear from its polarization tensor $\frac{1}{2}(\eta^{AC}\eta^{BD} + \eta^{AD}\eta^{BC}) - \frac{1}{D-2}\eta^{AB}\eta^{CD}$. This is the massless graviton of the theory [7] and it reduces to that computed in the ADD setup [4,5] when $f_R(0) = 1$, $f_Q(0) = 0$, $f_{RR}(0) = 0$ and $f_P(0) = 0$.

The second terms of $d_1(p^2)$ and $d_2(p^2)$ combine to give a $J = 2$ excitation with mass-squared m_1^2 and projector $\frac{1}{2}(\eta^{AC}\eta^{BD} + \eta^{AD}\eta^{BC}) - \frac{1}{D-1}\eta^{AB}\eta^{CD}$. The most spectacular aspect of this propagator is that it has negative residue, i.e., the excitation under concern is a ghost (negative norm states in Hilbert space). This can be cured by no choice of the model parameters because the sole and obvious choice of negative $f_R(0)$ converts massless graviton into a ghost—an absolutely unwanted situation since then theory possesses no Einsteinian limit at all. The existence of this tensorial ghost is a characteristic property of higher-curvature gravity consisting of Ricci and Riemann tensors [7], and it actually plays a rather affirmative role in canceling the divergences in loop calculations in the same sense as the Pauli–Villars regulation in quantum field theory does.

Finally, as suggested by the second and third lines of $d_1(p^2)$, the particle spectrum consists of three scalar excitations with poles $p^2 = m_1^2$, $p^2 = m_-^2$, $p^2 = m_+^2$ and projector $\eta^{AB}\eta^{CD}$. Their properties, whether they are ghosts (negative residue) or tachyons (negative pole mass-squared), depend on the model parameters. Obviously, as $m_1^2 \to \pm\infty$, which can be achieved by taking a special $f(\mathcal{R}, P, Q)$ with $f_P(0) = -4f_Q(0)$ or $f_P(0) = 0 = f_Q(0)$, the tensor ghost, the scalar field with mass m_1 as well as one combination of scalars having masses $m_\pm^2$ decouple from the system. What is left behind then is a massless $J = 2$ state and a massive scalar—the spectrum of $f(\mathcal{R})$ gravity [9]. The gravitational theory achieved this way, albeit non-Einstein, is obviously devoid of any ghosty degree of freedom.

Having determined the gravi-particle spectrum of $f(\mathcal{R}, P, Q)$ gravity in D dimensions, we start analyzing the consequences of the compactness of the extra space. Indeed, by letting extra space be torus-shaped with radius R as in the ADD mechanism, the matter stress tensor obeys the Kaluza–Klein expansion

$$\mathcal{T}_{AB}(x) = \sum_{n_1=-\infty}^{+\infty} \cdots \sum_{n_\delta=-\infty}^{+\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{\sqrt{V_\delta}} e^{-i(k\cdot z - \frac{\vec{n}\cdot\vec{y}}{R})} \delta_A^\mu \delta_B^\nu T_{\mu\nu}(k), \tag{12}$$

where $(n_1, \ldots, n_\delta)$ is a δ-tuple of integers. Given this Fourier decomposition of the stress tensor, the amplitude for an on-brane system a to make a transition into another on-brane system b becomes

$$\mathcal{A}(k^2) = \frac{1}{\bar{M}_{\rm Pl}^2} \sum_{\vec{n}} T_{\mu\nu}^{(a)}(k) \mathcal{D}^{\mu\nu\lambda\rho}\left(k^2 - \frac{\vec{n}\cdot\vec{n}}{R^2}\right) T_{\lambda\rho}^{(b)}(k), \tag{13}$$

where summation over Kaluza–Klein levels arises from propagation of gravi-particles off the brane. Conservation of energy and momentum implies that only the first two terms in the propagator (8) contributes to (13), and after performing summation the transition amplitude takes the form

$$\begin{aligned}
\mathcal{A}(k^2) &= \frac{S_{\delta-1}}{(2\pi)^\delta} \frac{1}{\bar{M}_D^4 f_R(0)} \left(\frac{\Lambda}{\bar{M}_D}\right)^{\delta-2} \\
&\times \left\{ \mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2}}\right) \left(T_{\mu\nu}^{(a)} T^{(b)\mu\nu} - \frac{1}{\delta+2} T_\mu^{(a)\mu} T_\nu^{(b)\nu}\right) \right. \\
&- \mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2 - m_1^2}}\right) \left(T_{\mu\nu}^{(a)} T^{(b)\mu\nu} - \frac{1}{\delta+3} T_\mu^{(a)\mu} T_\nu^{(b)\nu}\right) \\
&+ \frac{\xi^2 (m_0^2 + m_1^2)}{(\delta+2)(\xi(\delta+3)(m_0^2+m_1^2) - (\delta+2)m_0^2)} \left[(1-a)\mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2 - m_-^2}}\right) \right. \\
&\left.\left. + a\mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2 - m_+^2}}\right) - \frac{(\delta+2)m_0^2}{\xi^2(m_0^2+m_1^2)} \mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2 - m_1^2}}\right)\right] T_\mu^{(a)\mu} T_\nu^{(b)\nu} \right\}
\end{aligned} \tag{14}$$

which exhibits a huge overall enhancement $\mathcal{O}(\bar{M}_{\rm Pl}^2/\bar{M}_D^2)$ compared to (13) due to the contributions of finely-spaced Kaluza–Klein levels [1]. Here $S_{\delta-1} = (2\pi^{\delta/2})/\Gamma(\delta/2)$ is the surface area of δ-dimensional unit sphere and Λ (which is expected to be $\mathcal{O}(\bar{M}_D)$ since above $\bar{M}_D$ underlying quantum theory of gravity completes the classical treatment pursued here) is the ultraviolet cutoff needed to tame divergent summation over Kaluza–Klein levels. In fact, $\mathcal{A}(q^2)$ exhibits a strong dependence on Λ through $\mathcal{G}(\Lambda/\sqrt{q^2})$ whose explicit expression given in [4,5,9].

The first line of $\mathcal{A}(k^2)$ in (14), except for the overall $1/f_R(0)$ factor in front, is identical to virtual graviton exchange amplitude computed within the ADD setup. The stress tensors of the on-brane systems a and b contribute to the transition amplitude via their contractions $T_{\mu\nu}^{(a)} T^{(b)\mu\nu}$ and via their traces $T_\mu^{(a)\mu} T_\nu^{(b)\nu}$. While the former is effective for any two systems of particles [4,5], the latter can exist only for systems possessing conformal breaking [9].

The second line of (14), induced by the exchange of a massive graviton, is completely new in that it exists neither in ADD [4,5] nor in $f(\mathcal{R})$ gravity setups. The presence of the operator $T_{\mu\nu}^{(a)} T^{(b)\mu\nu}$ in this novel contribution proves particularly useful for distinguishing this general modification of gravity from $f(\mathcal{R})$ theory since the latter does not induce this particular operator structure.

The third and fourth lines of (14) are generated by exchanges of the three fundamental scalars in the system. Their contributions always involve traces of the stress tensors and thus for them to significantly influence a scattering process conformal invariance must be broken strongly (masses of the brane-localized fields must be a significant fraction of $\bar{M}_D$), as has been analyzed in detail elsewhere [9].

Generically, interactions of brane matter are contained in dimension-8 operators $T_{\mu\nu}^{(a)} T^{(b)\mu\nu}$ and $T_\mu^{(a)\mu} T_\nu^{(b)\nu}$ whose coefficients are independent of what particles are scattered on the brane but highly-sensitive to what gravitational theory is employed in the higher-dimensional bulk. Therefore, a comparative quantitative analysis of the higher-curvature gravity theory under concern and ADD setup can shed light on collider and other signatures of nonminimal curvature terms in the bulk. The $f(\mathcal{R}, P, Q)$ gravity

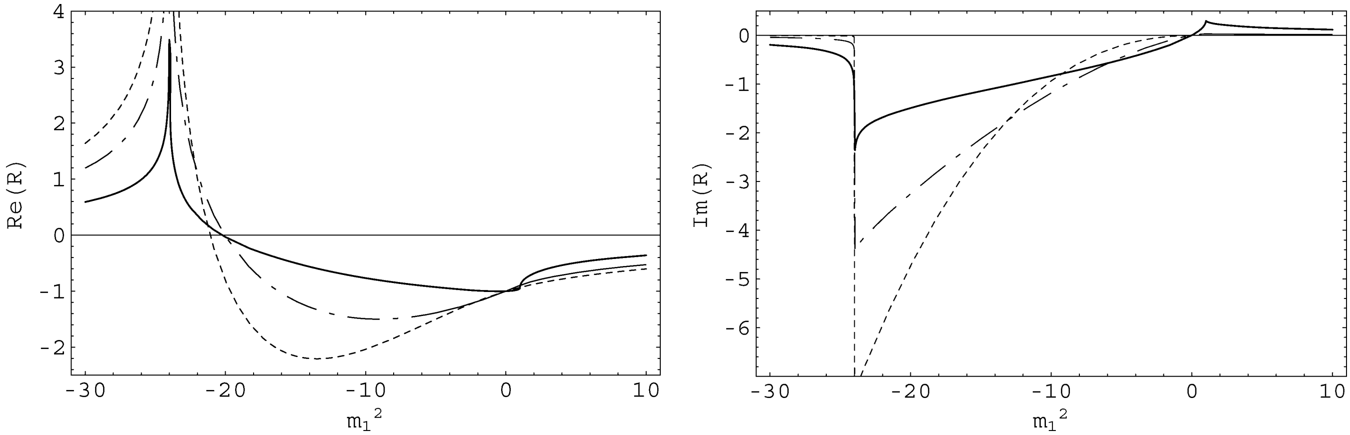

Fig. 1. The dependencies of $\mathrm{Re}[R(k^2)]$ (left panel) and $\mathrm{Im}[R(k^2)]$ (right panel) on m_1^2 for $k^2 = (1\ \mathrm{TeV})^2$, $\Lambda = \bar{M}_D = 5$ TeV, and $\delta = 3$ (solid curve), $\delta = 5$ (dot-dashed curve) and $\delta = 7$ (short-dashed curve).

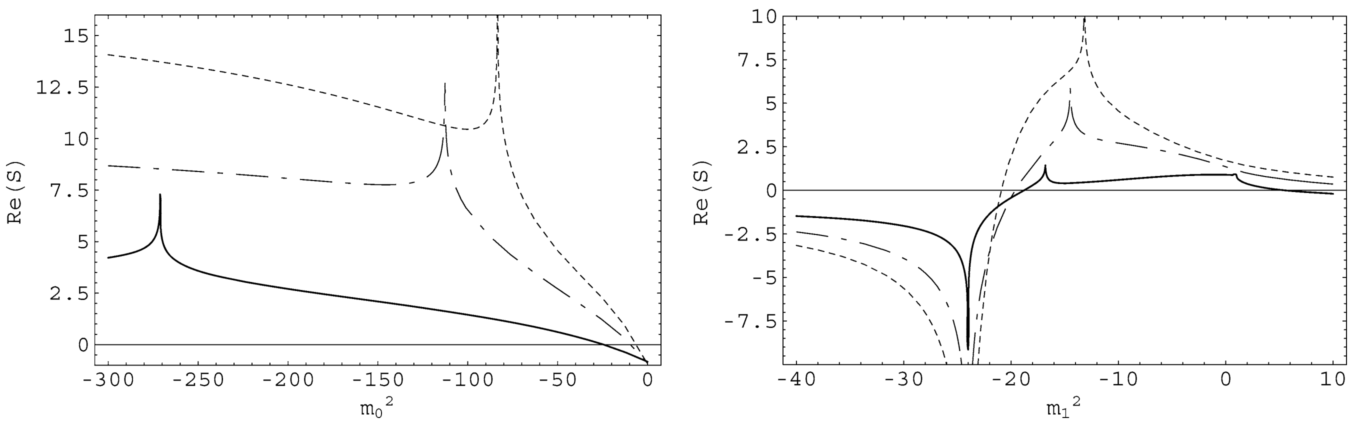

Fig. 2. The dependence of $\mathrm{Re}[S(k^2)]$ on m_0^2 for $m_1^2 = -10\ \mathrm{TeV}^2$ (left panel) and on m_1^2 for $m_0^2 = 5m_1^2$ (right panel). The parameters and labels are as in Fig. 1.

effects on $T^{(a)}_{\mu\nu} T^{(b)\mu\nu}$, can be quantified by analyzing the ratio

$$R(k^2) = -\frac{\mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2 - m_1^2}}\right)}{\mathcal{G}\left(\frac{\Lambda}{\sqrt{k^2}}\right)} \tag{15}$$

whose real and imaginary parts are plotted in Fig. 1 as functions of m_1^2 by taking, in accord with the future collider searches, $k^2 = (1\ \mathrm{TeV})^2$ and $\Lambda = \bar{M}_D = 5$ TeV for each number of extra dimensions considered: $\delta = 3$ (solid), $\delta = 5$ (dot-dashed) and $\delta = 7$ (short-dashed). These figures make it clear that massive graviton (a ghosty tensor mode special to $f(\mathcal{R}, P, Q)$ gravity) exchange significantly dominates, if not competes, the massless graviton (the only propagating mode in the ADD setup) exchange when the former is a tachyon with mass-squared $\sim -0.5\Lambda^2$ (excluding the rather narrow peak at $m_1^2 = -24\ \mathrm{TeV}^2$ which corresponds to resonating of the transition amplitude by Kaluza–Klein levels with mass-squared $= k^2 - m_1^2 = \Lambda^2$). The ghosty nature of the massive graviton affects only the sign of (15) whereas its tachyonic nature gives rise to a spectacular enhancement in $R(k^2)$ which in turn enables one to disentangle $f(\mathcal{R}, P, Q)$ gravity effects from those of the Einstein gravity in high-energy collider environment. From (11) it is clear that a negative m_1^2 implies a positive $f_P(0) + 4 f_Q(0)$.

The question of how $f(\mathcal{R}, P, Q)$ gravity influences the coefficient of $T^{(a)\mu}_{\mu} T^{(b)\nu}_{\nu}$ can be answered by analyzing the ratio

$$S(k^2) = \frac{\text{Coefficient of } T^{(a)\mu}_{\mu} T^{(b)\nu}_{\nu} \text{ from 2nd, 3rd and 4th lines of (14)}}{\text{Coefficient of } T^{(a)\mu}_{\mu} T^{(b)\nu}_{\nu} \text{ from 1st line of (14)}} \tag{16}$$

in a way similar to (15). This quantity does not have a direct meaning in interpreting the scattering rates of massive brane matter as they receive contributions from $T^{(a)}_{\mu\nu} T^{(b)\mu\nu}$, too. Nevertheless, it serves a useful tool to determine relative strengths of $f(\mathcal{R}, P, Q)$, $f(\mathcal{R})$ and Einstein gravity predictions. Depicted in Fig. 2 is the variation of $\mathrm{Re}[S(k^2)]$ with m_0^2 for $m_1^2 = -10\ \mathrm{TeV}^2$ (the left panel) and variation of the same quantity with m_1^2 for $m_0^2 = 5m_1^2$ (right panel). The rest of the parameters and labeling are as in Fig. 1. The left panel makes it clear that $f(\mathcal{R}, P, Q)$ gravity contributions (mediated by all three gravi-particles) completely dominate the ones found $f(\mathcal{R})$ and ADD setups for large negative m_0^2 where $\mathrm{Re}[S(k^2)]$ exhibits a plateau rather than getting diminished. The reason is that m_1^2 is fixed to $-10\ \mathrm{TeV}^2$ and associated gravi-scalars do not decouple from the spectrum. The right panel of Fig. 2 shows that $f(\mathcal{R}, P, Q)$ gravity contribution is particularly enhanced in negative m_1^2 domain especially when $m_1^2 \sim -10\ \mathrm{TeV}^2$. Clearly, $\mathrm{Re}[S(k^2)]$ is diminished at large $|m_1^2|$ since this time all gravi-scalars collectively decouple from the spectrum.

3. Effects of gravi-particle exchange on brane processes

In this section we group and analyze effects of higher-dimensional operators according to the virtualities of the gravi-particles involved.

Tree-level effects of virtual gravi-particles: The tree-level gravi-particle exchange, as has been detailed in the last section, gives rise to anomalous interactions among brane matter species [4,5,9]. The on-brane processes are entirely tree-level in such processes; however, they are sensitive to gravi-particle exchange due to rather high virtualities that gravi-particles obtain via their propagation through the extra dimensions. In general, tree-level gravi-particle exchange induces various modifications in scattering processes, and they may be detected at colliders or in other experiments [11]. At an e^+e^- collider, for instance, pair-productions of gauge bosons (e.g., $e^+e^- \to VV$ where $V = \gamma, Z, W$) and of fermions (e.g., $e^+e^- \to t\bar{t}$ or any other quark or lepton) form grounds for searching gravi-particle effects. In fact, existing results from LEP experiments already provide precise bounds from pair-productions of gauge bosons and fermions [12]. There also exist promising scattering processes at hadron (pp collisions at the LHC and $p\bar{p}$ collisions at Tevatron [13]) and lepton–hadron (ep collisions at HERA [14]) colliders.

For illustrating impact of $f(\mathcal{R}, P, Q)$ gravity on collider processes we here prefer to study $2 \to 2$ fermion scatterings. The scattering amplitude $\mathcal{A}(\psi_a(k_1)\bar{\psi}_a(k_2) \to \psi_b(q_1)\bar{\psi}_b(q_2))$ for two identical fermions can be directly obtained via the replacements

$$T^{(a)}_{\mu\nu}T^{(b)\mu\nu} \to \frac{1}{8}\big[(k_1+k_2)\cdot(q_1+q_2)\bar{\psi}_a(k_2)\gamma^{\mu}\psi_a(k_1)\bar{\psi}_b(q_2)\gamma_{\mu}\psi_b(q_1) + \bar{\psi}_a(k_2)(\not{q}_1+\not{q}_2)\psi_a(k_1)\bar{\psi}_b(q_2)(\not{k}_1+\not{k}_2)\psi_b(q_1)\big],$$
$$T^{(a)\mu}_{\mu}T^{(b)\nu}_{\nu} \to m_{\psi_a}m_{\psi_b}\bar{\psi}_a(k_2)\psi_a(k_1)\bar{\psi}_b(q_2)\psi_b(q_1) \tag{17}$$

in gravi-particle exchange amplitude (14). These replacements correspond to s-channel gravi-particle exchange with $k = k_1 - k_2 = q_2 - q_1$, and depending on the quantum numbers of ψ_a and ψ_b it could be necessary to include t and u channel contributions, too. The amplitude can compete in size with the ADD expectation for certain parameter values, especially when the scattering energy $\sqrt{k^2}$ becomes comparable with the new gravitational scales m_0 or m_1 [9]. A highly interesting aspect of (14) with the replacements (17) is that effects of the modified gravity survive even in the limit of massless fermions. This is, as one recalls form [9], not the case for $f(\mathcal{R})$ gravity to which only scatterings of the massive brane matter exhibit sensitivity. This property is important as its effects can be directly probed at high-energy colliders where colliding beams of matter are essentially massless. Indeed, $e^+e^- \to \bar{f}f, \gamma\gamma$, $q\bar{q} \to \bar{f}f, \gamma\gamma$ and especially $gg \to \gamma\gamma$ are golden modes to test if ADD expectation is modified by higher-curvature effects. In general, independent of if the brane matter is massive or massless, there are certain parameter values for which $f(\mathcal{R}, P, Q)$ gravity contributions get significantly enhanced and thus become more easily observable with respect to Einstein gravity as can be observed in Figs. 1 and 2.

The fermion scattering example above can be generalized to any other SM particle where relative enhancements/suppressions in their rates are always governed by (14) with supplementary illustrations given in the figures. A dedicated search for extra dimensions via virtual gravi-particle exchange processes requires a global analysis of various collider processes [12–14]. The most advantageous aspect of $f(\mathcal{R}, P, Q)$ gravity is the separability of massive ghosty graviton contribution from those of the remaining gravi-particles via the measurement of the scattering rates of massless particles.

Loop-level effects of virtual gravi-particles: Loop-level processes on the brane involve both brane matter and gravi-particles where the latter are now virtual both in ordinary and extra dimensions. These effects can give rise to corrections to the existing SM amplitudes as in, for instance, electroweak precision observables (particle self energies, interaction vertices, box diagrams) and rare decays [4,5,9]. In fact, the dedicated analysis of [15] shows that gravi-particle loop effects can become more important than their tree-level effects since they can induce potentially important dimension-6 operators with double gravi-particle exchange. This lower-dimension operator can arise in fermion, gauge boson as well as Higgs sectors of the SM.

For illustrating the impact of $f(\mathcal{R}, P, Q)$ gravity, consider dimension-6 four-fermion operator $(1/2)\bar{f}\gamma_{\mu}\gamma_5 f\bar{f}'\gamma^{\mu}\gamma_5 f'$ where f, f' standing for light quarks or leptons. Coefficient of this operator turns out to be quite sensitive to higher-curvature gravity. More precisely, for massless fermions, $f_R(0) = 1$, $m_1^2 = -14\ \mathrm{TeV}^2$, $\Lambda = 5$ TeV and $\delta = 3, 5, 7$ it is, respectively, 1.9, 2.6, and 2.7 times larger than the ADD prediction [15]. That this should be the case follows from Fig. 1. Clearly, existing experimental results on contact interactions, dijet and dilepton production processes as well as lepton–hadron scattering rates can put stringent limits from such dimension-6 operators.

In addition to their virtual effects just mentioned, the gravi-particles can decay into brane-matter or can be produced by scatterings among the brane-matter [4,5,9]. While the former plays a crucial role in cosmological and astrophysical contexts, the latter constitutes one of the most important signatures of extra dimensions at colliders: single missing energy signal [11]. However, as suggested by the figures, one hardly expects an observable difference from ADD expectations in such processes since significant enhancements occur mainly in tachyonic domains. The situation is similar to the one arrived for $f(\mathcal{R})$ gravity [9].

4. Conclusion

In this work we have discussed phenomenological implications of $f(\mathcal{R}, P, Q)$ gravity in spacetimes with large extra spatial dimensions. In Section 2 we have computed the propagator around flat background. Following the determinations of propagating degrees of freedom and virtual gravi-particle exchange amplitude, we have provided a comparative analysis of the contributions of $f(\mathcal{R}, P, Q)$ and Einstein gravity.

In Section 3 we have analyzed effects of virtual and real gravi-particles on the scatterings among the brane particles. Therein we have shown that there exist a number of collider and other processes can give important information on the nature of the gravitational theory in the higher-dimensional bulk.

The higher-curvature gravity theory discussed in this work possess various properties which help distinguishing it from the Einstein and $f(\mathcal{R}, P, Q)$ gravity theories, and it is after a global survey of laboratory, astrophysical and cosmological observables as in [1] that one can make sure of size and type of such modifications of Einstein gravity.

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