

Neutralino dark matter in the left-right supersymmetric modelDurmuş A. Demir,^{1,*} Mariana Frank,^{2,†} and Ismail Turan^{2,‡}¹*Department of Physics, Izmir Institute of Technology, IZTECH, TR35430, Izmir, Turkey*²*Department of Physics, Concordia University, 7141 Sherbrooke Street West, Montreal, Quebec, Canada H4B 1R6*

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We study the neutralino sector of the left-right supersymmetric model. In addition to the possibilities available in the minimal supersymmetric model, the neutralino states can be superpartners of the $U(1)_{B-L}$ gauge boson, the neutral $SU(2)_R$ gauge boson, or of the Higgs triplets. We analyze neutralino masses and determine the parameter regions for which the lightest neutralino can be one of the new pure states. We then calculate the relic density of the dark matter for each of these states and impose the constraints coming from the ρ parameter, the anomalous magnetic moment of the muon, $b \rightarrow s\gamma$, as well as general supersymmetric mass bounds. The lightest neutralino can be the bino, or the right-wino, or the neutral triplet Higgsino, all of which have different couplings to the standard model particles from the usual neutralinos. A light bino satisfies all the experimental constraints and would be the preferred dark matter candidate for light supersymmetric scalar masses, while the right-wino would be favored by intermediate supersymmetric mass scales. The neutral triplet Higgs fermion satisfies the experimental bounds only in a small region of the parameter space, for intermediate to heavy supersymmetric scalar masses.

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I. INTRODUCTION

Recent measurements from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [1] have shown that the cold dark matter (CDM) abundance in the universe is $\Omega_{\text{CDM}} h^2 = 0.1126^{+0.00805}_{-0.00904}$, where Ω_{CDM} is the density of the CDM species versus the critical density, and h is the present value of the Hubble parameter. This corresponds to about 22% of the energy density of the universe being present in the form of dark matter.

The question of its nature and composition has been an open problem for some time. A candidate for dark matter must be stable, or long-living; it must be electrically and color neutral, as required by astrophysical constraints; and it must be nonrelativistic, which means that it should be massive. Since the relic density can be connected to the (thermally averaged) annihilation cross section of a dark matter candidate, its value indicates that these particles are interacting weakly. That is, dark matter candidates must be weakly interacting massive particles. Scenarios beyond the standard model provide several such exotic particles. Supersymmetry, in particular, provides the lightest neutralino as the lightest supersymmetric particle (LSP), which must be stable if R -parity is conserved.¹

The minimal supersymmetric standard model (MSSM) Lagrangian, based on the symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is invariant under the R -parity discrete symmetry, under which all standard model particles are even, and their superpartners are odd. This parity is an essential element of the MSSM, and forbids baryon and

lepton number violating renormalizable couplings in the superpotential, which would lead to fast proton decay. Thus the R -parity is natural, and so is the possibility of having an LSP which emerges as a natural candidate for dark matter. In most supersymmetric scenarios, the LSP is the lightest neutralino [2].

Unfortunately there are problems with this CDM assignment in MSSM [3]. If one calculates the relic abundance of the lightest neutralino assumed to be a pure bino (the superpartner of the gauge boson of the $U(1)_Y$ gauge group), the relic density is too large (see [4] and references therein for a detailed discussion). To avoid this problem, one considers that there are several other supersymmetric particles of masses close to the LSP. If such sparticles exist, their coannihilation with the LSP leads to a reduction of the LSP relic abundance [5]. Extensive studies of such phenomena exist, and several publicly available codes such as DARKSUSY [6] and MICROMEGA [7], which include all the relevant coannihilation channels, are available for calculations within the MSSM. Cosmological and phenomenological constraints (such as $b \rightarrow s\gamma$ [8], $(g-2)_\mu$ [9], and Higgs mass bounds [10]) are also included. We could of course also abandon the pure bino state, and thus the constrained MSSM (CMSSM) which predicts it, and explore other versions of the lightest neutralino. However, this does not solve the problem because if one allows the lightest neutralino to be mostly left-wino, or Higgsino, the relic density becomes too small. Ways to avoid this exist as well, such as considering nonuniversal gaugino masses, abandoning the pure states and examining mixed states (such as bino-Higgsino), or more annihilations [11].

An alternative option is to explore scenarios beyond the MSSM. The extensions of the MSSM, motivated by the need to a dynamical solution to the naturalness problem associated with the μ parameter, offer novel CDM candidates not necessarily belonging to gaugino or Higgsino

*Electronic address: demir@physics.iztech.edu.tr†Electronic address: mfrank@vax2.concordia.ca‡Electronic address: ituran@physics.concordia.ca¹Gravitinos and axinos have also been studied as dark matter candidates, but we shall not consider this possibility here.

sectors of the MSSM. In next-to-minimal model (NMSSM) [12], for instance, the neutralino can be extremely light because it is dominated by its singlino component, which does not couple to SM particles (except for Higgs doublets). The existence of a very light CP -odd Higgs boson provides the possibility for this neutralino to annihilate sufficiently to avoid being overproduced in the early universe. Similar features are also found in $U(1)'$ models [13] or in more general extensions [14]. This raises the hope that models beyond MSSM, preferably comprising the neutrino masses as well, can provide viable scenarios for alternative neutralinos which have different features than those in MSSM.

We propose here to look at the left-right supersymmetric model (LRSUSY) for candidates for dark matter. The advantages of the LRSUSY are many, such as combining supersymmetry with right-handed symmetry, and thus providing a mechanism for neutrino masses with the seesaw within a supersymmetric scenario. From the point of view of dark matter, the LRSUSY model is interesting because, unlike in MSSM, the usual explicit R -parity violating terms are forbidden in the superpotential by the symmetries of the model [15].² Thus the model naturally predicts a lightest supersymmetrical particle.

The LRSUSY model emerges naturally in some superstring theories [17], or in the breaking of such SUSY GUT scenarios, such as $SO(10)$ [18]. LRSUSY is based on the symmetry group $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{V=B-L}$ and thus provides several new sources for exotic neutralinos, in the (neutral) partners of the $SU(2)_R$ or $U(1)_{B-L}$ gauge bosons, as well as the fermionic partners of several Higgs bosons. We analyze the candidates for dark matter in this model to see if they can avoid the problems that plague MSSM. After briefly reviewing the model in Sec. II, we perform a comprehensive study of neutralino mass eigenstates in Sec. III, followed by a calculation of relic density in Sec. IV. We look at the simplest scenario, that of pure states and avoid coannihilation with other supersymmetric particles by choosing the supersymmetric masses accordingly. We include constraints from the WMAP, $b \rightarrow s\gamma$, $\Delta\rho$, and muon $g - 2$ as well as from experimental bounds on supersymmetric masses from direct collider searches.

II. THE LRSUSY MODEL

The most general superpotential for the group $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{V=B-L}$ is [19]:

$$W = \mathbf{Y}_Q^{(i)} Q^T \Phi_i i\tau_2 Q^c + \mathbf{Y}_L^{(i)} L^T \Phi_i i\tau_2 L^c + i(\mathbf{Y}_{LR} L^T \tau_2 \delta_L L + \mathbf{Y}_{LR} L^{cT} \tau_2 \Delta_R L^c) + \mu_{LR} [\text{Tr}(\Delta_L \delta_L + \Delta_R \delta_R)] + \mu_{ij} \text{Tr}(i\tau_2 \Phi_i^T i\tau_2 \Phi_j) + W_{NR}, \quad (2.1)$$

where W_{NR} denotes (possible) nonrenormalizable terms

²Note however that the R -parity can be broken spontaneously [16].

arising from higher scale physics or Planck scale effects [20,21]. Here the matter fields are defined as

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \sim (3, 2, 1, 1/3), \quad Q^c = \begin{pmatrix} d^c \\ u^c \end{pmatrix} \sim (3^*, 1, 2, -1/3), \\ L = \begin{pmatrix} \nu \\ e \end{pmatrix} \sim (1, 2, 1, -1), \quad L^c = \begin{pmatrix} e^c \\ \nu^c \end{pmatrix} \sim (1, 1, 2, 1), \quad (2.2)$$

where the numbers in the brackets denote the quantum numbers under $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. The Higgs sector consists of the bidoublet and triplet Higgs superfields:

$$\Phi_1 = \begin{pmatrix} \phi_{11}^0 & \phi_{11}^+ \\ \phi_{12}^- & \phi_{12}^0 \end{pmatrix} \sim (1, 2, 2, 0), \\ \Phi_2 = \begin{pmatrix} \phi_{21}^0 & \phi_{21}^+ \\ \phi_{22}^- & \phi_{22}^0 \end{pmatrix} \sim (1, 2, 2, 0), \\ \Delta_L = \begin{pmatrix} \frac{\Delta_L^-}{\sqrt{2}} & \Delta_L^0 \\ \Delta_L^{--} & -\frac{\Delta_L^-}{\sqrt{2}} \end{pmatrix} \sim (1, 3, 1, -2), \\ \delta_L = \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix} \sim (1, 3, 1, 2), \\ \Delta_R = \begin{pmatrix} \frac{\Delta_R^-}{\sqrt{2}} & \Delta_R^0 \\ \Delta_R^{--} & -\frac{\Delta_R^-}{\sqrt{2}} \end{pmatrix} \sim (1, 1, 3, -2), \\ \delta_R = \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & -\frac{\delta_R^+}{\sqrt{2}} \end{pmatrix} \sim (1, 1, 3, 2), \quad (2.3)$$

where Δ_L and δ_R are introduced in the model to cancel the fermionic anomalies introduced by the fermionic partners of δ_L and Δ_R . The vev's of the Higgs fields in the LRSUSY can be chosen:

$$\langle \Phi_1 \rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \\ \langle \Delta_L \rangle = \begin{pmatrix} 0 & v_{\Delta_L} \\ 0 & 0 \end{pmatrix}, \quad \langle \delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_{\delta_L} & 0 \end{pmatrix}, \\ \langle \Delta_R \rangle = \begin{pmatrix} 0 & v_{\Delta_R} \\ 0 & 0 \end{pmatrix}, \quad \langle \delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_{\delta_R} & 0 \end{pmatrix}. \quad (2.4)$$

Some comments and explanations about the vev's chosen are required: κ_1 and κ_2 are the vev's of the MSSM-like Higgs bosons. They are responsible for giving masses to the quarks and leptons and they also contribute to M_{W_L} . We take the vev's of ϕ_{12}^0 and ϕ_{21}^0 to be zero because they induce FCNC at tree level (in both the leptonic and hadronic systems), as well as being responsible for $W_L - W_R$ mixing. The vev's could also have a phase which induces CP violation, which is severely restricted in the kaon system. The non-MSSM Higgs vev's, v_{δ_L} and v_{Δ_R} are responsible for neutrino masses. For one generation (see [22], also [20,23,24])

$$m_\nu = Y_{LR} v_{\delta_L} - \frac{(Y_L^{(\nu)})^2 \kappa_1^2}{Y_{LR} v_{\Delta_R}}, \quad (2.5)$$

where v_{δ_L} must be very small and v_{Δ_R} large, phenomenologically. In addition, v_{Δ_L} and v_{δ_L} enter in the formula for the mass of W_L (or the ρ parameter), while v_{Δ_R} , v_{δ_R} enter in the formula for the mass of W_R . It is thus justified to take v_{Δ_L} , v_{δ_L} to be negligibly small. For v_{Δ_R} there are two possibilities: either v_{Δ_R} is $\approx 10^{13}$ GeV [24,25], which supports the seesaw mechanism, leptogenesis and provides masses for the light neutrinos in agreement with experimental constraints, but offers no hope to see right-handed particles; or v_{Δ_R} is ≈ 1 –10 TeV, but one must introduce something else (generally an intermediate scale, or an extra symmetry) to make the neutrinos light [15,20,25]. Note

that both v_{Δ_R} and v_{δ_R} contribute to the mass of W_R [26], but only one needs to be heavy. Since v_{Δ_R} is responsible for heavy right-handed neutrino masses, it must be large. Thus v_{δ_R} is not constrained by the data. The mass term for neutralinos is given by

$$\mathcal{L}_N = -\frac{1}{2} \psi^{0T} Y \psi^0 + \text{H.c.}, \quad (2.6)$$

where

$\psi^0 = (-i\lambda_V, -i\lambda_L^0, \tilde{\phi}_{11}^0, \tilde{\phi}_{22}^0, \tilde{\Delta}_R^0, \tilde{\delta}_R^0, -i\lambda_R^0, \tilde{\phi}_{12}^0, \tilde{\phi}_{21}^0)^T$. Here λ_V is the $U(1)_{B-L}$ bino, λ_L^0 the left-handed neutral $[SU(2)_L]$ wino, and λ_R^0 the right-handed neutral $[SU(2)_R]$ wino. The rest of the fields are Higgsinos. The neutralino mass matrix Y is equal to [27]

$$Y = \begin{pmatrix} & -i\lambda_V & -i\lambda_L^0 & \tilde{\phi}_{11}^0 & \tilde{\phi}_{22}^0 & \tilde{\Delta}_R^0 & \tilde{\delta}_R^0 & -i\lambda_R^0 & \tilde{\phi}_{12}^0 & \tilde{\phi}_{21}^0 \\ -i\lambda_V & M_V & 0 & 0 & 0 & -\sqrt{2}g_V v_{\Delta} & \sqrt{2}g_V v_{\delta} & 0 & 0 & 0 \\ -i\lambda_L^0 & 0 & M_L & \frac{g_L \kappa_1}{\sqrt{2}} & -\frac{g_L \kappa_2}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ \tilde{\phi}_{11}^0 & 0 & \frac{g_L \kappa_1}{\sqrt{2}} & 0 & -\mu_1 & 0 & 0 & -\frac{g_R \kappa_1}{\sqrt{2}} & 0 & 0 \\ \tilde{\phi}_{22}^0 & 0 & -\frac{g_L \kappa_2}{\sqrt{2}} & -\mu_1 & 0 & 0 & 0 & \frac{g_R \kappa_2}{\sqrt{2}} & 0 & 0 \\ \tilde{\Delta}_R^0 & -\sqrt{2}g_V v_{\Delta} & 0 & 0 & 0 & 0 & \mu_2 & -\sqrt{2}g_R v_{\Delta} & 0 & 0 \\ \tilde{\delta}_R^0 & \sqrt{2}g_V v_{\delta} & 0 & 0 & 0 & \mu_2 & 0 & -\sqrt{2}g_R v_{\delta} & 0 & 0 \\ -i\lambda_R^0 & 0 & 0 & -\frac{g_R \kappa_1}{\sqrt{2}} & \frac{g_R \kappa_2}{\sqrt{2}} & \sqrt{2}g_R v_{\Delta} & -\sqrt{2}g_R v_{\delta} & M_R & 0 & 0 \\ \tilde{\phi}_{12}^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 \\ \tilde{\phi}_{21}^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 \end{pmatrix} \quad (2.7)$$

where $\mu_{ij} = \mu_i$, ($i \neq j$) and $\mu_{LR} \equiv \mu_2$ are assumed.³ The upper 4×4 part of the matrix contains MSSM-like states. There are still too many unknown parameters and for further simplification one can define the following:

$$g_L = g_R \equiv g, \quad g_V = \frac{e}{\sqrt{\cos 2\theta_W}}, \quad (2.8)$$

$$\sqrt{\kappa_1^2 + \kappa_2^2} = 174 \text{ GeV}, \quad \tan\beta = \frac{\kappa_2}{\kappa_1}.$$

If we assume that GUT relations between gaugino masses hold, we can simplify the parameters further, but for now we keep them general and discuss specific scenarios later.

Two roots of the characteristic equation for Y are already known, $\pm\mu_1$, and we are left with a 7th order equation to solve. There is no exact solution and a numerical or semi-exact approach is necessary. Whereas there are constraints on M_{W_R} and M_{Z_R} [28], there are no constraints on M_V , the

$U(1)_{B-L}$ gaugino mass, or on $M_{L(R)}$, the $SU(2)_{L(R)}$ gaugino mass parameter.

Basically the number of free parameters can be chosen as M_V , M_L , M_R , v_{Δ} , v_{δ} , μ_1 , μ_2 , and $\tan\beta$. So, we could look at under what circumstances the lightest neutralino becomes a mainly pure state (bino, left-wino, right-wino, or Higgsino) or a mixed state as bino-Higgsino and right-wino-Higgsino.

III. THE CLASSIFICATION OF SCENARIOS

Starting from the neutralino mass matrix, we can classify the following cases:⁴

- (1) The lightest neutralino is mostly bino (λ_V in our notation). Note that this is the $B - L$ gaugino, not to be confused with the $U(1)_Y$ MSSM bino. To obtain

³From now on, we drop the subscript ‘‘R’’ from v_{Δ_R} , v_{δ_R} .

⁴We set $v_{\Delta} = 1.5$ TeV for all the cases unless otherwise stated. Note also that there will be no rigorous model constraint analysis on the masses of the LSP in this section. We leave that to the next section.

this bino as the lightest state, we must have $M_V \approx 50\text{--}100$ GeV and $v_\delta \approx 50\text{--}100$ GeV (both light) while $\mu_2 \approx 1$ TeV or larger (heavy). Decreasing μ_2 to 200–500 GeV, the lightest state becomes a mixture of λ_V and $\tilde{\delta}_R^0$.

- (2) The lightest neutralino is mostly a right-handed wino (λ_R^0 in our notation). To obtain this, the masses of the other two gauginos must be larger than that of λ_R^0 : $M_V, M_L \approx 600$ TeV and $v_\delta \approx 50\text{--}100$ GeV is light; while $\mu_1 \approx 1$ TeV, $\mu_2 \approx 3\text{--}5$ TeV. Decreasing μ_2 to 1 TeV, the lightest state becomes a mixture of λ_R^0 , λ_V , and $\tilde{\delta}_R^0$.
- (3) The lightest neutralino is the right-handed Higgsino $\tilde{\delta}_R^0$. This scenario is obtained for a large range of parameters, as long as both μ_2 and v_δ are small, ≈ 200 GeV or larger, and (0, 100) GeV, respectively. In fact, this requirement is satisfied for a wide portion of the parameter space, as long as μ_2 and v_δ are smaller than other parameters in the theory. This scenario is interesting since $\tilde{\delta}_R^0$ does not couple to any standard model particles (or their SUSY partners). As explained previously, v_δ is not constrained by the data.
- (4) The lightest neutralino could be a mixture of λ_V , $\tilde{\delta}_R^0$, and $\tilde{\Delta}_R^0$. In this case we can get eigenvectors and eigenvalues analytically and calculate the relic density for the mixed state. This is a generalization of the first case, where both $\tilde{\Delta}_R^0$ and $\tilde{\delta}_R^0$ are included. Here we can take advantageous ratios of the vevs of $\tilde{\delta}_R^0$ and $\tilde{\Delta}_R^0$ to get only one combination of $\tilde{\delta}_R^0$ and $\tilde{\Delta}_R^0$ mixed with the bino. This is the case if we assume the vev's v_δ and v_Δ equal.
- (5) The lightest neutralino could be a mixture of λ_R^0 (the right-handed wino), and the non-MSSM Higgsinos $\tilde{\delta}_R^0$, $\tilde{\Delta}_R^0$. As opposed to the $B-L$ bino, the right-handed wino mixes with MSSM Higgsinos as well, which will be considered in a separate scenario. Again like in scenario (4), one can make the right-wino couple with one combination of $\tilde{\delta}_R^0$ and $\tilde{\Delta}_R^0$ if their vev's are assumed to be equal.
- (6) Finally, the lightest neutralino could be a mixture of λ_R^0 with only the MSSM Higgsinos, $\tilde{\phi}_{11}^0$ and $\tilde{\phi}_{22}^0$. This can be the case if both M_R and μ_1 are small compared to M_L, M_V, μ_2 , and the vev's v_Δ and v_δ . One could consider the $v_\Delta = v_\delta$ case and decouple $\tilde{\delta}_R^0$ and $\tilde{\Delta}_R^0$ by taking μ_2 to be large.

We are not interested in the MSSM lightest neutralino scenarios (in which the left-wino or MSSM Higgsino mixed with left-wino are the LSP), since these have already been studied. As far as we can tell, scenarios 1–6 are the important (most striking) possibilities.

A. The lightest neutralino state

The following four sets of figures further illustrate the scenarios mentioned above. We assume the composition of

the lightest state as (for the first three scenarios)

$$|\tilde{\chi}_1^0\rangle = N_{11}|\lambda_L^0\rangle + N_{12}|\lambda_R^0\rangle + N_{13}|\lambda_V\rangle + N_{14}|\tilde{\phi}_{11}^0\rangle + N_{15}|\tilde{\phi}_{22}^0\rangle + N_{16}|\tilde{\Delta}_R^0\rangle + N_{17}|\tilde{\delta}_R^0\rangle. \quad (3.1)$$

First we look at the possibilities for scenarios (1) and (3). Here we want the bino to be the lightest, so we take M_L, M_R , and μ_1 large. Accordingly, we need v_δ to be small. Varying μ_2 will take us from a mostly bino lightest state to a mostly $\tilde{\delta}_R^0$ lightest state.

In the left panel of Fig. 1, the difference between the bino and Higgsino compositions of the lightest neutralino state is shown as a function of μ_2 by choosing $(M_V, v_\delta) = (0, 0), (0, 100), (0, 400)$, and $(200, 400)$ GeV. The rest of the parameters are fixed as $M_L = M_R = 600$ GeV, $\mu_1 = 500$ GeV, $v_\Delta = 1.5$ TeV, and $\tan\beta = 2$. In the right panel, the mass of the lightest neutralino is given as a function of μ_2 for the same parameter values. As seen from the figure the lightest state is pure Higgsino $\tilde{\delta}_R^0$ for very small μ_2 values, regardless of the values of (M_V, v_δ) , as long as they are smaller than 500 GeV. The ratio is more sensitive to the vev of the right-Higgsino (v_δ) than to the $U(1)_{B-L}$ gaugino parameter M_V . At large μ_2 the state becomes pure bino from pure Higgsino and its mass shows a strong dependence on μ_2 . So, one can consider these two limiting cases as realizations of the scenarios (1) and (3), mentioned above. From the right panel, one can conclude that, except for very small values of μ_2 , the cases where M_V and v_δ are larger than 200 GeV predict a lightest neutralino with a mass in the range of 200–300 GeV. Otherwise its mass remains less than 150 GeV.

Next we look at scenarios (2) and (3). Here we want the right-wino to be the lightest neutralino, so we take M_L, M_V , and μ_1 large, and v_δ to be small. Varying μ_2 will take us from a mostly right-wino lightest state to a mostly $\tilde{\delta}_R^0$ lightest state.

In Fig. 2, we plot the difference between the bino and the Higgsino compositions of the lightest neutralino state as a function of μ_2 for various (M_R, v_δ) values, for $M_L = M_V = 600$ GeV, $\mu_1 = 500$ GeV, $v_\Delta = 1.5$ TeV, and $\tan\beta = 2$. On the right panel, the mass of the lightest neutralino is shown as a function of μ_2 for the same parameter values. The third diagram shows the bino composition of the lightest state. This case exhibits a very similar dependence on μ_2 as the bino-Higgsino case. The curves for various (M_R, v_δ) pairs indicate that the ratio is insensitive to their chosen values, as long as they are assumed to be less than 200 GeV. The bino composition becomes significant only around $\mu_2 = 1$ TeV, where the right-wino and Higgsino mix almost equally, and it is negligible as μ_2 becomes larger. The contribution of the bino with respect to that of the right-wino is significant for very small μ_2 values (~ 0 GeV) but such small values are excluded from the lower bound on the mass of the lightest chargino, which is around 90 TeV. The mass of the lightest state is less than 150 GeV for $v_\delta \leq 200$ GeV if M_R is very

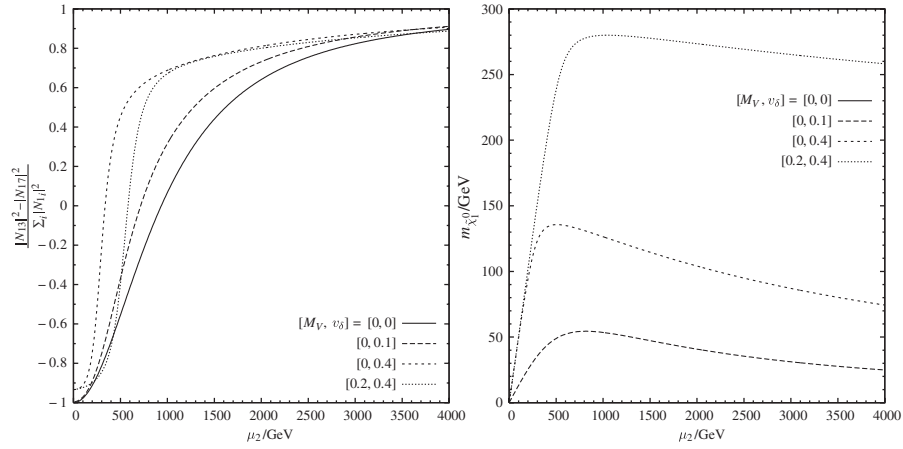


FIG. 1. On the left panel, the difference between bino and Higgsino compositions of the lightest neutralino state as a function of μ_2 for various (M_V, v_δ) values in TeV, for $M_L = M_R = 600$ GeV, $\mu_1 = 500$ GeV, $v_\Delta = 1.5$ TeV, and $\tan\beta = 2$. On the right panel, the mass of the lightest neutralino as a function of μ_2 for the same parameter values.

small, or for $(M_R, v_\delta) \leq (50, 50)$ GeV. Otherwise, it is larger than 200 GeV for μ_2 larger than 500 GeV. While the composition of the state is insensitive to M_R and v_δ , the

mass of the lightest neutralino is sensitive to both. Here one can recover scenarios (2) and (3) in the limiting cases and the mixed state $(|\lambda_R^0\rangle + |\tilde{\delta}_R^0\rangle)$ in between.

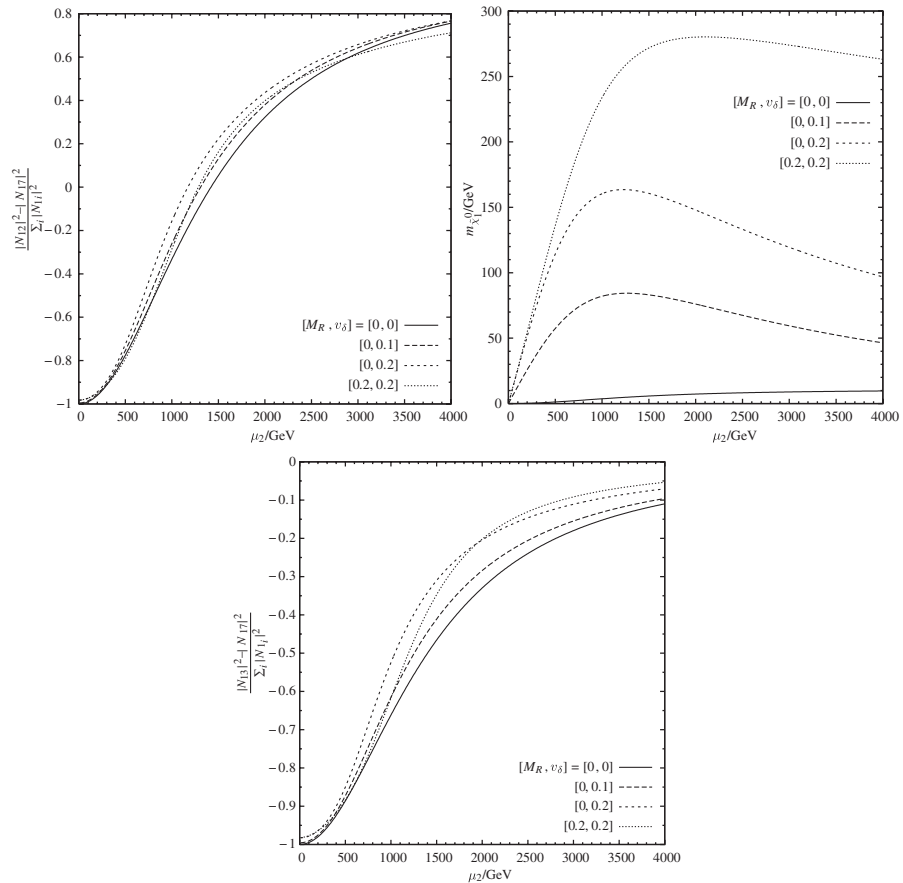


FIG. 2. On the left panel, the difference between right-wino and Higgsino compositions of the lightest neutralino state as a function of μ_2 for various (M_R, v_δ) values in TeV, for $M_L = M_V = 600$ GeV, $\mu_1 = 500$ GeV, $v_\Delta = 1.5$ TeV, and $\tan\beta = 2$. On the right panel, the mass of the lightest neutralino as a function of μ_2 for the same parameter values. The third diagram shows the difference between bino and Higgsino compositions of the lightest state.

Finally, we look at realizations of scenarios (4), (5), and (6). Here we are interested in mixtures of bino-(non-MSSM) Higgsino, right-wino-(non-MSSM) Higgsino, and right-wino-MSSM Higgsino. We need to decouple the MSSM particles for scenarios (4) and (5), so we take M_L and μ_1 to be large. In addition we take $v_\Delta = v_\delta$ (as opposed to the previous scenarios, where we took one of them small). We vary M_V , M_R and μ_2 to go from one case to another.

The realization of the last three scenarios discussed above requires some radical changes in the parameter set and has a distinct structure. We first decouple the $\tilde{\phi}_{12}^0$ and $\tilde{\phi}_{21}^0$ fields. Then, without loss of generality, we rotate the

basis

$$\begin{aligned} & \{|\lambda_L^0\rangle, |\lambda_R^0\rangle, |\lambda_V\rangle, |\tilde{\phi}_{11}^0\rangle, |\tilde{\phi}_{22}^0\rangle, |\tilde{\Delta}_R^0\rangle, |\tilde{\delta}_R^0\rangle\} \\ & \rightarrow \{|\lambda_L^0\rangle, |\lambda_R^0\rangle, |\lambda_V\rangle, |\tilde{\phi}_+^0\rangle, |\tilde{\phi}_-^0\rangle, |\tilde{\eta}_+^0\rangle, |\tilde{\eta}_-^0\rangle\} \end{aligned} \quad (3.2)$$

such that

$$|\tilde{\phi}_\pm^0\rangle \equiv \frac{|\tilde{\phi}_{11}^0\rangle \pm |\tilde{\phi}_{22}^0\rangle}{\sqrt{2}}, \quad |\tilde{\eta}_\pm^0\rangle \equiv \frac{|\tilde{\Delta}_R^0\rangle \pm |\tilde{\delta}_R^0\rangle}{\sqrt{2}}. \quad (3.3)$$

In this new basis the mass matrix Y in Eq. (2.7) becomes

$$Y' = \begin{pmatrix} & -i\lambda_L^0 & -i\lambda_R^0 & -i\lambda_V & \tilde{\phi}_+^0 & \tilde{\phi}_-^0 & \tilde{\eta}_-^0 \\ -i\lambda_L^0 & M_L & 0 & 0 & \frac{g_L}{2}\kappa^- & \frac{g_L}{2}\kappa^- & 0 \\ -i\lambda^0 & 0 & M_R & 0 & -\frac{g_R}{2}\kappa^+ & -\frac{g_R}{2}\kappa^+ & 2g_R v_R \\ -i\lambda_V & 0 & 0 & M_V & 0 & 0 & -2g_V v_R \\ \tilde{\phi}_+^0 & \frac{g_L}{2}\kappa^- & -\frac{g_R}{2}\kappa^- & 0 & -\mu_1 & 0 & 0 \\ \tilde{\phi}_-^0 & \frac{g_L}{2}\kappa^+ & -\frac{g_R}{2}\kappa^+ & 0 & 0 & \mu_1 & 0 \\ \tilde{\eta}_-^0 & 0 & 2g_R v_R & -2g_V v_R & 0 & 0 & \mu_2 \end{pmatrix}, \quad (3.4)$$

where we have assumed $v_\Delta = v_\delta \equiv v_R$, and further defined $\kappa^\pm = \kappa_1 \pm \kappa_2$. Under this assumption the rotated state $\tilde{\eta}_+^0$ decouples and only $\tilde{\eta}_-^0$ remains, which allows us to analyze scenarios (4), (5) and (6) more conveniently.

Figure 3 shows the difference between the right-wino (bino) and the rotated non-MSSM Higgsino ($\tilde{\eta}_-^0$) compositions of the lightest neutralino state as functions of μ_2 by choosing various (M_R , M_V) values in the left (right) panel. The rest of the parameters are taken as $M_L = 600$ GeV, $\mu_1 = 5$ TeV, $v_\Delta = v_\delta \equiv v_R = 1$ TeV, and $\tan\beta = 2$. Here $|N_{16}^-|^2$ is the amount of the rotated Higgsino field $\tilde{\eta}_-^0$ in the lightest neutralino state. One can divide the discussion into two regions—one with small $M_R \leq 50$ GeV and $M_V \leq 1$ GeV, and the other with larger M_R and M_V . In the first part, neither the right-wino nor the bino is the lightest neutralino till $\mu_2 \sim 500$ GeV. The decoupled state $\tilde{\eta}_+^0$ is the lightest state (which cannot be seen from the figures⁵). For $\mu_2 > 500$ GeV, the lightest state is mainly a right-wino or a right-wino-Higgsino mixture. Otherwise, for intermediate values of μ_2 , it is a mixture of right-wino, bino, and Higgsino $\tilde{\eta}_-^0$. For the second part where $M_R > 50$ GeV and $M_V > 1000$ GeV, the curves are horizontal lines passing through zero for

⁵Note that the horizontal line passing through zero does not always mean equal mixing. One could obtain such a result for vanishing individual contributions as well, which is indeed the case here.

both graphs until $\mu_2 \sim 2$ TeV. In that region of the parameter space the lightest state is pure left-wino. The right-wino-Higgsino mixture is possible as the lightest neutralino state only after that point. The bino-Higgsino lightest state is not realized in this region. From the third diagram in Fig. 3, one can say that, within the range considered for μ_2 , the mass of the lightest state is greater than 300 GeV for $\mu_2 \geq 500$ GeV. So, the scenarios (4) and (5) require the lightest state to be rather heavy unless μ_2 is too large. This takes us to pure right-wino scenario. Including all the possible model constraints in the next section will restrict our parameter space further.

In Fig. 4, the difference between the composition of the lightest neutralino as a right-wino, or a MSSM Higgsino is shown on the left panel as a function of M_R for various μ_1 values in the range 0 to 500 GeV. The other parameters are chosen as $M_L = M_V = 1$ TeV, $\mu_2 = 20$ TeV, $v_\Delta = v_\delta = 1$ TeV, and $\tan\beta = 2$. The ratio is sensitive to both M_R and μ_1 . For values of μ_1 smaller than 25 GeV, the lightest state is pure MSSM Higgsino and for values larger than 450 GeV the state almost is pure right-wino, if M_R is equal to, or less than, 150 GeV. Evenly mixed states are obtained for intermediate values of μ_1 . As M_R becomes equal to, or larger than, 400 GeV, the lightest state remains pure right-wino for the most part of the μ_1 range. As suggested by the right panel, the mass of the lightest state is very sensitive to the parameter M_R and can be quite large for large M_R and μ_1 values.

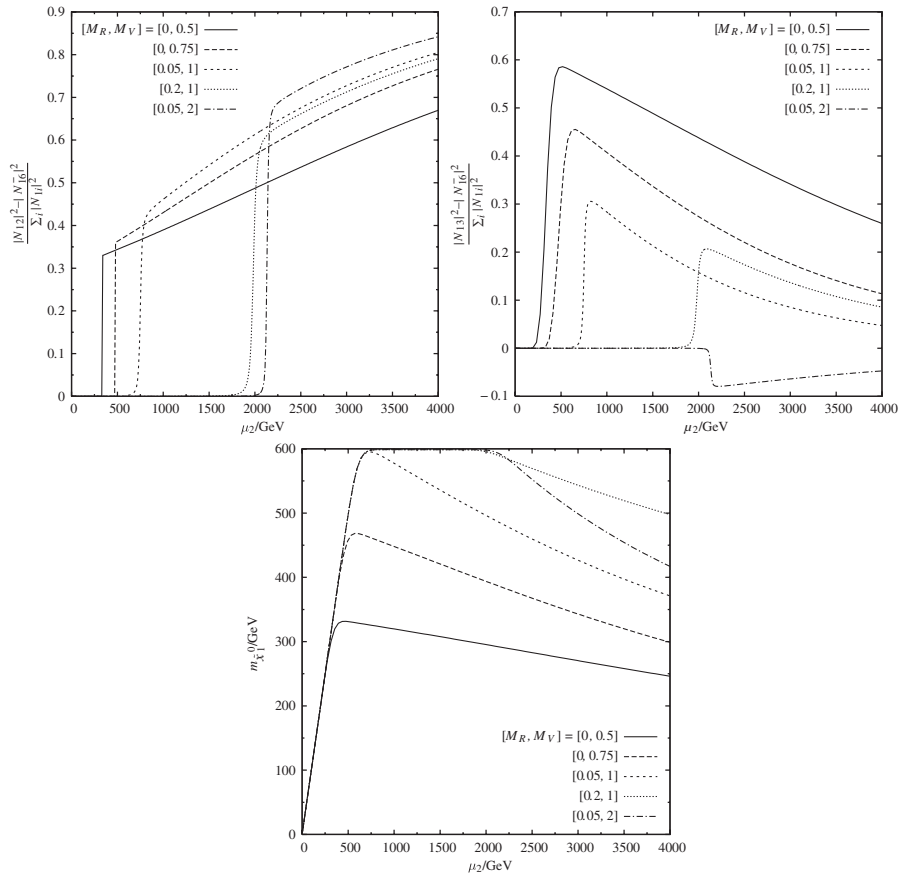


FIG. 3. On the left panel, the difference between right-wino and rotated non-MSSM-Higgsino compositions of the lightest neutralino state as a function of μ_2 for various (M_R, M_V) values in TeV, for $M_L = 600$ GeV, $\mu_1 = 5$ TeV, $v_\Delta = v_\delta = 1$ TeV, and $\tan\beta = 2$. On the right panel, the difference between the bino and the rotated non-MSSM-Higgsino compositions as functions of μ_2 for the same parameter values. The third diagram shows the mass of the lightest neutralino as a function of μ_2 .

This completes analyses of possible LSP candidates in LRSUSY for certain patterns of the model parameters. The figures illustrate the nature and purity of the lightest neutralino states as a function of various gaugino masses and

Higgs vevs. In the next section we will perform a detailed analysis of the relic abundance of LSP and its ability to explain the CDM in the universe.

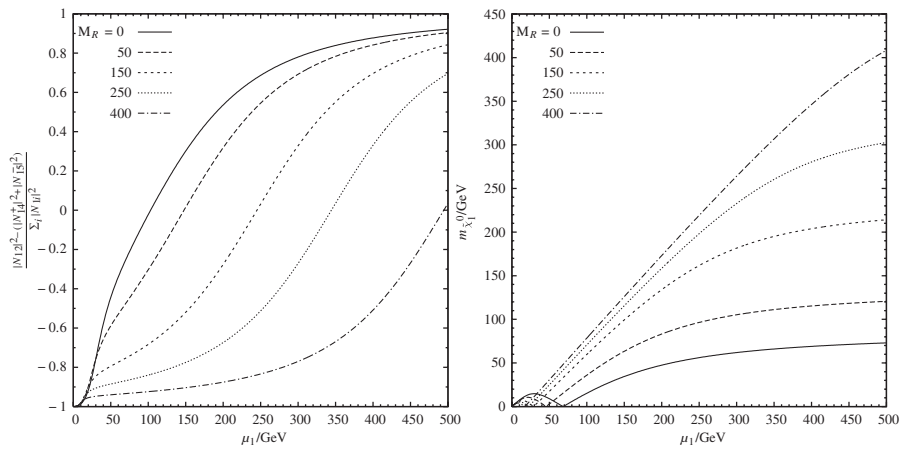


FIG. 4. On the left panel, the difference between right-wino and rotated MSSM-Higgsino compositions of the lightest neutralino state as functions of μ_1 at various M_R values in GeV, for $M_L = M_V = 1$ TeV, $\mu_2 = 20$ TeV, $v_\Delta = v_\delta = 1$ TeV, and $\tan\beta = 2$. On the right panel, the mass of the lightest neutralino as a function of μ_1 for the same parameter values.

IV. RELIC ABUNDANCE ANALYSIS OF CDM

In this section we calculate the relic density of cold dark matter within the LRSUSY framework. Before presenting our results we give some details of the procedure followed.

The time evolution of the number density n_i for a relic particle is given by the Boltzmann equation (taken within spatially flat Friedman-Robertson-Walker background). Furthermore, a single Boltzmann equation can be defined [5] for the total number density, $n \equiv \sum_i n_i$

$$\dot{n} + 3hn = -\langle \sigma_{\text{eff}} v \rangle [n^2 - n_{\text{eq}}^2], \quad (4.1)$$

where h is the Hubble parameter, v is the relative velocity of the annihilating particles, and n_{eq} is the number density corresponding to the sum of each species number density at thermal equilibrium (that is, the density in the early universe). Here σ_{eff} is the properly averaged effective cross section of the CDM candidate into ordinary particles (i.e., SM particles including the Higgs bosons) and will be defined below. Clearly, “ $\langle \rangle$ ” stands for the thermal average.

In this study we concentrate on regions of the parameter space where the coannihilation effects are not significant. The relativistic thermally averaged cross section times relative velocity reads as [29]

$$\langle \sigma_{\text{eff}} v \rangle(x) = \frac{\int_{4m_\chi^2}^{\infty} ds \sigma(s) \sqrt{s}(s/4 - m_\chi^2) K_1(\sqrt{s}/(m_\chi x))}{2m_\chi^5 x K_2(1/x)^2}, \quad (4.2)$$

where m_χ is the mass of the annihilating particle, the lightest neutralino in our consideration; K_1 and K_2 are modified Bessel functions; $x \equiv T/m_\chi$ is the dimensionless temperature parameter; \sqrt{s} is the center of mass energy of the annihilating neutralino pair; and $\sigma(s)$ is the cross section for the annihilation reaction $\chi_1^0 \bar{\chi}_1^0 \rightarrow X_{\text{SM}}$, where X_{SM} represents all the allowed two-body SM final states. We will not approximate Eq. (4.2) by expanding v in a Taylor series, as this approximation might not always be safe, especially for cases with fast varying integrands.

In general, there is no analytical solution to the Boltzmann equation, Eq. (4.1), which is a Riccati-type equation. Thus a numerical approach is required. There exist several different ways to proceed in the literature [7,29]. Here we define a freeze-out temperature parameter $x_F \equiv T_F/m_\chi$, which we use as an approximate solution to Eq. (4.1) [30] as

$$n_\chi(T_0) \sim \frac{1}{m_\chi M_{\text{Pl}}} \sqrt{\frac{4\pi^3 g^*}{45}} \left(\frac{T_\chi}{T_\gamma}\right)^3 T_\gamma^3 \frac{1}{\int_{x_0}^{x_F} dx \langle \sigma_{\text{eff}} v \rangle(x)}, \quad (4.3)$$

where M_{Pl} is the Planck mass, T_0 is today's temperature

(~ 2.742 K), so that x_0 can be approximated as zero; $T_\chi(T_\gamma)$ is the present temperature of the neutralino (cosmic microwave background), and g^* represents the SM effective number of degrees of freedom at the freeze-out temperature ($g^* \sim 81$ is used). Note that using an approximate solution of the Boltzmann equation instead of solving it numerically will introduce an uncertainty of up to 10% into our results.

The freeze-out temperature parameter x_F can be obtained from the following transcendental equation:

$$\frac{1}{x_F} = \log \left[\frac{m_\chi M_{\text{Pl}}}{2\pi^3} \sqrt{\frac{45}{2g^*}} \sqrt{x_F} \langle \sigma_{\text{eff}} v \rangle(x_F) \right], \quad (4.4)$$

which can be solved iteratively. As a starting point we need to choose a value for x_F . We used 1/25 in our calculations. The range for x_F changes between 1/25 to 1/15. Finally, the relic density of neutralinos at present time is defined as

$$\Omega_\chi h^2 = \frac{\rho(T_0)}{\rho_c}, \quad (4.5)$$

where $\rho(T_0) = m_\chi n_\chi(T_0)$ is the density of the neutralino, and $\rho_c = 3h^2 M_{\text{Pl}}^2 / 8\pi = 8.098 \times 10^{-47} \text{ GeV}^4$ is the critical density of the universe. On the left-hand side of the equation, h is the normalized Hubble expansion rate and its value today is 0.71. Thus, from the central value of $\Omega_\chi h^2$, Ω_χ is found to be about 22%.

To obtain our final results, a three-dimensional integration needs to be carried out numerically. The three parts are an integration over the Mandelstam variable t to compute $\sigma(s)$ for each subprocess in the kinematically allowed region, an integration over the center of mass energy squared s from the threshold to practically infinity, and finally an integration over x to compute the thermal averaging from the freeze-out temperature to today's (x_0), which we approximate to zero. MATHEMATICA is used for the computation and the matrix element calculations have been carried out with FEYNALC [31].

The Feynman diagrams contributing to the annihilation cross section are given in Fig. 5. We analyze here pure state contributions only, while any mixed case scenario can be calculated in a straightforward manner. Since our analysis concentrates on the non-MSSM scenarios, the MSSM contributions are not shown here. Depending of the center of mass energy available, the resonance problem in s -channel is handled using the Breit-Wigner prescription.

The mass of the Z_R boson in diagram (a) is $M_{Z_R} = \frac{g \cos \theta_w}{\sqrt{\cos 2\theta_w}} (v_\delta^2 + v_\Delta^2)^{1/2}$ [26]. In diagram (b) we sum over all left- and right-handed quarks and leptons in the final state. The sfermion \tilde{f} has a (mass)² given by, for U -type squarks and sneutrinos,

$$\mathcal{M}_{U_k}^2 = \begin{pmatrix} M_{\text{SUSY}}^2 + M_{Z_L}^2 (T_u^3 - Q_u \sin^2 \theta_W) \cos 2\beta & m_{u_k} (A - \mu \cot \beta) \\ m_{u_k} (A - \mu \cot \beta) & M_{\text{SUSY}}^2 + M_{Z_L}^2 Q_u \sin^2 \theta_W \cos 2\beta \end{pmatrix}, \quad (4.6)$$

and for D -type squarks and sleptons,

$$\mathcal{M}_{D_k}^2 = \begin{pmatrix} M_{\text{SUSY}}^2 + M_{Z_L}^2 (T_d^3 - Q_d \sin^2 \theta_W) \cos 2\beta & m_{d_k} (A - \mu \tan \beta) \\ m_{d_k} (A - \mu \tan \beta) & M_{\text{SUSY}}^2 + M_{Z_L}^2 Q_d \sin^2 \theta_W \cos 2\beta \end{pmatrix}. \quad (4.7)$$

Here M_{SUSY} represents the universal scalar mass, A and μ the trilinear and bilinear scalar parameters, respectively, $m_{u,d}$ are quark masses and the index k labels generations. It is customary to restrict the supersymmetric parameter space by drawing contours in the $M_{1/2} - M_{\text{SUSY}}$ plane, where $M_{1/2}$ is the relevant gaugino or Higgsino mass parameter and M_{SUSY} the relevant scalar mass. When the bino or right-wino are the LSP, we take their masses to be the significant gaugino mass. In diagram (c) the final state h^0 is the MSSM-like lightest Higgs boson, which we constrain to have a mass $M_{h^0} = 115$ GeV. In LRSUSY, its (mass)² is given by the lowest eigenvalue of the matrix [26]:

$$\mathcal{M}_{\phi_{22}, \phi_{11}}^2 = \begin{pmatrix} m_{\phi_1 \phi_2}^2 \cot \beta - g^2 v^2 & -m_{\phi_1 \phi_2}^2 \\ -m_{\phi_1 \phi_2}^2 & m_{\phi_1 \phi_2}^2 \tan \beta + g^2 v^2 \end{pmatrix}, \quad (4.8)$$

where $v^2 = v_\delta^2 + v_\Delta^2 + \kappa_1^2 + \kappa_2^2$ and $m_{\phi_1 \phi_2}$ is the Higgs mass parameter. Note that, while the cross sections for the decay of λ_V , λ_R^0 depend on M_{SUSY} , the one for the $\tilde{\delta}_R^0$ Higgsino does not; we must find another relevant parameter for that case.

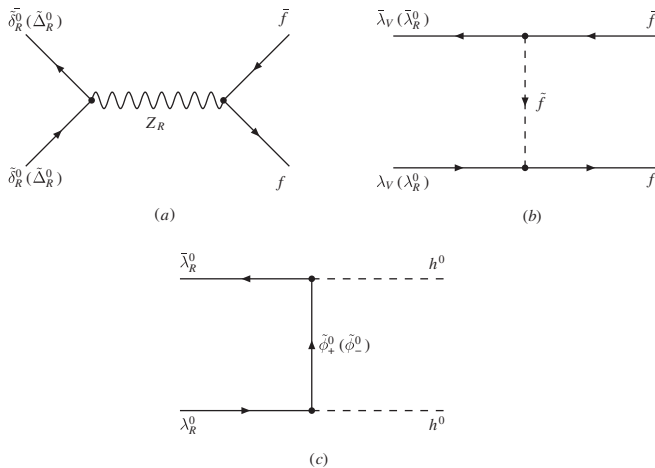


FIG. 5. The annihilation Feynman diagrams contributing $\tilde{\lambda}_1^0 \tilde{\lambda}_1^0 \rightarrow X_{\text{SM}}$ in the LRSUSY model. The MSSM contributions and u -channel diagrams are not shown here. Contributions are given in the basis of pure states whose mixtures can be easily constructed. Here f represent all fermions, and \tilde{f} all sfermions, h^0 is the SM-like Higgs particle, and $\tilde{\phi}_+^0$, $\tilde{\phi}_-^0$ are the rotated Higgsinos fields defined in the previous section.

V. NUMERICAL ANALYSIS

Based on our classification of some possible mass scenarios in Sec. III, we analyze the first three mass scenarios with the lightest neutralino as a pure bino, right-wino, or Higgsino state. The feasibility of the last three scenarios, assuming mixed gaugino-Higgsino states, can be analyzed in the light of the pure state predictions. Throughout the numerical calculations, we have chosen the parameters of the model such that the $\Delta\rho$ bound, taken as $\Delta\rho \leq 0.002$ [28], is always satisfied.

Figure 6 shows the relic density $\Omega_\chi h^2$ of the neutralino as a pure bino state in the $M_V - M_{\text{SUSY}}$ plane for $\mu_1 = 500$ GeV, $A = 200$ GeV, and $\tan\beta = 2$. There is no s -channel contribution and the t -channel fermion pair production $\lambda_V \tilde{\lambda}_V \rightarrow \tilde{f} f \tilde{f}$, ($f = u, c, t, d, b, s \dots$) is the only

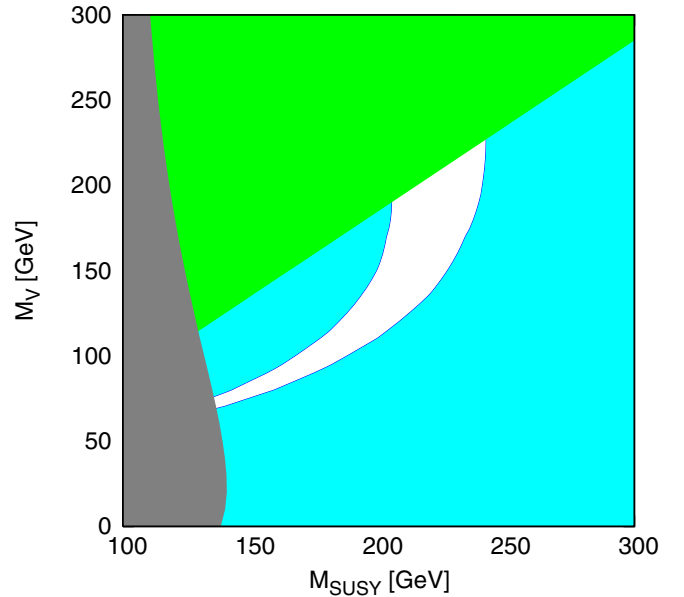


FIG. 6 (color online). The relic density of a pure bino state in the $M_V - M_{\text{SUSY}}$ plane for $\mu_1 = 500$ GeV, $A = 200$ GeV, and $\tan\beta = 2$, in the scenario in which the bino is the lightest neutralino. The upper triangular region in green is excluded by the lower bound requirement of the lightest neutralino mass from LEP2 and by the lower bound on the lightest sfermion mass constraint. The light blue region is excluded by WMAP measurements. The gray vertical strip on the left is excluded by $b \rightarrow s\gamma$. The white strip agrees with the WMAP at the 2σ level. The anomalous magnetic moment of the muon is satisfied within the entire region.

channel available.⁶ Because of the s -wave suppression, the fermion pair production for heavy fermions is dominant, but we include all possible channels. Unlike the MSSM bino, the $B - L$ bino does not couple to $W^+ W^-$ pairs or to Higgs pairs. For the exchanged sfermions, a flavor conserving scenario with a common scale M_{SUSY} is assumed. In the figure, the upper triangular region in green is excluded by the lower mass bound requirement on the lightest neutralino from LEP2 and by the lower bound of the lightest sfermion mass determined mainly by M_{SUSY} . (We calculated the masses of the sfermions and require that they are at least 15 GeV higher than the mass of the bino to avoid coannihilation channels.) The light blue regions are excluded by the WMAP measurements. In addition, the gray vertical strip on the left is excluded by the branching ratio of $b \rightarrow s\gamma$ upper bound, for which the formulas are given in the Appendix. We assumed that $2.4 \times 10^{-4} \leq BR(b \rightarrow s\gamma) \leq 4.1 \times 10^{-4}$ range is allowed. In numerical calculation, we used the following set of parameter values, consistent with pure bino scenario: $M_L = M_R = 3$ TeV, $m_{\tilde{g}} = 400$ GeV, $A = 200$ GeV, $\mu_1 = 500$ GeV, $\mu_2 = 4$ TeV, $\nu_{\delta} = 100$ GeV, and $\nu_{\Delta} = 1.5$ TeV.⁷ As seen from the figure the $BR(b \rightarrow s\gamma)$ constraint excludes the light M_{SUSY} (≤ 150 GeV) region. We also take into account the anomalous magnetic moment of muon $a_{\mu} = (g - 2)_{\mu}/2$, where the analytical formulas are given in the Appendix. The dominant chargino-sneutrino and neutralino-smuon loop contributions are included. In numerical computation, like $BR(b \rightarrow s\gamma)$, the same parameter set is assumed and the range for the deviation from the SM, $-0.53 \times 10^{-10} \leq \Delta a_{\mu} \leq 44.7 \times 10^{-10}$, at 95% C.L. is used based on $e^+ e^-$ data (see [32] and references therein). Δa_{μ} depends mainly on the chosen values of A , μ_1 , $\tan\beta$, and M_{SUSY} . We observe that it remains in the allowed range throughout the entire (M_{SUSY}, M_V) interval considered, as long as $\mu_1 \tan\beta - A \geq 0$ holds.

Consequently, the white strip is the only region satisfying the WMAP at the 2σ level. A realization of such a scenario requires light sfermion masses in the range ~ 150 – 240 GeV with an acceptable LSP mass in the region ~ 60 – 220 GeV. In the light blue region on the right, the relic density $\Omega_{\chi} h^2|_{\chi=\lambda_V}$ becomes larger than experimentally allowed values, while in the left painted region it is smaller than experimental requirements. This scenario can only be considered feasible in frameworks where there is at least one light sfermion, like stop \tilde{t} , or stau $\tilde{\tau}$.

Next, in Fig. 7 we show the relic density $\Omega_{\chi} h^2$ of the neutralino as a pure right-wino state in the $M_R - M_{\text{SUSY}}$ plane for $\mu_1 = 1$ TeV, $A = 200$ GeV, and $\tan\beta = 2$. The

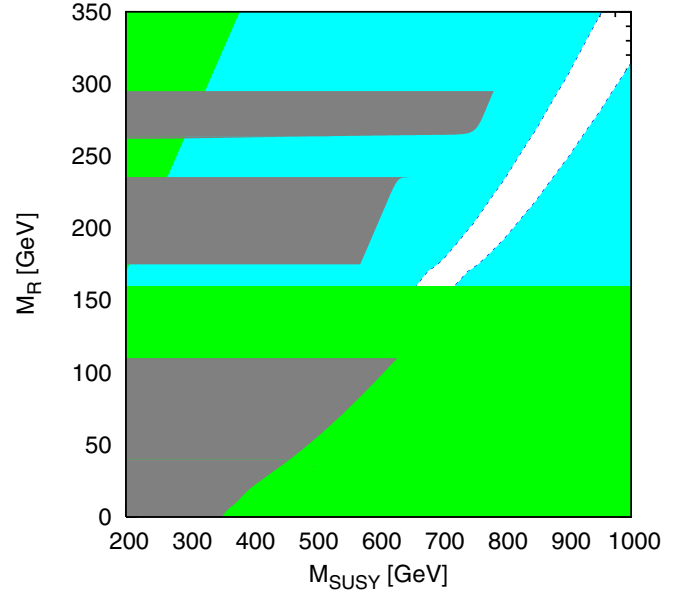


FIG. 7 (color online). The relic density of a pure right-wino state in the $M_R - M_{\text{SUSY}}$ plane for $\mu_1 = 1$ TeV, $A = 200$ GeV, and $\tan\beta = 2$ in the scenario in which the right-wino is the lightest neutralino. The left upper small triangular and the lower rectangular regions in green are excluded by the lower bound mass requirement on the lightest neutralino from LEP2 and by the lower bound on the lightest sfermion mass. The light blue region is excluded by WMAP measurements. The gray region is excluded by $b \rightarrow s\gamma$. The white strip agrees with the WMAP at the 2σ level. The anomalous magnetic moment of the muon is satisfied within the entire region.

right-wino as a neutralino is a completely new possibility peculiar to the LRSUSY model. There is no s -channel contribution and the possible annihilation contributions are $\lambda_R^0 \bar{\lambda}_R^0 \rightarrow \tilde{f} f \tilde{f}$, ($f = u, c, t, d, b, s \dots$) and $\lambda_R^0 \bar{\lambda}_R^0 \rightarrow \tilde{\phi}^{\pm} h^0 h^0$, when kinematically allowed. An important feature in this case is that there exists an additional condition—the LEP lower bound for the lightest chargino, which we take as 90 GeV. We also ensure that the lightest chargino remains always heavier than the lightest neutralino, at least 15 GeV heavier, to avoid significant contributions from coannihilation channels. For the mixing in the chargino sector, we assume all parameters (except M_R and ν_{δ}) large enough to obtain a pure right-wino state. Using $\nu_{\delta} = 30$ GeV, a 100 GeV or larger mass is obtained for the right-wino, while fulfilling the bound on the mass of the lightest chargino. In order to obtain a neutralino lighter than the lightest chargino, M_R should be at least 160 GeV or larger; hence, the lower rectangular excluded region (in green) in Fig. 7. For illustrative purposes only, we have included, in Table I, the values of the masses of the LSP, as well as the masses of the second lightest neutralino, the lightest chargino, and the lightest sfermion, for all three scenarios investigated. As in the case of pure bino, the mass upper bound comes from sfermion sector. After implementing WMAP constraint, the only surviving region

⁶We recall that the bino here is the fermionic partner of the $U(1)_{B-L}$ gauge boson and is different from the bino in the MSSM.

⁷We ensure at all times that the bino is not just the lightest neutralino, but a pure state.

TABLE I. Representative values for the masses of the LSP (χ_1^0), the next-lightest neutralino (χ_2^0), lightest chargino (χ_1^\pm), and the lightest sfermion (\tilde{f}_1) for the three scenarios described in this section. The masses are given in GeV. The parameter sets are $(M_L, M_R, M_V, \mu_1, \mu_2, M_{\text{SUSY}}, v_\delta) = (1, 1, 0.13, 1, 2, 0.2, 0.05)$ TeV for Set (1), $(M_L, M_R, M_V, \mu_1, \mu_2, M_{\text{SUSY}}, v_\delta) = (1, 0.2, 1, 1, 4, 0.8, 0.01)$ TeV for Set (2), and $(M_L, M_R, M_V, \mu_1, \mu_2, M_{\text{SUSY}}, v_\delta) = (2, 2, 2, 2, 0.85, *, 0.1)$ TeV for Set (3). For all three sets, $\tan\beta = 2$, $A = 0.2$ TeV, and $v_\Delta = 1.5$ TeV are used. “*” indicates that the entry depends on M_{SUSY} which can be taken arbitrarily large.

Scenario	LSP	$M_{\chi_1^0}$	$M_{\chi_2^0}$	$M_{\chi_1^\pm}$	$M_{\tilde{f}_1}$	$\Omega_\chi h^2$	Parameter set
1	bino	130.6	472.2	500.0	176.2	0.106	Set (1)
2	<i>R</i> -wino	177.1	924.5	196.4	713.4	0.121	Set (2)
3	$\tilde{\delta}_R^0$ Higgsino	412.5	1426.5	713.4	*	0.115	Set (3)

at the 2σ level is shown in the white strip. This scenario satisfies the WMAP conditions for a relatively large M_{SUSY} scale, 600 GeV or larger. Note that the upper bound for the mass of the right-wino is closely related to the chosen values for the other gaugino masses, the vev's, and μ_2 , assumed heavy. For larger M_R values, the lightest neutralino will not longer be a pure state. But one can see from Fig. 7 that it is not difficult to satisfy the WMAP result. Here too the constraints $b \rightarrow s\gamma$ and $(g-2)_\mu$ of the muon are taken into account with the parameter set, $M_L = M_V = 1.5$ TeV, $m_{\tilde{g}} = 400$ GeV, $A = 200$ GeV, $\mu_1 = 500$ GeV, $\mu_2 = 4$ TeV, $v_\delta = 100$ GeV, and $v_\Delta = 1.5$ TeV. The $b \rightarrow s\gamma$ constraint for this case excludes mainly $M_{\text{SUSY}} \leq 600$ GeV region but practically does not effect the WMAP allowed region. $(g-2)_\mu$ does not constrain the parameter space, as a light right-wino does not contribute signifi-

cantly to the muon anomalous magnetic moment, and the other neutralinos are heavy.

In the third scenario we investigate the possibility that the $\tilde{\delta}_R^0$ is the lightest neutralino. This an interesting scenario, as $\tilde{\delta}_R^0$ is only introduced to cancel anomalies in the fermionic sector, and, because of its $B-L$ quantum number, its direct interactions with matter are forbidden. The relic density prediction for such scenario is shown in Fig. 8, as a function of the Higgsino mass $M_{\tilde{\delta}_R^0}$, for $v_\delta = 100$ GeV, $v_\Delta = 1.5$ TeV. Only the s -channel contribution is available for the annihilation process $\tilde{\delta}_R^0 \tilde{\delta}_R^0 \rightarrow Z_R f \bar{f}$, ($f = u, c, t, d, b, s \dots$). Note that, for this process, the cross section and thus the relic density is formally independent of M_{SUSY} , since $\tilde{\delta}_R^0$ cannot interact with sfermions. The $\Delta\rho$ bound does not allow values for v_Δ smaller than 1.5 TeV, once we assume a relatively small v_δ . Then the mass of the exchange particle Z_R becomes heavy, around 1.7 TeV, yielding suppressed cross sections. This is why we obtain a large relic density $\Omega_\chi h^2|_{\chi=\tilde{\delta}_R^0}$, which is inversely proportional to the thermally averaged cross section. Only larger masses for the pure Higgsino lead to smaller relic density values. In that case, as well as having more phase space available, one is closer to the Z_R resonance, which increases the annihilation cross section. So, as seen from the figure, the WMAP at the 2σ level is satisfied if the mass of the Higgsino lies in 400–450 GeV narrow range. Neither $b \rightarrow s\gamma$ nor $(g-2)_\mu$ receive contributions from a $\tilde{\delta}_R^0$ Higgsino. Unfortunately the WMAP allowed region at 2σ level restricts the LSP $\tilde{\delta}_R^0$ mass to a very small interval.

We will summarize our result in the next section.

VI. CONCLUSION

In this study we have considered the neutralino sector of the LRSUSY model and concentrated on the lightest neutralino state as dark matter, motivated from the fact that WMAP result requires considerations of beyond MSSM models. Now, we would like to summarize our findings.

For the pure states in the LRSUSY model; if the masses of the supersymmetric partners (in both bosonic and fermionic sectors) are very small, the lightest neutralino is

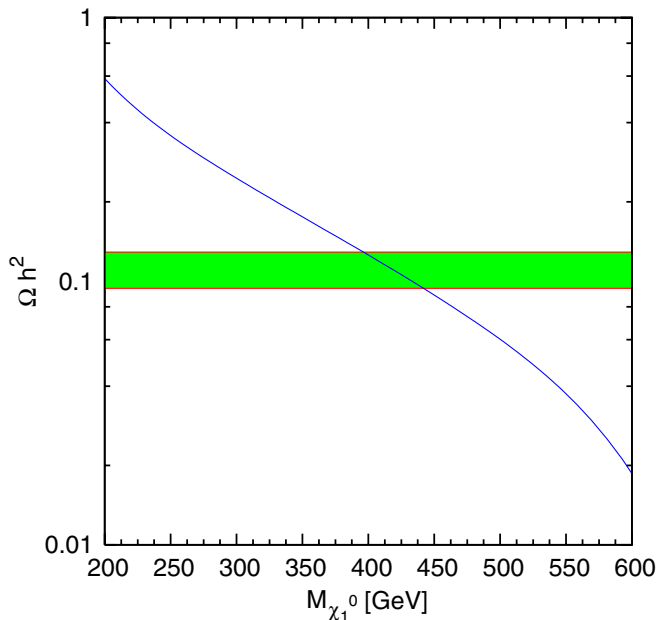


FIG. 8 (color online). The relic density of a pure Higgsino $\tilde{\delta}_R^0$ as a function of its mass for $v_\delta = 100$ GeV, $v_\Delta = 1.5$ TeV, in the scenario in which the Higgsino is the lightest neutralino. The green strip is the allowed region by the WMAP at the 2σ level.

most likely to be a pure $B - L$ bino. This covers regions with $M_{\chi_1^0} \sim 60\text{--}220$ GeV, and $M_{\text{SUSY}} \sim 150\text{--}250$ GeV. For intermediate to larger values for the masses of the supersymmetric partners, the lightest neutralino is most likely to be pure right-wino; this covers regions with $M_{\chi_1^0} \sim 150\text{--}350$ GeV, and $M_{\text{SUSY}} \sim 600\text{--}1000$ GeV. Finally if the mass of the lightest neutralino is large, in the 400–450 GeV region, the state could be pure $\tilde{\delta}_R^0$ -Higgsino. The mass region is severely restricted for pure Higgsino masses, but completely independent of M_{SUSY} , as long as $M_{\text{SUSY}} > 500$ GeV, so that none of the sfermions are lighter than the lightest neutralino.

The analysis above shows that the lightest neutralino as pure bino shares some features with the similar scenario in MSSM. Both are possible only if the scalars are light, especially for $M_V \sim 60$ GeV. Outside this region, one obtains a relic density that is too large. This similarity is not unexpected: although the bino here is the fermionic partner of the $B - L$ gauge boson, and not the hypercharge gauge boson, the same dominant decay channels are open to both bins, so the cross sections are of the same order of magnitude.

The situation changes when we analyze the case in which the lightest neutralino is the right-wino. In MSSM, the case where the lightest neutralino is the left-wino requires a left-wino mass of ~ 2.5 TeV to satisfy dark matter limits. For our case, right-wino masses in the 150–350 GeV regions are in good agreement with the relic density constraints. The reason for this manifest difference is that the MSSM left-wino cross section is dominated by the decay into $W_L^+ W_L^-$ pairs, which is not available for the right-wino.

The s -channel is available only for the $\tilde{\delta}_R^0$ -Higgsino, but this channel is suppressed by the large mass of the Z_R boson propagator. The cross section is too small for light neutralinos, and the relic density is too large. It is less likely that the lightest neutralino would be the $\tilde{\delta}_R^0$ -Higgsino, since only a very small mass range satisfies the dark matter constraints. In MSSM, the LSP is even less likely to be the (MSSM) Higgsino (see the recent study [33]). In LRSUSY, its annihilation cross section into W_L and Z_L gauge boson pairs is unsuppressed, and the relic density is too small, unless $\mu_1 > 1$ TeV.

Our scenarios are very different from those present in the NMSSM or in other extensions of the MSSM, where a very light bino, or a singlino, or their mixture could be the LSP [12–14]. There the cross section is dominated by a light CP -odd Higgs boson or additional Higgs resonances, and these particles only couple to no SM particles except for the Higgs doublets.

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APPENDIX: CONSTRAINTS

We include, for completeness, the constraints on the parameter space coming from the anomalous magnetic moment of the muon Δa_μ and the branching ratio for $b \rightarrow s\gamma$. Constraints coming from $B_s \rightarrow \mu^+ \mu^-$ are more important than those coming from $b \rightarrow s\gamma$ only for $\tan\beta > 11$ [34], which does not affect the calculation for our choice of parameters. For larger $\tan\beta$ values, we expect the values for the masses of the supersymmetric partners to shift to larger values.

In addition to $b \rightarrow s\gamma$ and Δa_μ , we have included constraints from the LEP limits on supersymmetric masses; we have constrained the mass of the next-to-lightest supersymmetric particle to be at least 15 GeV heavier than the lightest neutralino mass (to avoid coannihilations); and we have restricted the Higgs vev's so the ρ parameter

$$\begin{aligned} \rho^{-1} &= \frac{M_{Z_L}^2 \cos^2 \theta_W}{M_{W_L}^2} \\ &= 1 - \frac{1}{4} \frac{\cos^2 2\theta_W}{\cos^4 \theta_W} \frac{\kappa_1^2 + \kappa_2^2}{v_\Delta^2 + v_\delta^2} + \mathcal{O}\left[\left(\frac{\kappa_1^2 + \kappa_2^2}{v_\Delta^2 + v_\delta^2}\right)^2\right] \end{aligned} \quad (\text{A1})$$

is consistent with its experimental limits $\rho = 0.9998_{-0.0010}^{+0.0025}$ at the 2σ level [28].

1. $(g - 2)_\mu$

In addition to the relic density, another experimental result that can constrain the $M_{1/2} - M_{\text{SUSY}}$ parameter region is the BNL E821 measurement of the muon anomalous magnetic moment $a_\mu \equiv (g - 2)_\mu/2$ [35]:

$$a_\mu(\text{exp}) = 11\,659\,208(6) \times 10^{-10}, \quad (\text{A2})$$

which indicates that the anomalous magnetic moment may need additional contributions beyond the SM to be consistent with the experimental values. There are however uncertainties in how big the deviation from SM really is. A 2.4σ deviation is obtained if the SM prediction of hadronic contributions from $e^+ e^-$ data is used; if instead we use the data from indirect hadronic τ decays the disagreement with the muon anomalous magnetic moment is reduced to a 0.9σ deviation. We will use, from the 95% C.L. range [32],

$$-0.53 \times 10^{-10} \leq \Delta a_\mu \leq 44.7 \times 10^{-10}. \quad (\text{A3})$$

The dominant contributions to a_μ in LRSUSY come from the chargino-muon sneutrino and neutralino-smuon loops.

These are [36] $a_\mu = a_\mu^{\chi^\pm} + a_\mu^{\chi^0}$, where

$$a_{L\mu}^{\chi^\pm} = \sum_{k=1}^4 \frac{m_\mu}{16\pi^2} M_{\chi_k^\pm} g Y_\mu \operatorname{Re}[V_{k1} U_{k3}] \frac{F_3(x_{k\mu})}{m_{\tilde{\nu}_\mu}^2} \quad (\text{A4})$$

is the chargino contribution. The dominant neutralino contribution coming from neutralino-left slepton graphs is

$$\begin{aligned} a_{L\mu}^{\chi^0} = & \sum_{k=1}^7 \frac{m_\mu}{16\pi^2} M_{\chi_k^0} g \left[\sqrt{2} Y_\mu (N_{k1}) \right. \\ & + \tan^2 \theta_W N_{k3} N_{5k} \frac{F_4(y_{k\mu_L})}{m_{\tilde{\mu}_L}^2} + g(N_{2k} \\ & - 2 \tan^2 \theta_W N_{k3})(N_{k1} + \tan^2 \theta_W N_{k3}) \\ & \left. \times \frac{m_\mu (A_\mu - \mu_1 \tan \beta)}{m_{\tilde{\mu}_R}^2} \frac{F_4(y_{k\mu_L})}{m_{\tilde{\mu}_L}^2} \right] \quad (\text{A5}) \end{aligned}$$

and, from the neutralino-right slepton graphs,

$$\begin{aligned} a_{R\mu}^{\chi^0} = & \sum_{k=1}^7 \frac{m_\mu}{16\pi^2} M_{\chi_k^0} g \left[\frac{1}{\sqrt{2}} Y_\mu (N_{k2}) \right. \\ & - 2 \tan^2 \theta_W N_{k3} N_{k5} \frac{F_4(y_{k\mu_R})}{m_{\tilde{\mu}_R}^2} + g(N_{k1} \\ & + \tan^2 \theta_W N_{k3})(N_{k2} - 2 \tan^2 \theta_W N_{k3}) \\ & \left. \times \frac{m_\mu (A_\mu - \mu_1 \tan \beta)}{m_{\tilde{\mu}_L}^2} \frac{F_4(y_{k\mu_R})}{m_{\tilde{\mu}_R}^2} \right] \quad (\text{A6}) \end{aligned}$$

with the loop functions $F_3(x)$, $F_4(x)$ given in Eq. (A22) and where $x_{k\mu} = M_{\chi_k^\pm}^2/m_{\tilde{\nu}_\mu}^2$, $y_{k\mu_{L,R}} = M_{\chi_k^0}^2/m_{\tilde{\mu}_{L,R}}^2$, and U , V (and N) are matrices that diagonalize the chargino (and neutralino) mass matrices.

2. $b \rightarrow s\gamma$

The inclusive decay width for the process $b \rightarrow s\gamma$ is given by

$$\Gamma(b \rightarrow s\gamma) = \frac{m_b^5 G_F^2 \alpha}{32\pi^4} (\hat{M}_{\gamma L}^2 + \hat{M}_{\gamma R}^2), \quad (\text{A7})$$

where \hat{M} means evolving down to the decay scale $\mu = m_b$. The branching ratio can be expressed as

$$BR(b \rightarrow s\gamma) = \frac{\Gamma(b \rightarrow s\gamma)}{\Gamma_{SL}} BR_{SL}, \quad (\text{A8})$$

where the semileptonic branching ratio $BR_{SL} = BR(b \rightarrow c e \bar{\nu}) = (10.49 \pm 0.46)\%$ and

$$\Gamma_{SL} = \frac{m_b^5 G_F^2 |(K_{CKM})_{cb}|^2}{192\pi^3} g(z), \quad (\text{A9})$$

where $z = m_c^2/m_b^2$ and $g(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \log z$. The experimental measurement from CLEO can be expressed as [37]

$$BR(b \rightarrow s\gamma) = (3.21 \pm 0.43 \pm 0.27_{-0.10}^{+0.18}) \times 10^{-4}. \quad (\text{A10})$$

The SM contribution is

$$\begin{aligned} A_{SM} = & \frac{\pi \alpha_W}{2\sqrt{2} G_F M_W^2} (K_{CKM})_{ts}^* (K_{CKM})_{tb} 3x_{tW} [Q_u F_1(x_{tW}) \\ & + F_2(x_{tW})]. \quad (\text{A11}) \end{aligned}$$

The matrix elements responsible for the $b \rightarrow s\gamma$ decay acquire the following contributions from the supersymmetric sector of the model [38]. For b_L decay,

$$M_{\gamma_R} = A_{\tilde{g}}^R + A_{\chi^-}^R + A_{\chi^0}^R \quad (\text{A12})$$

with the gluino, chargino, and neutralino contributions given by

$$\begin{aligned} A_{\tilde{g}}^R = & -\frac{\sqrt{2}\pi\alpha_s}{G_F} Q_d C(R) \sum_{k=1}^6 \frac{1}{m_{\tilde{d}_k}^2} \left\{ \Gamma_{DL}^{kb} \Gamma_{DL}^{*ks} F_2(x_{\tilde{g}\tilde{d}_k}) \right. \\ & \left. - \frac{m_{\tilde{g}}}{m_b} \Gamma_{DR}^{kb} \Gamma_{DL}^{*ks} F_4(x_{\tilde{g}\tilde{d}_k}) \right\}, \quad (\text{A13}) \end{aligned}$$

$$\begin{aligned} A_{\chi^-}^R = & -\frac{\pi\alpha_W}{\sqrt{2}G_F} \sum_{j=1}^5 \sum_{k=1}^6 \frac{1}{m_{\tilde{u}_k}^2} \left\{ (G_{UL}^{jkb} - H_{UR}^{jkb})(G_{UL}^{*jks} - H_{UR}^{*jks}) \right. \\ & \times [F_1(x_{\chi_j \tilde{u}_k}) + Q_u F_2(x_{\chi_j \tilde{u}_k})] + \frac{m_{\chi_j}}{m_b} (G_{UR}^{jkb} - H_{UL}^{jkb}) \\ & \left. \times (G_{UL}^{*jks} - H_{UR}^{*jks}) [F_3(x_{\chi_j \tilde{u}_k}) + Q_u F_4(x_{\chi_j \tilde{u}_k})] \right\}, \quad (\text{A14}) \end{aligned}$$

$$\begin{aligned} A_{\chi^0}^R = & -\frac{\pi\alpha_W}{\sqrt{2}G_F} Q_d \sum_{j=1}^9 \sum_{k=1}^6 \frac{1}{m_{\tilde{d}_k}^2} \left\{ (\sqrt{2}G_{ODL}^{jkb} - H_{ODR}^{jkb}) \right. \\ & \times (\sqrt{2}G_{ODL}^{*jks} - H_{ODR}^{*jks}) F_2(x_{\chi_j^0 \tilde{d}_k}) \\ & + \frac{m_{\chi_j^0}}{m_b} (\sqrt{2}G_{ODR}^{jkb} - H_{ODL}^{jkb}) \\ & \left. \times (\sqrt{2}G_{ODL}^{*jks} - H_{ODR}^{*jks}) F_4(x_{\chi_j^0 \tilde{d}_k}) \right\}, \quad (\text{A15}) \end{aligned}$$

and, for the decay of b_R ,

$$M_{\gamma_L} = A_{\tilde{g}}^L + A_{\chi^-}^L + A_{\chi^0}^L \quad (\text{A16})$$

again, with the following gluino, chargino, and neutralino contributions:

$$\begin{aligned} A_{\tilde{g}}^L = & -\frac{\sqrt{2}\pi\alpha_s}{G_F} Q_d C(R) \sum_{k=1}^6 \frac{1}{m_{\tilde{d}_k}^2} \left\{ \Gamma_{DR}^{kb} \Gamma_{DR}^{*ks} F_2(x_{\tilde{g}\tilde{d}_k}) \right. \\ & \left. - \frac{m_{\tilde{g}}}{m_b} \Gamma_{DL}^{kb} \Gamma_{DR}^{*ks} F_4(x_{\tilde{g}\tilde{d}_k}) \right\}, \quad (\text{A17}) \end{aligned}$$

$$\begin{aligned} A_{\chi^-}^L = & -\frac{\pi\alpha_W}{\sqrt{2}G_F} \sum_{j=1}^5 \sum_{k=1}^6 \frac{1}{m_{\tilde{u}_k}^2} \left\{ (G_{UR}^{jkb} - H_{UL}^{jkb})(G_{UR}^{*jks} - H_{UL}^{*jks}) \right. \\ & \times [F_1(x_{\chi_j \tilde{u}_k}) + Q_u F_2(x_{\chi_j \tilde{u}_k})] \\ & + \frac{m_{\chi_j}}{m_b} (G_{UL}^{jkb} - H_{UR}^{jkb})(G_{UR}^{*jks} - H_{UL}^{*jks}) \\ & \left. \times [F_3(x_{\chi_j \tilde{u}_k}) + Q_u F_4(x_{\chi_j \tilde{u}_k})] \right\}, \quad (\text{A18}) \end{aligned}$$

$$\begin{aligned}
A_{\chi^0}^L = & -\frac{\pi\alpha_W}{\sqrt{2}G_F} Q_d \sum_{j=1}^9 \sum_{k=1}^6 \frac{1}{m_{\tilde{d}_k}^2} \left\{ (\sqrt{2}G_{0DR}^{jkb} - H_{0DL}^{jkb}) \right. \\
& \times (\sqrt{2}G_{0DR}^{*jks} - H_{0DL}^{*jks}) F_2(x_{\chi_j^0 \tilde{d}_k}) \\
& + \frac{m_{\chi_j^0}}{m_b} (\sqrt{2}G_{0DL}^{jkb} - H_{0DR}^{jkb}) \\
& \left. \times (\sqrt{2}G_{0DR}^{*jks} - H_{0DL}^{*jks}) F_4(x_{\chi_j^0 \tilde{d}_k}) \right\}, \quad (A19)
\end{aligned}$$

where the chargino-quark-squark mixing matrices G and H are defined as

$$\begin{aligned}
G_{UL}^{jki} &= V_{j1}^* (\Gamma_{UL})_{ki}, & G_{UR}^{jki} &= U_{j2} (\Gamma_{UR})_{ki} \\
H_{UL}^{jki} &= \frac{1}{\sqrt{2}m_W} \left(\frac{m_{u_i}}{\sin\beta} (K_{CKM})_{il} U_{j3} + \frac{m_{d_l}}{\cos\beta} \delta_{il} U_{j4} \right) (\Gamma_{UL})_{kb}, \\
H_{UR}^{jki} &= \frac{1}{\sqrt{2}m_W} \left(\frac{m_{u_i}}{\sin\beta} (K_{CKM})_{il} V_{j3}^* + \frac{m_{d_l}}{\cos\beta} \delta_{il} V_{j4}^* \right) (\Gamma_{UR})_{kb}, \quad (A20)
\end{aligned}$$

and where the neutralino-quark-squark mixing matrices G_0 and H_0 are defined as

$$\begin{aligned}
G_{0DL}^{jki} &= \left\{ -\left[\tan^2\theta_W \left(T_d^3 - Q_d - \frac{Q_u + Q_d}{2} \right) \right. \right. \\
& \quad \left. \left. + \frac{Q_u + Q_d}{2} \right] N_{j1}^* - \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W} \tan\theta_W \left(T_d^3 - Q_d \right. \right. \\
& \quad \left. \left. - \frac{Q_u + Q_d}{2} \right) N_{j2}^* + T_d^3 N_{j3}^* \right\} (\Gamma_{DL})_{ki}, \\
G_{0DR}^{jki} &= \left\{ -\frac{\cos 2\theta_W}{\cos^2\theta_W} (T_d^3 - 2Q_d \sin^2\theta_W) N_{j1} \right. \\
& \quad \left. + \sqrt{\cos 2\theta_W} \sin\theta_W \left[Q_d + \frac{1}{\cos^2\theta_W} \right. \right. \\
& \quad \left. \left. \times (T_d^3 - 2Q_d \sin^2\theta_W) \right] N_{j2} + 2\sin^2\theta_W Q_d N_{j3} \right\} (\Gamma_{DR})_{ki}, \\
H_{0DL}^{jki} &= \frac{1}{\sqrt{2}m_W} \left(\frac{m_{u_i}}{\sin\beta} N_{j4} - \frac{m_{d_l}}{\cos\beta} N_{j5} \right) (\Gamma_{DL})_{ki}, \\
H_{0DR}^{jki} &= \frac{1}{\sqrt{2}m_W} \left(\frac{m_{u_i}}{\sin\beta} N_{j4}^* + \frac{m_{d_l}}{\cos\beta} N_{j5}^* \right) (\Gamma_{DR})_{ki}, \quad (A21)
\end{aligned}$$

and where the matrices $\Gamma_{U,D}$ diagonalize the squark mass matrices in the up and down sectors. The functions appearing in the expressions above are

$$\begin{aligned}
F_1(x) &= \frac{1}{12(x-1)^4} (x^3 - 6x^2 + 3x + 2 + 6x \log x), \\
F_2(x) &= \frac{1}{12(x-1)^4} (2x^3 + 3x^2 - 6x + 1 - 6x^2 \log x), \\
F_3(x) &= \frac{1}{2(x-1)^3} (x^2 - 4x + 3 + 2 \log x), \\
F_4(x) &= \frac{1}{2(x-1)^3} (x^2 - 1 - 2x \log x). \quad (A22)
\end{aligned}$$

The convention $x_{ab} = m_a^2/m_b^2$ is used. $C(R) = 4/3$ is the quadratic Casimir operator of the fundamental representation of $SU(3)_C$.

In order to compare the results obtained with experimental branching ratios, QCD corrections must be taken into account. We assume the renormalization group evolution pattern. There is no mixing between left- and right-handed contributions:

$$\begin{aligned}
A^\gamma(m_b) &= \eta^{-16/23} \left\{ A^\gamma(M_W) + A_0^\gamma \left[\frac{116}{135} (\eta^{10/23} - 1) \right. \right. \\
& \quad \left. \left. + \frac{58}{189} (\eta^{28/23} - 1) \right] \right\}, \quad (A23)
\end{aligned}$$

where $\eta = \alpha_s(m_b)/\alpha_s(M_W)$ and $A_0^\gamma = \frac{\pi\alpha_W}{2\sqrt{2}G_F M_W^2} \times (K_{CKM})_{is}^* (K_{CKM})_{tb}$. We choose the renormalization scale to be $\mu = m_b = 4.2$ GeV.

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- [1] H. V. Peiris *et al.*, *Astrophys. J. Suppl. Ser.* **148**, 213 (2003); M. R. Nolte *et al.*, *Astrophys. J.* **608**, 10 (2004).
[2] H. Goldberg, *Phys. Rev. Lett.* **50**, 1419 (1983); J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, *Nucl. Phys.* **B238**, 453 (1984).
[3] K. A. Olive, hep-ph/0412054; C. Boehm, A. Djouadi, and M. Drees, *Phys. Rev. D* **62**, 035012 (2000); J. R. Ellis, K. A. Olive, Y. Santoso, and V. C. Spanos, *Phys. Lett. B* **565**, 176 (2003).
[4] N. Arkani-Hamed, A. Delgado, and G. F. Giudice, *Nucl. Phys.* **B741**, 108 (2006).
[5] K. Griest and D. Seckel, *Phys. Rev. D* **43**, 3191 (1991).
[6] P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, M. Schelke, and E. A. Baltz, *J. Cosmol. Astropart. Phys.* **07** (2004) 008.
[7] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov, *Comput. Phys. Commun.* **174**, 577 (2006); *Comput. Phys. Commun.* **149**, 103 (2002).
[8] M. E. Gomez, G. Lazarides, and C. Pallis, *Phys. Lett. B* **487**, 313 (2000).
[9] U. Chattopadhyay, A. Corsetti, and P. Nath, *Phys. Rev. D* **68**, 035005 (2003); R. Arnowitt, B. Dutta, B. Hu, and Y. Santoso, *Phys. Lett. B* **505**, 177 (2001).
[10] V. D. Barger and C. Kao, *Phys. Rep.* **307**, 207 (1998).
[11] H. Baer, T. Krupovnickas, A. Mustafayev, E. K. Park, S.

- Profumo, and X. Tata, hep-ph/0511034; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev, and X. Tata, J. High Energy Phys. 07 (2005) 065.
- [12] G. Belanger, F. Boudjema, C. Hugonie, A. Pukhov, and A. Semenov, J. Cosmol. Astropart. Phys. 09 (2005) 001; V. Barger, P. Langacker, and H. S. Lee, AIP Conf. Proc. **805**, 306 (2005); V. Barger, C. Kao, P. Langacker, and H. S. Lee, Phys. Lett. B **600**, 104 (2004); J.F. Gunion, D. Hooper, and B. McElrath, Phys. Rev. D **73**, 015011 (2006).
- [13] B. de Carlos and J.R. Espinosa, Phys. Lett. B **407**, 12 (1997).
- [14] V. Barger, P. Langacker, and H. S. Lee, Phys. Lett. B **630**, 85 (2005).
- [15] C. S. Aulakh, B. Bajc, A. Melfo, A. Rasin, and G. Senjanovic, Phys. Lett. B **460**, 325 (1999); C. S. Aulakh, A. Melfo, A. Rasin, and G. Senjanovic, Phys. Lett. B **459**, 557 (1999).
- [16] K. Huitu and J. Maalampi, Phys. Lett. B **344**, 217 (1995).
- [17] M. Frank, M. Sher, and I. Turan, Phys. Rev. D **71**, 113002 (2005); M. Frank, I. Turan, and M. Sher, Phys. Rev. D **71**, 113001 (2005).
- [18] B. Dutta, Y. Mimura, and R. N. Mohapatra, Phys. Rev. D **72**, 075009 (2005).
- [19] R. M. Francis, M. Frank, and C. S. Kalman, Phys. Rev. D **43**, 2369 (1991).
- [20] C. S. Aulakh, A. Melfo, and G. Senjanovic, Phys. Rev. D **57**, 4174 (1998).
- [21] Z. Chacko and R. N. Mohapatra, Phys. Rev. D **58**, 015003 (1998); B. Dutta and R. N. Mohapatra, Phys. Rev. D **59**, 015018 (1999).
- [22] R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).
- [23] C. S. Aulakh, K. Benakli, and G. Senjanovic, Phys. Rev. Lett. **79**, 2188 (1997).
- [24] M. Frank, Phys. Lett. B **540**, 269 (2002).
- [25] C. S. Aulakh, A. Melfo, A. Rasin, and G. Senjanovic, Phys. Rev. D **58**, 115007 (1998).
- [26] M. Frank and P. Pnevmonidis, Phys. Rev. D **67**, 015010 (2003).
- [27] M. Frank and I. Turan, Phys. Rev. D **72**, 035008 (2005).
- [28] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
- [29] P. Gondolo and G. Gelmini, Nucl. Phys. **B360**, 145 (1991).
- [30] H. Baer, C. Balazs, A. Belyaev, and J. O’Farrill, J. Cosmol. Astropart. Phys. 09 (2003) 007; H. Baer, C. Balazs, and A. Belyaev, J. High Energy Phys. 03 (2002) 042.
- [31] R. Mertig and J. Kublbeck, prepared for International Workshop on Software Engineering, Artificial Intelligence and Expert Systems for High-energy and Nuclear Physics, Lyon, France, 1990; J. Kublbeck, H. Eck, and R. Mertig, Nucl. Phys. A, Proc. Suppl. **29**, 204 (1992).
- [32] G. Lazarides, hep-ph/0601016.
- [33] U. Chattopadhyay, D. Choudhury, M. Drees, P. Konar, and D. P. Roy, Phys. Lett. B **632**, 114 (2006).
- [34] M. Frank and S. Nie, Phys. Rev. D **66**, 055001 (2002).
- [35] G. W. Bennett *et al.* (Muon g-2 Collaboration), Phys. Rev. Lett. **92**, 161802 (2004).
- [36] M. Frank, Phys. Rev. D **59**, 013003 (1999).
- [37] S. Chen *et al.* (CLEO Collaboration), Phys. Rev. Lett. **87**, 251807 (2001).
- [38] M. Frank and S. Nie, Phys. Rev. D **65**, 114006 (2002).