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The implementation of nonlinear dynamical systems with wavelet network

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Abstract

A dynamic wavelet network circuit implementation for modelling the nonlinear dynamical networks has been proposed in this study. The dynamical wavelet network includes static wavelet network with Mexican hat wavelet function, the voltage-controlled switches and capacitors. The circuit simulations have been done in Spice for the period-1 limit cycle, the spiral and double scroll attractors of the Chua's circuit.

Keywords: Wavelets; Circuit realization; Function approximation

1. Introduction

The time–frequency domain methods have attracted many researchers from different branches of science in order to analyze the behavior of the nonlinear dynamical systems. The wavelets [1–3], the Wigner–Ville distribution [4–6] have been used to analyze the chaotic systems. Also, there are studies on the modelling of the nonlinear dynamical systems with artificial neural networks. The wavelet network which uses both the learning and generalization ability of feedforward neural networks and time–frequency domain localization property of wavelet decomposition has been proposed in [7]. The wavelet network has been used in the identification of static and dynamical systems [8–10]. The circuit implementation of a wavelet network for static systems has been done in [11] using sigmoidal wavelet function proposed in [12]. Also, the wavelet network circuit for identification of the static networks using Mexican hat wavelet has been given in [13]. In this study, a circuit implementation procedure for modelling dynamical systems using the wavelet network with Mexican Hat mother wavelet in [13–15] has been proposed. After introducing the brief descriptions of the wavelet analysis and the wavelet network in Section 2, the dynamical wavelet network modelling has been explained in Section 3. The circuit structure for the dynamical wavelet network has been proposed in Section 4. Finally, the examples of dynamical system modelling have been given in Section 5.

2. Wavelet analysis and wavelet network

The wavelets are the family of the signals that is produced by the translations and the dilations of a mother wavelet satisfying the admissibility condition. The properties of the wavelet transform (WT) and the fundamentals of the wavelet networks will be given in the sequel.

2.1. Wavelet analysis

The continuous wavelet transform coefficients of a signal $s(t) \in L^2(\mathbb{R})$ are determined for the different scales and

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translations as follows:

$$W_s(a,b;\Psi)\triangleq\int_{-\infty}^{\infty} s(t)\Psi^*_{a,b}(t)\,\mathrm{d}t, \tag{1}$$

where a and b are the dilation (scale) and translation coefficients, respectively, and $*$ denotes the complex conjugate; the scaled and translated wavelet is obtained as

$$\Psi_{a,b}(t)\triangleq\frac{1}{\sqrt{a}}\Psi\left(\frac{t-b}{a}\right), \quad a\in\mathbb{R}^+, \quad b\in\mathbb{R}, \tag{2}$$

where $\Psi(\cdot)$ is the mother wavelet. The mother wavelet must satisfy the admissibility condition as

$$c_\psi=\int_0^{\infty}|\hat{\Psi}(\omega)|^2\frac{\mathrm{d}\omega}{|\omega|}<\infty, \tag{3}$$

where $\hat{\Psi}(\omega)$ is the Fourier transform of the mother wavelet [16].

For the numerical computations, the discrete samples of the continuous wavelet transform have been considered and the scaled and translated wavelets have been defined at the dyadic grid as

$$\Psi_{mn}(t)=a_0^{-m/2}\,\Psi(a_0^{-m}t-nb_0), \quad m,n\in\mathbb{Z}, \tag{4}$$

where $a_0, b_0\in\mathbb{R}$ are the dilation and translation coefficients, respectively, and WT coefficients are defined as

$$c_{mn}\triangleq W_s(a_m,b_n;\Psi), \tag{5}$$

where $a_m\triangleq a_0^{-m}$, $b_n\triangleq nb_0$.

If the wavelets defined in Eq. (4) are chosen such as to constitute a Riesz basis for every $s(t)\in L^2(\mathbb{R})$

$$A\|s\|_2^2\leqslant\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}|c_{mn}|^2\leqslant B\|s\|_2^2, \tag{6}$$

where $0<A\leqslant B<\infty$ are called as lower and upper frame bounds, respectively, then the signal can be decomposed using the wavelet basis as

$$s(t)=\sum_{-\infty}^{\infty}c_{mn}\Psi_{mn}(t). \tag{7}$$

2.2. Wavelet network

A network called wavelet network combining both feed-forward neural networks and wavelet decomposition has been presented in [7].

When the input–output pairs measured from the system to be modelled is given as

$$\{x(t_k),\,y(t_k)\,|\,y(t_k)=f(x(t_k))+\varepsilon_k, \quad k=1,2,\ldots,K, \quad f(\cdot):\mathbb{R}^d\to\mathbb{R}\}, \tag{8}$$

where ε_k is a random variable representing the measurement noise, then the problem is to minimize the mean square error between the actual output and the output of the wavelet network

$$\mathrm{MSE}\triangleq\tfrac{1}{2}E\left\{(y-f_w(x))^2\right\}, \tag{9}$$

where E represents the expected value and the output of the wavelet network $f_w(\cdot):\mathbb{R}^d\to\mathbb{R}$ is defined as

$$f_w(x)=\sum_{i=1}^{N}w_i\Psi_d(D_i(x-b_i))+c^{\mathrm{T}}x+\overline{b}, \tag{10}$$

where N is the number of *d-dimensional* wavelons, w_i is the wavelet coefficient for each *d-dimensional* wavelon, $D_i=\mathrm{diag}(d_{11}^1,\ldots,d_{dd}^1)\in\mathbb{R}^{d\times d}$ is the diagonal dilation matrix whose diagonal elements are $d_{jj}^i=1/a_{ij}$ where a_{ij} is the dilation coefficient, $\Psi_d(\cdot):\mathbb{R}^d\to\mathbb{R}$ is the mother wavelet function, and $b_i\in\mathbb{R}^d$ is the translation coefficient vector, $c\in\mathbb{R}^d$ represents the coefficient of the linear term and $\overline{b}$ is the bias term to approximate the functions with nonzero mean. The block diagram of the static wavelet network is shown in Fig. 1. In the wavelet network circuit, the dilation of the wavelons are modelled by adding DC voltages to the input and scaling has been implemented by op-amp amplifiers by adjusting the gain of the amplifier according to the scaling parameter. Similarly, the linear term has been obtained by op-amp amplifiers and the bias term is obtained by adding DC voltage. The adding operation has also been implemented by op-amp adders. The optimum parameter set and the number of wavelons are to be determined for the construction of the wavelet network. The selection of suitable wavelons is implemented by the "Stepwise Selection by Orthogonalization" algorithm proposed in [9].

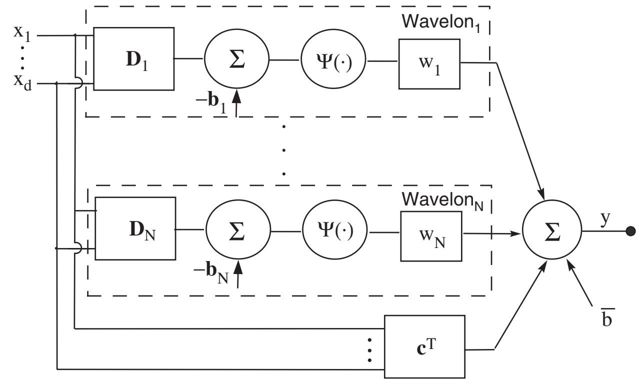

Fig. 1. The block diagram of the static wavelet network.

3. Dynamical system modelling with wavelet network

The wavelet networks have also been used in the identification of the dynamical systems or in the prediction

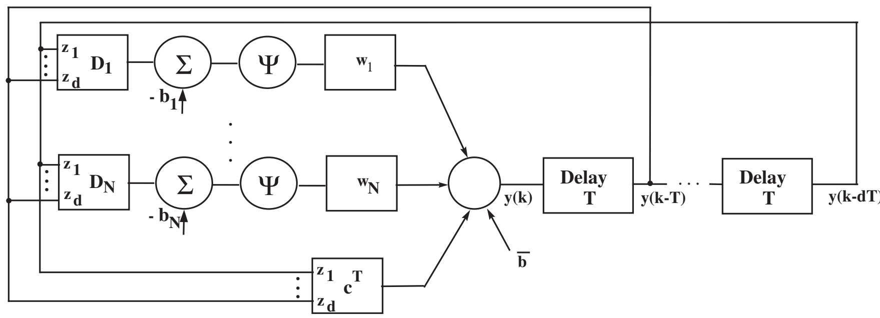

Fig. 2. The block diagram of the dynamical wavelet network.

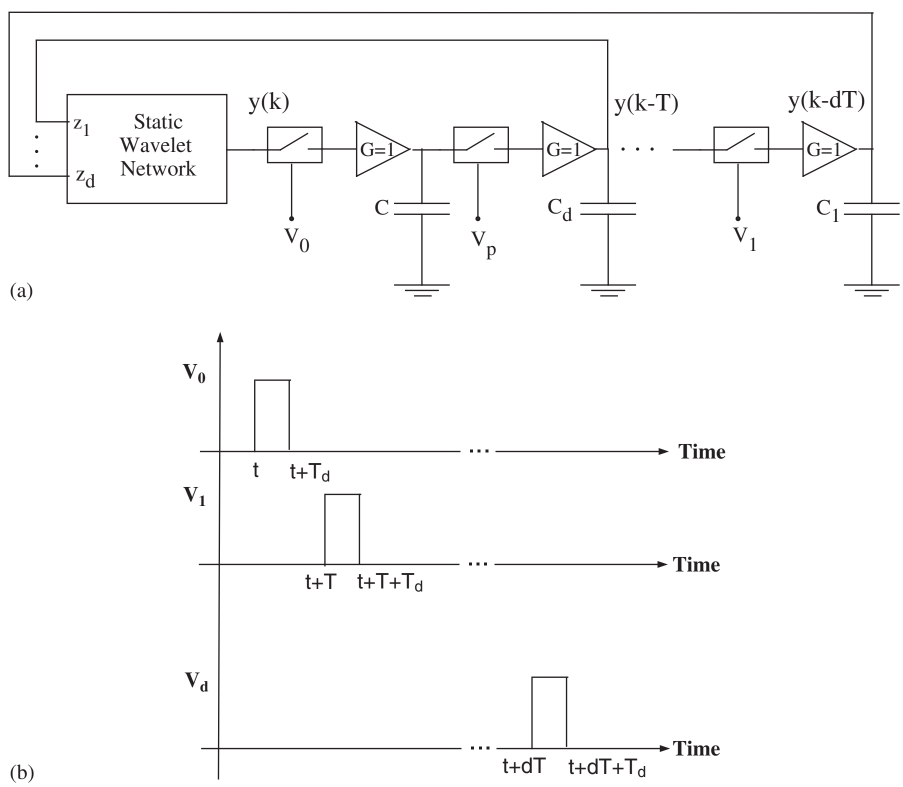

Fig. 3. (a) Dynamical wavelet network circuit and (b) the triggering pulses for voltage-controlled switches.

of the future outputs of the systems. In order to build a wavelet network model, the model parameters are fit to the given time series. Assume that a random process generating $y(k)$, $x(k) \in \mathbb{R}^d \rightarrow \mathbb{R}$, $k = 1, 2, \ldots, K$ and the relationship between them is as following:

$$y(k) = F(x(k)) + \varepsilon_k, \quad F(\cdot) : \mathbb{R}^d \rightarrow \mathbb{R}, \tag{11}$$

where ε_k is a random variable. According to the Taken's time-delay embedding theorem [17], the multidimensional dynamical structure of the system can be retrieved from single scalar variable observed from the system. Let a vector $z \in \mathbb{R}^d$ is constructed from the observations as

$$z(k) = [y(k-T)\ y(k-2T) \ldots y(k-dT)], \tag{12}$$

where d is the embedding dimension and T is the embedding delay. The

$$y(k) = \tilde{F}(z(k)) \tag{13}$$

follows the dynamical evolution of the original system. Therefore, the next state of the system is predicted from the previous observations.

The system evolution is approximated by some arbitrary set of basis functions for the modelling or identification of the nonlinear dynamical systems [8,18–20]. The purpose is to represent Eq. (13) with the suitable wavelet network. Since the observations can be expressed as a function of past measurements, the past values are used as inputs and the present values are used as output for the wavelet network

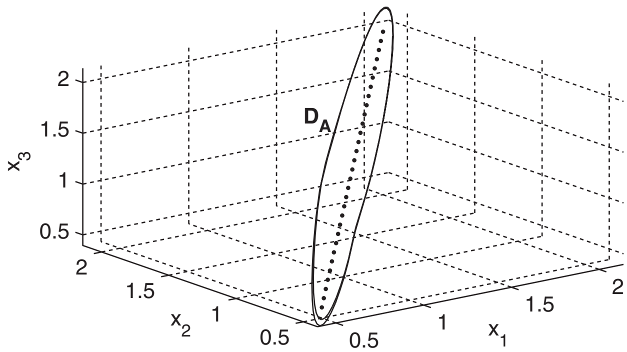

Fig. 4. The basin of attraction of period-1 limit cycle of the Chua's circuit.

to approximate the function $F(\cdot)$. The output of the wavelet network F_w is

$$y(k) = F_w(z(k)) = \sum_{i=1}^{N} w_i \Psi_d(D_i z(k) - b_i) + c^{\mathrm{T}} x + \overline{f}. \tag{14}$$

The block diagram of the dynamical wavelet network is shown in Fig. 2.

4. The circuit implementation of dynamical wavelet network

The wavelet network can be used for modelling the nonlinear dynamical systems as explained in Section 3. The circuit of the dynamical wavelet network shown in Fig. 2 has been implemented using the static wavelet network with Mexican hat mother wavelet as a main block and also using the delay blocks which have been realized by switched capacitors as shown in Fig. 3a.

The capacitors are used as the memory elements which store the delayed versions of the output of the wavelet network. The voltage-controlled switches are controlled by the external pulse generators which are triggered sequentially as shown in Fig. 3b where the duration of the pulses satisfies $T_d < T$. The inputs of static wavelet network consists of the voltages on the capacitors.

5. Applications

The Chua's circuit with periodic limit cycle, the spiral and double scroll attractors have been chosen as the dynamical model examples. The dynamics of the Chua's circuit [21] is described by the following differential set of equations

$$\begin{aligned} \dot{x} &= \alpha(y - h(x)), \\ \dot{y} &= x - y + z, \\ \dot{z} &= -\beta y, \end{aligned} \tag{15}$$

where α and β are the parameters defined by the circuit components and the piecewise linear characteristic is as

$$h(x) = (m_1 - m_0)(|x+1| - |x-1|), \tag{16}$$

where $m_1 = -\frac{1}{7}$, $m_0 = -\frac{2}{7}$.

Example 1 (Period 1 limit cycle of Chua's circuit). The time series of the period 1 limit cycle has been obtained by numerically integrating the system equation of the Chua's circuit for $N=500$ samples with sampling period $\tau_s = 0.01$ s and by selecting the parameters $\alpha = 8$ and $\beta = 14.2857143$. The time-series has been embedded with embedding dimension $d_e = 3$ and embedding delay $T = \tau_s$. The wavelet network contains three 3-*dimensional* wavelons. The observed limit-cycle is stable almost everywhere except on the set with zero measure $B = D - D_A$ where $D = \{x = [x_1, x_2, x_3] | -5 \leqslant x_i \leqslant 5 \quad i = 1, 2, 3\}$ and x_1, x_2, x_3 are the time-delay embedded coordinates, $D_A = \{x \in D_{in} | x_1 = x_2 = x_3\}$ and D_{in} represents the set inside the attractor. The basin of attraction of the wavelet network has been investigated by comparing the Hausdorff distances [22] of the attractors to the original attractor. An illustration of the basin of the attraction of the wavelet network is shown in Fig. 4. The parameters of the wavelet network have been given in Table 1.

The wavelet network trained for the Period 1 Limit Cycle of Chua's circuit has been implemented in Spice by using the ideal circuit components with the given structure in Fig. 3. The output of the circuit has been filtered by a low-pass filter in order to eliminate the high-frequency switching effects. The filtered output is shown in Fig. 5. The limit cycle from the circuit also have same basin of attraction characteristics with the wavelet network simulated in MATLAB.

Table 1. The wavelet network parameters of Example 1

i	D_i	b_i	w_i
1	diag(2.0585, 6.3006, 5.4508)	$[0.5305\ 1.0902\ 1.0910]^{\mathrm{T}}$	−0.0132
2	diag(3.4354, 2.3726, 5.3374)	$[0.8998\ 0.8716\ 1.1062]^{\mathrm{T}}$	0.0106
3	diag(5.9255, 10.5644, 5.1070)	$[0.7516\ 0.8955\ 0.8895]^{\mathrm{T}}$	−0.0057

$c = [0.9993 - 2.9832\ 2.9835]^{\mathrm{T}} \quad \overline{b} = 0.0012$

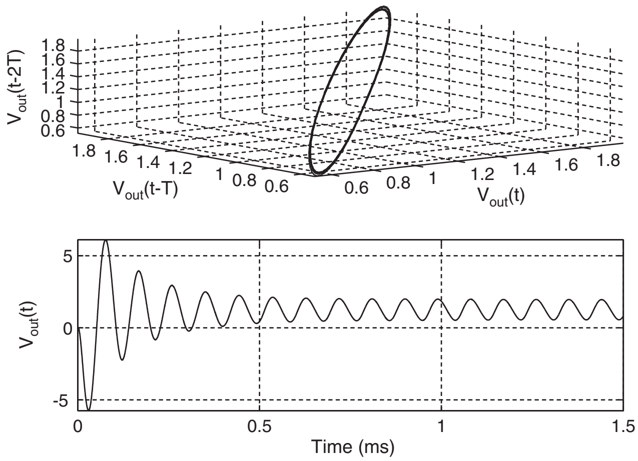

Fig. 5. The output of dynamical wavelet network circuit for Period-1 limit cycle.

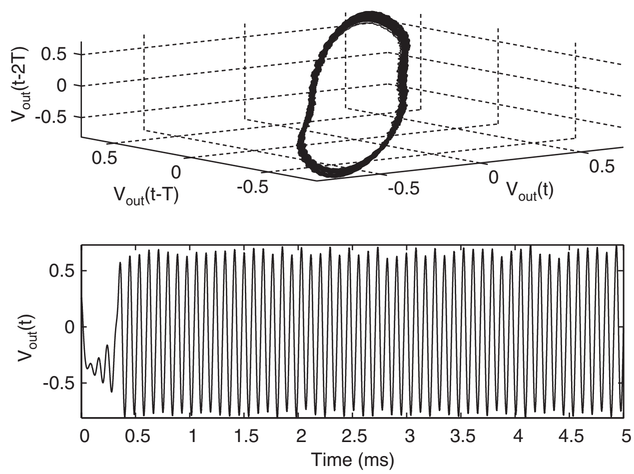

Fig. 6. The output of dynamical wavelet network circuit for spiral attractor.

Example 2 (Spiral attractor of Chua's circuit). The time series of the spiral attractor of Chua's circuit has been obtained for $N=1000$ samples for $\tau_s=0.05$ s and selecting the parameters $\alpha = 8.50000425$ and $\beta = 14.2857143$. The embedding dimension and the embedding delay have been chosen as $d_e=3$ and $T=\tau_s$, respectively. The wavelet network contains 5 wavelons. The output of the circuit is shown in Fig. 6 and the parameters of the wavelet network have been given in Table 2.

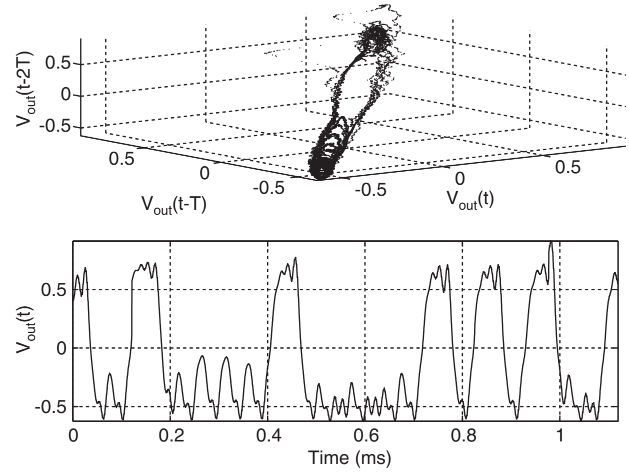

Fig. 7. The output of dynamical wavelet network circuit for double-scroll attractor.

Example 3 (Double-scroll attractor of Chua's circuit). When the bifurcation parameter α of the Chua's circuit has been chosen as 9, the double scroll attractor has been obtained in the phase space. The time series has been obtained for $N = 2000$ samples for $\tau_s = 0.1$ s. The embedding dimension and the embedding delay have been chosen as $d_e = 3$ and $T = 3\tau_s$, respectively. The wavelet network contains 6 wavelons and the parameters of the wavelet network have been given in Table 3. The output of the circuit is shown in Fig. 7.

6. Conclusions

The circuit implementation of the dynamical wavelet network has been introduced by using the Mexican Hat mother wavelet circuit proposed in [13]. Because of the difficulties in obtaining the correct values of the components the

Table 2. The wavelet network parameters of Example 2

i	D_i	b_i	w_i
1	diag(2.3338, 1.9983, 2.3056)	$[0.5985\ 1.0037\ 0.6188]^T$	0.0065
2	diag(6.1175, 5.4661, 6.2254)	$[0.5557\ 0.2928\ 0.0139]^T$	0.0117
3	diag(13.9630, 14.0703, 14.0315)	$[0.2929\ 0.2838\ 0.3219]^T$	0.0019
4	diag(1.2247, 1.9068, 1.2169)	$[0.2264\ 0.6518\ 1.0349]^T$	0.0058
5	diag(3.4556, 3.5469, 3.9565)	$[0.2620\ 0.7019\ 0.3844]^T$	0.0074

$c = [0.8564 - 2.6954\ 2.8359]^T \quad \bar{b} = -4.9823e - 004$

Table 3. The wavelet network parameters of Example 3

i	D_i	b_i	w_i
1	diag(2.3981, 3.3344, 3.7859)	$[0.6150\ 0.0001\ 0.3245]^T$	−0.2477
2	diag(2.3406, 1.8408, 3.8478)	$[0.4715\ 0.9353\ 0.4970]^T$	−0.1256
3	diag(2.0044, 0.0000006, 2.2383)	$[-0.5015\ 0.1297\ -0.3632]^T$	0.0841
4	diag(16.6680, 16.7195, 16.7027)	$[0.2234\ 0.4278\ 0.3927]^T$	−1.3303
5	diag(33.3401, 33.3050, 33.2771)	$[0.2277\ 0.4123\ 0.4021]^T$	−6.2826
6	diag(8.2909, 8.3315, 8.1242)	$[0.2089\ 0.4498\ 0.3834]^T$	−0.3437

$c = [0.3248\ -1.4898\ 1.9562]^T \quad \bar{b} = 0.0487$

proposed method may not be suitable for the implementation of the large circuits. On the other hand, it is suitable for VLSI manufacturing because of the systematic procedure and also the VLSI manufacturing allows the accurate realization. The circuit simulations for the period-1 limit cycle, the spiral and double-scroll attractor of the Chua's circuit have been accomplished and the attractors have been embedded in the phase space.

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