DESIGN OF TRANSFORMABLE DOUBLY CURVED SURFACE COMPOSED OF SCISSOR LINKAGE MECHANISMS

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ABSTRACT

DESIGN OF TRANSFORMABLE DOUBLY CURVED SURFACE COMPOSED OF SCISSOR LINKAGE MECHANISMS

This study focuses on obtaining multi-loop scissor linkages capable of defining doubly curved surfaces that can transform from an initial form to a desired final form. A review of the existing literature reveals a limited number of studies addressing curvature transformation between different states. Most transformable designs are restricted to planar movements, while three-dimensional forms are typically achieved through the translational repetition of planar scissor linkages. Additionally, when scissor linkages are arranged in a grid, they often exhibit multiple degrees of freedom, making controlled transformations challenging. To address these limitations, this research introduces a novel geometric design method for multi-loop planar scissor linkages that enables transformation between predefined curves. The proposed approach utilizes quadrilateral loops to construct planar scissor linkages capable of achieving distinct curved forms. Furthermore, the study extends this concept to spatial scissor linkages that can dynamically alter their curvature between predefined surface geometries. The research employs simulation and modeling as primary methods. Computer simulations are used to develop the proposed models, while 3D-printed prototypes are produced to analyze their geometric behavior. These tools facilitate a comprehensive investigation of the transformability and structural performance of the designed mechanisms.

ÖZET

MAKAS ÇUBUK MEKANİZMALARDAN OLUŞAN BİÇİM DEĞİŞTİREBİLEN ÇİFT EĞRİLİKLİ YÜZEYLERİN TASARIMI

Bu çalışma, başlangıç formundan istenen nihai forma dönüşebilen ve çift eğrilikli yüzeyler tanımlayabilen, çok döngülü makas mekanizmaları elde etmeye odaklanmaktadır. Mevcut literatür incelendiğinde farklı durumlar arasındaki eğrilik dönüşümünü ele alan sınırlı sayıda çalışma ortaya çıkmaktadır. Tek serbestlik derecesine sahip dönüştürülebilir tasarımların çoğu düzlemsel hareketlerle sınırlı iken üç boyutlu formlar ise genellikle düzlemsel makas mekanizmalarının ötelemeli tekrarı yoluyla elde edilebilmektedir. Buna ek olarak, düzlemsel makas mekanizmalarıyla ızgarada düzeninde bir yüzey tanımlayabilen formlar oluşturulduğunda, genellikle birden fazla serbestlik derecesine sahip olmaktadır ve bu da form dönüşümlerinin kontrolünü zorlaştırır. Bu tezde, belirtilen eksikliklerden yola çıkarak, önceden tanımlanmış eğriler arasında dönüşüme olanak tanıyan düzlemsel makas mekanizmaları için yeni bir geometrik tasarım yöntemi sunulmaktadır. Önerilen yaklaşım, farklı kavisli formlar elde edebilen düzlemsel makas mekanizmalarını oluşturmak için dörtgen döngülerden yararlanmaktadır. Bu çalışma, belirtilen yöntemden yola çıkarak, önceden tanımlanmış yüzey geometrileri arasında eğriliklerini dinamik olarak değiştirebilen uzaysal makas mekanizmalarını bu konsepte dahil etmektedir. Araştırma, birincil yöntemler olarak simülasyon ve modellemeyi kullanmaktadır. Önerilen modelleri geliştirmek için bilgisayar simülasyonları kullanılırken, geometrik davranışlarını analiz etmek için 3 boyutlu yazdırılmış prototipler üretilmiştir. Bu araçlar, tasarlanan mekanizmaların dönüştürülebilirliğinin ve yapısal performanslarının kapsamlı bir şekilde incelenmesini kolaylaştırmaktadır.

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CHAPTER 1

INTRODUCTION

Deployable and transformable structures have gained attention due to advancements in construction technology, robotics, architectural computing, and material science. This interest stems from a growing demand for structures with enhanced functional flexibility, adaptability, sustainability, and structural performances.

Transformable structures could be able to alter their shape and adapt in response to a variety of circumstances, such as shifting environmental conditions, climatic conditions, functional requirements, and emergency scenarios. Temmerman (2012) distinguishes transformable structures as deployable and demountable structures. Deployable structures are those prefabricated structures that include kinematic mechanisms that enable them to extend from a compact configuration to a predetermined, expanded form in which they are stable and carry loads (Gantes 1991).

Scissor mechanisms are the most commonly used mechanisms for deployable structures. In the early 1960s, Piñero pioneered a scissor mechanism that made it possible to create surfaces with deployable structures with his design Travelling Theatre (Teatro Ambulante) (Castanon and Ramos 2023). After him, many researchers, especially Escrig, Valcarcel, Hoberman, Gantes, and Pellegrino, worked on deployable scissor structures. Deployable scissor structures change their form only between open and closed states.

In the present study, different from deployable scissor linkages, transformable scissor linkages are investigated in terms of their ability to change their form from concave to convex vice versa.

1.1. Problem Statement

In the literature, there are various studies based on scissor linkage mechanisms. Most of these studies have focused on deployable structures Along with the planar studies, there are both synclastic and anticlastic studies that define spatial doubly curved surfaces (Escrig 1985; Escrig and Valcárcel 1987). On the other hand, transformable studies with scissor linkage mechanisms are limited in the literature (Zhang et al. 2016; Yar et al. 2017; Gür, Korkmaz, and Kiper 2019; Maden, Ölmez, et al. 2019; Sarısayın et al. 2022). Existing studies have not been able to go beyond the planar forms. For example, Zhang et. al. (2016) presented a new classification of scissor units. He produced planar models that can transform from a given shape to the target shape. Yar et al.(2017), Gür, Korkmaz, and Kiper (2017), Gür et al. (2018), Maden, Ölmez, et al. (2019), Maden, Akgün, et al. (2019), Gür, Korkmaz, and Kiper (2019) took a different approach, which is obtaining planar scissor linkages with quadrilateral loops. They have obtained transformable planar scissor linkages that can change their form between convex and concave forms consisting of a rhombus, parallelogram, antiparallelogram, and dart loops.

The main research problem of this study is to design spatial scissor linkages by changing the curvature to define a transformable surface. In order to solve this problem, the condition that the planar scissor linkages should be able to change their form between the two predefined curves must be proved. It can be observed that planar scissor linkages transform between circular concave and convex curves as a result of the loop assembly method. But, the existing method is not useful to obtain scissor linkages that can transform predefined curves (Kiper et al. 2022). In this direction, a new geometric loop assembly method is presented in this study to solve this problem. Then, the design of transformable spatial scissor linkages is presented.

1.2. Objectives of the Research

The main objective of this thesis is to propose a new design methodology for generating transformable scissor linkages that can change its form between concave and convex. To reach this objective, a new geometric construction method is developed based on the loop-assembly method, which enables planar transformable multi-loop scissor linkages that can transform between two predefined curves. After obtaining planar scissor linkages, it is aimed to design transformable surfaces that can change their form between two desired states.

1.3. Methodology

The study contains both architectural, geometrical, and mechanical aspects. At the beginning of the research, it is essential to understand geometrical and kinematic properties and motion characteristics of scissor mechanisms. In this way, a detailed literature survey is conducted to indicate the geometrical properties of starting from one scissor unit to deployable scissor linkages. Additionally, the basic surface geometries literature is examined in detail to understand the transformation of surface profiles. During the research, simulation, and modeling are used. Both planar animated drawings and solid models are studied in computer environments with Solidworks®. Following these steps, models are produced using the 3D printer Ultimaker2+.

1.4. Organization of the Thesis

The first chapter of this thesis introduces the study, describes the main concerns it tries to answer, indicates the scope, and presents the methodology. The thesis continues with a literature review of the main areas which construct the foundation of the thesis.

Chapter 2 gives a correlated literature review of scissor structures and surface geometries of scissor structures.

Chapter 3 continues with the basic principles of scissor linkages, such as scissor units, geometrical construction methods of scissor linkages, and their motion characteristics.

Chapter 4 addresses essential geometrical definitions needed to describe the form and surface geometry of scissor linkages.

Chapter 5 contains the geometrical background of the surface profile, which is based on conic sections. Scissor linkages obtained with loop assemblies based on frieze patterns are examined according to their result motion characteristics and geometrical aspects.

Chapter 6 presents a new geometrical construction method to overcome the missing part of surface profile transformations. In addition, transformable scissor linkages are presented.

Chapter 7 concludes the dissertation with main achievements and recommendations for future work.

CHAPTER 2

REVIEW OF LITERATURE

This chapter provides a literature review of scissor linkages in both planar and spatial aspects. Existing studies are examined in terms of surface geometries.

2.1. Review of the Scissor Structures

Scissor linkages are one of the most common types of deployable and transformable structural mechanisms due to their simple geometries and easy design techniques. That's why we see them in applications from engineering to architecture. The idea of using scissors mechanisms in deployable structures goes back to Piñero. He created domes and space grids using this concept in the early 1960s. (Piñero 1961a; 1961b; 1962). Since then, many people have built on Piñero's work, creating new scissor units and structures. Researchers have also studied the key geometric, kinematic, and structural principles of various scissor structures.

In this section, all studies related to scissor elements existing in the literature will be examined under two sections: planar and spatial. This literature review aims to examine the studies made with scissor systems based on form changes. In addition, planar and spatial studies will be divided into subheadings within themselves to classify the literature properly.

2.1.1. Planar Scissor Linkages

In this section, planar scissor linkages are investigated under two headings: open chain and closed chain planar scissor linkages.

2.1.1.1. Open Chain Planar Scissor Linkages

Chuck Hoberman's discovery of angulated elements in 1990 changed how we design scissor mechanisms. These elements, made of two connected angled bars, let structures deploy radially from the center to the edge with a single degree of freedom (DoF). Hoberman used the loop-assembly method to construct open-chain linkages (Figure 2.1 and Figure 2.2) (Hoberman 1990).

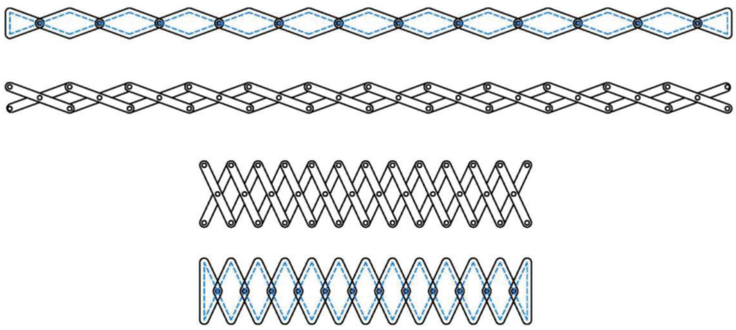


Figure 2.1. Rhombus loop assembly in a direction and resulting scissor linkage
(Source: Hoberman, Demaine, and Rus 2013)

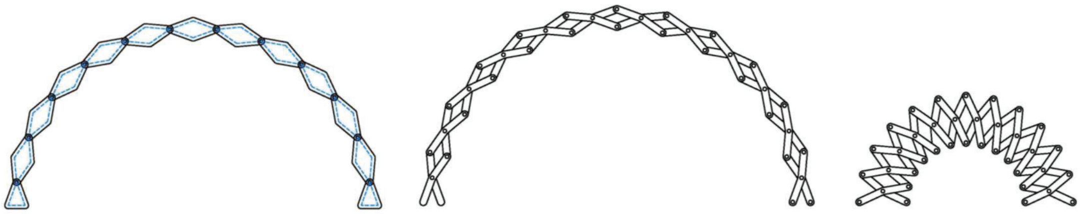


Figure 2.2. a) Hinged rhombs on an arc b) Open and deployed forms of resulting linkage
(Source: Hoberman, Demaine, and Rus 2013)

Kokawa's Cable Scissor Arch study (1997) focused on a planar open chain scissor structure. Its key feature is the ability to change curvature without changing the span. The structure uses two scissor assemblies and zigzag cables with pulleys. Figure 2.3 shows its deployment sequence.

Figure 2.3. Transformation sequence of Cable-scissor arch
(Source: Kokawa 1997)

The Hoberman Arch is a 72x36 foot changing curtain used at the 2002 Winter Olympics. It's made of rigid panels on six rings of angled pieces. The panels slide over each other by attaching to the angled elements. (Figure 2.4).

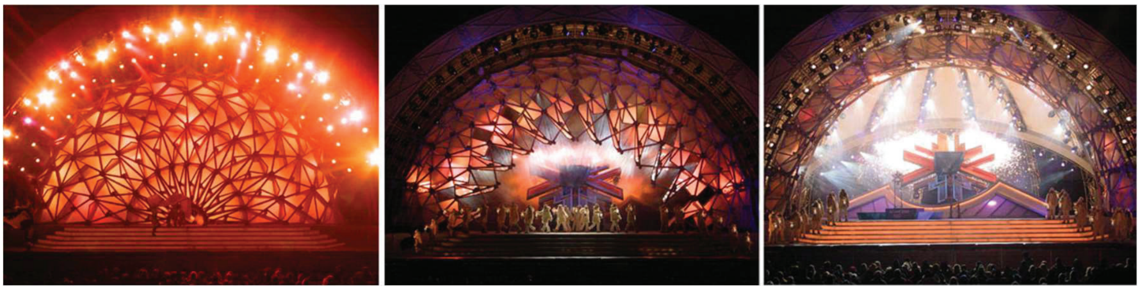


Figure 2.4. Hoberman Arch in the 2002 Winter Olympics
(Source: © Hoberman 2021)

Mele et al. (2010) designed a foldable roof for a tennis court (Figure 2.5). They used scissor-like arches with angled parts. Instead of one arch, they used two halves pinned to the seating areas and connected in the middle when closed. The arches work with a foldable membrane. Cables and pulleys control the change from one shape to another.

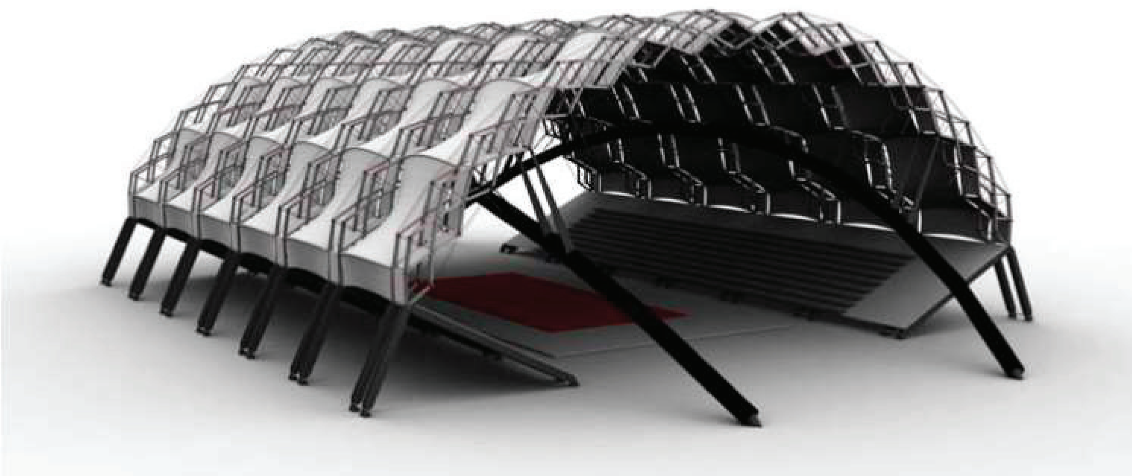
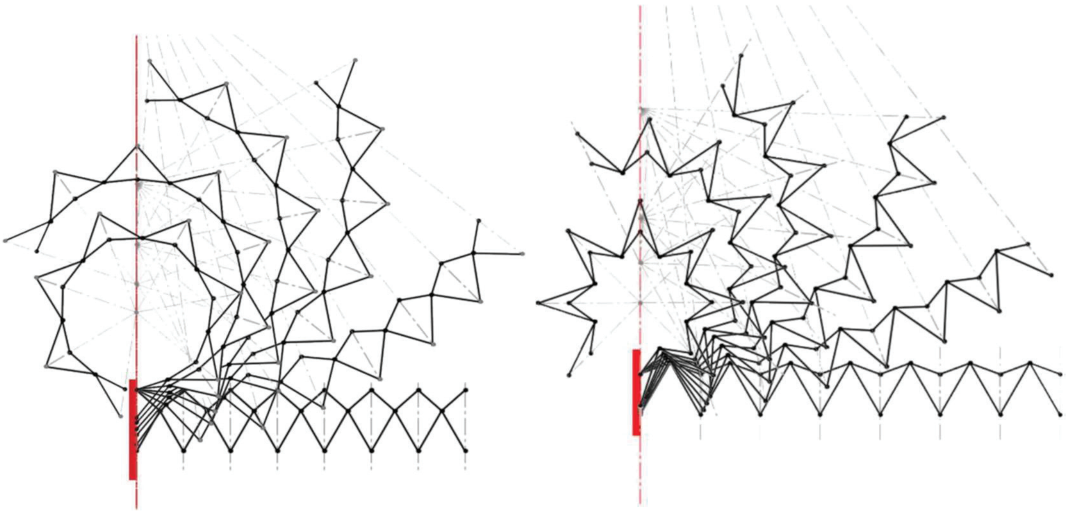


Figure 2.5. A scissor-hinged retractable membrane roof for a tennis arena
(Source: Mele et al. 2010)

Bouleau and Guscetti (2016) used angulated scissor elements to design a transformable bridge called Jet d’Eau movable footbridge in Geneva. They aimed to create a curved shape that can change into a flat one. (Figure 2.6).

Figure 2.6. Jet d’Eau movable footbridge
(Source: Bouleau and Guscetti 2016)

Yar et al. (2017) proposed planar scissor linkages that can transform between convex and concave configurations by changing the curvature. These linkages are obtained by kite and dart loop assemblies. (Figure 2.7).



a) b)

Figure 2.7. Transformable scissor linkages obtained with a) kite and b) dart loops (Source: Yar et al. 2017)

Similar to Yar et al. (2017), Gür, Korkmaz and Kiper (2019) presented deployable and transformable planar structural mechanisms using loop assembly method and anti-parallelogram loops (Figure 2.8). To be able to produce arrays methodically, they used symmetry operations.

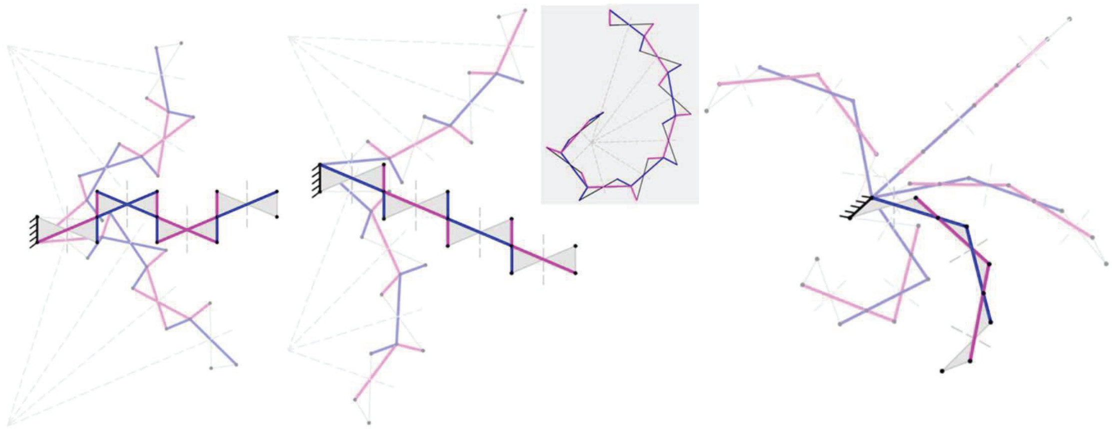


Figure 2.8. a) transformable linkage consists of angulated links, b) transformable linkage consists of ternary and quaternary links, c) transformable linkage consists of polar scissor units (Source: Gür, Korkmaz, and Kiper 2019)

2.1.1.2. Closed Chain Planar Scissor Linkages

In addition to open chain examples explained before, Hoberman (1990) also proposed closed-chain examples with the "loop-assembly" method (Figure 2.9). In a lecture at MIT in 2013, he proposed that regular and irregular closed-chain deployable scissor linkages can be obtained with this method (Hoberman, Demaine, and Rus 2013).

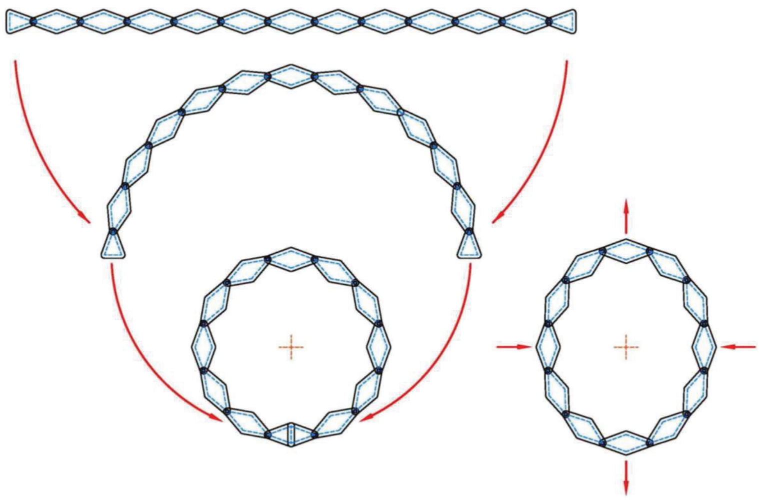


Figure 2.9. Forming hinged rhombi from open to closed chain (Source: Hoberman, Demaine, and Rus 2013)

You and Pellegrino (1997) studied angulated scissor units in detail. Building on Hoberman's work, they found that angulated parts (made from equal or similar triangles with any number of parallelograms between) always open at the same angle. (Figure 2.10).

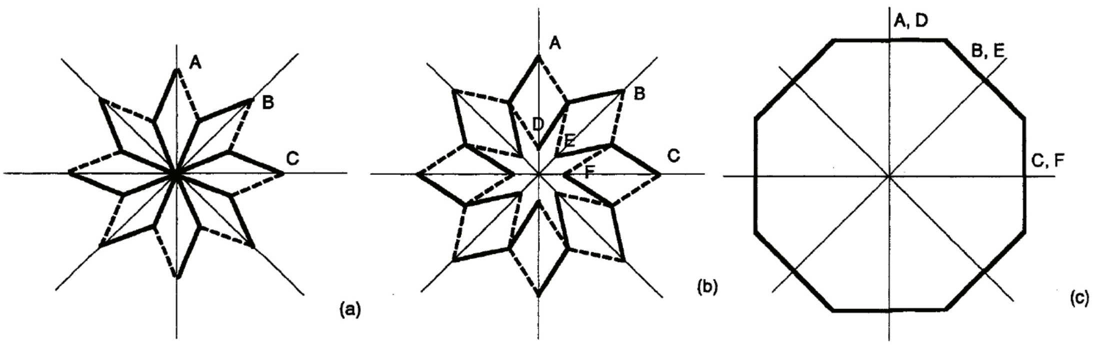


Figure 2.10. Foldable ring linkage formed by identical angulated rods with a kink angle of 135°. (a) "Expanded" and (c) "retracted" configurations
(Source: You and Pellegrino 1997)

They also found that using rods with multiple angles decreases the number of components and makes the connections simpler (Figure 2.11).

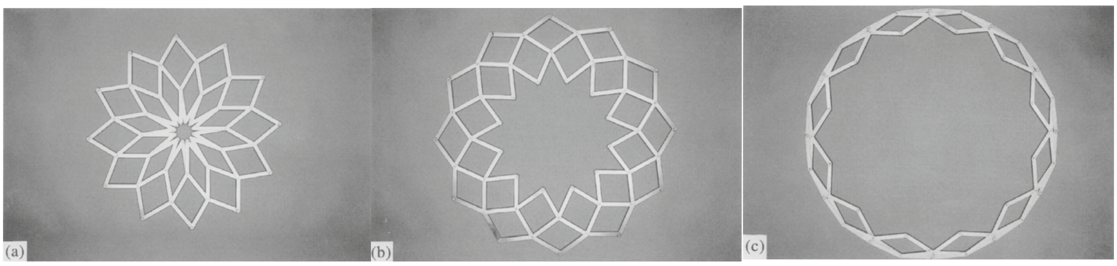


Figure 2.11. Model structure built from 24 identical three-segment, multi-angulated rods
(Source: You and Pellegrino 1997)

Bai et al. (2014) developed a method to create mechanisms for scaling polygons. They used four types of tetragon elements (rhombus, kite, parallelogram, and general tetragon element). With these elements, one can easily build mechanisms to scale polygons. They provided a table that shows how to use these elements to create scaling mechanisms quickly. Zhang et al. (2016) developed a method to design scissor-like linkages on a flat surface that transforms a starting shape into a target shape with high accuracy. (Figure 2.12).

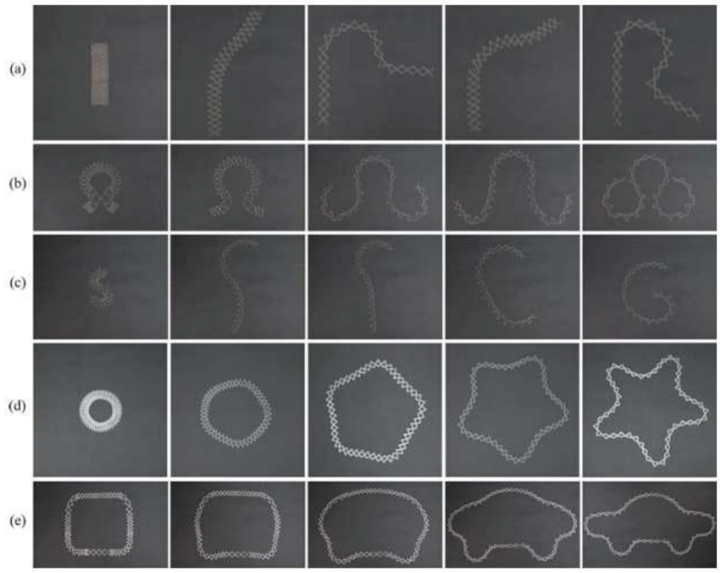


Figure 2.12. Photos of the real objects fabricated by 3D printing with SLA material (Source: Zhang et al. 2016)

Gür, Korkmaz, and Kiper (2019) also showed how to build deployable planar scissor structural mechanisms using a loop assembly method and anti-parallelogram loops (Figure 2.13). They used symmetry operations to create arrays in a systematic way.

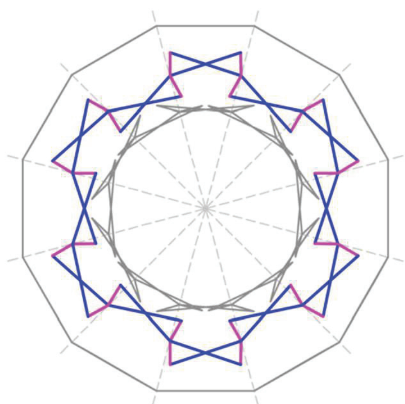


Figure 2.13. Deployable ring-like linkage consists of angulated links. (Source: Gür, Korkmaz, and Kiper 2019)

2.1.2. Spatial Scissor Structures

In this section, spatial scissors structures are investigated according to their surfaces as monoclastic, synclastic and anticlastic.

2.1.2.1. Monoclastic Scissor Structures

The geometric condition for the deployability of scissor mechanisms composed of translational and polar scissor-like elements (SLE) was demonstrated by Escrig (1985). The study showed how to modify the intermediate hinge's location inside the scissor units to produce curvature in the grid. Later, Escrig and Valcárcel (1987; 1993) showed how to create three-dimensional structures. They placed scissor units facing different directions on a grid to achieve this. They presented two-way and three-way cylindrical grids with two-way and three-way spherical grids. As a result of two-way cylindrical and spherical grids, they obtained a monoclastic, single curvature, scissor structure as shown in Figure 2.14 and Figure 2.15. Escrig and Sánchez (2006) demonstrated conditions for adapting a deployable mesh to these grids.

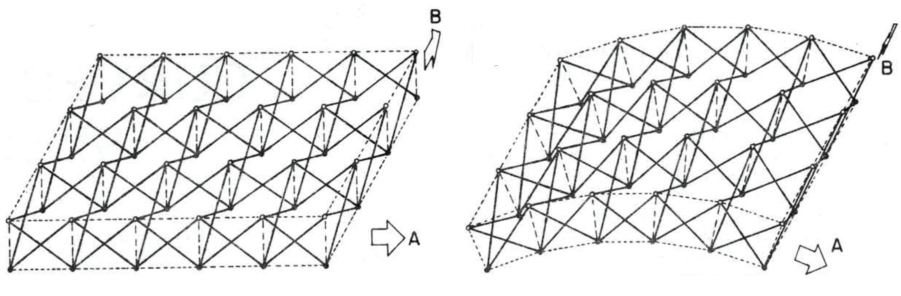


Figure 2.14. Sequence two-way cylindrical grid with translational unit (Escrig and Valcárcel 1987)

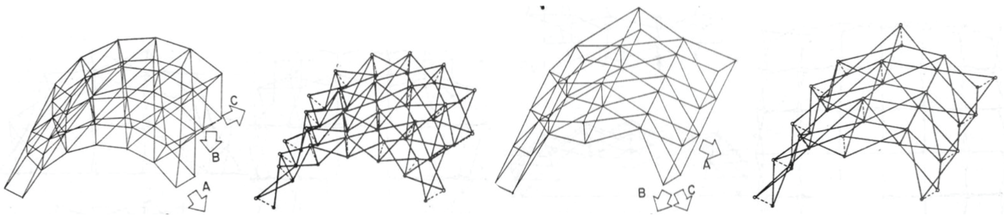


Figure 2.15. Two examples of the three-way spherical grid with translational units (Escrig and Valcárcel 1987)

Langbecker (1999) expanded on Escrig's foldability conditions for SLEs. He studied how different scissor structure shapes (translational, cylindrical, spherical) could be deployed. Langbecker also created many models of curved, foldable structures like barrel vaults and synclastic shapes using the right SLEs, as in Figure 2.16 (Langbecker and Albermani 2000).

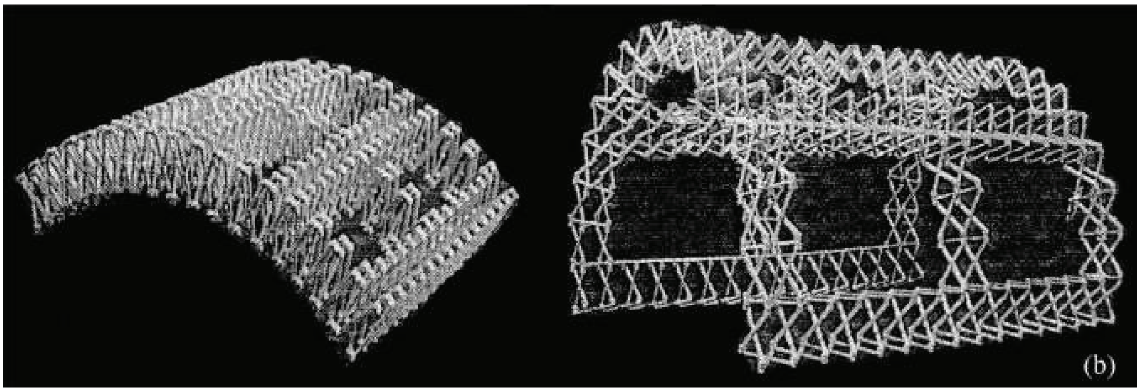


Figure 2.16. Single-curved foldable barrel vault.
(Langbecker and Albermani 2000)

A well-known use of scissor units is the deployable roof for the San Pablo Sports Centre swimming pool in Seville, Spain, designed by Escrig and Sánchez (2006). The roof has two matching curved rhomboid structures made of equal SLE grids covered in a thin fabric. It opens from a folded to an unfolded state (Figure 2.17).

Figure 2.17. Deployable roof structure composed of SLEs in Seville
(Source: Escrig and Sánchez 2006)

Rippmann (2007) designed a scissor unit with adjustable hinge points (Figure 2.18). His design allows for various shapes by repositioning the hinges. While appearing flexible, the system is single-DoF; units must be fully disassembled and rebuilt to change shapes.

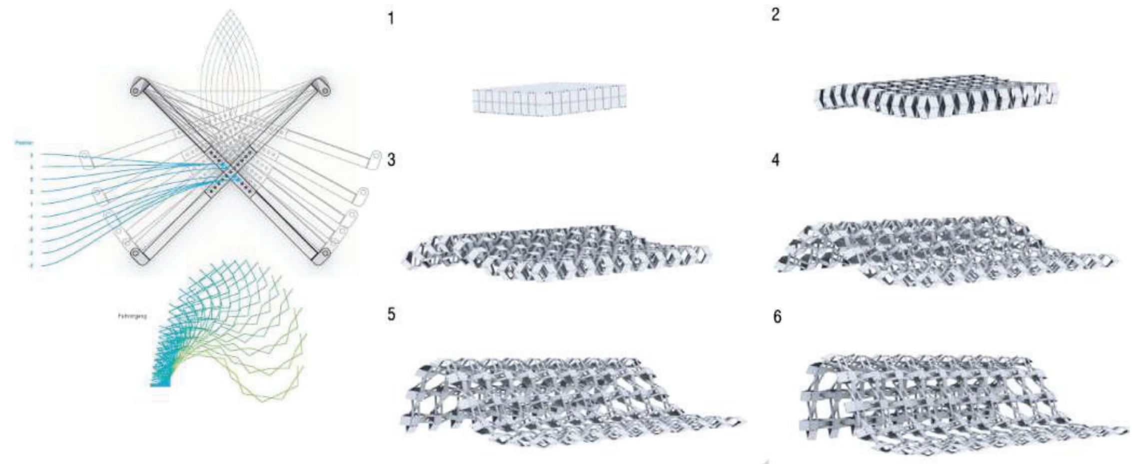


Figure 2.18. Scissor unit with various hinge points and the structure constructed with.
(Source: Rippmann 2007)

Alegria Mira, Thrall, and Temmerman (2014) introduced a concept for a foldable scissor shelter, including design, analysis, and testing of a full-scale prototype (Figure 2.19). They shared the conceptual design and analysis using a parametric finite element method. The prototype was tested to assess deployment, support, and stiffening. For the final design, they presented a single curved shelter structure with scissor elements and a membrane that can meet the requirements of the European Standards.

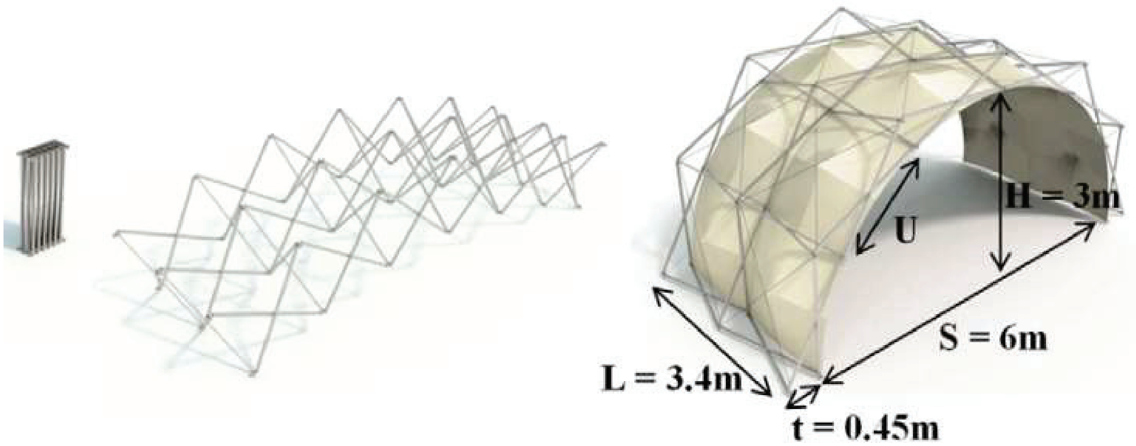


Figure 2.19. The conceptual design for a 4-person shelter comprised of deploying scissor arches with a membrane.
(Source: Alegria Mira, Thrall, and Temmerman 2014)

Roovers and Temmerman (2014a) provided an overview of single-curvature deployable scissor grids. They classified single-curved double-layer scissor grid structures by grid cell geometry and listed parameters for scissor units, grids, and kinematic behavior (Figure 2.20).

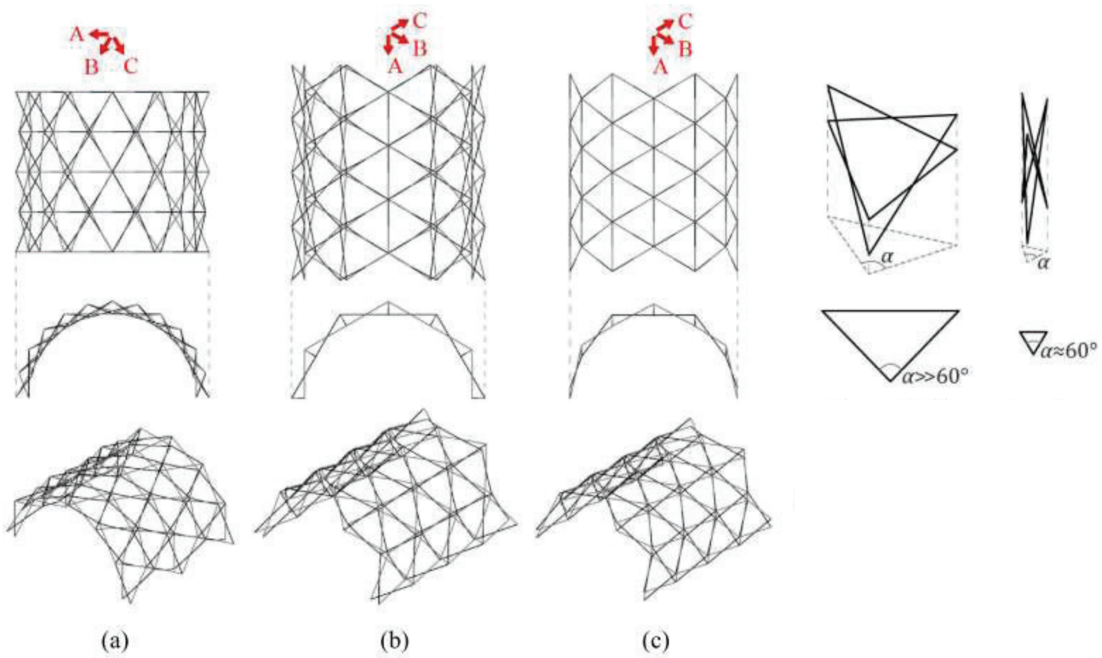


Figure 2.20. Three types of triangulated single curved scissor grids consisting of: a) polar units; b) polar and plane-translational units; c) plane- and curved-translational units Source: Roovers and Temmerman 2014a).

Maden, Ölmez et. al. (2019) designed a dynamic shelter that can change form to adapt to different needs. Here's how they created it: First, they used a loop assembly method to build the structure. Next, they created a customizable model in Grasshopper® to study the loops' geometry and create a flexible design tool. They analyzed the geometry to develop the structure. The shelter can transform into various shapes, specifically from flat to S-shapes and reverse S-shapes. (Figure 2.21).

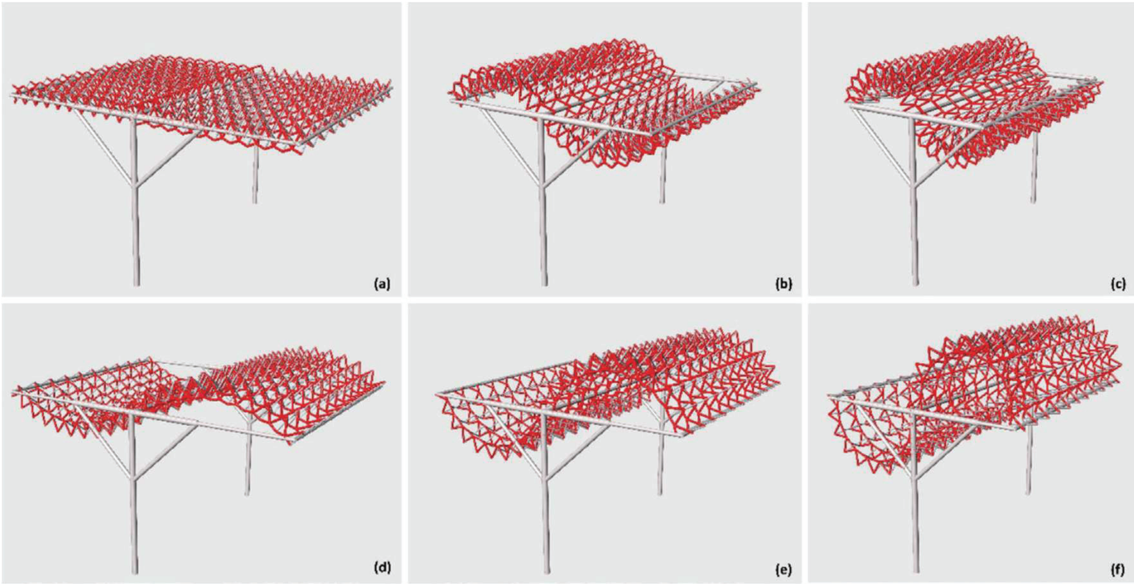


Figure 2.21. The transformation process of dynamic shelter structure.
(Source: Maden, Ölmez, et al. 2019)

2.1.2.2. Synclastic Scissor Structures

Escrig and Valcarcel (1987; 1993) designed spherical scissor structures with two-way and three-way grids. Many curved geometric models were proposed for these grids (Figure 2.22, Figure 2.23, Figure 2.24, Figure 2.25). Later, Escrig and Sánchez (2006) studied two-way grids on a sphere's surface. These grids need crossbars or cables to stabilize them when they open due to non-triangulation instability.

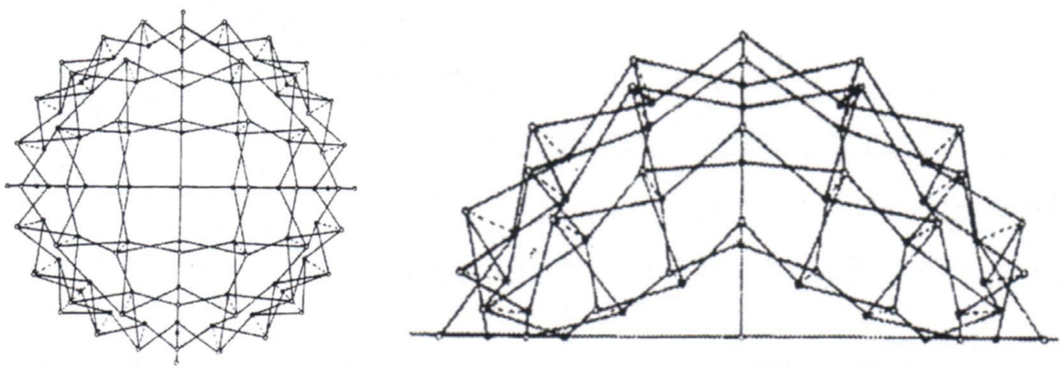


Figure 2.22. Top view and side elevation of a two-way spherical grid with identical polar units (Source: Escrig and Valcárcel 1987)

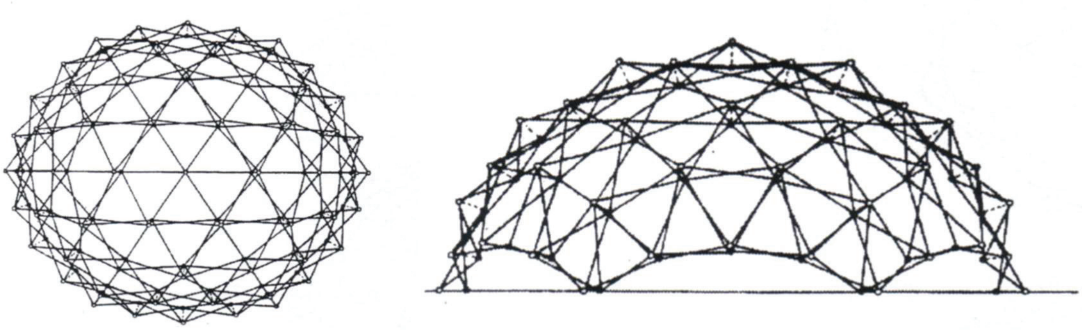


Figure 2.23. Top view and side elevation of a three-way spherical grid with polar units (Source: Escrig and Valcárcel 1987)

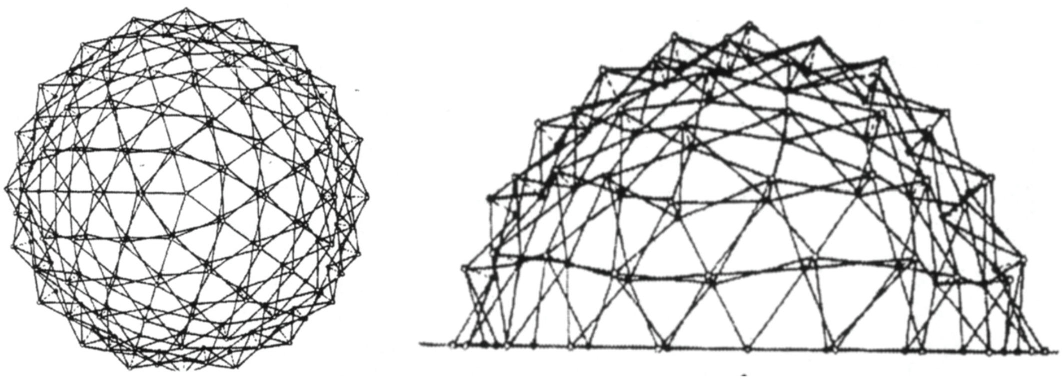


Figure 2.24. Top view and side elevation of a geodesic dome with polar units (Source: Escrig and Valcárcel 1987)

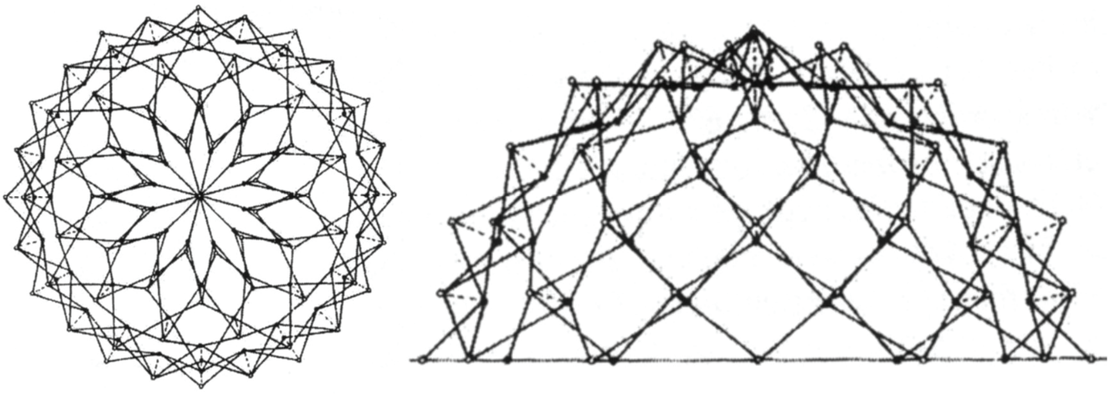


Figure 2.25. Top view and side elevation of a lamella dome with identical polar units (Source: Escrig and Valcárcel 1987)

By using the angulated elements, Hoberman (1990) used angulated elements to create expanding synclastic structures like the Geodesic Dome (Figure 2.26), Sphere, Icosahedron, and Iris Dome.

Figure 2.26. Hoberman's kinetic sculpture: Expanding Geodesic Dome.
(Source: © Hoberman 2021)

Another example is the Iris Dome (collapsible dome), made up of interconnected spirals that can pull back toward the edges (Figure 2.27) (Hoberman 1993).

Figure 2.27. Iris Dome
(Source: © Hoberman 2021)

Charis Gantes (1996; 2001) has done extensive research on deployable structures that can lock into place in two different positions. He has created design methods for flat and curved grids, as well as structures with any shape. He studied how to scissor structures can "snap-through" into place due to differences in member lengths at certain angles. He developed ways to design these structures to be deployed and expanded smoothly without stress(Gantes et al. 1993).

Hoberman's concepts encouraged researchers to take things a step further and conduct thorough research on angulated scissor structures. You and Pellegrino (1997) created a multi-angulated rod design that simplified the structure and its joints by reducing the number of parts. In order to create a deployable structure that is installed on pinned columns, Kassabian, You, and Pellegrino (1999) employed multi-angulated parts (Figure 2.28).

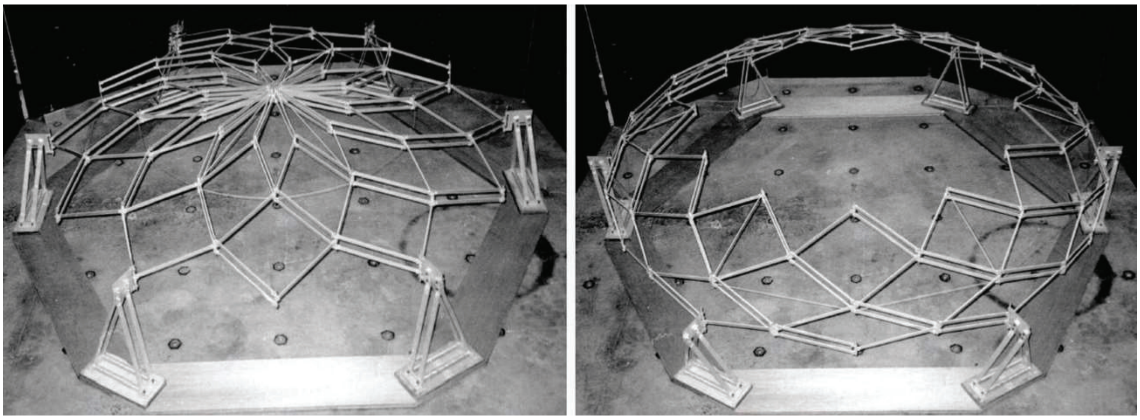


Figure 2.28. Deployable structure composed of multi-angulated elements.
(Source: Kassabian, You, and Pellegrino 1999)

In addition to monoclastic studies, Langbecker and Albermani (2000) presented synclastic scissor structures with translational elements, as in Figure 2.29.

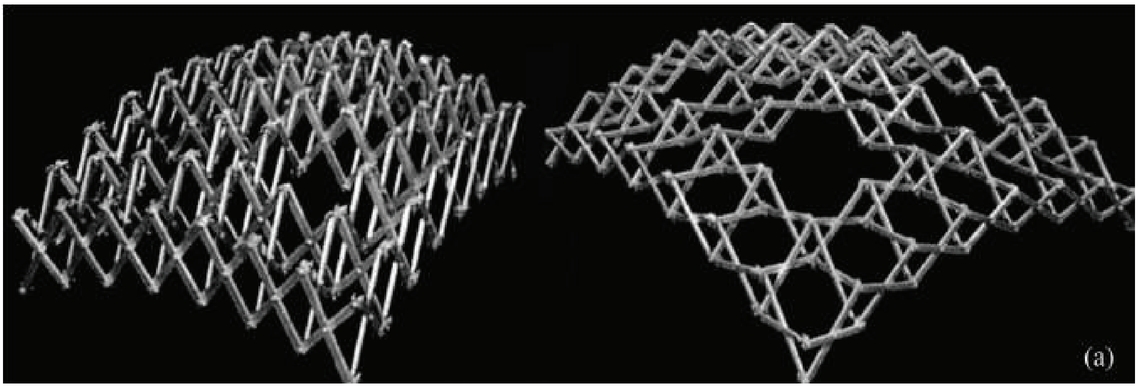


Figure 2.29. Double-curved synclastic scissor structure.
(Source: Langbecker and Albermani 2000)

Kokawa (2000) presented 3-dimensional multi-angulated scissor element as the main structural element of a new type of called “Retractable Loop-Dome”. He describes the geometry of the 3-dimesional multi-angulated scissor element determined by cutting a sphere with a plane passing through the apex. In order to provide double curved, synclastic shell-like behavior of the structure, he adds an expandable ring consisting of expandable rods that form a regular polygon to both the inner and outer circles of the dome (Figure 2.30).

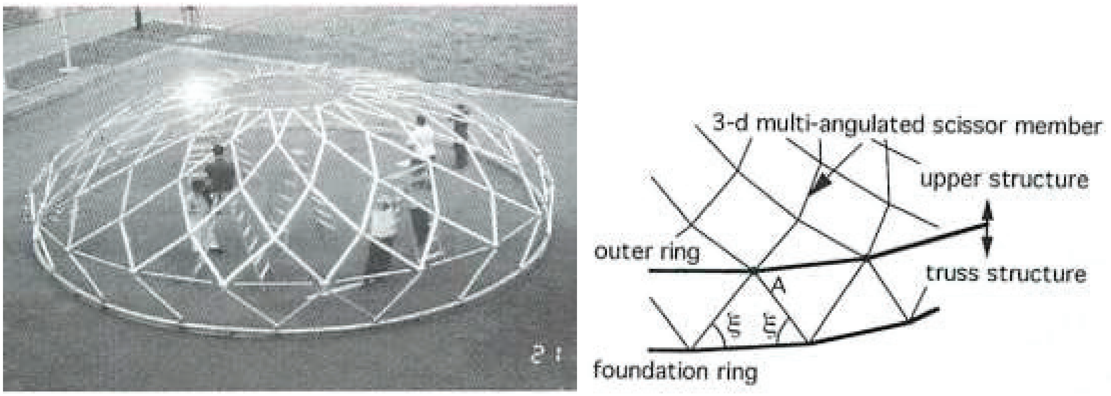


Figure 2.30. Model of retractable loop-dome and its outer and foundation rings
(Source: Kokawa 2000)

Based on polar units, Roovers and Temmerman (2017b) studied the geometry of both spherical and non-spherical scissor grids (Figure 2.31). They developed a way to adapt spherical scissor grids into non-spherical ones without changing how they move. The shape and motion of these modified scissor units and grids were explained by the researchers. In addition, they produced a joint model that maintains the geometry's compatibility.

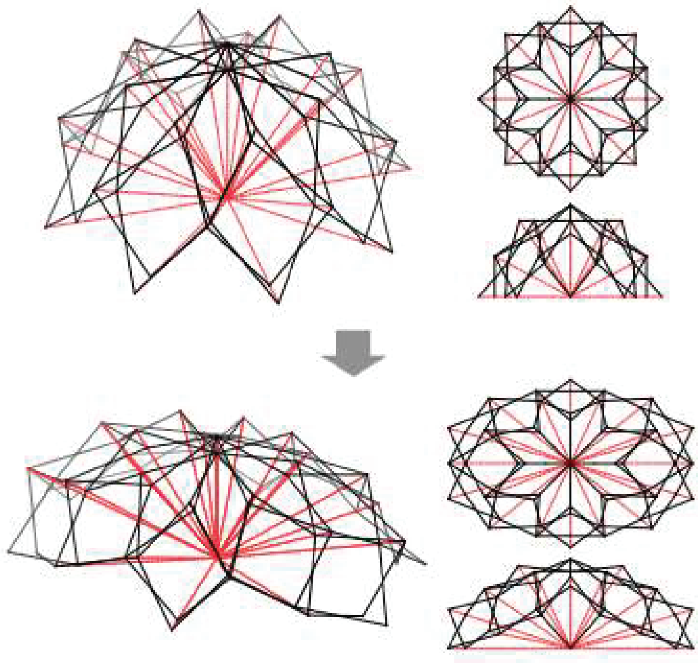


Figure 2.31. An example of the generalization of a spherical foldable scissor grid with a lamella rhomboid pattern, which is translated into an ellipsoidal grid with the same pattern and foldable deployment behavior.
(Source: Roovers and Temmerman 2017b)

Roovers and Temmerman (2017a) presented new proposals together with the existing ones in the literature in order to form an overview of the geometric potential and kinematic behavior of deployable scissor grids consisting of translational scissor units. The team examined deployable scissor grids with one or two curves, built from translational scissor units (Figure 2.32). They also developed a technique for creating inwardly and outwardly (double) curved anticlastic scissor grids. The scissor designs could be extended with joints in a variety of ways without compromising their ability to open and close.

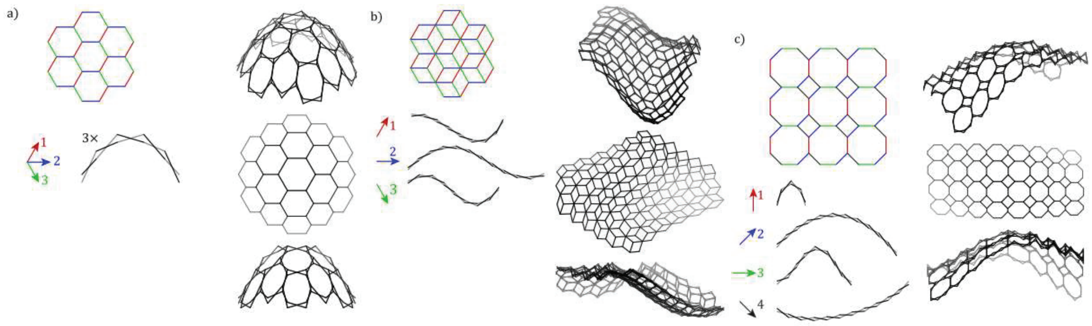


Figure 2.32. Various double-layer grids of translational units generated by tiling the scissor units of multiple planar linkages along a pattern: a) hexagonal tiling; b) rhombille tiling; c) truncated square tiling.
(Source: Roovers and Temmerman 2017a)

2.1.2.3. Anticlastic Scissor Structures

Hoberman created two deployable anticlastic constructions with angulated components in addition to synclastic models. One is the Expanding Helicoid from 1998 (Figure 2.33). Its surface is composed of two spiral forms, like the double helix of DNA. Later, a kinetic version was built for Discovery World's biotech exhibit in Milwaukee. When completely expanded, it unfolds from a bundle to a spiral that is 12 meters tall and 3.6 meters broad.

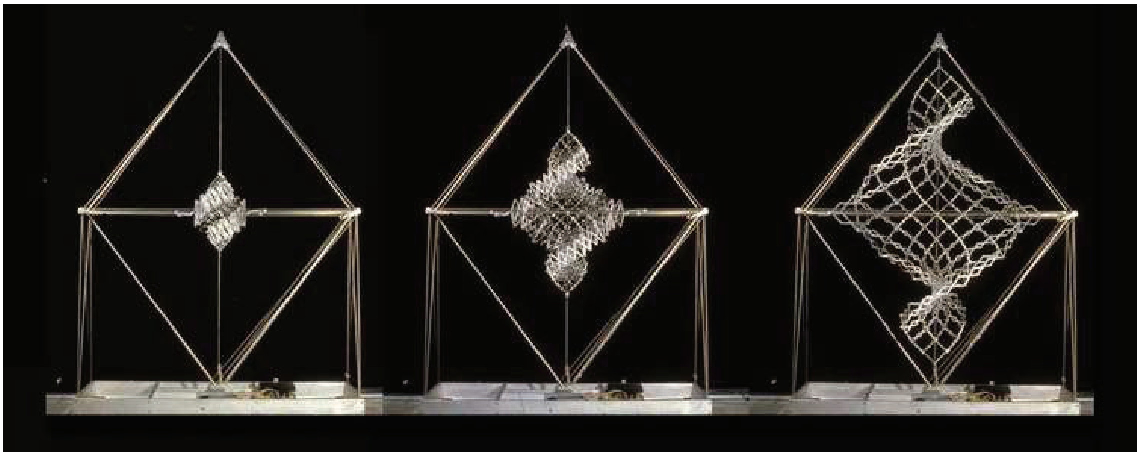


Figure 2.33. Expanding Helicoid
(Source: © Hoberman 2021)

Hoberman's Expanding Hypar for the California Science Center is another anticlastic example (Figure 2.34). Using angulated parts for such anticlastic structures was a new idea in deployable frames. However, none have been used in structures due to their complex mechanics and structure (Hoberman 2021).

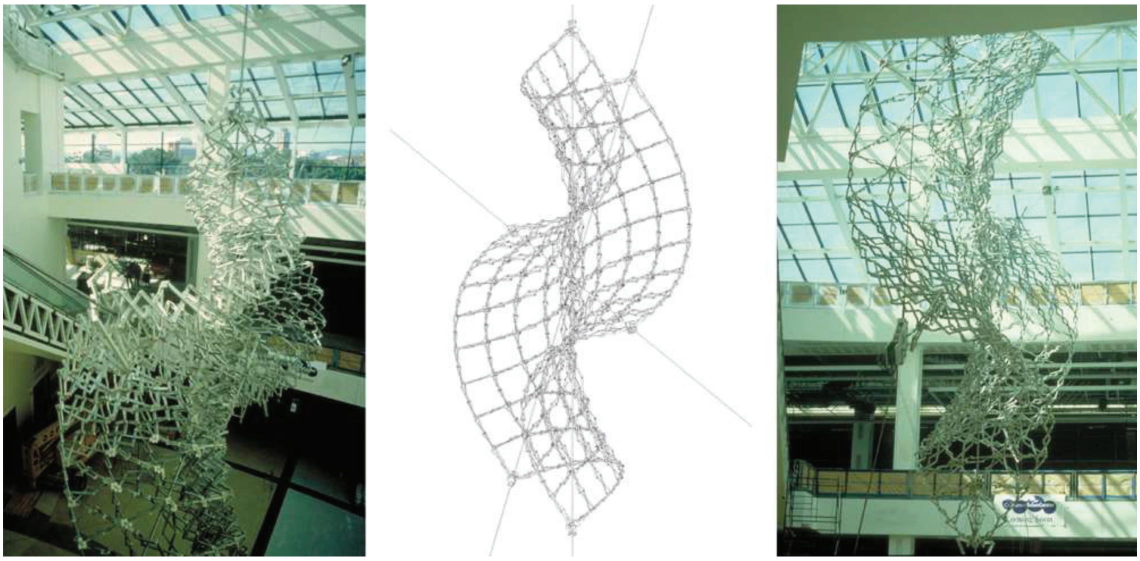


Figure 2.34. Expanding Hypar
(Source: © Hoberman 2021)

Mohamad Alkhayer (2007) studied how to use angulated elements for deployable curved shapes. They suggested many types of deployable hyperboloids (Figure 2.35) by using regular and semi-regular tessellation patterns.

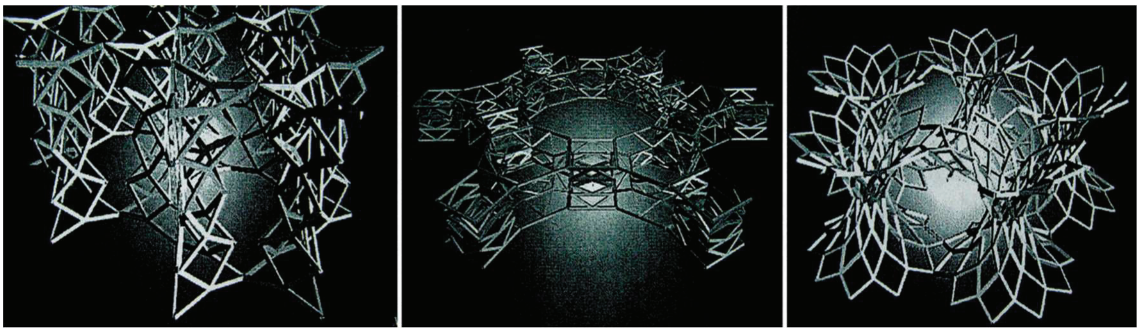


Figure 2.35. Deployable polygonal hyperboloids
(Source: Alkhayer 2007)

Langbecker and Albermani (2000) created several single DoF anticlastic structures shaped like hyperbolic paraboloids using compatible scissor units that can move together (Figure 2.36), in addition to single-curve foldable barrel vaults and double-curve synclastic structures.

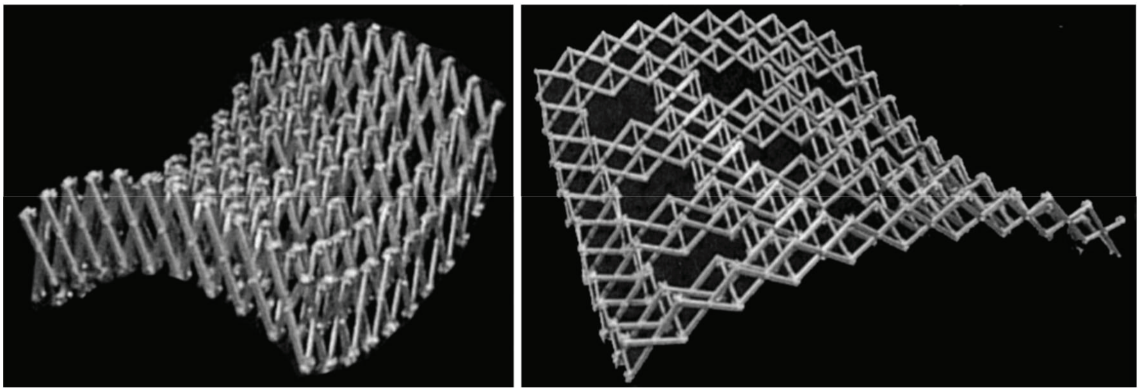


Figure 2.36. Double-curved anticlastic hypar structure.
(Source: Langbecker and Albermani 2000)

Petrova (2008) explored how to create double-curved structures with any type of curve. She studied different types of curvature to design more complex shapes for modern buildings. Petrova developed a method to create freely shaped double-curved surfaces that can be folded up. Using rhombic scissor units, she created many anticlastic structures where the surface shape and curvature can be freely chosen. (Figure 2.37).

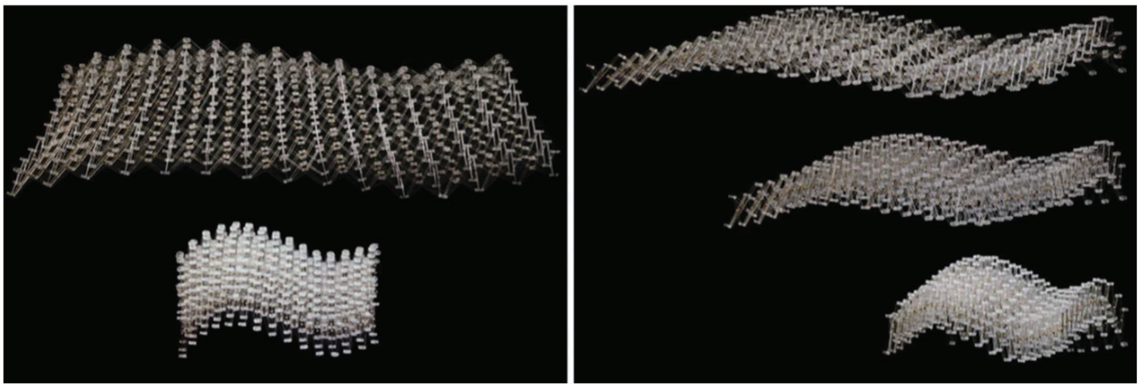


Figure 2.37. Arbitrary double-curved translational structures
(Source: Petrova 2008)

Akgün (2010) used a novel kind of SLEs to develop new transformable structures. By putting revolute joints at various locations along a bar, he produced three distinct kinds of modified SLE (M-SLE). Using six scissor arches, Akgün also created an adaptable roof for an exhibition hall (Figure 2.38). The design allows for a wide range of shape changes, transforming from arches to various curves.

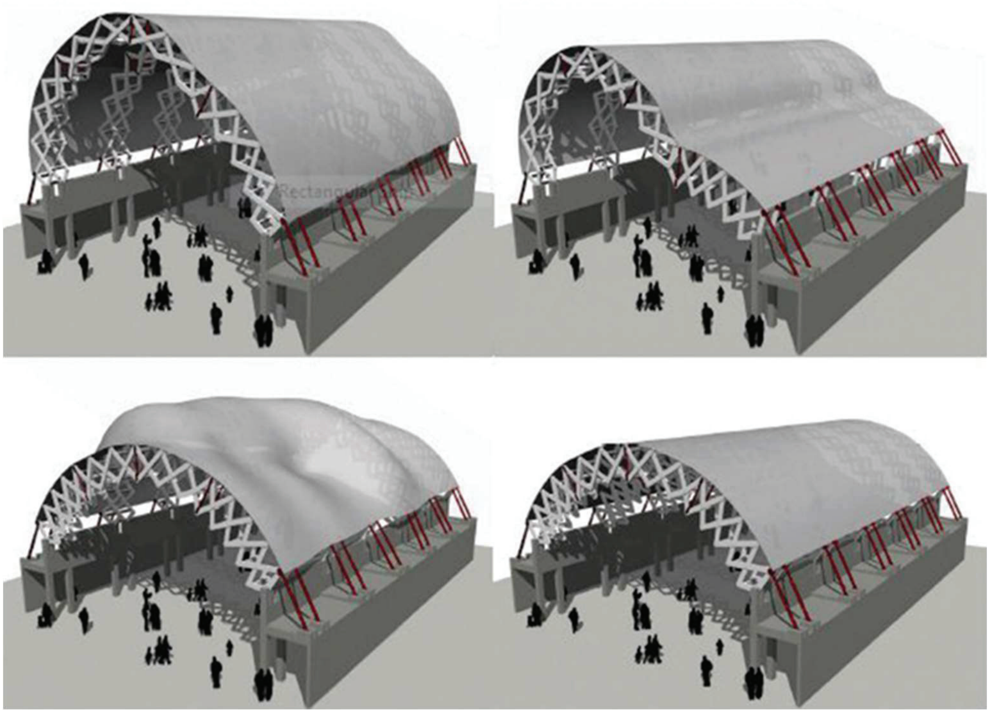


Figure 2.38. Adaptive roof structure composed of scissor arches.
(Source: Akgün et al. 2010)

As another transformable example, Akgün also created a 4-DoF spatial scissor structure. It consists of 25 spatial SLEs (S-SLEs), 4 modified spatial SLEs (MS-SLEs), 20 hybrid spatial SLEs (HS-SLEs), and 8 special SLEs (Figure 2.39) (Akgün et al. 2011).

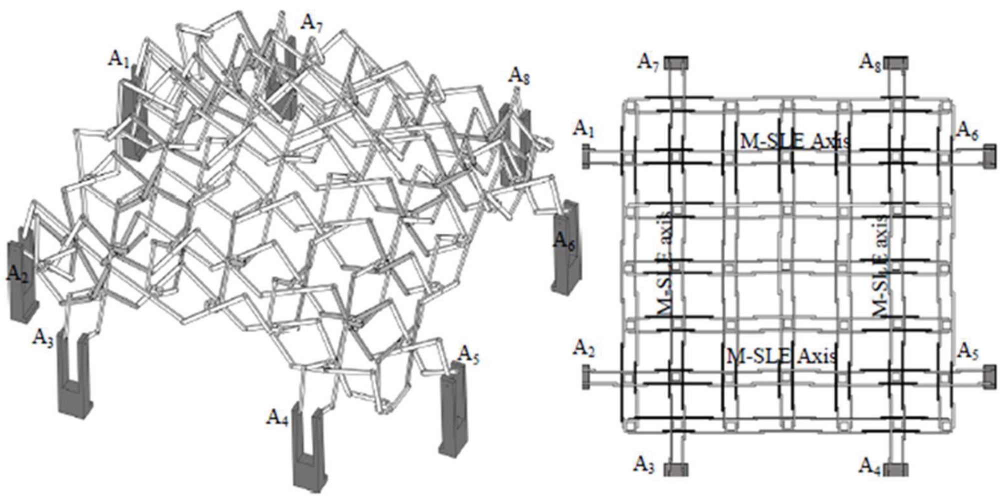


Figure 2.39. Perspective and top view of the proposed scissor-hinge shell structure
(Source: Akgün et al. 2011)

Roovers, Alegria Mira and Temmerman (2013) explored new geometric shapes to find innovative uses for angulated elements. They studied Hoberman's Expanding Helicoid instead of simple curved surfaces like cylinders or spheres. The researchers developed a method to convert any continuous surface into an angulated scissor grid. After testing various surfaces, they created a single DoF deployable catenoid structure. (Figure 2.40).

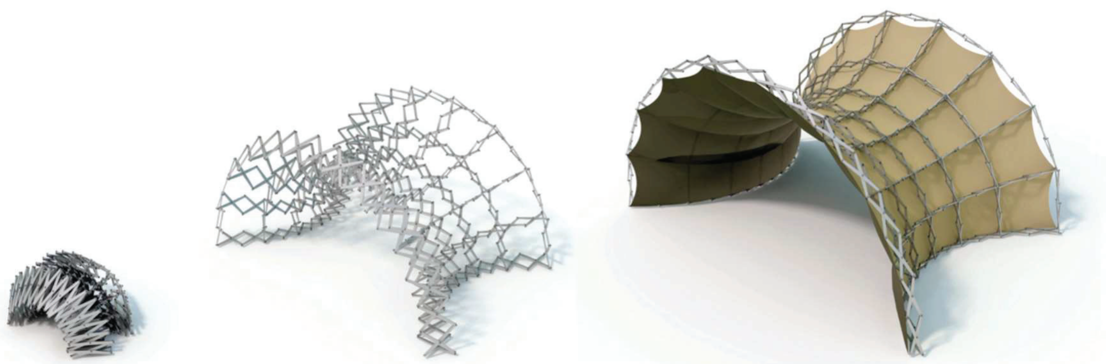


Figure 2.40. Deployable catenoid
(Source: Roovers, Alegria Mira, and Temmerman 2013)

Roovers and Temmerman (2017a) reviewed existing and new proposals to understand the geometric potential and movement of deployable scissor grids made up of translational scissor units. They looked at both single and double curved grids (Figure 2.41). The researchers also developed a method for double curved anticlastic scissor grids. They suggested ways to add joints to the line models without changing how they deploy.

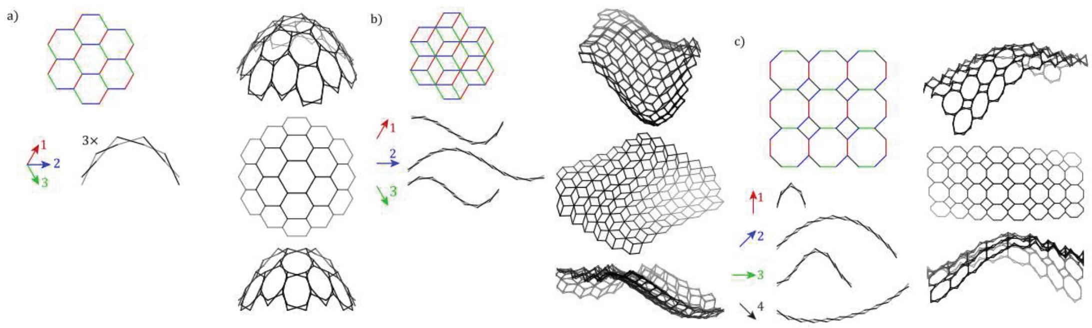


Figure 2.41 Various double-layer grids of translational units generated by tiling the scissor units of multiple planar linkages along a pattern: a) hexagonal tiling; b) rhombille tiling; c) truncated square tiling.
(Source: Roovers and Temmerman 2017a)

2.2. Surface Based Evaluation of Scissor Structures

When Table 2.1 is examined, it is inferred that the studies in the literature are concentrated on monoclastic and synclastic surfaces, especially cylindrical surfaces on monoclastic surfaces and spherical surfaces on synclastic surfaces. It is seen that there are fewer studies on anticlastic surfaces compared to monoclastic and synclastic surfaces. The most preferred form is hyperbolic paraboloid on anticlastic surfaces.
In the next

Table 2.2 brief description of the distinguished studies based on different surface morphologies by using scissor structures are given.

Table 2.1. Classification of the deployable scissor linkages according to their surface geometry.

	Surfaces											
	Single Curvature Surfaces			Double Curvature Surfaces								Free- Form Surfaces
	Monoclastic Surfaces			Synclastic Surfaces				Anticlastic Surfaces				
	Cylinder	Cone	Truncated Cone	Sphere-Hemisphere	Paraboloid	Ellipsoid	Double Surface Hyperboloid	Hyperboloid	Hyperbolic paraboloid	Catenoid	Conoid	
(Escrig 1984)	●											
(Escrig 1985)	●			●								
(Zanardo 1986)	●				●							
(Escrig and Valcárcel 1987)	●			●								
(Merchan 1987)	●											
(Gantes et al. 1989)				●								
(Hoberman 1990)				●								
(Hoberman 1991)				●								
(Escrig and Valcárcel 1993)	●			●								

continue on next page

Table 2.1. (cont.) Classification of the deployable scissor linkages according to their surface geometry.

	Surfaces											
	Single Curvature Surfaces			Double Curvature Surfaces								Free- Form Surfaces
	Monoclastic Surfaces			Synclastic Surfaces				Anticlastic Surfaces				
	Cylinder	Cone	Truncated Cone	Sphere-Hemisphere	Paraboloid	Ellipsoid	Double Surface Hyperboloid	Hyperboloid	Hyperbolic paraboloid	Catenoid	Conoid	
(Sánchez-Cuenca 1996)	•			•					•			•
(Alkhayer and Lalvani 2000)								•				
(Atake 2000)	•			•								
(Langbecker and Albermani 2000)				•					•			
(Langbecker and Albermani 2001)	•											
(Temmerman 2007)	•			•								
(Petrova 2008)									•			•

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Table 2.1. (cont.) Classification of the deployable scissor linkages according to their surface geometry.

	Surfaces										
	Single Curvature Surfaces			Double Curvature Surfaces							Free- Form Surfaces
	Monoclastic Surfaces			Synclastic Surfaces			Anticlastic Surfaces				
	Cylinder	Cone	Truncated Cone	Sphere-Hemisphere	Paraboloid	Ellipsoid	Double Surface Hyperboloid	Hyperboloid	Hyperbolic paraboloid	Catenoid	Conoid
(Babaei and Sanaei 2009)	●										
(Akgün 2010)											●
(Roovers 2012)				●	●			●	●	●	●
(Roovers, Alegria Mira, and Temmerman 2013)										●	
(Roovers and Temmerman 2014a)	●										
(Roovers and Temmerman 2014b)	●	●									
(Roovers and Temmerman 2015)				●							

continue on next page

Table 2.1. (cont.) Classification of the deployable scissor linkages according to their surface geometry.

	Surfaces										
	Single Curvature Surfaces			Double Curvature Surfaces							Free- Form Surfaces
	Monoclastic Surfaces			Synclastic Surfaces				Anticlastic Surfaces			
	Cylinder	Cone	Truncated Cone	Sphere-Hemisphere	Paraboloid	Ellipsoid	Double Surface Hyperboloid	Hyperboloid	Hyperbolic paraboloid	Catenoid	Conoid	
(Roovers and Temmerman 2017a)				●					●	●		●
(Roovers and Temmerman 2017b)	●	●	●	●	●				●	●		●
(Osmani et al. 2017)									●			
(Torres 2017)				●								
(Torres Londoño and Matos 2018)	●											
(López 2017)	●			●				●	●			
(Maden, Ölmez, et al. 2019)	●											
(García-Mora and Sánchez-Sánchez 2020)									●			

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Table 2.2. Summary of the studies related to different surface morphologies by using scissor structures.

Study	**Main Findings**
(Escrig 1984)	The study examines how to create expandable structures using flexible modules with spatial joints. It categorizes these structures into main types and discusses ways to modify them to create curved arches, grids, and surfaces. Visual aids are used to illustrate each model.
(Escrig 1985)	The study explores expandable space structures, highlighting their importance for easy transport and assembly. It covers various structures, their applications, construction, and analyses.
(Zanardo 1986)	The study describes two types of systems with rigid, pin-jointed elements: one expands on a cylindrical surface, and the other on a double-curved surface like a paraboloid. Both are patented by the National Research Council.
(Escrig and Valcárcel 1987)	Expandable bar structures can be customized but must meet strict unfolding requirements. The text classifies design models, introduces structural analysis methods, and includes several analyses.
(Gantes et al. 1989)	The study covers deployable-collapsible structures such as emergency shelters and space-station parts, focusing on compact storage and easy setup. It highlights self-stabilizing structures that remain stable and stress-free when deployed or collapsed. The complex deployment phase is analyzed using finite element methods to study load-displacement curves and internal forces, considering shape, member size, cross-sectional properties, and kinematic assumptions.
(Hoberman 1990)	The invention involves self-supporting structures that can expand or collapse while keeping their curved shape. These structures use "loop-assemblies" made of angled struts joined as scissors-pairs. These pairs connect to others or hubs, forming a closed loop. The design keeps key angles constant during folding and unfolding, maintaining the structure's shape.
(Merchan 1987)	The thesis examines deployable structures with scissor-hinged and sliding mechanisms, introducing new geometries for better construction and utility. These structures transition from compact to expanded forms, providing: - Relocatable spaces - Access to remote areas - Quick setup for shelters/equipment - Weather adaptability - Use in high-risk environments - Construction aid Types: 1. Strut Structures: Utilize scissor-hinged, tensile, and sliding mechanisms with components under compression, tension, or bending, connected by joints/hinges. 2. Surface Structures: Include folded, inflatable, and telescopic mechanisms, carrying stresses via surfaces, using tension in pressurized/inflatable forms or flexibly joined surfaces. Though costly, these structures offer adaptability, mobility, and labor savings.

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Table 2.2. (cont). Summary of the studies related to different surface morphologies by using scissor structures.

Study	Main Findings
(Hoberman 1991)	The invention allows self-supporting structures to maintain their curved shape while expanding or collapsing. An alternate version functions like an iris, with the center retracting while the perimeter remains constant. Both versions use "loop assemblies" of angulated, pivotally connected struts forming scissor pairs in a closed loop, preserving critical angles and geometry during movement.
(Escrig and Valcárcel 1993)	The study discusses "expendable structures," which are mechanisms adaptable to various geometric shapes. It highlights scissors mechanisms crucial for designing diverse structures like masts, arches, spatial structures, and cylindrical and spherical bars.
(Alkhayer and Lalvani 2000)	The study covers anticlastic structures in their deployed forms, focusing on scissor structures. It mentions work on polygonal hyperboloids using angulated elements and tessellation methods.
(Atake 2000)	The presentation introduces a scissors technique for frameworks with three or more main units, each with two rigid diagonals forming a rotating quadrangular side. Units connect via secondary and tertiary axes to form a ring structure. Complex frameworks are created by joining structures with couplers or shared units, using three-axis pin joints.
(Langbecker and Albermani 2000)	The study covers a study on foldable lattice structures with varying Gaussian curvature made from scissors units, including: - Kinematic Analysis: Geometric design constraints. - Non-linear Structural Analysis: Structural response under static load. It examines member sizes, depth-to-span ratio, rise-to-span ratio, grid density, and geometric imperfections to understand their impact on performance.
(Langbecker and Albermani 2001)	The study focuses on foldable barrel vaults (FBVs) from polar or angulated scissor units, examining: 1. Kinematic Analysis: Geometric constraints. 2. Non-linear Structural Analysis: Response to static load. 3. Load Cases: Effects of wind. 4. Parameters: Member sizes, depth-to-span ratio, and imperfections. It aims to understand FBVs' behavior under various loads and design parameters.

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Table 2.2. (cont). Summary of the studies related to different surface morphologies by using scissor structures.

Study	Main Findings
(Temmerman 2007)	Deployable structures transition from compact to expanded forms and include: 1. Spatial Bar Structures: Hinged bars. 2. Foldable Plate Structures: Hinged plates. 3. Tensegrity Structures: Bars and cables. 4. Membrane Structures: Tensioned membranes. This dissertation focuses on: - Scissor Structures: Foldable lattice frameworks. - Foldable Plate Structures: Jointed plates forming curves. It aims to develop deployable bar structures for mobile architecture, addressing design, kinematics, structural feasibility, and evaluating configuration strengths and weaknesses.
(Petrova 2008)	The study emphasizes the need for adaptable, sustainable architecture due to resource constraints. A solution is deployable space frames using scissor-like elements (SLEs) or X-frames. Key points: 1. Adaptable Architecture: Must be multifunctional and sustainable. 2. Deployable Space Frames: SLEs/X-frames expand from compact to larger forms. 3. Underutilization: X-frames are underused due to design challenges. 4. Research Aim: Enhance SLEs for complex shapes. 5. Elliptical Gears: Create various shapes and curvatures. 6. Double-Curved Surfaces: Improve performance with foldable surfaces. 7. Shape-Changing Structures: Change shape without extra mechanisms. 8. Testing: Verify kinematics and load-bearing with FEM models. The study aims to design flexible, adaptable structures efficiently and simply.
(García-Mora and Sánchez-Sánchez 2020)	The study examines the evolution and limitations of deployable structures since Da Vinci, focusing on straight scissors. It introduces a new geometric method for designing bistable and non-bistable deployable structures, applicable to various architectural designs, including iconic buildings like the Sydney Opera House and everyday structures. An example is the Manantiales Restaurant in Xochimilco, designed without geometric issues.

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Table 2.2. (cont). Summary of the studies related to different surface morphologies by using scissor structures.

Study	Main Findings
(Babaei and Sanaei 2009)	This paper examines the design of foldable structures using scissor-like elements without geometric constraints. Key points include: 1. Review: Overview of foldable structure research. 2. Geometric Design: Creating various shapes, focusing on barrels. 3. Configuration Processing: Forming geometries with arithmetic, geometric progressions, and algebraic equations. 4. Architectural Requirements: Designing free-form structures. 5. Changeability and Prototyping: Using pivot connections for variable geometry with a prototype. 6. Real-Scale Construction: Methods for constructing foldable structures. 7. Performance Analysis: Graph of maximum displacement vs. structure height. 8. Design-Construct Methodology: Approach for designing and constructing foldable structures.
(Akgün 2010)	The dissertation introduces a flexible scissor-hinge structure that outperforms current models in versatility. Traditional structures switch between preset forms without changing shape. This new design uses a modified scissor-like element (M-SLE) to transform between straight and curved forms while maintaining span length. The research involves geometric, kinematic, and static analyses, using computer simulations and prototypes to understand the structure's behavior.
(Roovers, Alegria Mira, and Temmerman 2013)	The study talks about scissor structures that can switch between compact and expanded forms, ideal for architectural applications. It explores new geometries, especially the angulated scissor unit, and presents a theory for transforming any surface into such a structure. The focus is on the math behind this theory and methods to make these structures more compact.
(Roovers and Temmerman 2014a)	The study describes new scissor structures that can easily switch between compact and expanded forms, useful for architectural purposes. Traditional designs often struggle with stability during deployment. The new approach uses translational scissor units, offering a stress-free deployment and stable, single-curvature shapes. This innovation enables mobile, self-supporting shelters. A scale model is promising, and a full-scale prototype is in the works.
(Roovers and Temmerman 2014b)	The study discusses deployable scissor structures that quickly transition between compact and expanded states for architectural uses. The authors compile a comprehensive overview of their geometrical and kinematic properties, focusing on single-curvature scissor grids like barrel vaults. Configurations are classified by grid cell geometry and other parameters, aiding designers. Future research will extend this classification to other geometries.
(Roovers and Temmerman 2015)	The study covers deployable scissor structures, known for easily switching states, ideal for architectural uses. Although traditionally complex to design, the paper presents a simpler method using circle packing and digital tools, making the design process more intuitive and interactive.
(Roovers and Temmerman 2017a)	The study examines deployable scissor grids, which are networks of bars that change configurations. Double curvature is prized for aesthetics, functionality, and strength. Previously, polar scissor units were restricted to spherical shapes. The authors introduce a generalized polar scissor unit that can form non-spherical grids while retaining movement patterns, broadening design and application possibilities. The paper details the geometry, movement, and a joint model for these units, using scale models for demonstration.

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Table 2.2. (cont). Summary of the studies related to different surface morphologies by using scissor structures.

Study	Main Findings
(Roovers and Temmerman 2017b)	The study examines deployable scissor grids for mobile and temporary uses. Traditional designs are complex and inflexible, but translational scissor units enable versatile curved grids with a simple 2D design, deploying smoothly without stress. The paper expands understanding and potential by introducing new designs and compiling existing ones. It reviews the geometric and kinematic properties of these grids, detailing necessary math concepts from single units to large assemblies, and discusses adding joints in line models without affecting deployment, enhancing design and usability.
(Osmani et al. 2017)	The study discusses pantographic structures, foldable frameworks of scissor link units with rotating bars. Typically flat, cylindrical, or spherical, they are not used in hyperbolic parabolic (HP) shapes, which combine convex and concave parabolas. The paper describes methods for creating HP surfaces with pantographic structures using: 1. Two border scissors. 2. Four border scissors. 3. All-scissor HP Pantographic Structures. It compares the effectiveness of these methods.
(Torres 2017)	The paper presents a deployable arch design using a scissor-system based on regular polygon geometry. Key points include: 1. Design: Articulated bars form a scissor-system. 2. Geometry: Arch shape from inscribed regular polygons. 3. Characteristics: Number of bars, pivot positions, distances, bar length, and open-geometry angle. 4. Goal: Create a deployable half-dome with semi-arches. 5. Comparison: Simplifies assembly and reduces deployment time compared to traditional arches needing external supports.
(Torres Londoño and Matos 2018)	The Deployable Structures Workshop at PUC Minas, led by the SMiA research group and Hugo Alkmim de Matos and Natalia Torres with LEFAD, focuses on scissor-type modular designs. Students use 3D printed connectors to build models and end the workshop by constructing a wooden pavilion, gaining hands-on experience with deployable structures.
(Maden, Ölmez et al. 2019)	The study covers dynamic building envelopes in architecture, systems that adjust shapes based on environmental conditions and occupant needs. Key points are: 1. Dynamic Systems: Reversibly change shape. 2. Innovative Solutions: Fold, expand, or curl. 3. Dynamic Shelter: Multiple shape options. 4. Loop Assembly Method: Development technique. 5. Parametric Model: Grasshopper-based analysis and changes. 6. Structural Mechanism: Geometric analysis allows various forms with a single DOF. The study highlights the flexibility and innovation of dynamic building envelopes.

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CHAPTER 3

BASIC PRINCIPLES OF SCISSOR LINKAGES

3.1. Basic Scissor Units

All scissor linkages are made up of the serial or parallel multiplication of identical, various, or random scissor units. A scissor unit is composed of links hinged with an intermediate joint (Figure 3.1). Scissor units are connected by two pairs of hinges. According to the literature, there are different identifications of scissor units. Some researchers gave a specific name to scissor units, while some of them did not. First, Piñero (1965), Zeigler (1976), and You and Pellegrino (1997) simply describe rods. Then scissor units are defined as struts (Clarke 1984; Escrig 1985), scissors (Atake 2000), pantograph (Hanaor and Levy 2001), and mostly scissor-like elements (Gantes 1991; Temmerman 2007; Mele 2008; Akgün et al. 2011; Maden, Korkmaz, and Akgün et al. 2011; Zhang et al. 2016).

In most of the studies, basic scissor units are classified into three main types: translational unit, polar unit, and angulated unit as in the Figure 3.1. The imaginary line passing through hinges is called as unit line (Temmerman 2007). Both translational and polar units comprise straight links. The translational unit has an intermediate joint in the middle of the links, while the polar unit has an intermediate joint away from the middle of the link. The angulated unit has kinked links that are less than 180^0. According to this classification, it is observed that unit lines, for translational units, are parallel to each other, for polar unit and angulated units, are intersecting. But this explanation is insufficient to express geometrical motion characteristics of scissor linkages (Maden, Akgün, et al. 2019).

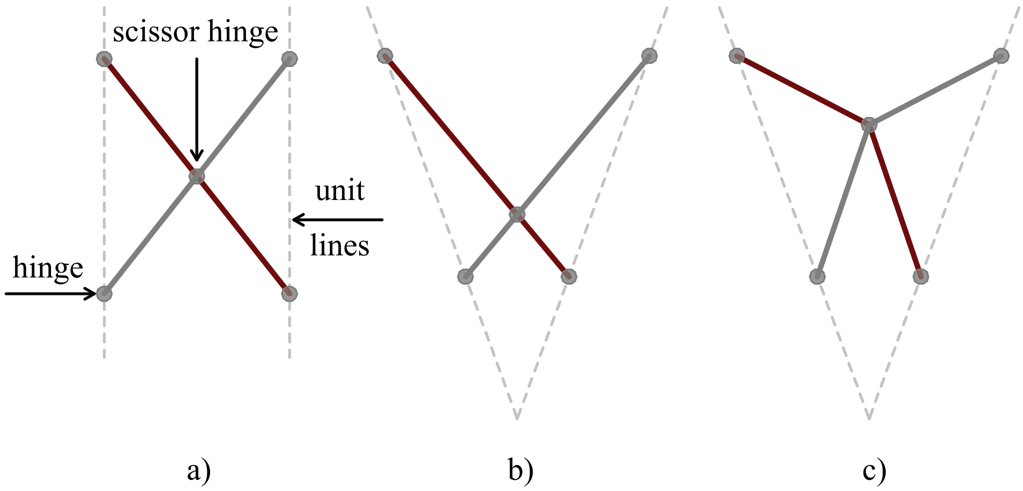


Figure 3.1. Basic scissor unit a) Translational unit b) Polar unit c) Angulated unit (Reproduced from: Maden, Akgün, et al. 2019)

There are different classifications and definitions for the geometry of the primary scissor units. However, in this study, definition of Zhang et al. (2016) is taking consideration. The imaginary line joining two hinges on one side is called a "normal line" passing through the side of the polygon that linkage defines. There are three particular types of scissor units to construct scissor linkages. They are classified by the angles between their two normal lines that are parallel (Figure 3.2a), isogonal (Figure 3.2b) and symmetric (Figure 3.2c). A parallel unit has parallel normal lines; and during the motion, they remain parallel to each other. An isogonal unit has intersecting normal lines but the angle between them does not change during the motion. A symmetric unit consists of two symmetric arms. It has intersecting normal lines too and during the motion the angle between normal lines changes.

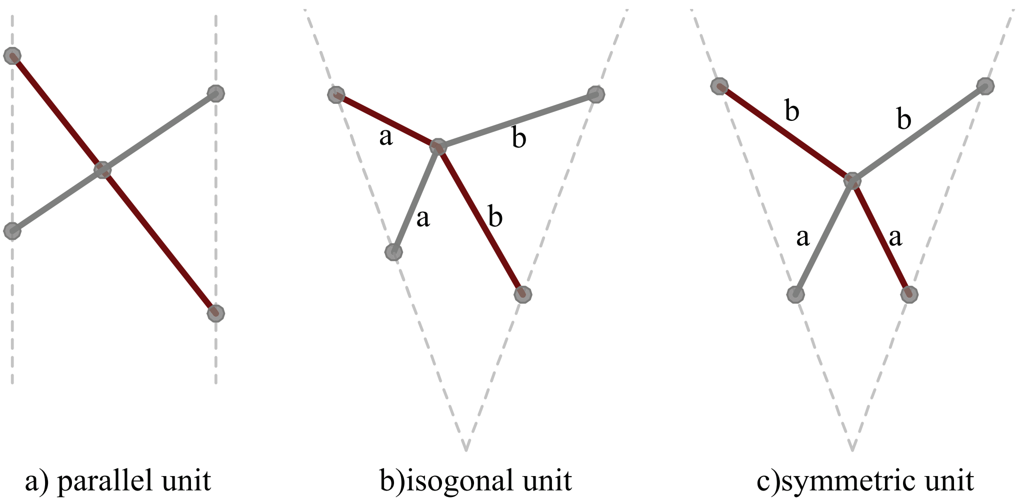


Figure 3.2. Three types of basic scissor units
(Reproduced from: Zhang et al. 2016)

Although there are lots of studies in the literature that classify scissor units as transformable, polar and angulated (Temmerman 2007; Maden, Korkmaz, and Akgün 2011), this is not an exact classification that can meet the motion characteristics of scissor structures formed with these units. Such as, the resulting motion of the polar scissor units can give us translational motion (Figure 3.3).

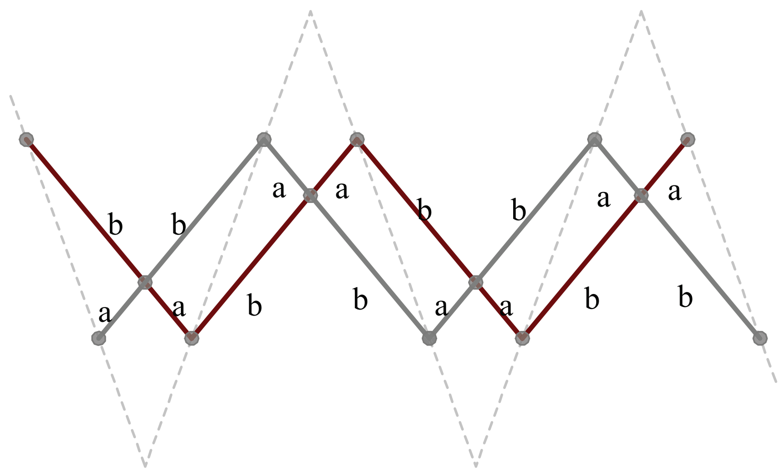


Figure 3.3. Polar scissor unis forming a linkage making translational movement
(Reproduced from: Maden, Akgün, et al. 2019)

3.2. Geometric Construction Methods of Scissor Linkages

Scissor linkages are basically formed by repeating units or loops along a direction by connecting them from their end joints. There are two main methods to design scissor linkages: 1) unit assembly method, 2) loop assembly method.

3.2.1. Unit-Assembly Method

In literature, most of the studies about scissor linkages are based on unit-assembly method. According to the unit-assembly method, scissor linkages are formed by attaching scissor units to each other along a direction. This method is based on the aforementioned scissor units that are translational, polar, and angulated. And it is based on deployability of scissor linkages.

There are some deployability constraints for foldability of planar scissor linkages composed of translational and polar units revealed by Escrig (1985). In the compact form it is desired to be capable of being stored of the scissor linkage. Theoretically, scissor linkage will have one dimension in compact form based on the equation: $a_i + b_i = a_{i+1} + b_{i+1}$. According to this equation, the sum of the lengths of bars on both sides of the unit line should be equal in order to provide the deployability (Figure 3.4). However, Escrig's equation can be only applied to units composed of straight bars, and not to those of angulated bars.

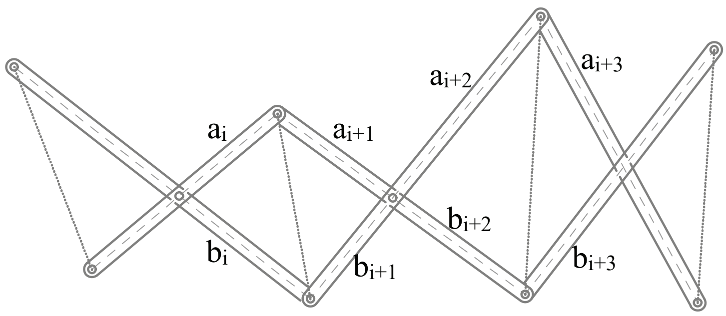


Figure 3.4. Deployability of scissor linkage
(Reproduced from: Maden, Korkmaz, and Akgün 2011)

Deployability conditions of scissor linkages composed angulated units are examined by You and Pellegrino (1997). They classified angulated scissor units into three groups: Hoberman’s angulated element, Type-I generalized angulated element (GAE) and Type-II GAE. General type of Hoberman's element is formed by identical angulated rods: $AE = DE$, $BE = CE$, $\psi = \phi$, ADE and BCE are similar isosceles triangles (Figure 3.5).

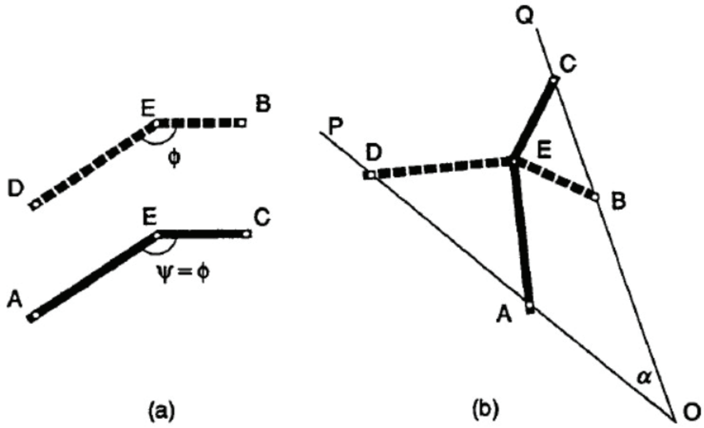


Figure 3.5. General type of Hoberman's element
(Source: You and Pellegrino 1997)

Simplest Type I GAE is formed by angulated rods with equal semi-length but different kink angles: $AE = DE$, $BE =$ CE, $\psi \neq \phi$, ADE and BCE are isosceles triangles (Figure 3.6)

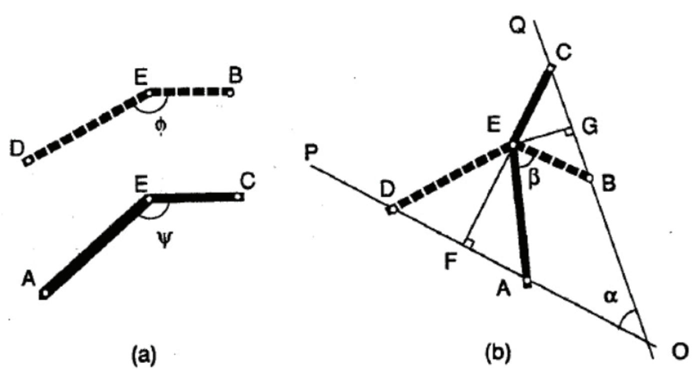


Figure 3.6. Simplest Type I GAE
(Source: You and Pellegrino 1997)

Simplest Type II GAE is formed by angulated rods with proportional semi-lengths and equal kink angles *AE/DE = CE/BE*, $\psi=\phi$, *ADE* and *BCE* are similar triangles (Figure 3.7).

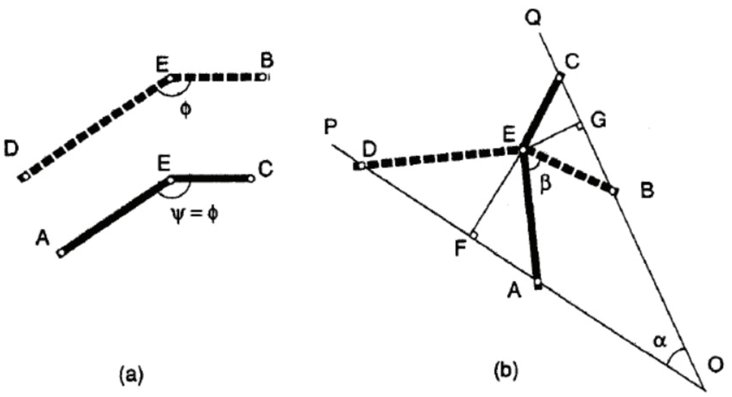


Figure 3.7. Simplest Type II GAE
(Source: You and Pellegrino 1997)

They analyzed these three types of scissor units and their assemblies. A generalized angulated element consists of linked angulated rods forming a chain of parallelograms, ending in either isosceles triangles (Type I GAE) or similar triangles (Type II GAE). An assembly of generalized angulated elements provides a constant angle as the linkage is folded or expanded. Although they used scissor units to form a scissor linkage, they needed to consider loops emerging between units to provide the deployability towards the center of a perimeter (Figure 3.8) (You and Pellegrino 1997).

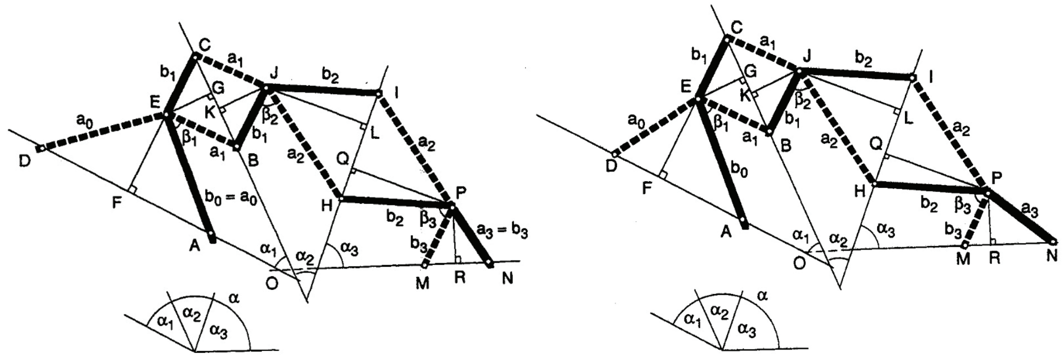


Figure 3.8. Assembly of Type I GAEs left and assembly of Type II GAEs right.
(Source: You and Pellegrino 1997)

3.2.2. Loop-Assembly Method

Loop assembly method is another method to form scissor linkages in the literature. Briefly, it is the method of obtaining the scissor linkages, whose desired form is given, by assembling 4-bar loops to each other at their vertex (Maden, Akgün, et al. 2019).

"Loop assembly" is defined by Hoberman (1990) for the first time in his patented work. After that, in a lecture at MIT in 2013, he presented scissor linkages obtained from hinged rhombi both in regular and irregular geometries (Figure 3.9) (Hoberman, Demaine, and Rus 2013). The simplest linkage, which is a lazy-tong mechanism, is formed by repeating rhombi on a line. Repeating rhombi on a curve results in radially deployable linkage. In addition, repeating rhombi on an arbitrary geometry results scaling deployable closed loop linkage.

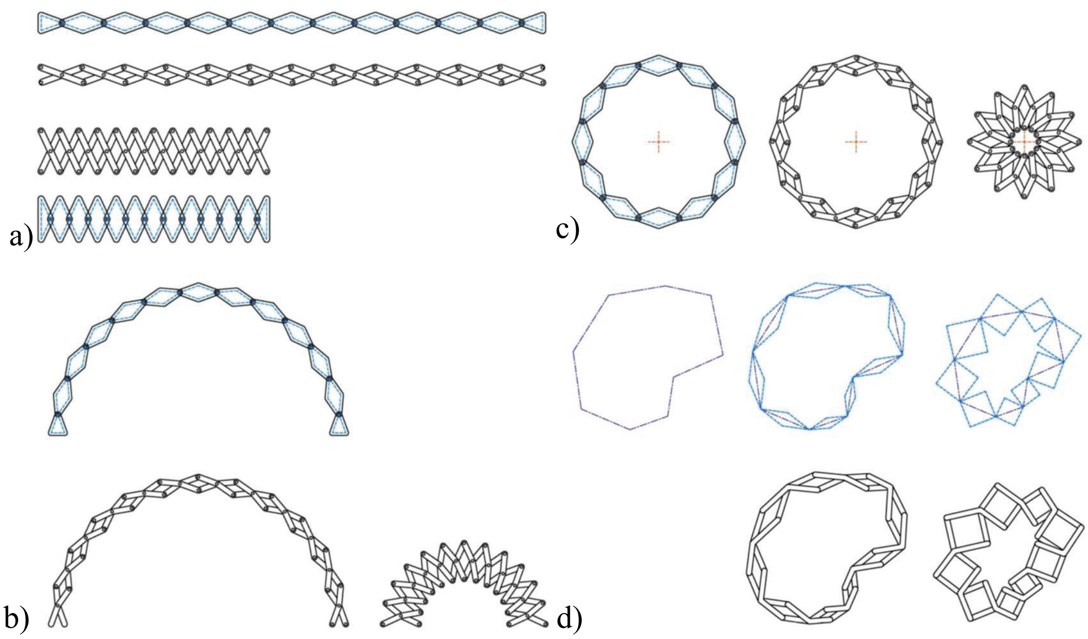


Figure 3.9. a) Rhombi on a line b) rhombi on an arc c) rhombi on a circle d) rhombi on an irregular geometry (Source: Hoberman, Demaine, and Rus 2013)

It can be observed that all scissor linkages comprise of 4-bar loops that are rhombus, parallelogram, and kite (Figure 3.10). Kite and parallelogram loops are convex quadrilateral. The shapes can be either convex or concave. A convex shape is usually called a kite (Figure 3.11a). A concave kite is known as a dart (Figure 3.11c), and a concave parallelogram is called an anti-parallelogram (Figure 3.12c) (Usiskin et al. 2008).

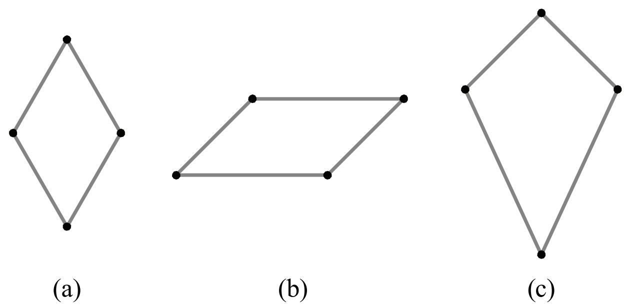


Figure 3.10. a) Rhombus loop, b) Parallelogram loop, c) Kite loop.
(Reproduced from: Usiskin et al. 2008)

When quadrilaterals are viewed as a 4-bar loop, both kite and dart shapes can be seen as different forms of the same 4-bar loop. In a special configuration (Figure 3.11b) where the two shorter sides are aligned, a kite can transform into a dart, just as a parallelogram can change into an anti-parallelogram (Figure 3.12b).

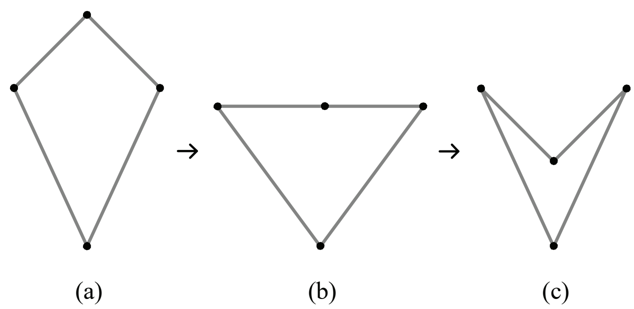
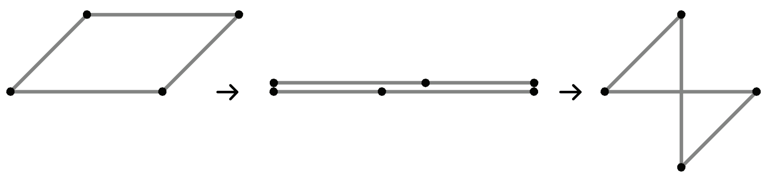


Figure 3.11. Assembly mode change of a kite loop (a) into a dart loop (c) through the singular configuration (b) (Reproduced from: Usiskin et al. 2008)

In the dead center position, where all links are aligned, a parallelogram can switch to an anti-parallelogram. An anti-parallelogram has two equal short sides and two equal crossing long sides. This linkage has four hinges at the ends but none at the crossing point. It has one line of symmetry passing through the crossing point (Figure 3.12).



(a) (b) (c)

Figure 3.12. Assembly mode change of a parallelogram loop (a) into a antiparallelogram loop (c) through the singular configuration (b)
(Reproduced from: Usiskin et al. 2008)

There are limited studies about loop assembly method in the literature. First, Hoberman (1990) revealed this method in his patent. He describes his invention as "a method for constructing reversibly expandable truss-structures that provides for an extremely wide variety of geometries". He also mentioned this method during a lecture at MIT as identical hinged rhombs to construct scaling deployable structures (Hoberman, Demaine, and Rus 2013). Then, You and Pellegrino (1997) mentioned about constructing angulated scissor linkage with parallelogram loops. They did not exactly use the loop assembly method, but their study brought a new perspective by considering parallelograms. Liao and Li (2005) examined scalable planar graph design. Also, Kiper and Söylemez (2010) made a contribution with using Cardan motion for scaling arbitrary polygonal forms. Bai et al. (2014) used four basic tetragon elements that are rhombus, parallelogram, kite and general tetragon element, to synthesize polygon-scaling mechanisms. Recently, in the scope of the Horizon 2020 project, this method was worked out in detail (Yar et al. 2017; Gür, Korkmaz, and Kiper 2017; Gür et al. 2018; Maden, Ölmez, et al. 2019; Maden, Akgün, et al. 2019; Gür, Korkmaz, and Kiper 2019). As a contribution to the literature, they dealt with not only rhombus or parallelogram loops but also antiparallelogram, kite, and its concave version dart loop. They discovered both angular and scalable deployable linkages with antiparallelogram, kite, and dart loops and linkages that can transform between convex and concave forms with antiparallelogram, kite, and dart loops.

3.3. Motion Characteristics of Planar Scissor Linkages

According to the literature, it is better to define geometry of motion of scissor linkages according to the transformation of the curve that the linkage represents rather than defining based on units. In this context, the type of transformation should be defined according to how the curvature changes. Change of the curvature can be observed according to relative positions of normal lines related to the curve during the motion.

In the literature, it is observed that there are three types of motion for scissor linkages: scaling deployment, angular deployment and transformation (Maden, Akgün, et al. 2019).

3.3.1. Scaling Deployment

In a scaling deployment, curves of linkages, both in compact and deployed forms, are identical but only differ in a scale. Scaling deployment can be examined in two situations: First is the radial deployment (changing the radius) (Figure 3.13). The second one is translational deployment (changing the span) which is scaling through a direction (Figure 3.14). In both situations, the angle between normal lines does not change during the deployment. For the translational deployment, normal lines remain parallel to each other. Thus, the angle between normal lines approach to zero, in other words radius tends to go infinity c.

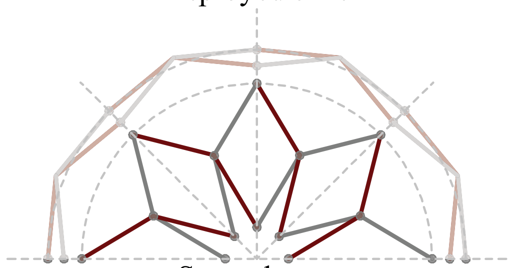


Figure 3.13. Radial deployment.

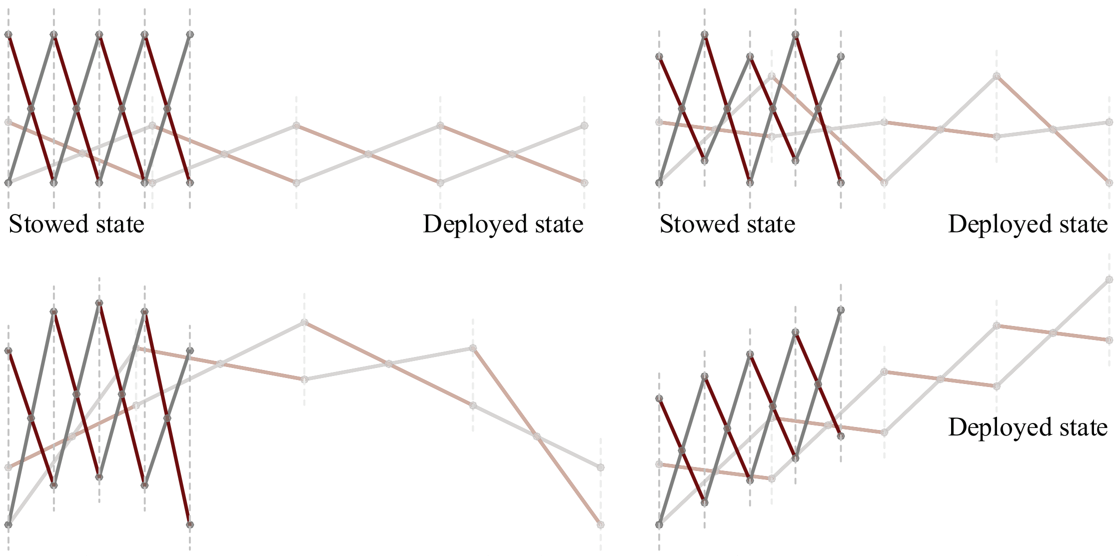


Figure 3.14. Types of linear deployment.

3.3.2. Angular Deployment

During angular deployment, the initial and final radii can remain the same. While the radius changes slightly during movement, the main difference is in the angle. The angle between normal lines changes during the motion, depending on it also the length of circle arc changes as in the Figure 3.15.

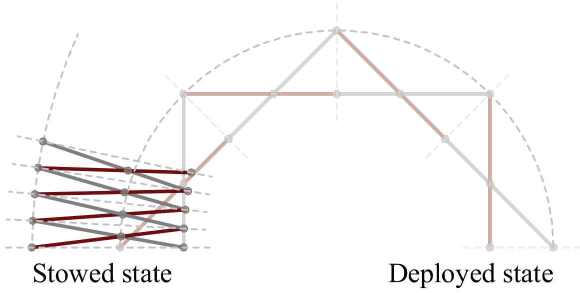


Figure 3.15. Angular deployment.

3.3.3. Transformation

Literature examples show that deployable scissor linkages often have similar compact and deployed forms. Therefore, there are scaling and angular deployable linkages. Linkages with different compact and deployed forms are called transformable (Maden, Akgün, et al. 2019). Transformable linkages are capable of transforming between an initial curve and another desired curve (Figure 3.16).

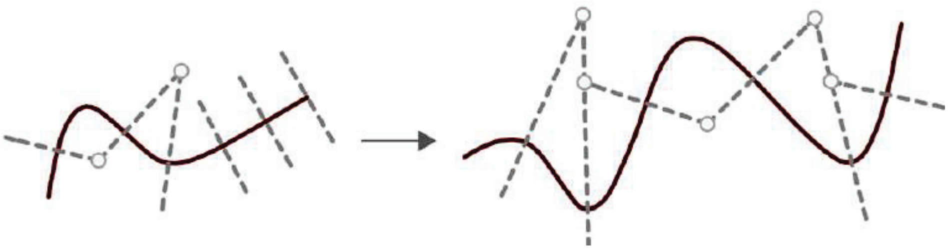


Figure 3.16. Transformation
(Source: Maden, Akgün, et al. 2019)

CHAPTER 4

CURVATURE OF SURFACES

It is essential to understand the surface geometry that has an important role in constructing and defining scissor linkages. This section concerns some geometrical definitions necessary to describe the geometry of form and motion of scissor linkages. In this scope, first, it needs to be concentrated on basic geometrical definitions and representation tools. Terms of curve and surface will be clarified. Then, the term curvature will be examined in detail.

In surface geometry, curves and surfaces are the two main inseparable components used to define a surface in geometrical aspects. In detail, the curves could be described as profiles to build surfaces. In other words, the form of the profile curve usually affects the final shape of the emerging surface strongly.

As a result, once the curves are investigated, it simultaneously leads to the understanding of surfaces unambiguously. Mathematically, there are two concepts: tangent plane and surface normal. They are complementary approaches for describing surfaces and curves (Figure 4.1) (Blaauwendraad and Hoefakker 2014).

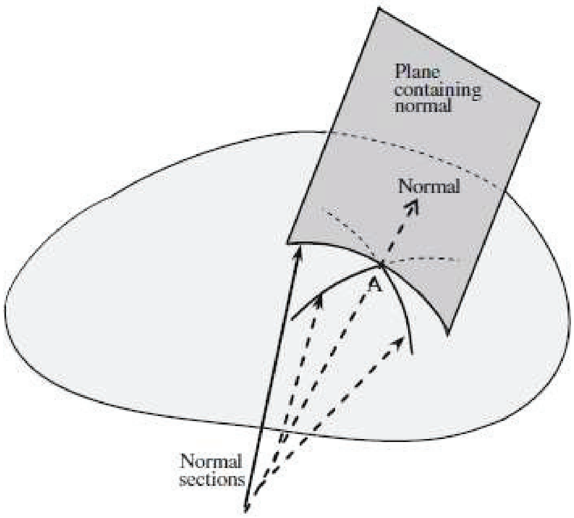


Figure 4.1. The tangent plane provides insight into the characteristics of surfaces.
(Source: Blaauwendraad and Hoeafkker 2014)

4.1. Introduction of Curves and Surfaces

4.1.1. Curve and Curve Tangent

A curve can be thought of as a connected one-dimensional series of points. Different parts of a curve could be covered by the mentioned points. For instance, all points of a circle lie in a plane, and these kinds of curves are called planar curves as opposed to spatial curves (Figure 4.2) (Pottmann et al. 2007).

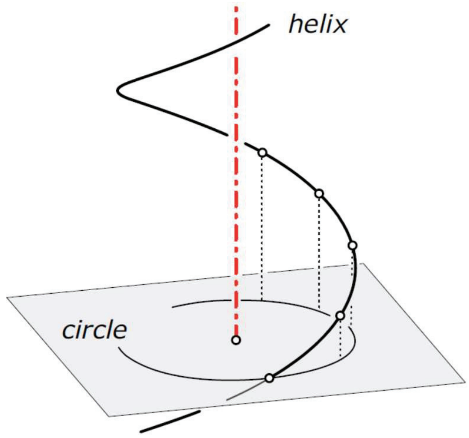


Figure 4.2. Comparison of circle (planar curve) and helix (spatial curve)
(Source: Pottmann et al. 2007)

A smooth curve *c* (Figure 4.3) can be approximated by a tangent line *T* at a point *p*. This line *l* can be found by connecting two close points *p* and *q* on the curve *c* or by using a limit process or the first derivative. The normal intersects the curve at a right angle.

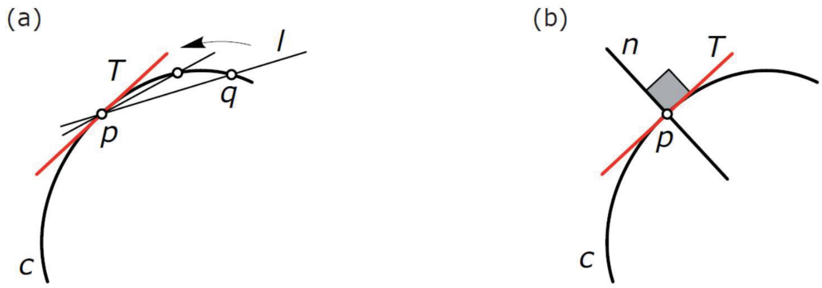


Figure 4.3. a) A tangent T touches a curve c, b) The normal of a planar curve intersects the curve at a right angle. (Source: Pottmann et al. 2007)

Variations in the tangent vector impact the length of the arc along the curve (Hyde et al. 1997). Curvature measures how much the tangent direction changes when one slightly moves the point on the curve. One needs to choose the right parametrization to define curvature using a standard parametrization of the curve. (Lastra 2021).

4.1.2. Discretizing of Curves

We can use a simple approach to understand how curves work. Instead of using complicated calculus, we can break down a curve into a series of connected line segments, like the sides of a polygon. This is shown in Figure 4.4. Let's call the polygon "*P*" and its vertices (the corners of the polygon) c_1, c_2, c_3, and so on. We also say that all the sides of the polygon have the same length, *L*. Given a smooth curve "*c*" and a length "*L*", we can create a polygon P where all the vertices lie on the curve. This polygon is also called a discrete curve. In many real-world situations, curves are actually treated as polygons (Pottmann et al. 2007).

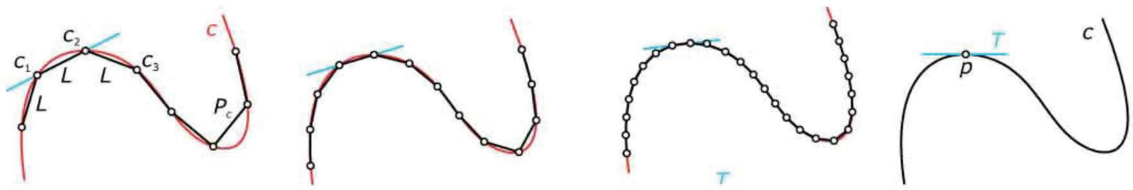


Figure 4.4. Evaluation of a discrete curve.
(Source: Pottmann et al. 2007)

4.1.3. Curvature of Surface

In the middle of the 17th century, Isaac Newton discovered that the idea of curvature extended from Newtonian calculus. Significant calculus difficulties like figuring out the area under curves and the perimeter of planar curves were solved by the concept of curvature. In particular, the previously stated innovative method helped to calculate the "quadrature" or area of a circle. As a useful metric, Newton defined "crookedness" (also known as curvature) as the radius of the circle of greatest fit to any given planar curve at all points on the curve. (Figure 4.5) (Hyde et al. 1997).

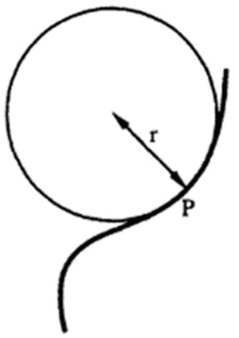


Figure 4.5. The curvature of a planar curve at a point P.
(Source: Hyde et al. 1997)

Basically, curvature means the amount by which a curve or a surface deviates from being straight or flat, respectively. A curve in the plane, the simplest case of curvature, could vary at every point along the curve but may be quantified at each point as the inverse of the radius (K = 1/r) of the circle that most closely fits the curve locally at that point (Hyde et al. 1997).

The curvature of the surface is a complex entity but can be understood as a generalization of the curvature of planar curves. At a specific location on the surface, the principal curvatures, k_1, and k_2, might be used to implement surface curvature. As seen in Figure 4.6 the principal curvatures represent the highest and minimum values of all the normal curvatures at that location. The principal directions are those that match these principal curvatures (Hyde et al. 1997).

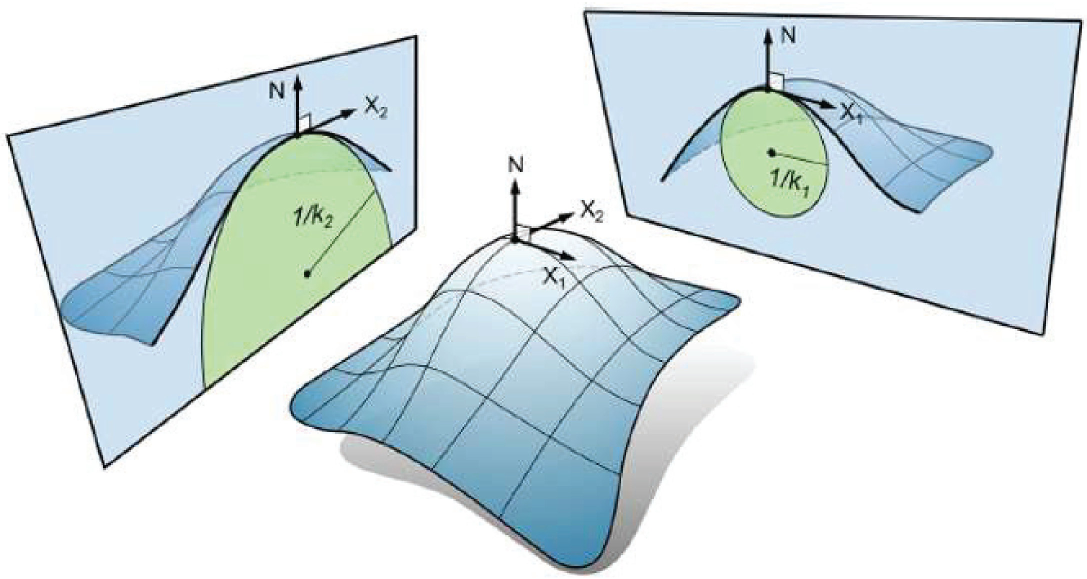


Figure 4.6. Measures of 3D surface curvature represent the greatest and lowest normal curvatures k_1 and k_2 using orthogonal unit vectors X_1 and X_2 (Source: Nabavi and Fossen 2021)

The most efficient approach to describe how the surface curves around a point on the surface is to use the principal curvatures and directions. It is important to note that the umbilical point (where all normal curvatures are equal) is where the major curvatures cannot be individually established. In differential geometry of three-dimensional surfaces, locally spherical points on a surface are referred to as umbilicus or umbilical points (Figure 4.7). At specific points, the normal curvatures are equal in all directions. Hence, both principal curvatures are equal, making every tangent vector a principal direction. Only two surfaces, the plane and the sphere, are entirely made up of these points (Hyde et al. 1997; Hilbert and Cohn-Vossen 1990). One can calculate the mean curvature *H* by averaging the principal curvatures $H = (k_1 + k_2)/2$

A flat plane has a mean curvature (*H*) of 0 since all its normal curvatures are 0. However, if the plane is bent into a wavy shape, H becomes non-zero in some areas because one of the principal curvatures is no longer 0 (Callens and Zadpoor 2018).

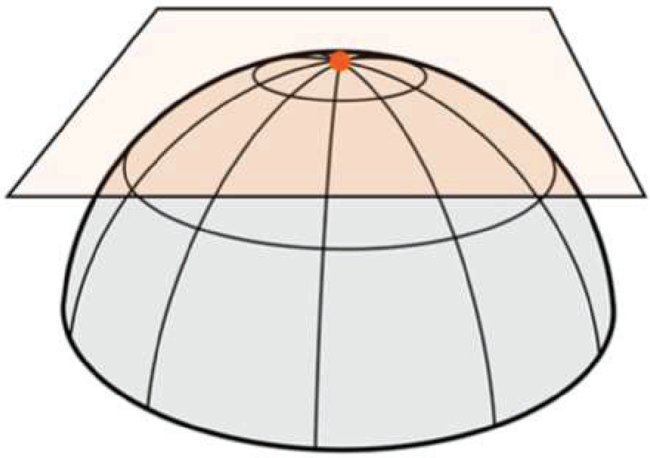


Figure 4.7. Umbilical point
(Source: Nabavi and Fossen 2021)

Another important metric, in addition to median curvature, is the "Gaussian curvature K." The principal curvatures are multiplied to find it: $K = k_1 * k_2$

Mean, and Gaussian curvatures provide distinct perspectives on the curvature of the surface. The mean curvature of a surface is extrinsic, meaning it is determined by the surface's location in 3D space from an external viewpoint. The Gaussian curvature is an inherent property that solely relies on measurements made within the surface. Therefore, a two-dimensional resident could ascertain it (Hilbert and Cohn-Vossen 1990; Calladine 1983). To interpret the difference between the Gaussian and mean curvature, a surface that is extrinsically curved but remains intrinsically flat would be a good example. First, imagine bending a flat plane into a wavy shape. Then, the surface now has non-zero mean curvature, but its Gaussian curvature is still zero because one of the principal curvatures is zero. This shows that bending a surface without stretching doesn't change its Gaussian curvature, unlike mean curvature (Pressley 2010). A *developable surface* is the special name shown in Figure 4.8 has zero Gaussian curvature everywhere. Developable surfaces are those that have zero Gaussian curvature because their intrinsic curvature stays zero even when it's bent to change their extrinsic curvature. Figure 4.8 shows three types of developable surfaces: generalized cylinders, generalized cones, and tangent developable to a space curve. These surfaces are formed by bending a flat plane (two-dimensional) without stretching or tearing (Nabavi and Fossen 2021). This is feasible because Gaussian curvature doesn't change when a surface is bent (Hilbert and Cohn-Vossen 1990; Toponogov and Rovenski 2006; Callens and Zadpoor 2018).

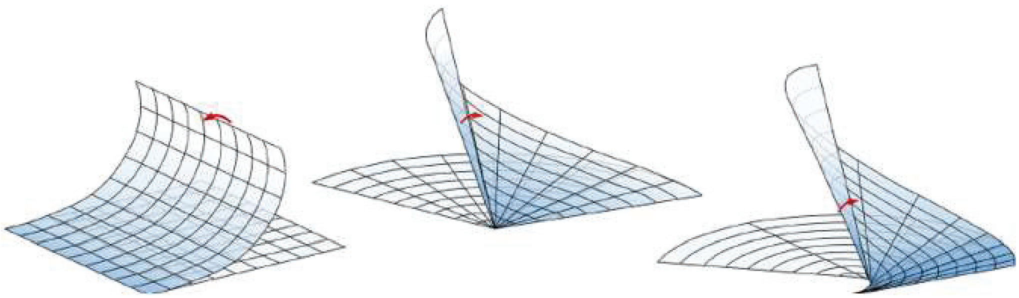


Figure 4.8. Three classes of developable surfaces (a) generalized cylinder(b) generalized cones (c) tangent developable (Source: Nabavi and Fossen 2021)

The Gaussian curvature of a surface can be defined in two ways: extrinsically (based on how it curves in 3D space) or intrinsically (based on measurements within the surface itself). The intrinsic approach can be visualized by considering an imaginary triangle drawn on the various surfaces. In detail, in Figure 4.8, three types of developable surfaces are based on just bending, which can be flattened into the plane through bending again. For a developable surface (like a cylinder, cone, or tangent developable all in K=0), this triangle would behave as if drawn on a flat plane, with angles summing to 180 degrees. These surfaces can be bent into a plane without stretching. Intrinsically curved surfaces (like a sphere $K > 0$, saddle $K < 0$, or vase varying K) can't be flattened without stretching. On these, the sum of a triangle's angles is more or less than 180 degrees (Weeks 2001; Calladine 1983; Toponogov and Rovenski 2006). This shows the surface has positive, negative, or zero curvature. (Weeks 2001). Measuring angles in a triangle tells you the Gaussian curvature of the surface, but it can't distinguish a flat plane from a cylinder, as both have zero intrinsic curvature (Callens and Zadpoor 2018).

Bending without stretching or shrinking is called an isometric deformation so that it leaves the metric unaffected. The metric is the short version of the term metric tensor that describes the distances between the neighboring points on a surface (Sharon and Efrati 2010). In the case of a flat plane, the metric tensor known as “Euclidean metric” could be represented physically as an imaginary regular grid consisting of equally spaced and perpendicular lines (Marder, Deegan, and Sharon 2007; Kamien 2007). As opposed to the previously mentioned situation in Figure 4.8, when the flat plane is subjected to pure bending, the grid is not distorted if the plane is deformed into, for example, a bell-shaped surface, meaning the distances and angles on the surface change, making the metric "non-Euclidean". Changing the Gaussian curvature of a surface requires changing its metric, which can't be done by bending alone. You need to stretch or shrink the surface. (Callens and Zadpoor 2018).

There are two methods to conceptualize surface curvature: intrinsic (Gaussian curvature) and extrinsic (mean curvature). Certain surfaces, such as a sheet of paper curled into a tube, may appear curved from the outside but flat on the inside, known as a developable surface. To make a flat surface have extrinsic curvature, one can simply bend it without stretching or shrinking it. But to give a surface intrinsic curvature, bending alone is insufficient; one needs to change the distances between points on it, which means stretching or shrinking it (Callens and Zadpoor 2018).

4.2. Types of Surfaces

Surface classes use a simple "kinematic" method. This involves smoothly moving a profile curve. This section will examine the types of surfaces and their generation. According to their generation methods, they will first be analyzed in three groups: rotational, translational, and ruled surfaces, as in Figure 4.9 (Blaauwendraad and Hoefakker 2014).

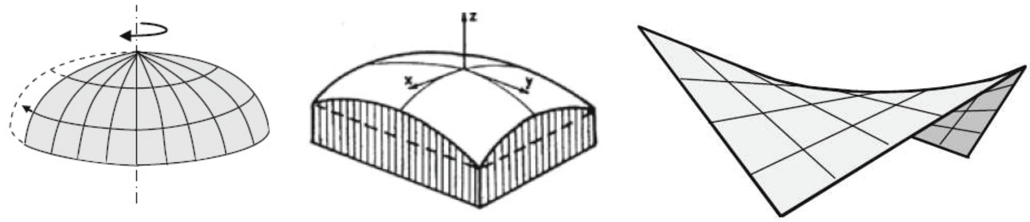


Figure 4.9. Illustrations of rotational surface, translational surface, and ruled surface. (Source: Blaauwendraad and Hoefakker 2014)

This classification considers the profile curve's shape, grouping curved surfaces into three types: single curvature, double curvature, and freeform.

Rotational surfaces are obtained by rotating planar or spatial curve (*c*) about an axis (*A*). They are also called surfaces of revolution. Every point (*p*) on generating curve (*c*) defines a circle (*cp*) through the surface. These circles lie on supporting planes (S_c) orthogonally to the axis (*A*) thus they parallel to each other (Figure 4.10) (Pottmann et al. 2007).

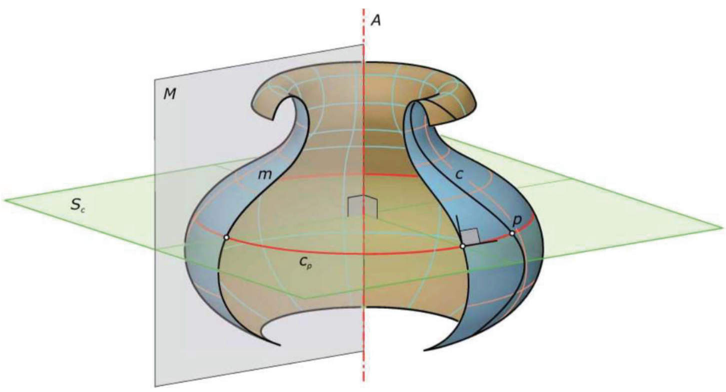


Figure 4.10. Rotational surface
(Source : Pottmann et al. 2007)

The intersection of the rotational surface and the orthogonal plane (*M*) passing through the axis of rotation (A) gives meridian curves (*m*). Orthogonal planes intersect at right angle with supporting planes. This means meridian curves and parallel circles also intersect at right angles (Pottmann et al. 2007).

It is better to define the final shape of the rotational surface with meridian curves rather than arbitrarily generating curves. Meridian curves are symmetric to the rotational axis. The same rotational surface can be obtained by rotating the half meridian 360 degrees while rotating to the entire meridian 180 degrees (Figure 4.11) (Pottmann et al. 2007).

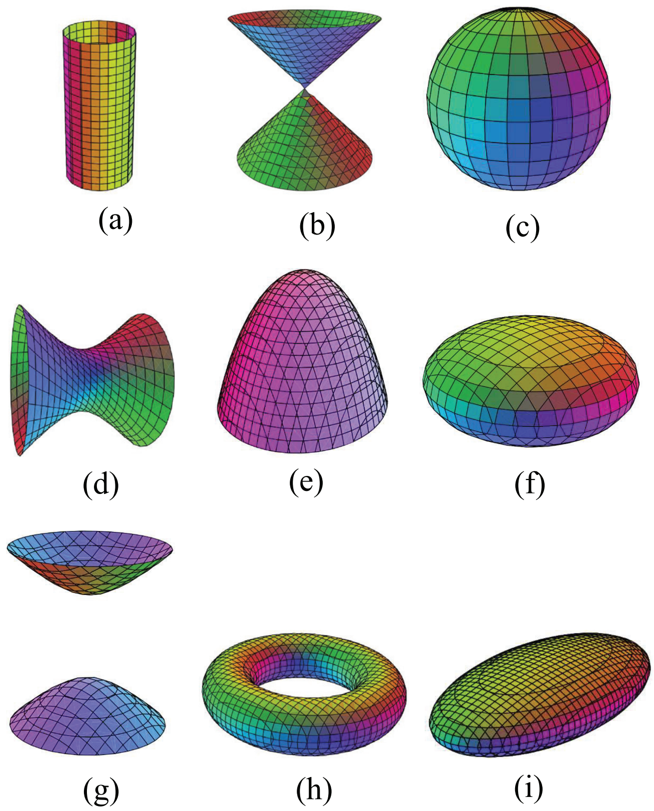


Figure 4.11. Examples of rotational surfaces (a) cylinder, (b) cone, (c) sphere, (d) one-sheet rotational hyperboloid, (e) rotational paraboloid, (f) oblate rotational ellipsoid, (g) two-sheet rotational hyperboloid, (h) torus, (i) prolate rotational ellipsoid. (Generated with CalcPlot3D).

Translational surfaces are generated by translating a profile curve (*k*) along the path curve (*l*). Thus, a translational surface contains a set of parallel curves (*kp*) that are congruent with the profile curve (*k*). The same translational surface can be generated by changing the roles of profile and path curves (Figure 4.12).

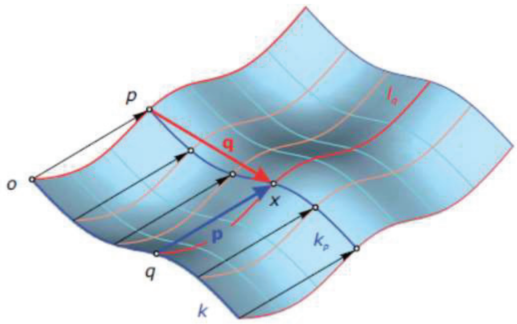


Figure 4.12. Translational surface
(Source: Pottmann et al. 2007)

The simplest version of translational surfaces is cylinder. With cylinder, elliptic paraboloid and hyperbolic paraboloid could be generated as translational surfaces (Figure 4.13).

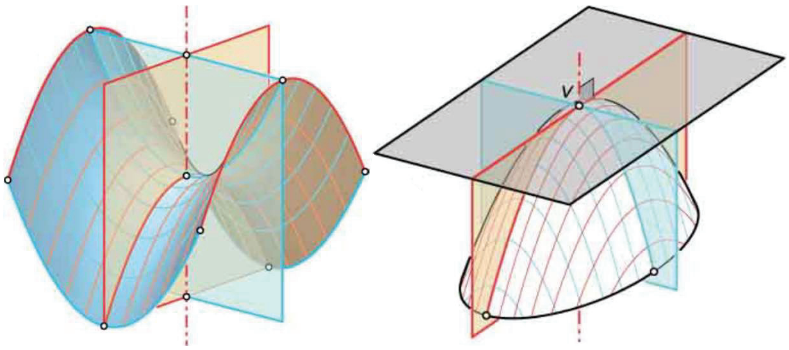


Figure 4.13. Hyperbolic paraboloid and ellipsoid as translational surface.
(Source: Pottmann et al. 2007).

Ruled surfaces are created by moving a straight line (*g*) along a curve (c_1) at a right angle. They contain many straight lines called generators. The direction of the straight-line segment should be indicated (*d*). The direction can vary continuously when moving along the directrix curve (c_1). Ruled surfaces can also be generated by joining points on two parametrized curves (Figure 4.14). All defined ruled surfaces are generated with this method (Pottmann et al. 2007).

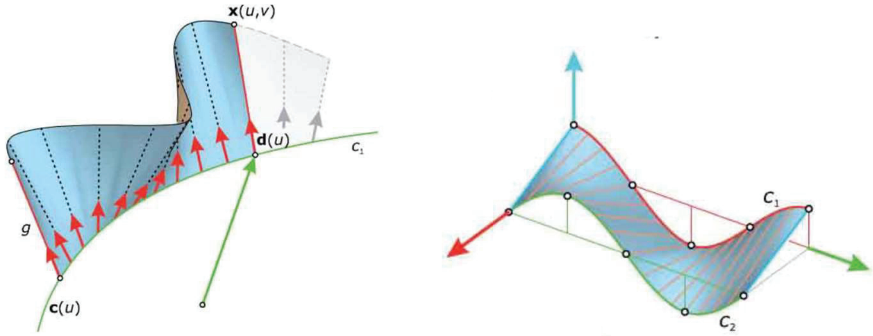


Figure 4.14. Free form ruled surfaces generated by a) moving line segment a long directrix, b) joining two points on parametrized curves.
(Source: Pottmann et al. 2007)

Cylinder, cone, one-sheet hyperboloid, and hyperbolic paraboloid have straight lines on them, so they can be made as ruled surfaces (Figure 4.15).

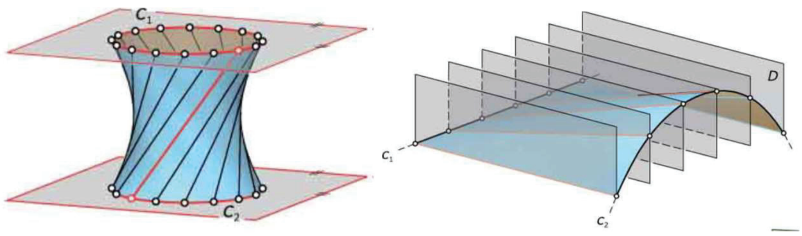


Figure 4.15. One-sheet rotational hyperbolic paraboloid and conoid.
(Source: Pottmann et al. 2007)

4.2.1. Single Curvature Surfaces

Single curvature surfaces are ruled surfaces created by moving a curve along a straight line (Türkçü 2017). All developable surfaces are ruled, but not all ruled surfaces are developable. A developable surface is one that can be flattened into a plane without stretching or shrinking (Figure 4.16). This means its Gaussian curvature is zero everywhere, so it's also called a single-curved or monoclastic surface (Pottmann et al. 2007).

There are three kinds of developable surfaces: cylinders, cones, and surfaces made by touching curves in space. These are special kinds of ruled surfaces because they have a tangent plane along the entire line, not just at one point. That's why they are called developable ruled surfaces. Also, these surfaces have no curvature, so their image on a sphere is just a line. So, points on a developable surface are either parabolic or flat (Pottmann et al. 2007).

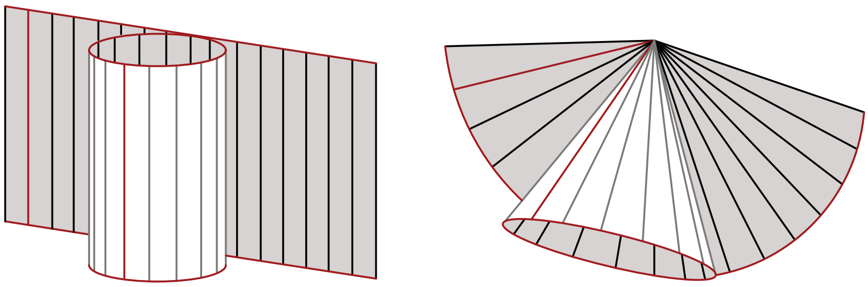


Figure 4.16. Cylinder and cone as developable surfaces.
(Source: Pottmann et al. 2007)

4.2.2. Double Curvature Surfaces

According to Siegel's classification, double curvature surfaces can be classified into two types: synclastic and anticlastic. (Türkçü 2017).

Synclastic surfaces have positive Gaussian curvature. These surfaces are not cut by their tangent planes. They are classified into two groups based on how they are created: rotation or translation (Türkçü 2017). Examples of rotational synclastic surfaces include spheres, paraboloids, ellipsoids, two-sheet rotational hyperboloids, and the outer face of a torus (Figure 4.17). These surfaces, especially spheres and parts of spheres, are often used in architecture. The term "dome" refers to a sphere (Türkçü 2017). An elliptic paraboloid is an example of a translational synclastic surface. It is created by translating one parabola along another parabola. Both parabolas must be open to the same side and have parallel axes. The axis of an elliptic paraboloid is where the two symmetry planes intersect (Pottmann et al. 2007).

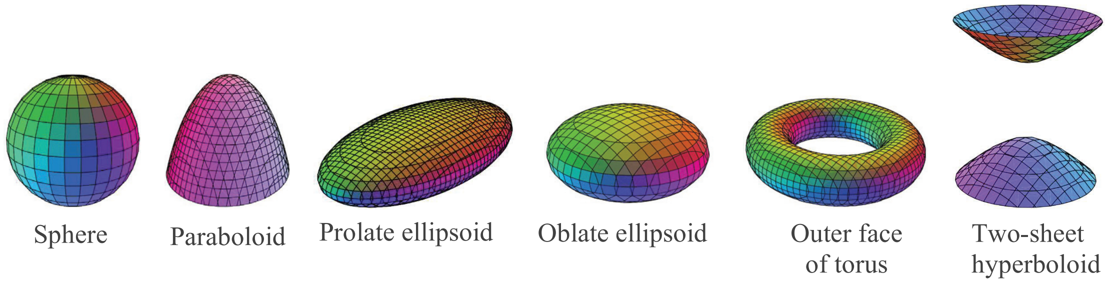


Figure 4.17. Examples of synclastic surfaces
(Generated with CalcPlot3D)

Anticlastic surfaces have negative curvature. They can be categorized into three types based on their creation: rotational, translational, and ruled surfaces (Türkçü 2017). Examples of rotational anticlastic surfaces (Figure 4.18) include the one-sheet hyperboloid, catenoid, and the inner side of a torus. Translational anticlastic surfaces are saddle-shaped, like hyperbolic sections and paraboloids. The one-sheet hyperboloid, hyperbolic paraboloid, and conoid are examples of ruled anticlastic surfaces.

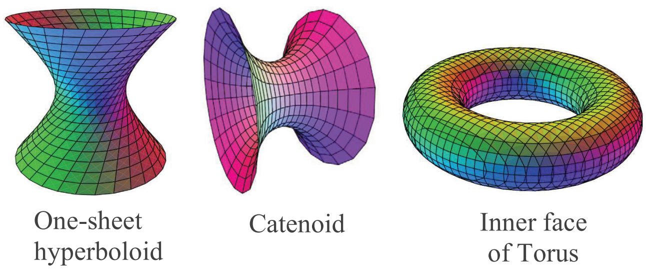


Figure 4.18. Rotational anticlastic surfaces
(Generated with CalcPlot3D)

A conoid is created by a movable line that always stays parallel to a specific plane (the director plane) and slides along two end curves called directrices. The most common conoid shapes happen when one of the directrices is a straight line, like in Figure 4.19. We usually picture the straight-line directrix and the plane holding the curved directrix as both being at the right angles of the director plane. We also assume the curved directrix is symmetrical around its vertical axis. The type of curve used as the directrix determines whether one can get a parabolic, circular, or catenary conoid. Parabolic conoids are the most common. Architects like conoidal surfaces because they let in lots of natural light while keeping structural costs low. Plus, the formwork for this type of surface is easy to make with straight planks (Türkçü 2017).

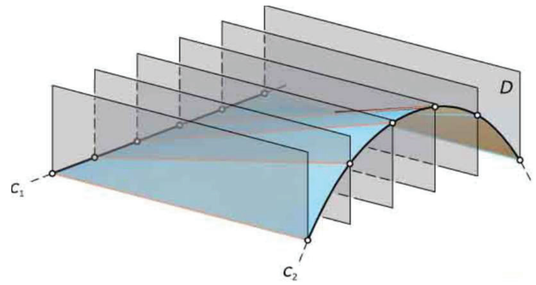


Figure 4.19. All rulings of a conoid are parallel to the director plane (D) and intersect a straight line c_1 and, c_2 are directrices (Source: Pottmann et al. 2007)

All surfaces are summarized in Figure 4.20 below.

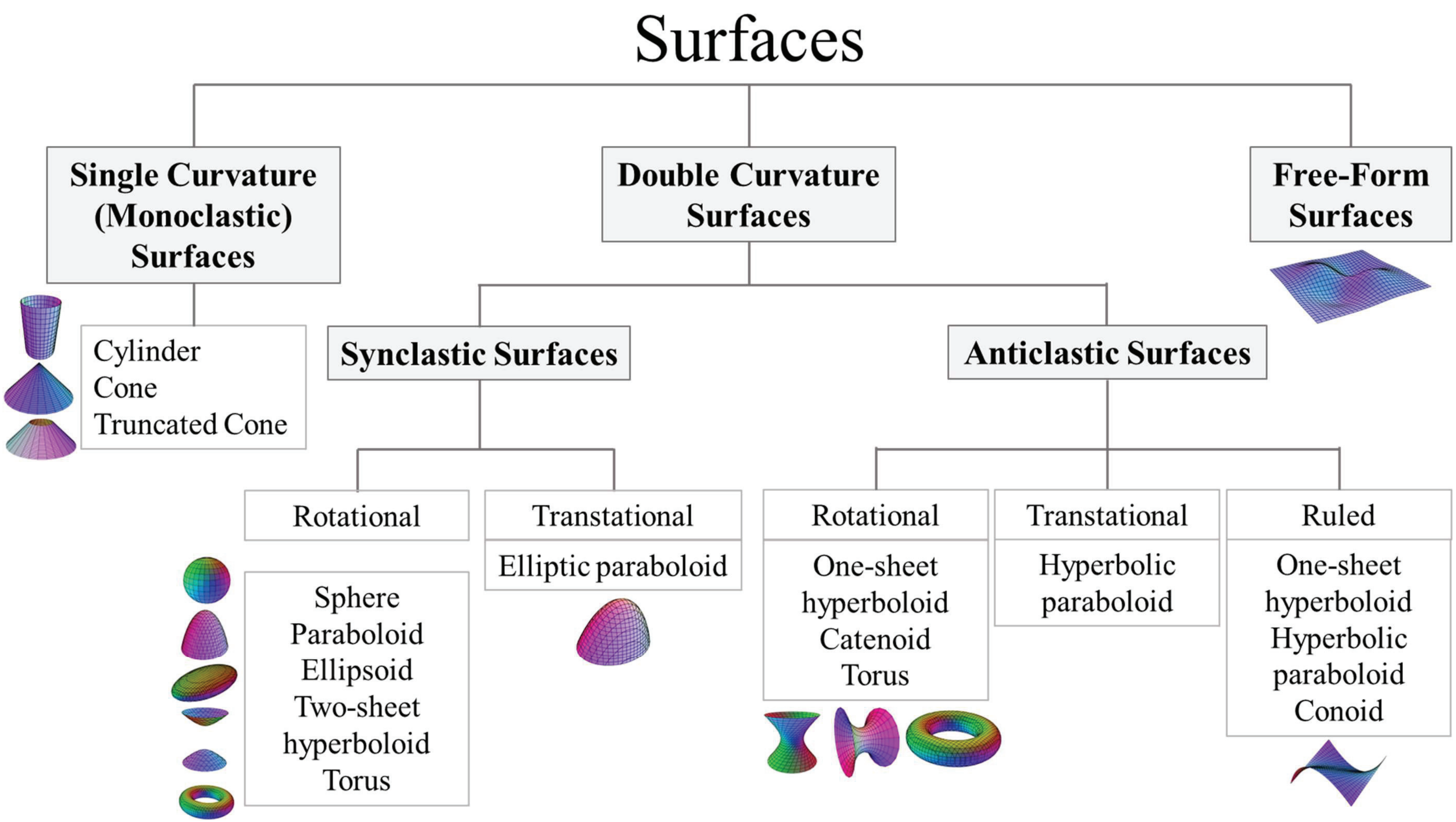


Figure 4.20 Classification of Surfaces
(Reproduced from: Türkçü 2017)

CHAPTER 5

CONIC SECTIONS AND TRANSFORMATIONS OF LOOP ASSEMBLIES

When the studies carried out so far are examined, it is seen that three types of motions have been carried out in the literature. These are scaling deployment, angular deployment, and transformation (Maden, Akgün, et al. 2019). In the literature, some spatial examples exist besides the planar examples for the first two motion types. However, studies that can evolve to the spatial for transformation are insufficient. For this purpose, in this study, it is aimed to design spatial linkages that can pass from convex form to concave form. In this direction, it should start with planar linkages that have the potential to reach the desired spatial forms. The loop-assembly method is used to observe the motion types in the scissor linkages easily.

5.1. Conoid Surface and Conic Sections as a Surface Profile

Conic sections are used to describe the surface geometry. Conic sections are obtained as a result of the intersection of a plane and a double right circular cone. A line or a curve can be obtained by cutting a surface with a plane. By changing the angle and location of the intersection plane but not passing through the vertices of the cone, different types of conic sections can be produced, as follows in Figure 5.1a below. If $\alpha<\beta\leq 90°$, then the plane intersects the vertex precisely at a *point*; (b) If $\alpha=\beta$, when the plane is parallel to the edge of the cone, an intersection of the plane with a cone forms a *straight line*, (c) If $0\leq\beta<\alpha$, when the plane passing through the vertex, the intersection being formed is a *crossed pair of straight lines*, (d) If $\beta=90°$ when the plane is parallel to the base plane of the cone, the conic section being formed is a *circle*. (e) If $\alpha<\beta<90°$ the conic section is an *ellipse*; (f) If $\alpha=\beta$, when the plane is parallel to the edge of the cone but not passing through the vertex, the conic section being formed is a *parabola,* (g) If $0 \leq \beta < \alpha$ the plane intersects both nappes and the conic section which is known as a hyperbola.

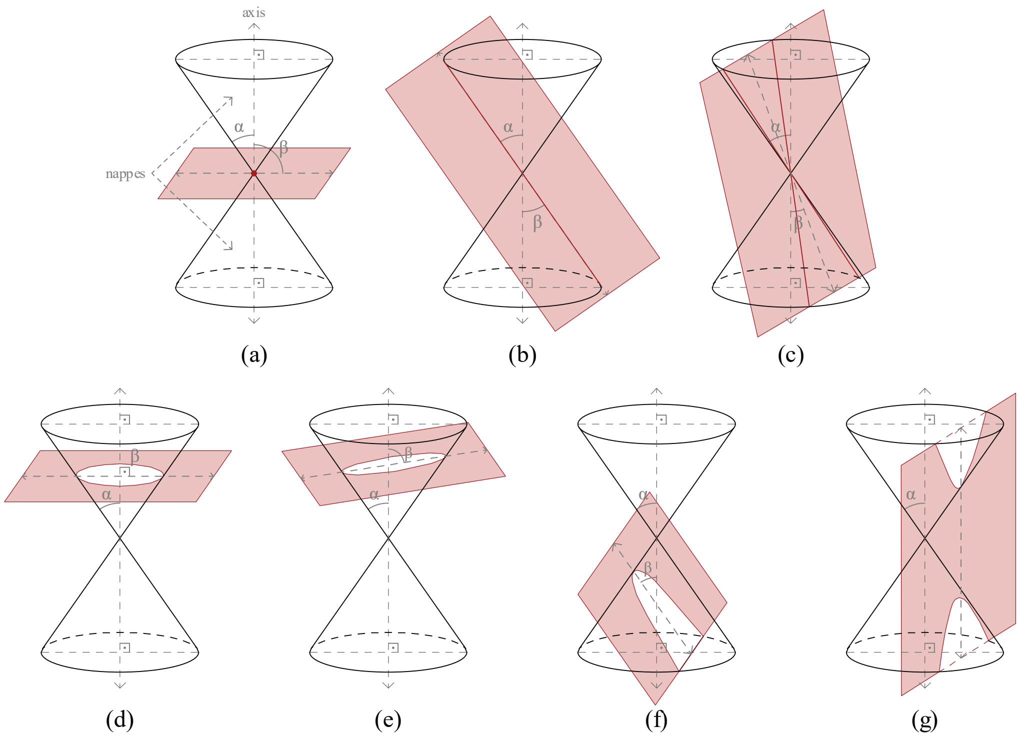


Figure 5.1. Conic sections
(Reproduced from: Lastra 2021)

Catalan surfaces, named after the Belgian mathematician Eugène Charles Catalan, are ruled surfaces with rectilinear generatrixes parallel to any fixed plane, which is called the directrix plane of the surface (Figure 5.2). In the case when both directrix lines are curved, a cylindroid can be obtained; when one directrix is a straight line, a conoid can be obtained (Krivoshapko and Ivanov 2015).

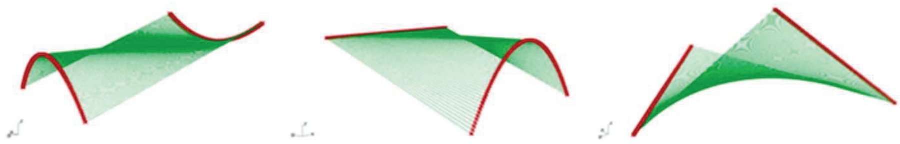


Figure 5.2. Typical examples of Catalan surfaces, respectively from the left to right: a cylindroid, a conoid; a hyperbolic paraboloid (Source: Dzwierzynska 2019)

A conoid is a ruled surface that can be defined by three elements: plane *p*, straight line *a* (axis of the conoid), and curve *c*. The conoid is then formed by all straight lines (rulings) which are parallel to the plane *p* and intersect axis *a* and curve *c*. If *a* and *p* are perpendicular to each other, then the conoid is called the right conoid (Figure 5.3). Therefore, all forming rulings of the surface are perpendicular to axis a. It also means that if axis *a* and control curve *c* have been specified, there is no need to specify control plane *p*, because it must be perpendicular to axis a and its absolute position is not important (Dolezal 2011).

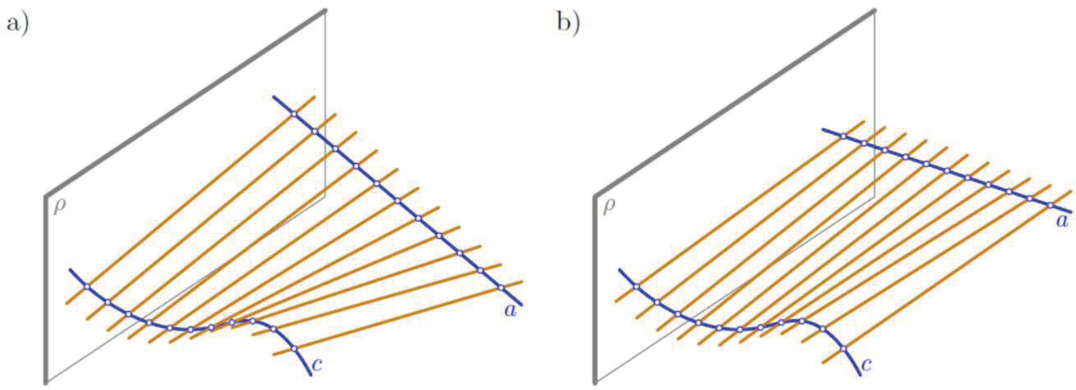


Figure 5.3. Illustration of a) general, and b) right conoid (Source: Dolezal 2011)

5.2. Loop Assemblies Based on Conic Sections and Frieze Patterns

Sections taken from the conoid surface perpendicular to the director plane give curved directrix. Starting from the conic sections, while one of the directrix of a conoid is a straight line, the other one can be half circle, parabola, ellipse, or S-shaped curve. Possible transformations of directrices of a conoid one another are illustrated in Figure 5.4:

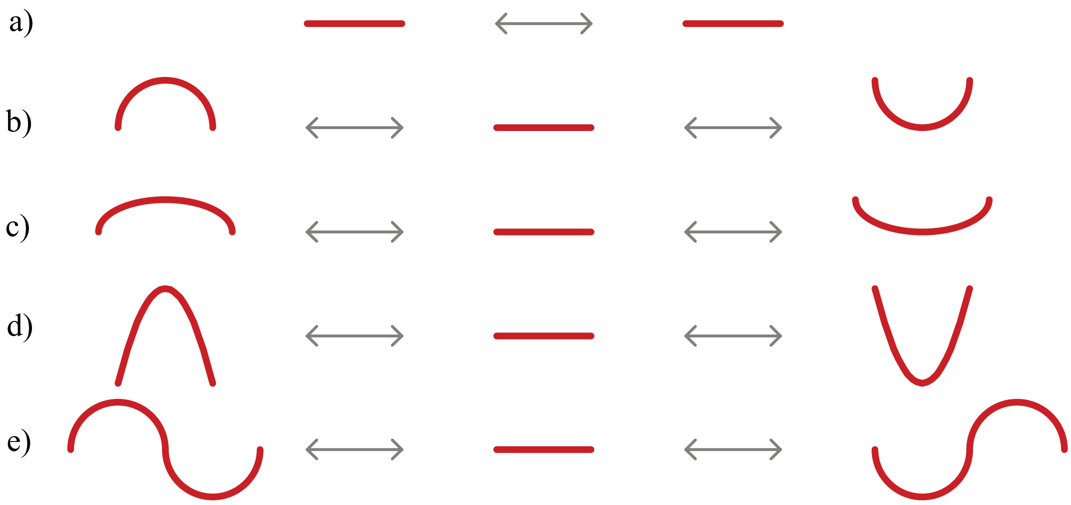


Figure 5.4. Transformations of conic section profiles.

From this point of view, planar mechanisms that can give the directrix forming the conoid with antiparallelogram loops has been examined. According to Figure 5.4 starting from top to the bottom, linear to linear, linear to circular arc, linear to elliptical arc, linear to parabolic arc, linear to S-shaped curve and circular arc to circular arc that is derived from linear to circular arc, are the cases which was accomplished successfully as a result of loop assemblies. Whereas the rest of the cases elliptical arc to elliptical arc, parabolic arc to parabolic arc, circular arc to elliptical arc, circular to parabolic arc, and elliptical arc to parabolic arc in Figure 5.4 need to be considered with different approach.

Assuming that there are three generalized cases, they are:

- Linkages that maintain its linear form (Figure 5.4a),
- Linkages that can form linear to concave and/or convex arcs (Figure 5.4 b, c, d),
- Linkages starting with linear form to where convex and concave states transform into each other (S-shaped) (Figure 5.4 e).

There should be a systematic way to assemble antiparallelogram loops to each other on a line or on a curve (Kiper et al. 2022). For this reason, frieze patterns are used to classify the outputs of anti-parallelogram loop assemblies (Figure 5.5). Frieze patterns are the patterns along a line, and they are obtained as combination of translation operation (T) with other symmetry operations: identity (I), half turn (or 180° rotation) (R), horizontal reflection about the line (H), vertical reflection about a normal to the line (V) and glide reflection (G) operations (Conway, Burgiel, and Goodman-Strauss 2008).

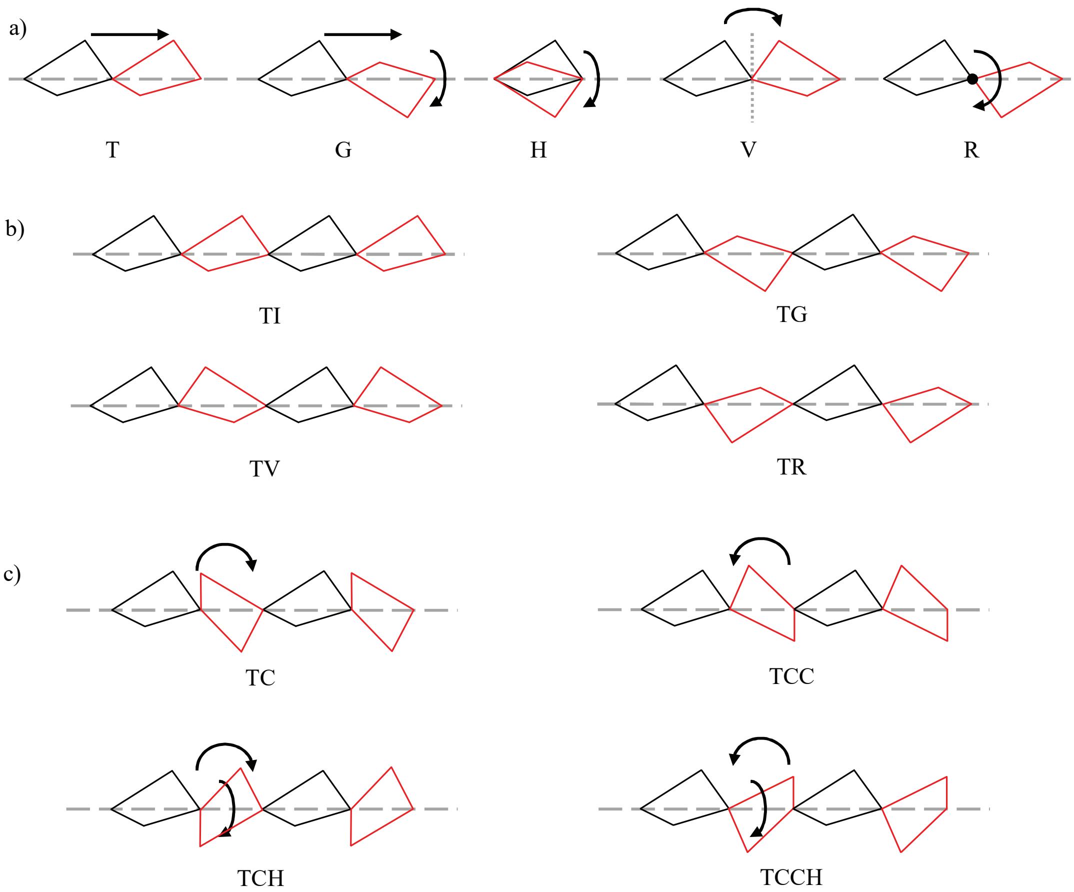
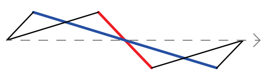
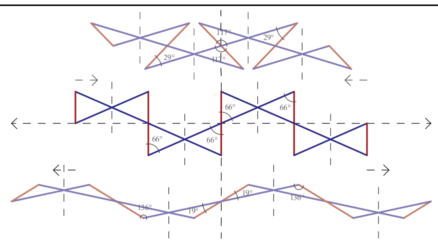
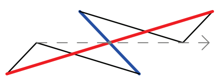
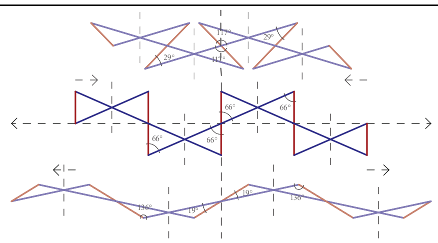
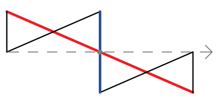
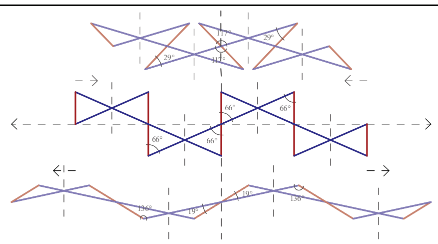


Figure 5.5. a) Basic isometry operations, b) the first four frieze patterns, c) long-short diagonal connections (Source: Kiper et al. 2022)

5.2.1. Antiparallelogram Loop Assemblies in Linear Direction

The cases where antiparallelogram loops are multiplied on a straight line are examined. Frieze patterns are used as the basis for the reproduction process. The scissor linkages illustrated in Table 5.1 are obtained by Translation+Glide Reflection (TG) and Translation+180° Rotation (TR) patterns which give the same results. There are three cases according to the way the loops are assembled as seen in the middle column in Table 5.1. Scissor units consisting of straight arms are obtained from the way the loops are connected. The resulting linkages are identical with each other in the way they can make translation movements along a direction. Thus, normal lines are parallel to each other during motion.

Table 5.1. Antiparallelogram loop assemblies from linear-to-linear forms

Pattern Type	Loop Assembly	Deployment Process
TG, TR		
TG, TR		
TG, TR		

5.2.2. Antiparallelogram Loop Assemblies from Linear to Circular Concave and Convex Forms

When the loops are multiplied based on the Translation+Identity (TI) and Translation+Vertical Reflection (TV) patterns, the same results are obtained. There are two linkages that can transform from concave to convex seen in the Table 5.2 obtained by TI and TV frieze patterns. In Linkage 1, a scissor unit consisting of angulated elements is obtained from the way the loops are connected. It is observed that the angle between the normal lines remains constant when the linkage comes to a position to form a semicircle arc in concave and convex forms. However, the radii of the circular arcs are not equal.

For the Linkage 2, scissor units are composed of angulated arms. The radii are equal at the intermediate positions for the concave and convex forms. Angles between normal lines differ for concave and convex cases that define a semicircular arc. The Radii are also not the same. It is seen that the angles in the loops for the two cases are not the same when the radii in the intermediate states are equal.

The scissor linkages obtained by Translation+Glide Reflection (TG) and Translation+180° Rotation (TR) patterns are illustrated as 3, 4 and 5 in Table 5.2. They define circular arcs with the same values for both concave and convex forms. While the radii are the same in the intermediate cases, the angles within the loops are the same depending on their positions for both inside and outside of the curve. In the case of concave and convex, the radii have the same values when they define a semicircular arc. Normal lines intersect at the origin of the circle. The angles between the normal lines are also the same.

In addition to Frieze patterns, antiparallelogram loops can be multiplied by connecting the long and short diagonals (LS) of the loop. Such combinations are obtained by rotating the loop clockwise (C) or counterclockwise (CC), or by combining the rotation with a horizontal reflection. When the long-short diagonal connections are combined with the Translation (T) operation, four possible patterns emerge: TC, TCC, TCH, and TCCH. (Kiper et al. 2022).

From how loops are connected, Linkage 6 consists of a scissor unit made up of angulated arms. In the concave and convex forms of linkage 6, it is observed that the angle between the normal lines is the same when concave and convex states define the semicircular forms. However, the radii of the circular arcs are not equal to each other. In linkages with equal radii in the intermediate states, it is seen that the angles in the loops for the two states are not the same.

Cross-rectangle version of antiparallelogram loop assemblies based on TC and TCC patterns gives the same result with TI and TV patterns can be seen as Linkage 2 in the Table 5.2. For the concave and convex forms, the radii are equal at the intermediate positions. Angles between normal lines differ for concave and convex forms that define a semicircular arc. Radius values are also not the same.

As an outcome of TCH/TCCH patterns, scissor linkages 7 and 8 are formed. The linkage 7 has scissor units consisting of one angulated and one straight arms. Convex and concave forms define curves with different radii but the same angles between normal lines. For the linkage 8, there are scissor units consisting of angulated arms. Concave and convex forms define different semi-circles with different radii and angles between normal lines. In addition, the cross-rectangle version of antiparallelogram loop assemblies based on TCH and TCCH patterns gives the same result with TG and TR patterns, which can be seen as Linkage 5 in the Table 5.2. For the concave and convex forms, the radii are equal at the intermediate positions. The radii and angles between normal lines are the same for concave and convex forms where they define a semicircular arc.

Table 5.2. Antiparallelogram loop assemblies from linear to circular concave/convex forms

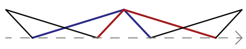
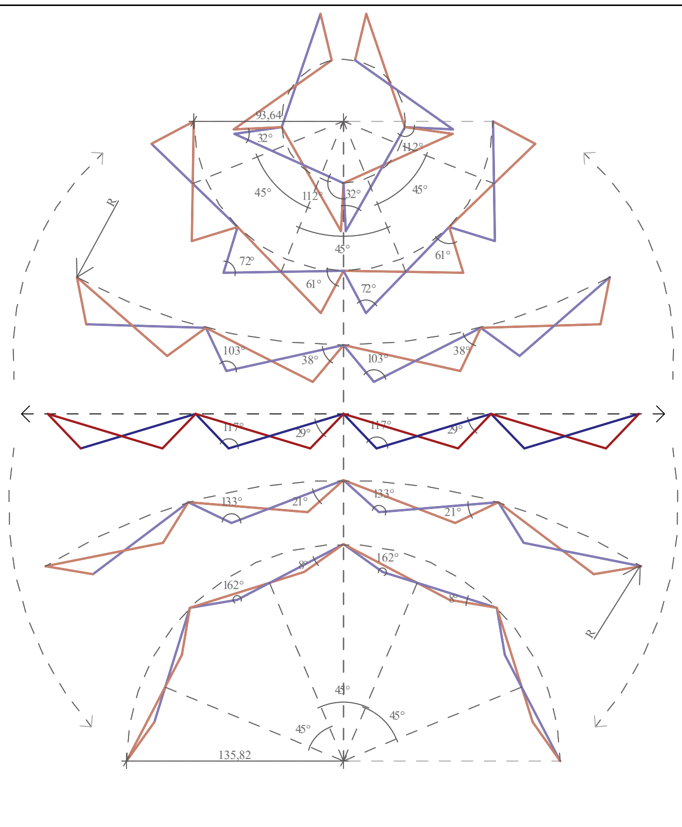
Pattern Type	Loop Assembly	Deployment Process
TI TV		 (1)

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Table 5.2. (cont.) Antiparallelogram loop assemblies from linear to circular concave/convex forms

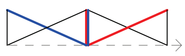
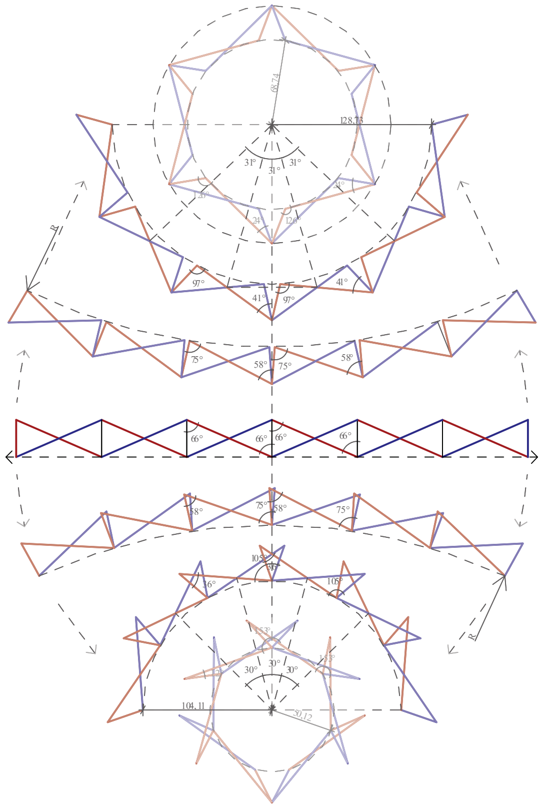
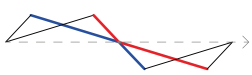
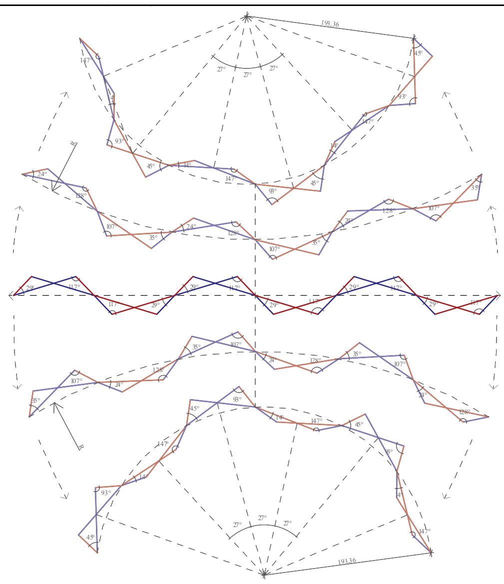
TI TV TC TCC		 (2)
TG TR		 (3)

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Table 5.2. (cont.) Antiparallelogram loop assemblies from linear to circular concave/convex forms

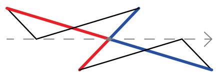
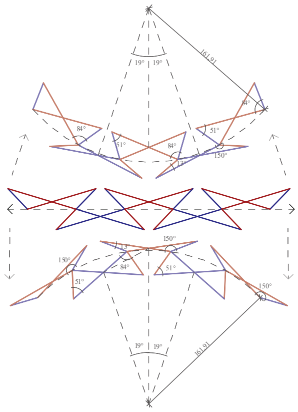
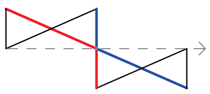
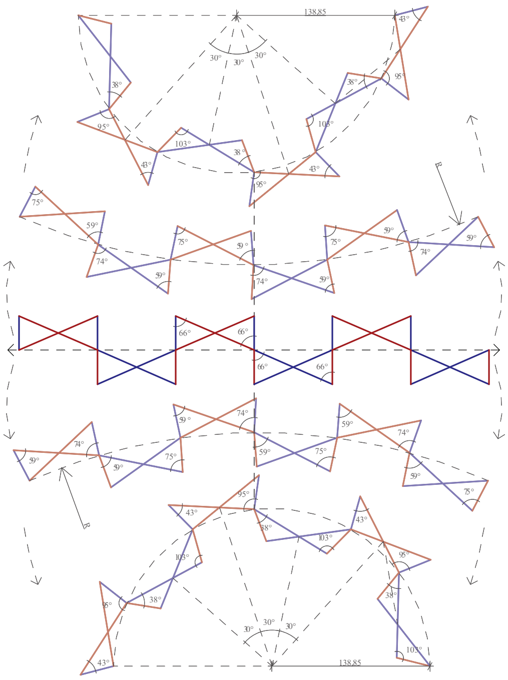
TG TR		 (4)
TG TR TCH TCCH		 (5)

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Table 5.2. (cont.) Antiparallelogram loop assemblies from linear to circular concave/convex forms

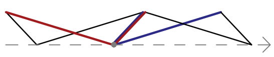
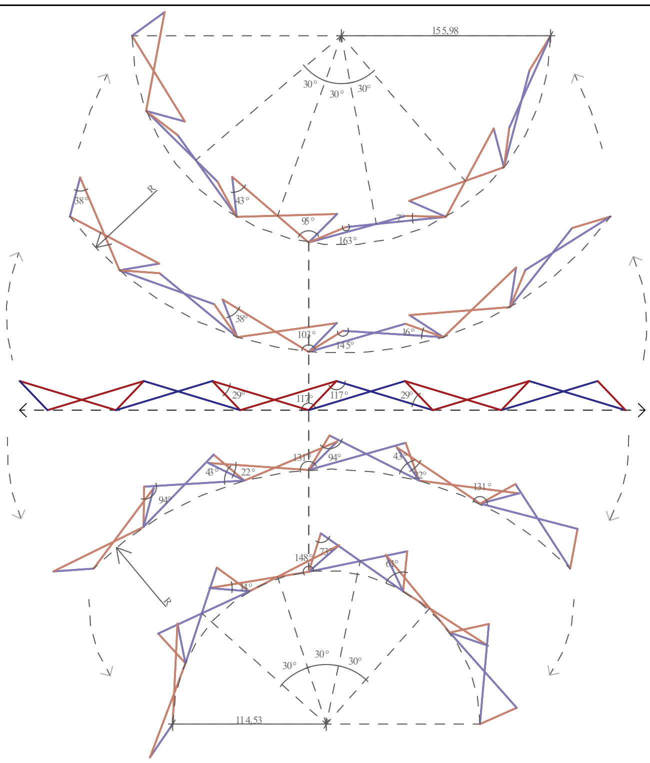
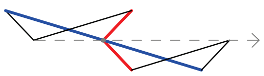
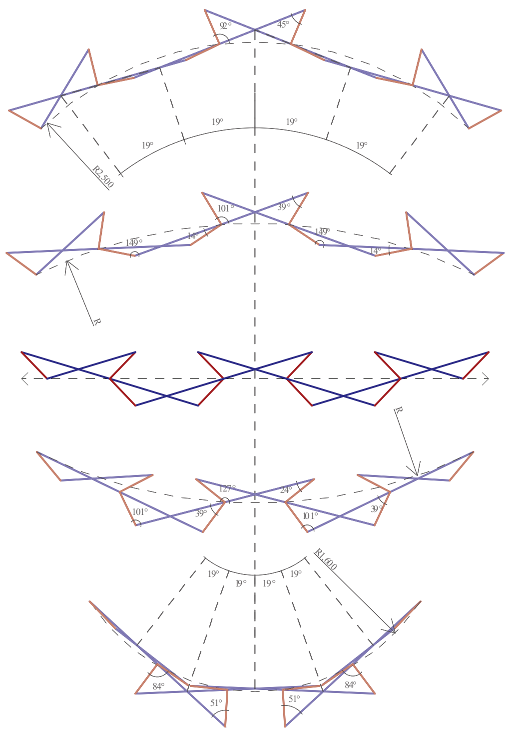
TC TCC		 (6)
TCH TCCH		 (7)

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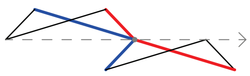
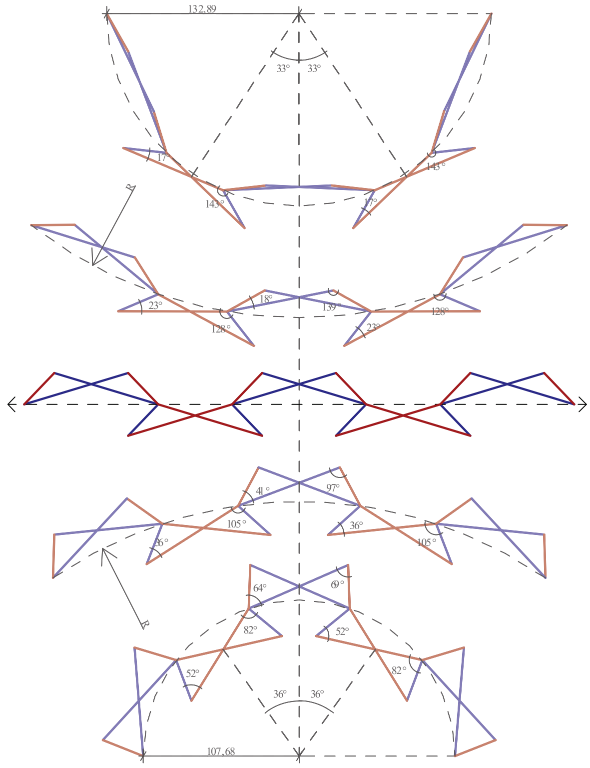
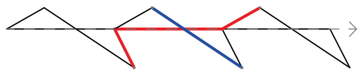
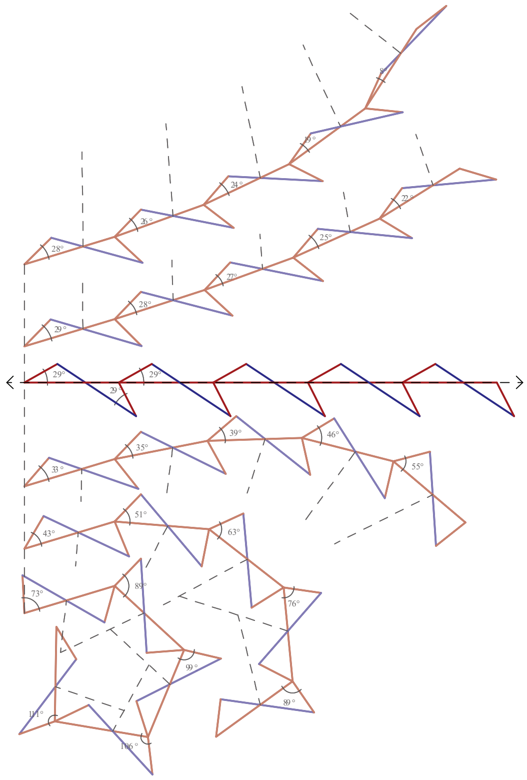
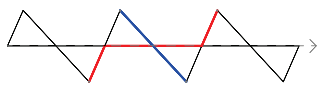
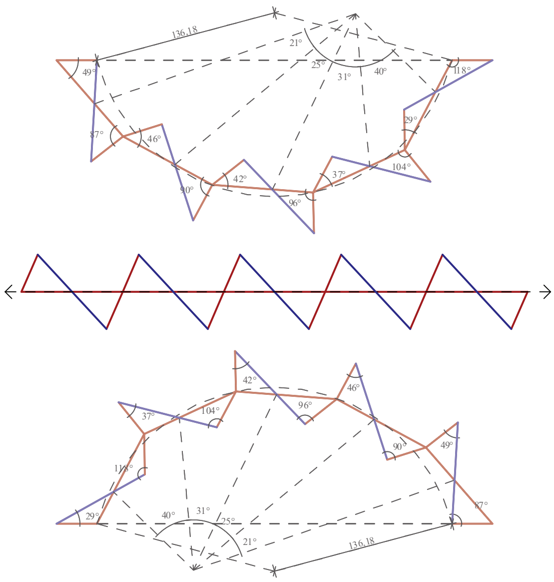
Table 5.2. (cont.) Antiparallelogram loop assemblies from linear to circular concave/convex forms

TCH TCCH		 (8)

In the Table 5.3 the first linkage has a different transformation capability from all discussed linkages. Unlike other linkages, it consists of binary and quaternary links. Concave and convex forms of the assembly are different from each other. Angles between normal lines are also different from each other. In concave form, it is observed that the linkage bends inward.

The second linkage, unlike other linkages, is obtained by connecting the long sides of the antiparallelogram loop. In this linkage, binary and quaternary links are revealed. When we want to define a circle passing through three points in concave and convex form, this circular arc does not hold the corners of all antiparallelogram loops. The normal lines of the loops intersect at a different point than the center of the circle. The convex and concave forms of the linkage are equivalent to each other.

Table 5.3. Antiparallelogram loop assemblies by connecting the long side of the antiparallelogram loops.

Loop Assembly	Deployment Process
	
	

5.2.3. Antiparallelogram Loop Assemblies from Linear to S-shaped Forms

So far, scissor linkages that can transform from linear to convex and concave circular geometry have been examined. It is observed that scissor linkages transforming concave to convex forms are able to provide S shaped geometries from line. Combining two identical modules with straight scissor arms at the connection point provides the motion from line to S form. All possible scissor linkages are illustrated in Table 5.4.

The first scissor linkage is formed based on TI/TV frieze patterns. It consists of a combination of two identical modules. In linear form, the modules are symmetrical to each other. The cases at the top and bottom of the Linkage 1 in Table 5.4 define circular arcs of the same radius. For both top and bottom cases, antiparallelogram loops have identical configurations. The second linkage can be obtained from both TI, TV, TC, and TCC patterns.

The third and fourth linkages are formed using TG/TR frieze patterns. Different from the previous linkages, in linear form, the modules are identical. They have similar characteristics due to their symmetrical features. For both Linkage 3 and Linkage 4, the top and bottom cases describe circular arcs of the same radius. They are symmetrical to each other. For both top and bottom cases, antiparallelogram loops have identical configurations. The only difference between Linkages 3 and 4 is that antiparallelogram loops attach with long diagonals in Linkage 3, while antiparallelogram loops attach with short diagonals in Linkage 4.

Linkage 5 is not only formed based on TG/TR but also TCH/TCCH frieze patterns. In both top and bottom cases, there are linkages with identical semicircles with the same radii.

Linkage 6 is formed based on TC/TCC frieze patterns. Modules are composed of antiparallelogram loop assemblies with long-short diagonal connections. The top and bottom cases have different loop forms at the same radius value.

Linkages 7 and 8 are formed based on TCH/TCCH frieze patterns. TCH/TCCH patterns provide two conditions: the Linkage 7 is composed of one straight and one angulated scissors arm, while the Linkage 8 is composed of both angulated scissors arms. For both linkages, deployment capacity is different between the two positions.

Table 5.4. Antiparallelogram loop assemblies from linear to S forms.

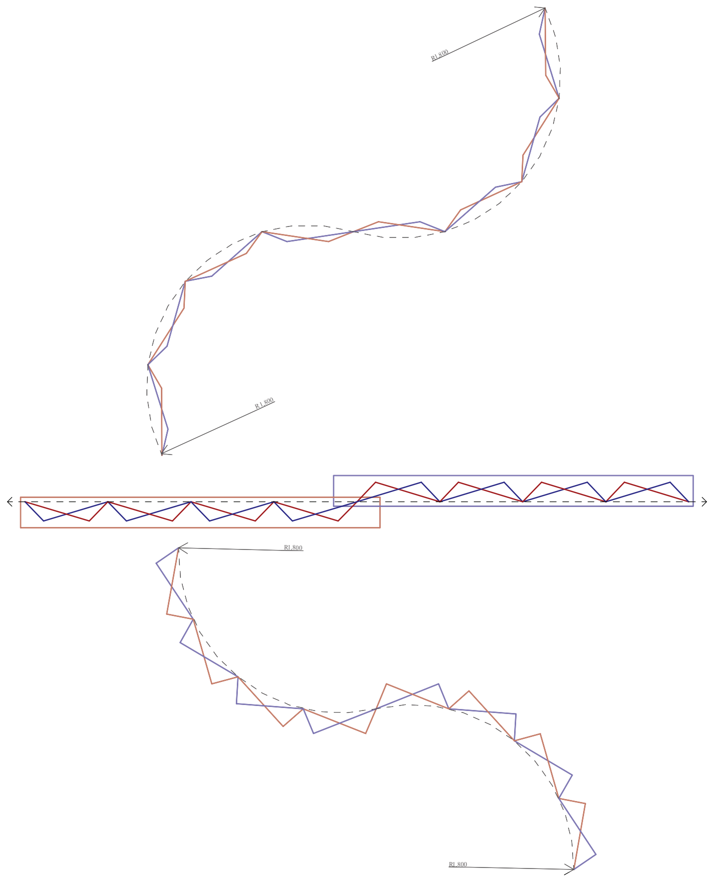
Pattern Type	Loop Assembly and Deployment
TI TV	 (1)

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Table 5.4. (cont.) Antiparallelogram loop assemblies from linear to S forms.

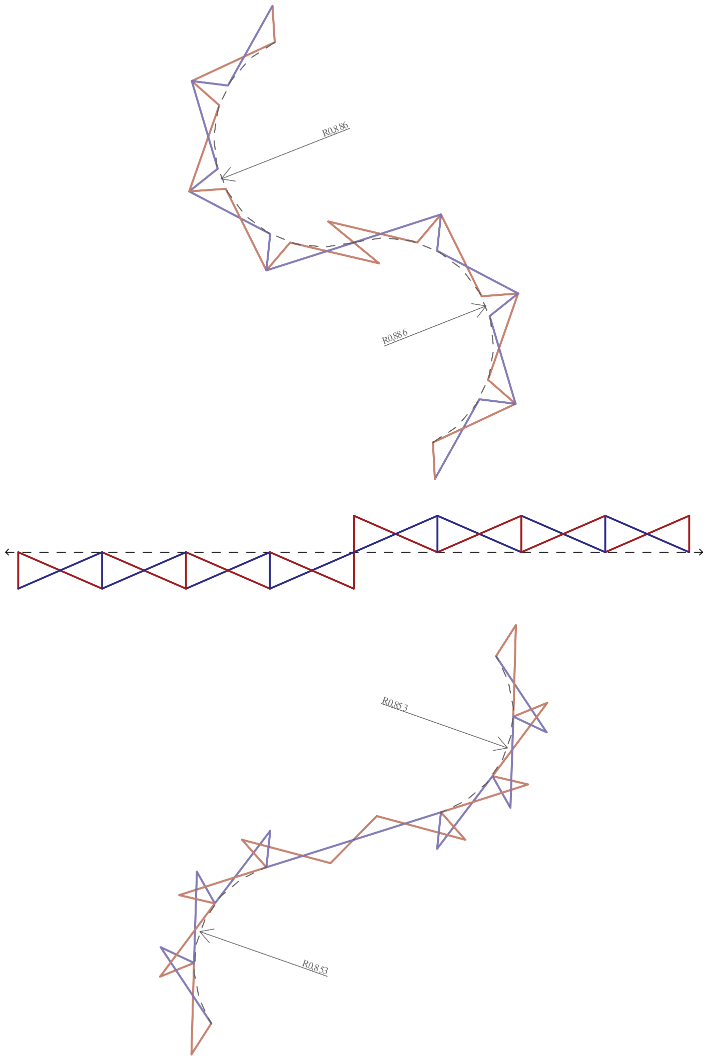
TI TV TC TCC	 (2)

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Table 5.4. (cont.) Antiparallelogram loop assemblies from linear to S forms.

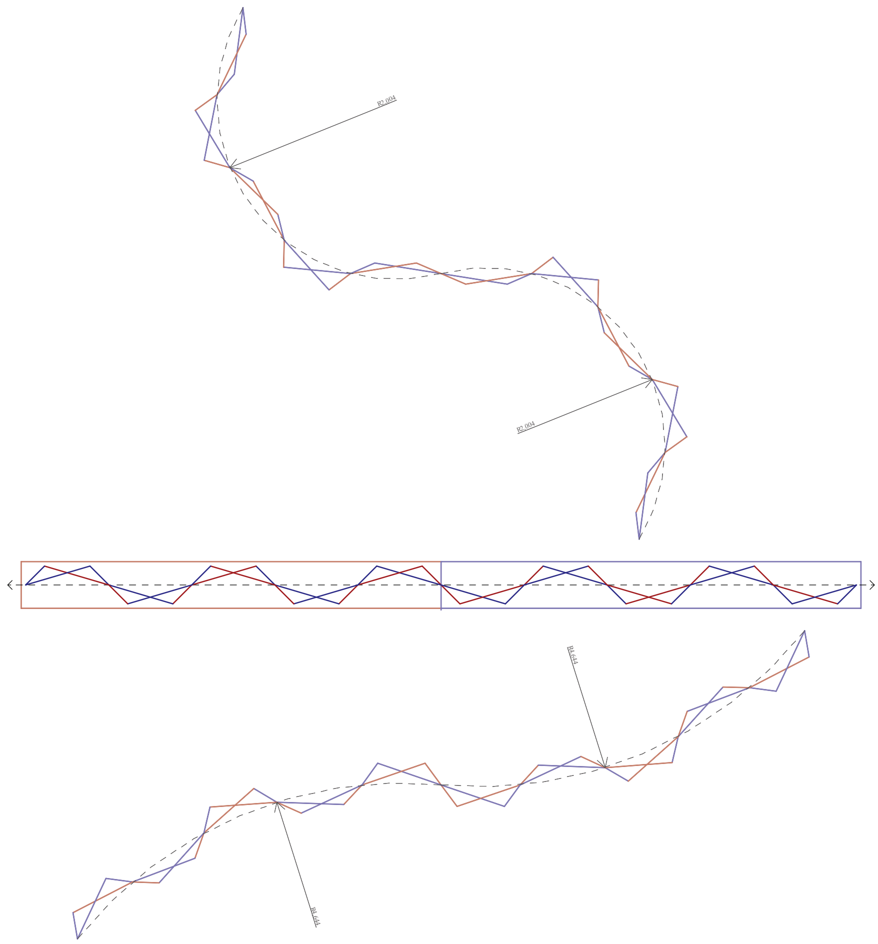
TG TR	 (3)

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Table 5.4. (cont.) Antiparallelogram loop assemblies from linear to S forms.

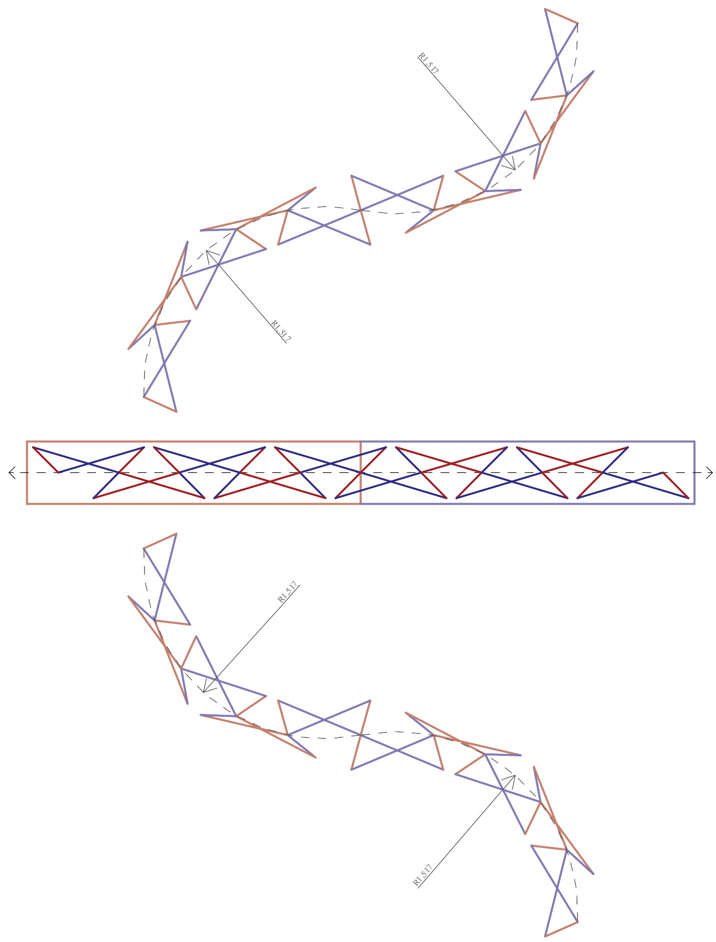
TG TR	 (4)

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Table 5.4. (cont.) Antiparallelogram loop assemblies from linear to S forms.

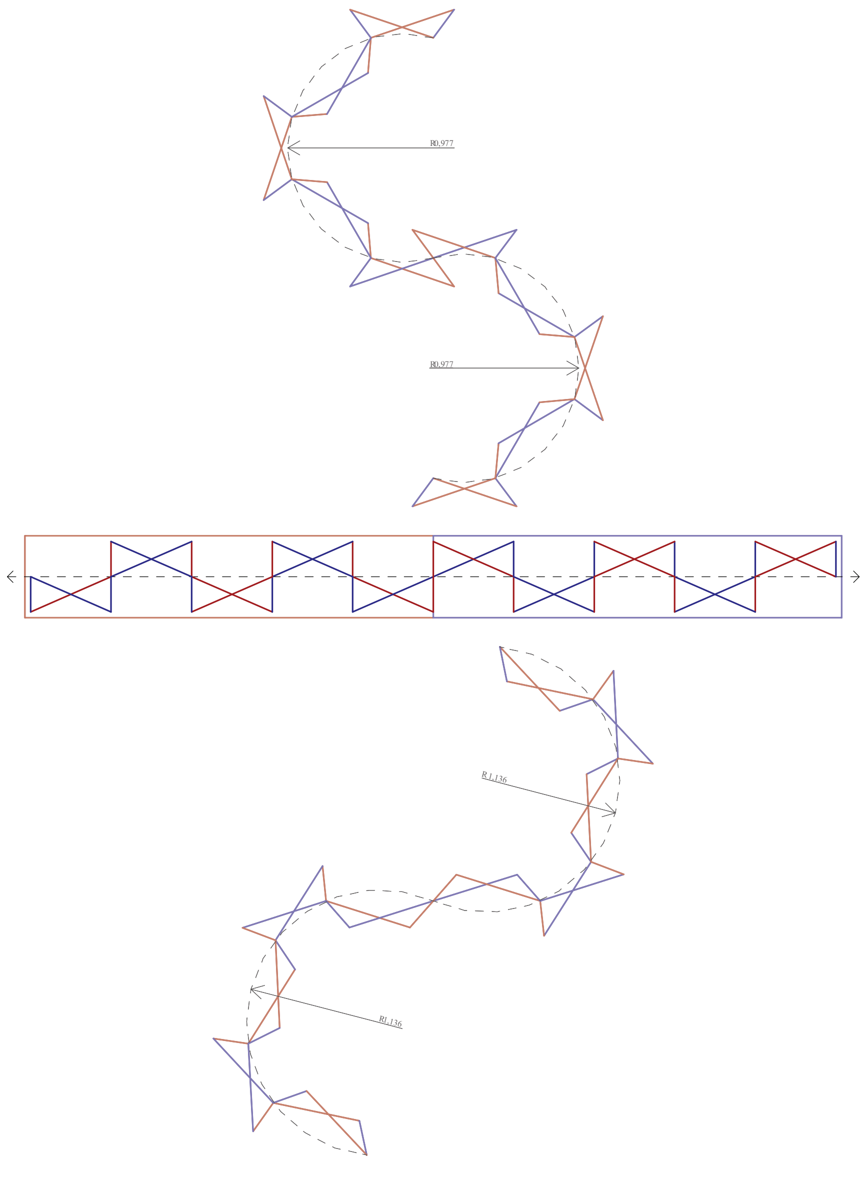
TG TR TCH TCCH	 (5)

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Table 5.4. (cont.) Antiparallelogram loop assemblies from linear to S forms.

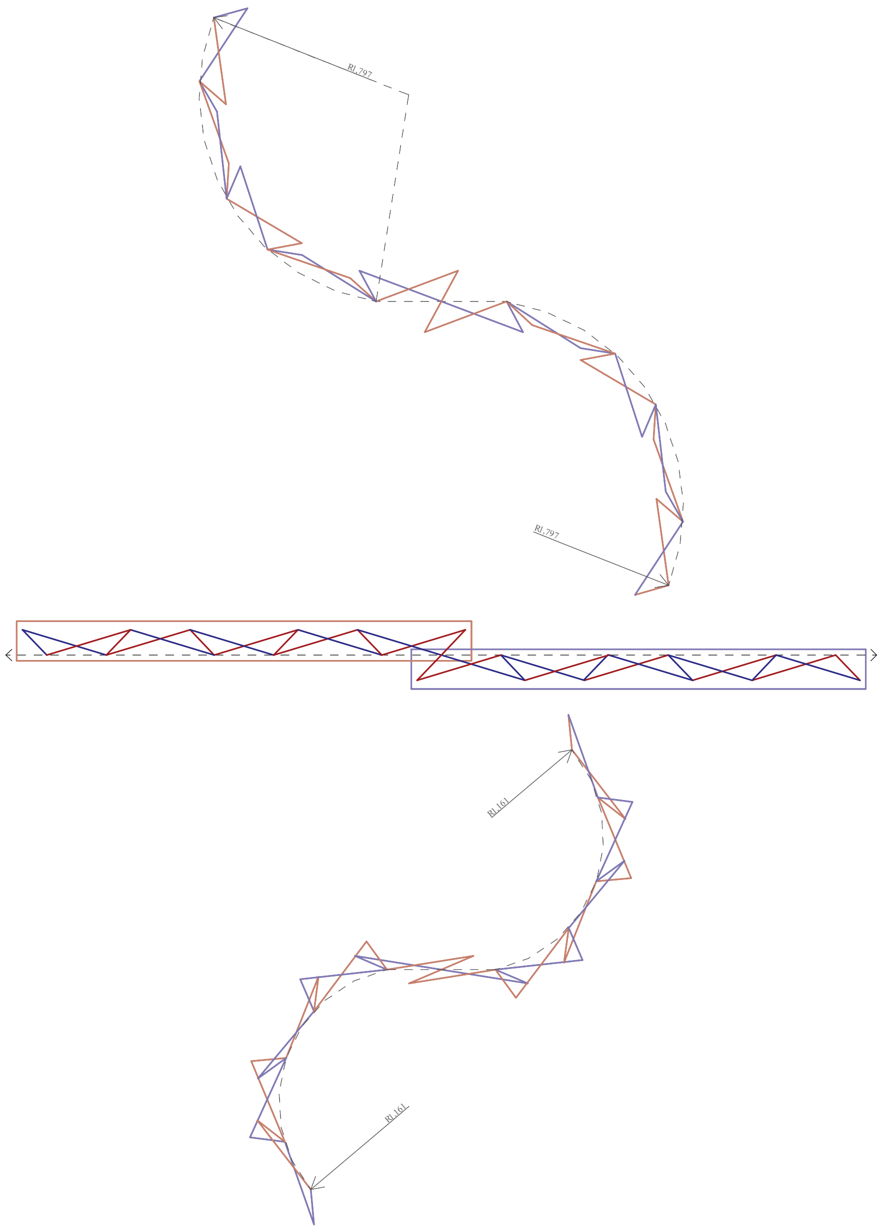
TC TCC	 (6)

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Table 5.4. (cont.) Antiparallelogram loop assemblies from linear to S forms.

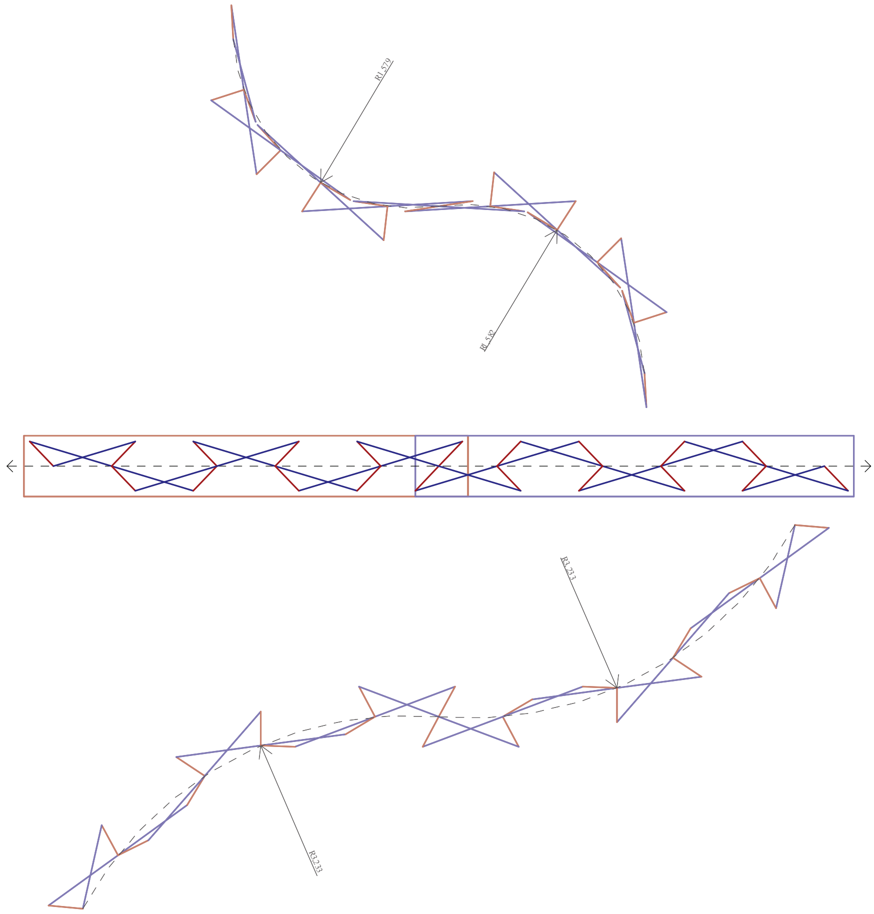
TCH TCCH	 (7)

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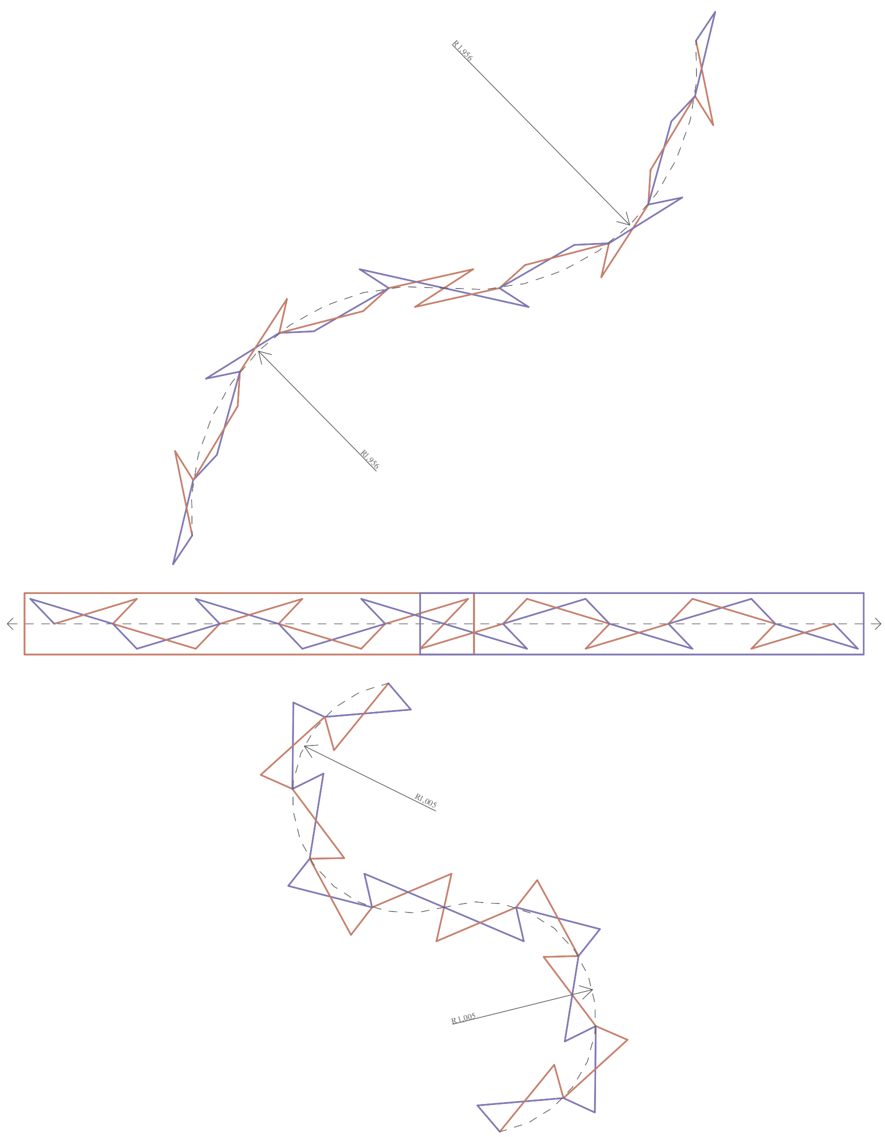
Table 5.4. (cont.) Antiparallelogram loop assemblies from linear to S forms.

	 (8)

Common features of these linkages are:

- They have two symmetrical linkage modules formed by the identical antiparallelogram loops,
- These linkage modules are connected to each other by parallel units,
- During the motion, linkage modules become concave and convex states of each other.

CHAPTER 6

PROPOSED SPATIAL TRANSFORMABLE SCISSOR LINKAGES

This chapter gives the methods for obtaining transformable scissor linkages and design principles. First, a novel geometrical loop assembly method is presented to form transformable multi-loop planar scissor linkages. Then, transformable monoclastic and synclastic scissor linkages are presented. The design process of a single degree of freedom transformable linkage is explained in phases, starting from a module and ending with the entire spatial linkage.

6.1. A Novel Geometrical Loop Assembly Method of Transformable Planar Multi-Loop Linkages

Transformable linkages are capable of transforming from the initial curve to another desired curve. Until so far, loops were replicated using frieze patterns, and the results were observed (Yar et al. 2017; Gür, Korkmaz, and Kiper 2019; Kiper et al. 2022; Sarısayın et al. 2022; Atlamaz et al. 2022). In examples that could move from one curve to another, the curve given by the linkages was always a circular curve. In order to obtain multi-loop planar scissor linkages that enable two different curves to transform into each other, a new perspective was required instead of multiplying the loops using frieze patterns and examining the results to be obtained. In addition, while creating multi-loop planar scissor linkages according to the given curves, a tool is required that allows them to take their final forms simultaneously and interconnectedly.

For this purpose, a novel geometrical construction method is performed in Solidworks® with an algorithmic arrangement. SolidWorks® is a convenient modeling tool to apply this method because it can animate two-dimensional drawings. So as to model multi-loop planar scissor linkages that can transform from one given curve to another in Solidworks®, the step-by-step implementation of this method is given in Table 6.1 with their illustrations.

6.1.1. Algorithmic Explanation of the Proposed Method

Algorithms are the fundamental components of computation, forming the backbone of the digital age. They are systematic, well-defined procedures or sets of instructions that are employed to address specific problems or carry out particular computational tasks. Algorithms provide the foundational structure for diverse computational operations and serve as the core elements that power the effective functioning of modern digital technologies (Cormen et al. 2009).

Algorithms can also be regarded as mechanisms for addressing well-defined computational problems. The problem statement outlines the desired input-output relationship in general terms, while the algorithm specifies a particular computational procedure for realizing that relationship. Algorithms can take on a variety of forms, from simple step-by-step instructions for daily tasks to complex mathematical procedures for solving engineering problems (Cormen et al. 2009).

To model multi-loop planar scissor linkages that can transform from one given curve to another in Solidworks®, the proposed method is explained in the Table 6.1 below in the simplest and most general way. First, it is necessary to decide which patterned linkage will be obtained. Table 6.1 shows the example of the steps to be followed to obtain TG / TR patterned linkage.

Table 6.1. Algorithmic explanation of a new geometrical loop assembly method.

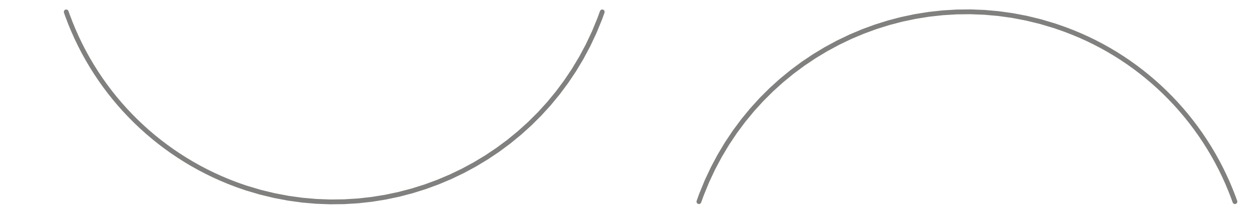
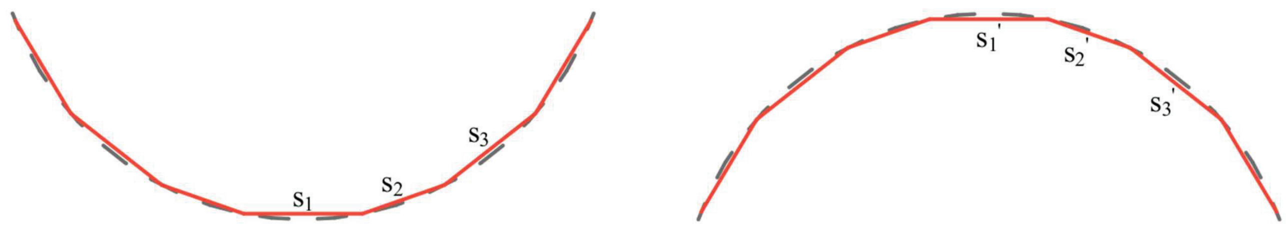
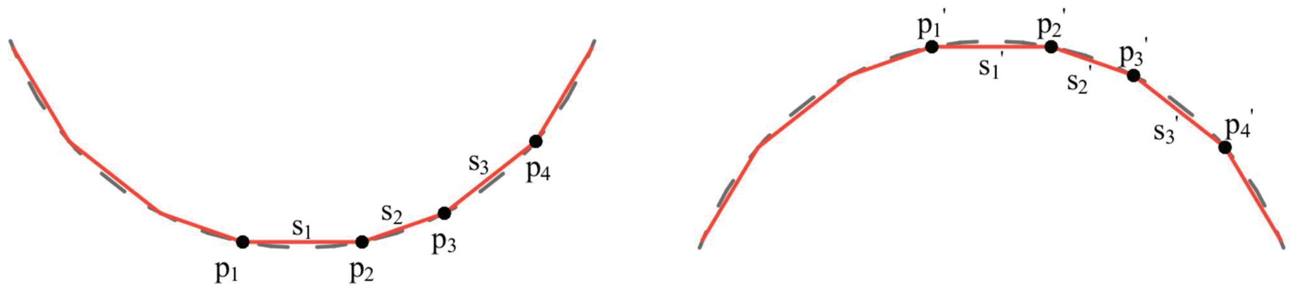
Step 1. Define two curves for initial and final forms.	
Step 2. Discretize curves into n segments (s_1/s_1', s_2/ s_2', s_3/s_3',…, s_n/s_n')	
Step 3. Start with the corner of the segments (p_1/p_1', p_2/p_2', p_3/p_3',).	

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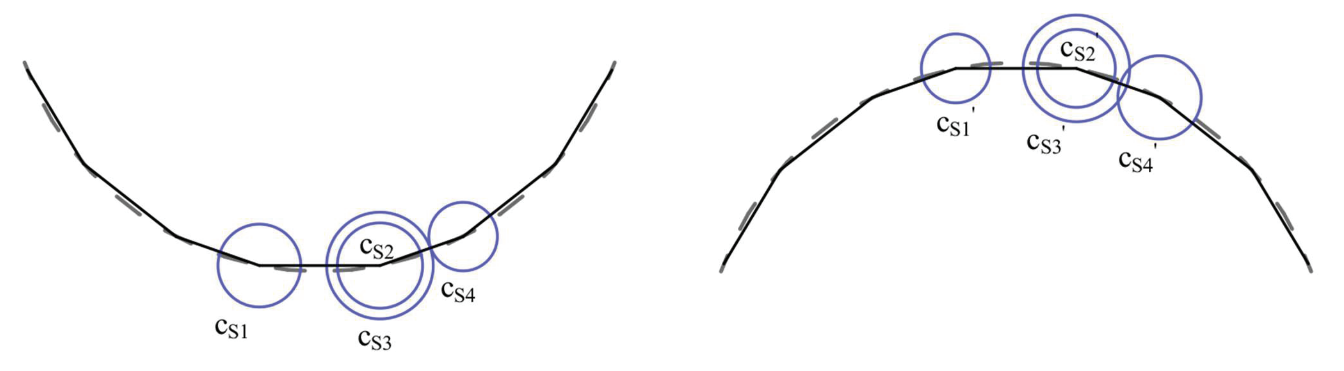
Table 6.1. (cont.) Algorithmic explanation of a new geometrical loop assembly method.

Step 4. Draw circles (c_{S1}/c_{S1}', c_{S2}/c_{S2}', c_{S3}/c_{S3}') on the corners of the segments (p_1/p_1', p_2/p_2', p_3/p_3',).

*All circles can be drawn with equal radii. In this case, it is sufficient to have only one circle at each point.



Step 5. Draw circles (c_{L1}/c_{L1}', c_{L2}/c_{L2}', c_{L3}/c_{L3}') on the corners of the segments (p_1/p_1', p_2/p_2', p_3/p_3',).

*All circles can be drawn with equal radii. In this case, it is sufficient to have only one circle at each point.

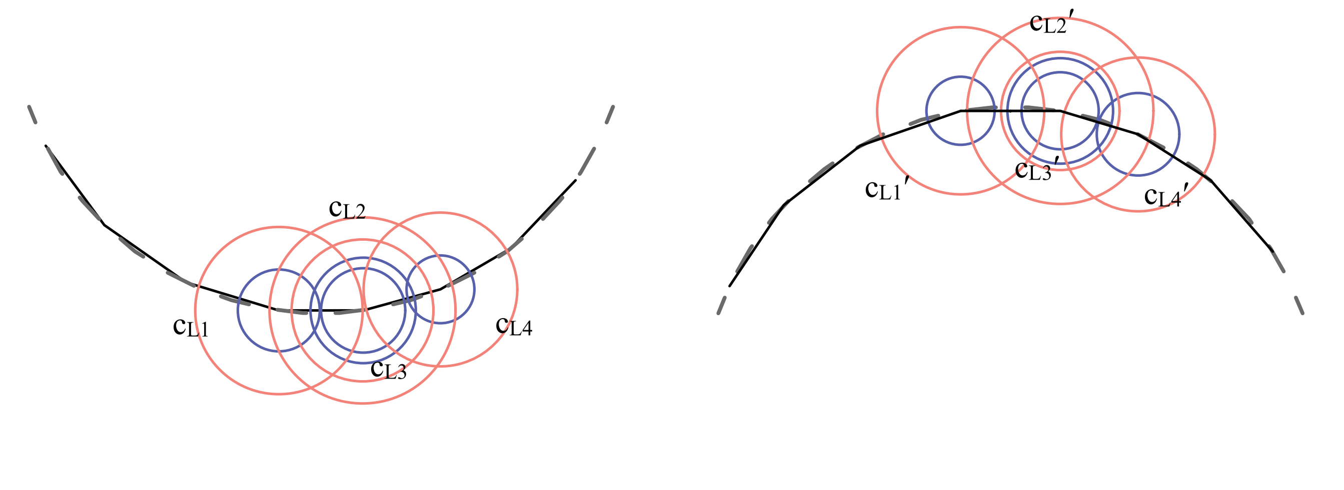


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Table 6.1. (cont.) Algorithmic explanation of a new geometrical loop assembly method.

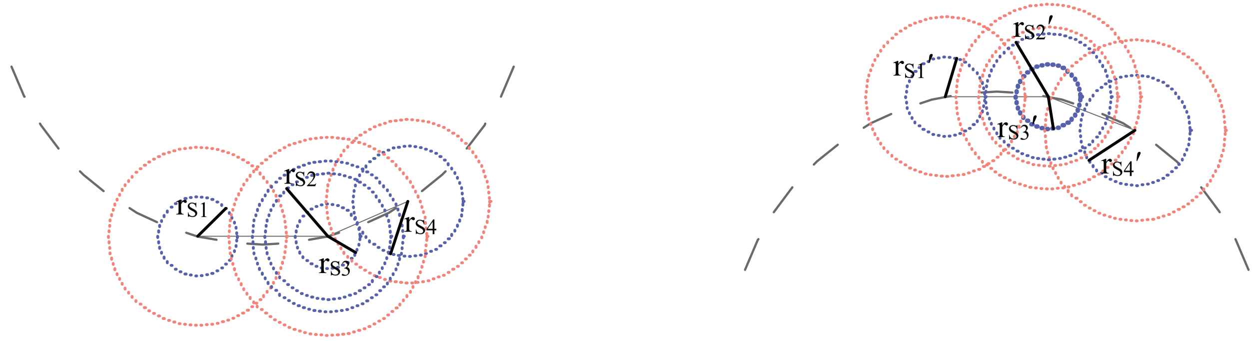
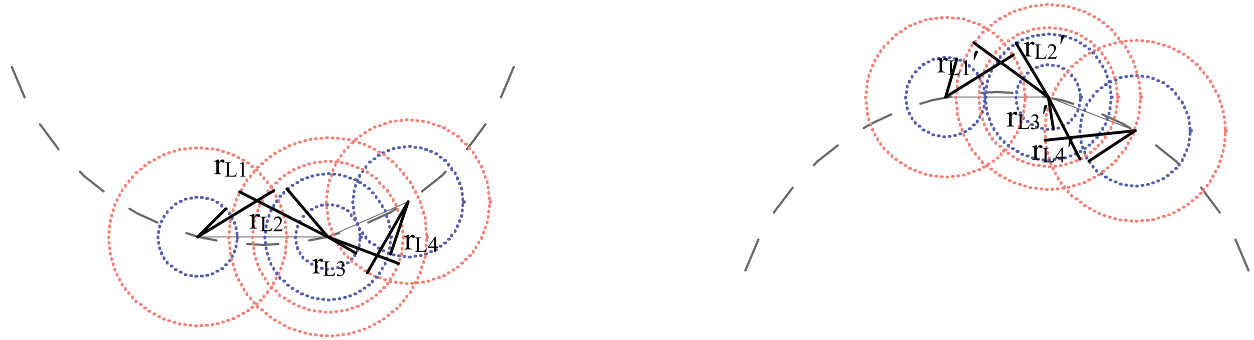
Step 6. Draw radii (r_{S1}/r_{S1}', r_{S2}/r_{S2}', r_{S3}/r_{S3}', r_{S4}/r_{S4}') to define short arm of the loops: r_{S1}/r_{S1}' is from p_1/p_1' to c_{S1}/c_{S1}' r_{S2}/r_{S2}' is from p_2/p_2' to c_{S2}/c_{S2}' r_{S3}/r_{S3}' is from p_3/p_3' to c_{S3}/c_{S3}' r_{S4}/r_{S4}' is from p_4/p_4' to c_{S4}/c_{S4}'	
Step 7. Draw radii (r_{L1}/r_{L1}', r_{L2}/r_{L2}', r_{L3}/r_{L3}', r_{L4}/r_{L4}') to define long arms of the loops: r_{L1}/r_{L1}' is from p_1/p_1' to c_{L1}/c_{L1}' r_{L2}/r_{L2}' is from p_2/p_2' to c_{L2}/c_{L2}' r_{L3}/r_{L3}' is from p_3/p_3' to c_{L3}/c_{L3}' r_{L4}/r_{L4}' is from p_4/p_4' to c_{L4}/c_{L4}'	

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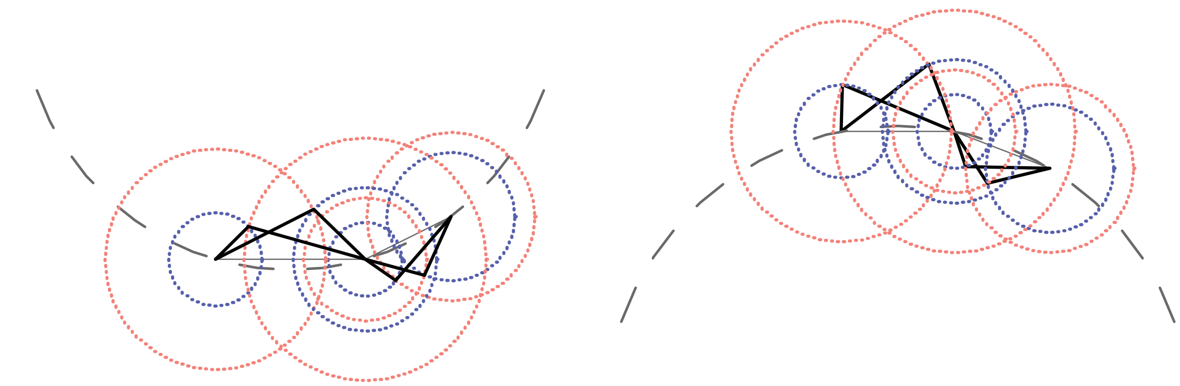
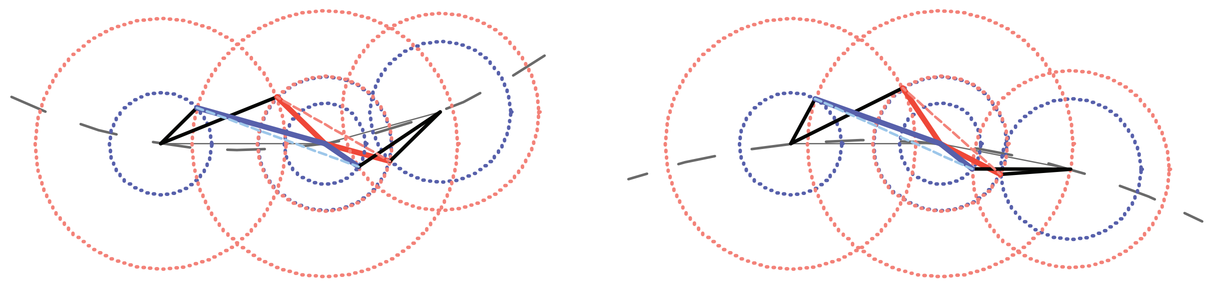
Table 6.1. (cont.) Algorithmic explanation of a new geometrical loop assembly method.

Step 8. Coincident radii with each other: r_{S1} with r_{L2} / $r_{S1}'$with r_{L2}' r_{S2} with r_{L1} / $r_{S2}'$with r_{L1}' r_{S3} with r_{L4} / $r_{S3}'$with r_{L4}' r_{S4} with r_{L3} / $r_{S4}'$with r_{L3}'	
Step 9. Draw auxiliary lines (l_1/l_1', l_2/l_2') and equalize them to each other: l_1 is from the intersection of the r_{S1} and r_{L2} to r_{S3} and r_{L4}, l_1' is from the intersection of the r_{S1}' and r_{L2}' to r_{S3}' and r_{L4}', l_2 is from the intersection of the r_{S2} and r_{L1} to r_{S4} and r_{L3}, l_2' is from the intersection of the r_{S2}' and r_{L1}' to r_{S4}' and r_{L3}'.	
Step 10. Repeat all these steps for other points, respectively.	

Additional information about the steps of the method:

Step 1: Any two desired curves can be determined for the initial final states. The curvatures of these two curves in the initial state do not have to be the same.

Step 2: The number of segments must be equal for both states. Because the number of loops they will have in the following stages will also be the same, however, the dimensions of the segments do not have to be equal in themselves or mutually.

Step 3: Here, it starts with a segment that can be accepted as the center but can be started from the desired segment.

Step 4: When drawing the circles that will define the short side of the loops, if it is desired that all the short sides are equal, the circles can be determined equal to each other. In Table 6.1 circles are drawn randomly.

Step 5: In this step, larger circles are drawn randomly to define the long side of the loops. If it is desired that all the long sides are equal, the circles can be determined equal to each other. It should be noted that $C_S < C_L$.

Steps 6 & 7: While applying these steps, the Frieze pattern of the linkage to be obtained should be considered. Short arms of loops are obtained with radii of C_{S1}, C_{S2}, …, and C_{Sn}. Long arms of loops are obtained with radii of C_{L1}, C_{L2}, …, C_{Ln}.

Step 9: With the help of the cosine theorem, the kink angle of one angulated arm is defined for both initial and final states. In this step of the Table 6.1 r_{L2} and r_{S3} and r_{L2}’ and r_{S3}’ form one of the angulated arms for initial and final states. Assuming that an angulated arm is the two sides of a triangle, the imaginary third side should be equal for both initial and final states (l_1 and l_1’).

It has been observed that when curves are moved from the points where they can be manipulated, the two situations mutually adapt to each other and take new forms.

6.1.2. Implementation of the Proposed Method for Transformable Planar Scissor Linkages

At first, there are two given curves. Segments where the loops will be placed are drawn on curves arbitrarily (S_n, S_n') (Figure 6.1).

Circles that will define the short arm of the loops (C_{Sn}, C_{Sn}') are drawn centered on the corners of these segments. Also, bigger circles (C_{Ln}, C_{Ln}') that will define the long arm of the loops are drawn, taking the corner points of these segments as centers (Figure 6.1).

The short arm of one of the loops is drawn from the center of the small circle to its periphery. Likewise, the long arm of the loop is drawn from the center of the same bigger circle to its periphery. At this point, one short arm and long arm of a loop coincide with their endpoints. So, the smaller circle of the short arm and the bigger circle of the long arm intersect at this point (Figure 6.1).

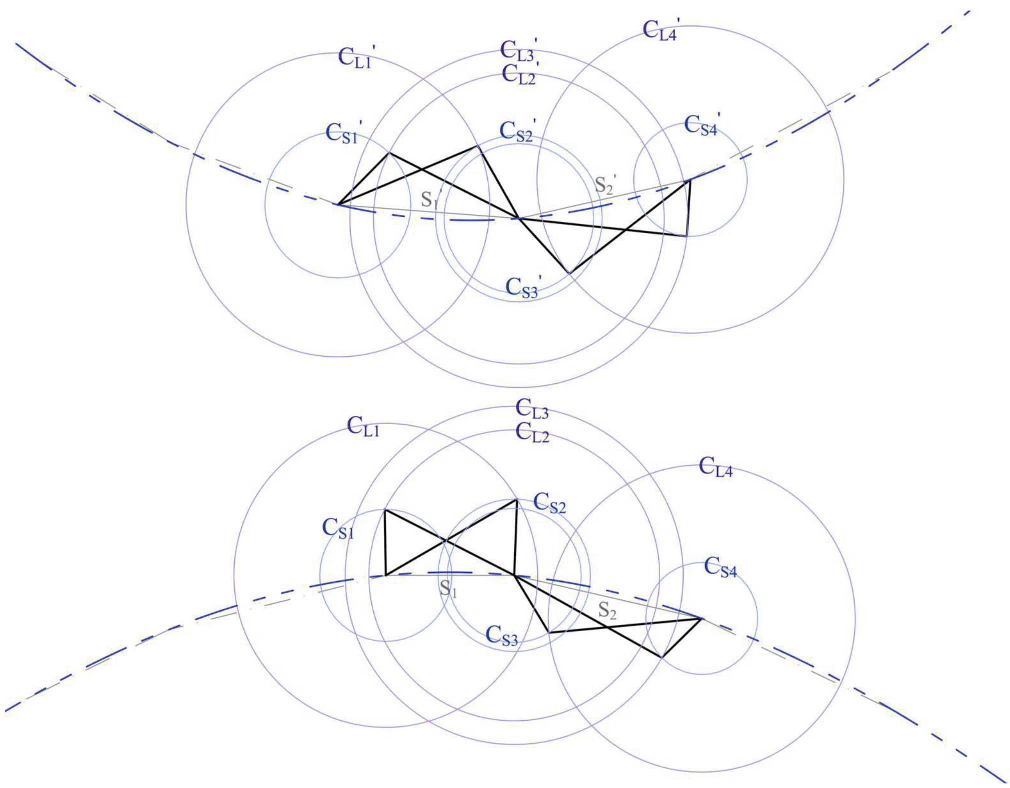


Figure 6.1. Forming antiparallelogram loops by matching the parameters for two cases.

The cosine theorem is used to ensure that the kink angle between the short and long arms that will form the angulated element is mutually equal for all situations. In this direction, a third arm (a_n) is drawn, forming a triangle with the arms forming the angulated element (Figure 6.2).

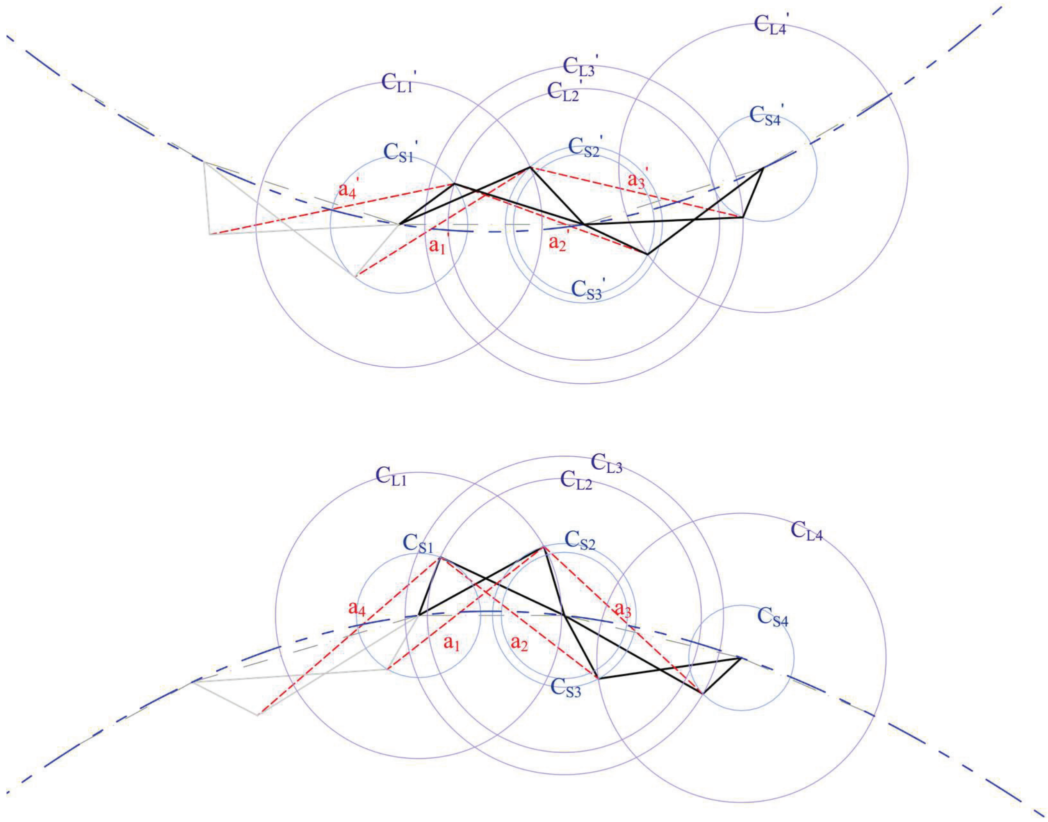


Figure 6.2. Equalization of kink angles of angulated elements.

The same procedures are applied to other circles. For each curve, smaller and larger circles are mutually equalized for concave and convex forms. As a result, we can obtain multi-loop planar scissor linkage composed of angulated elements that can change their form between concave and convex curves (Figure 6.3).

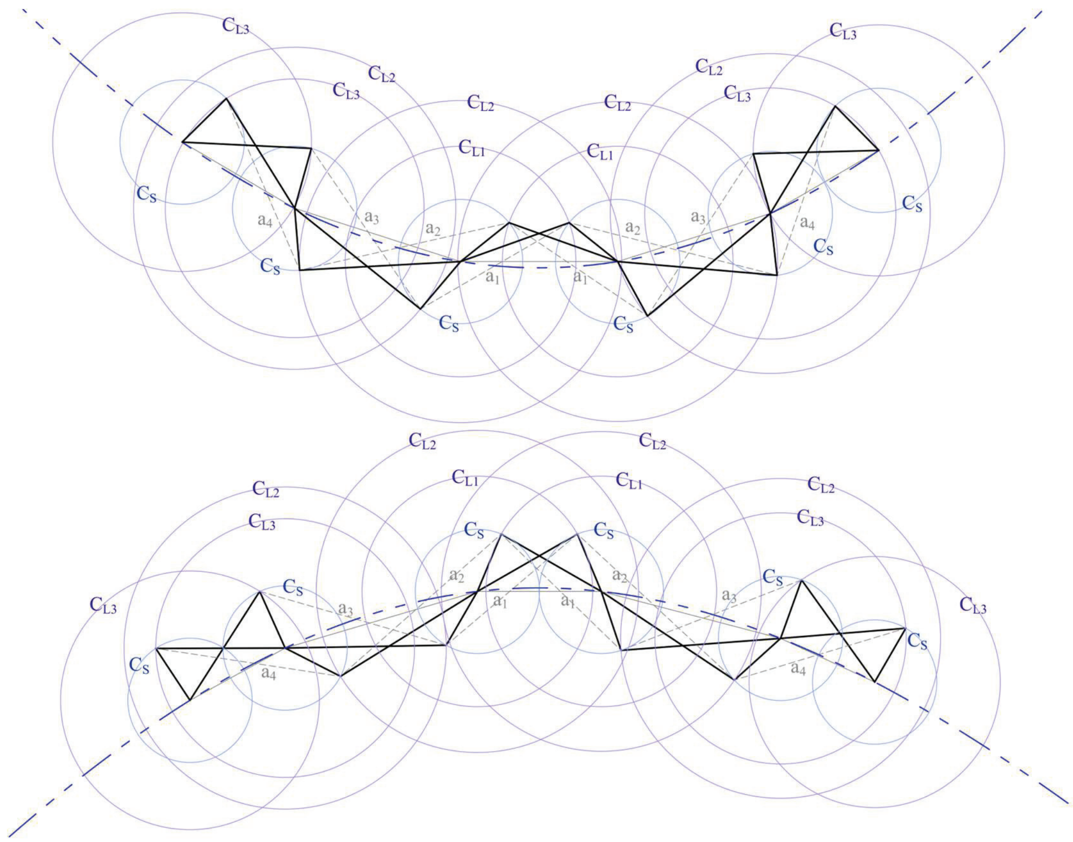


Figure 6.3. Planar scissor linkage is obtained with antiparallelogram loops that can define concave and convex conic curves.

When the linkages obtained with the loop-assembly method are examined, it is seen that transformable scissor linkages are formed with antiparallelogram, kite, and dart loops (Kiper et al. 2022). According to the loop assembly method, the result form is a circular curve. Using this new loop assembly method, linkages defining concave and convex parabola and ellipse curves are obtained with both antiparallelogram, kite, and dart loops. While loops with different long and short arm lengths can be obtained by using circles of various sizes, short and long arms, each in equal sizes within themselves, can be obtained by equalizing the circles to each other concave and convex forms. It can be seen that this condition is valid for antiparallelogram, kite, and dart loops. (Figure 6.4 and Figure 6.5).

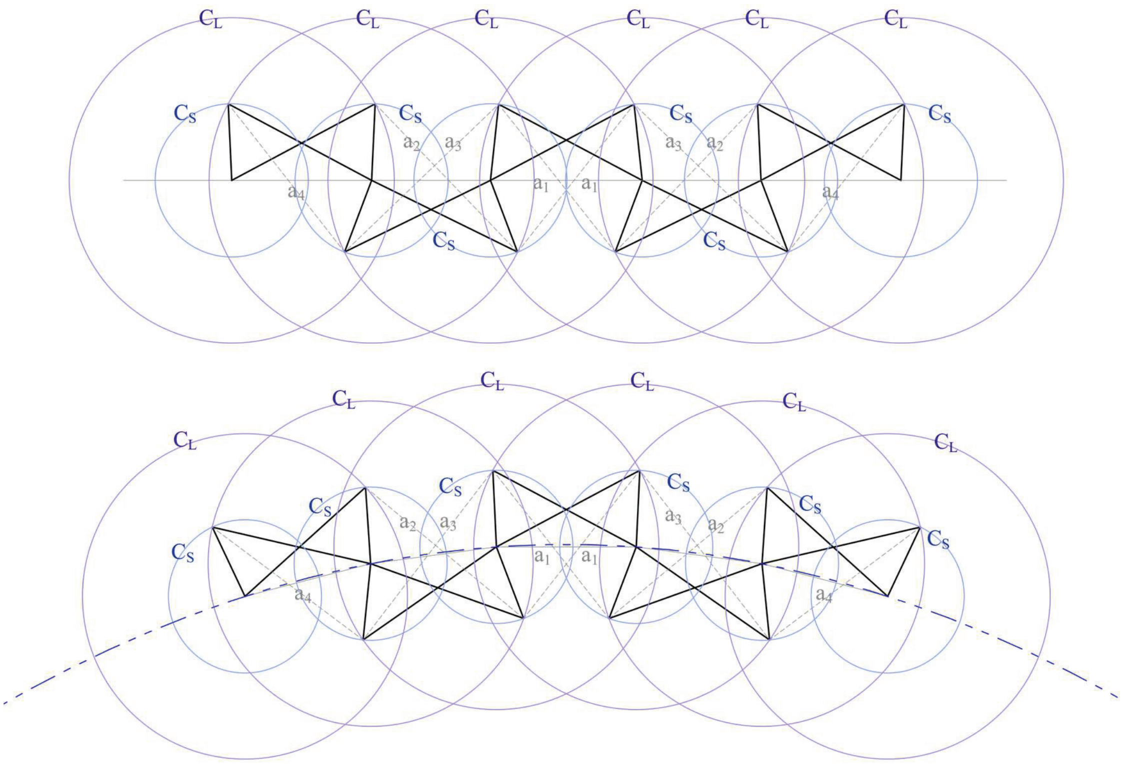


Figure 6.4. Transformation of a planar linkage composed of antiparallelogram loops from linear to parabola.

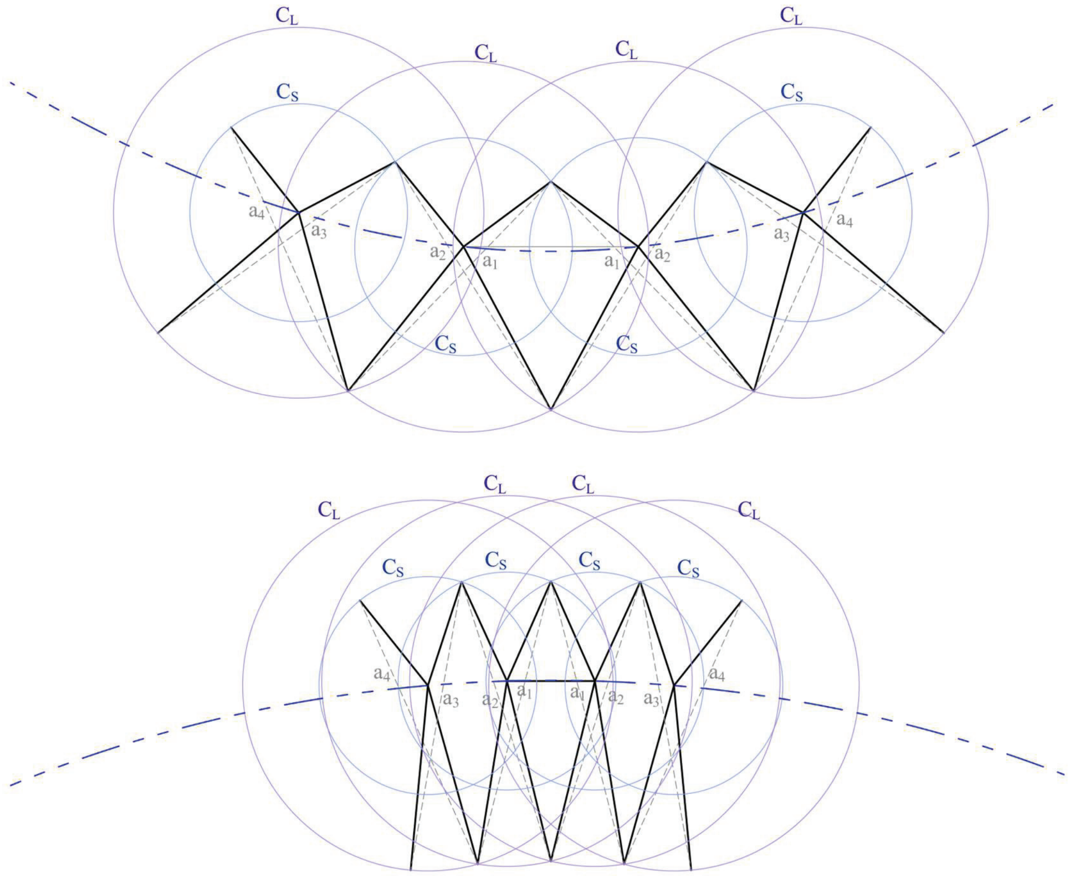


Figure 6.5. Transformation of a planar linkage composed of kite loops concave to convex conic curves.

Concave and convex forms can be matched mutually via this new method perfectly for rhombus and parallelogram loops in SolidWorks® as seen below in Figure 6.6.

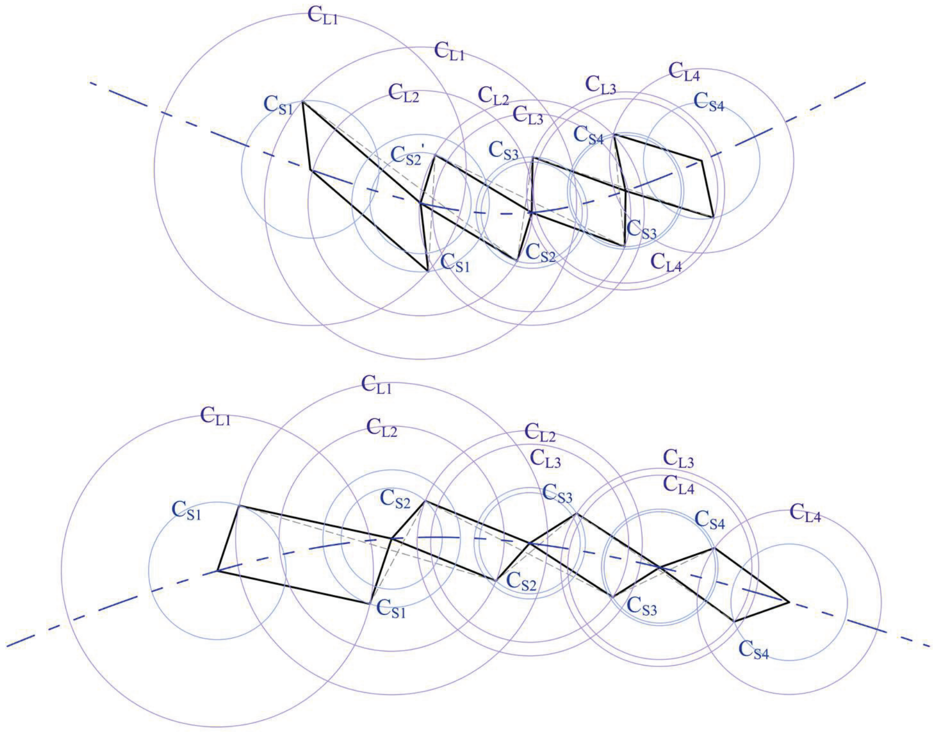


Figure 6.6. Concave and convex forms of planar scissor linkage that are formed with parallelogram loops.

However, when the linkage is modeled from the assembly, the desired transformation cannot be achieved. As seen in Figure 6.7 opposite respective angles of sides a_2, a_3, and a_4 in the convex form are opposite to those in concave form. To achieve this transformation, assembly mode change is required for the linkage. As it is understood, this method cannot be applied to rhombus and parallelogram loops.

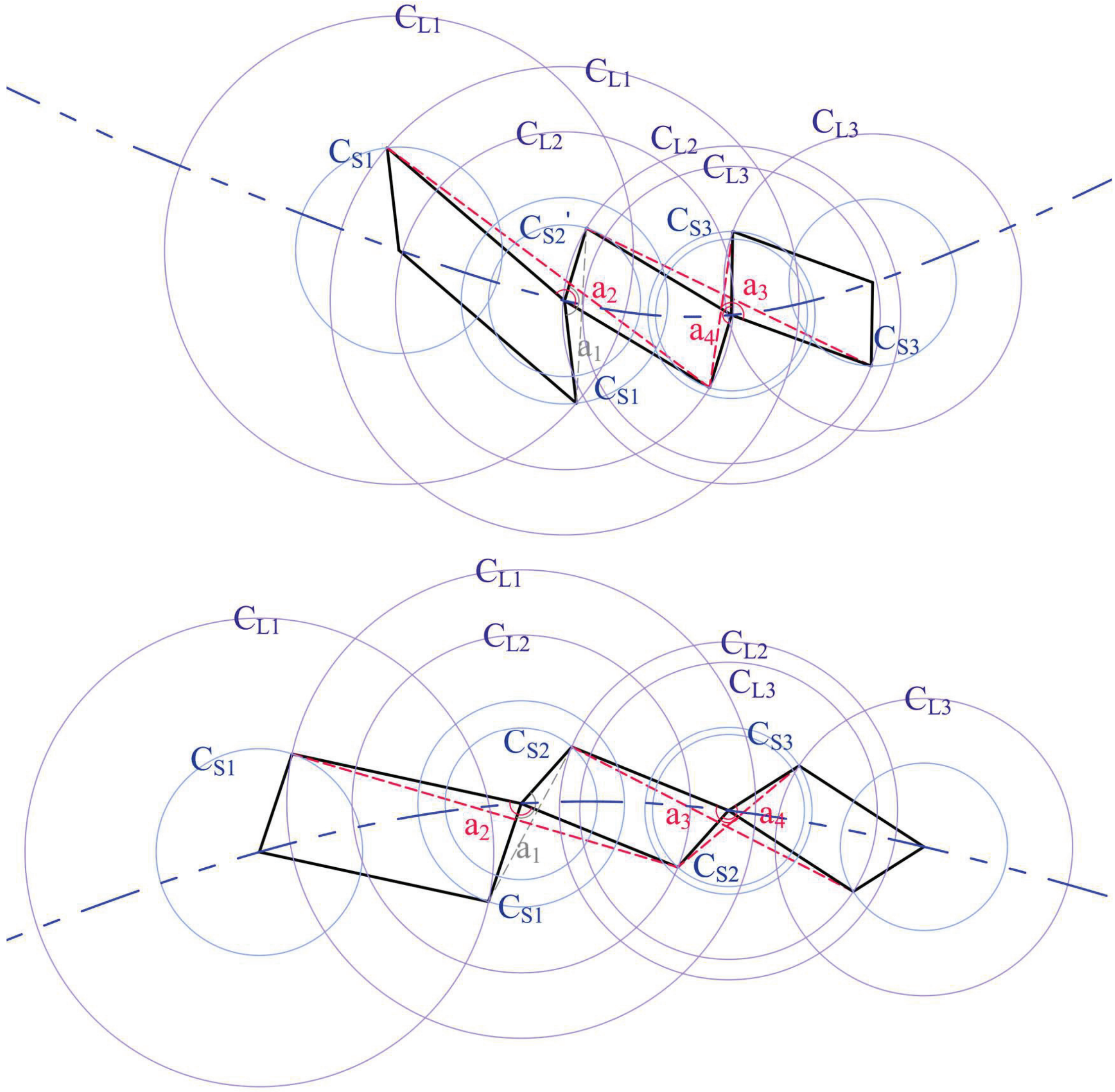


Figure 6.7 Concave and convex forms of parallelogram loop assemblies on a conic curve.

6.2. Proposed Transformable Scissor Linkages

In this section, the design methods of transformable spatial scissor linkages are examined under two headings according to the formation of monoclastic and synclastic surfaces.

6.2.1. Design of Transformable Monoclastic Scissor Linkages

According to the Table 2.1 in chapter 2, there is no study about conoid surfaces, with both deployable and transformable scissor structures. In this section, models to create the conoid surface are mentioned.

The section taken from the conoid surface perpendicular to the director plane (yz plane in Figure 6.8), gives curved directrix. Based on this, planar scissor linkages are assumed as a directrix of the surface. As seen in the Figure 6.8while one scissor linkage defines the axis of a conoid the other linkage defines the directrix of a conoid.

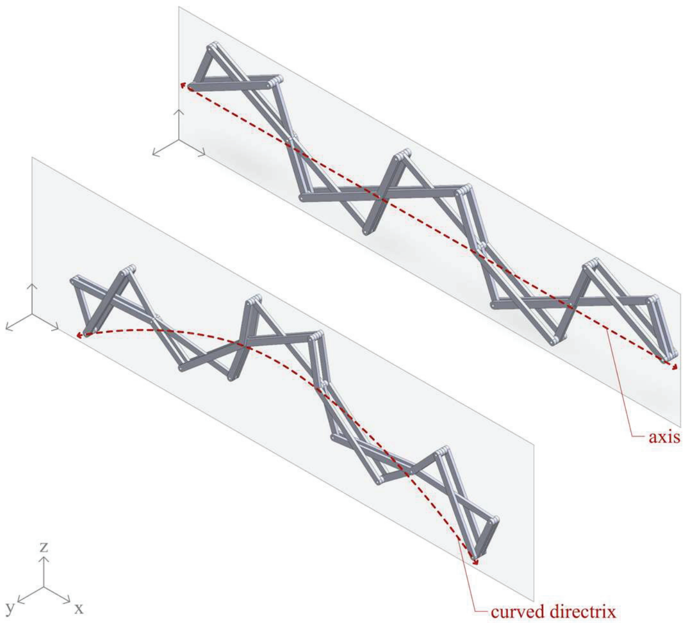


Figure 6.8 Scissor linkages defining a conoid axis and directrix.

Based on the aforementioned information, some models were tried to construct a conoidal surface. The first model can be seen in Figure 6.9. Two planar scissor linkages are connecting with middle-hinged straight bars where joint axes are perpendicular to xz plane. They are connected by a hinge which has joint axes on both y and z axes. The resulting form is vault which is monoclastic surface.

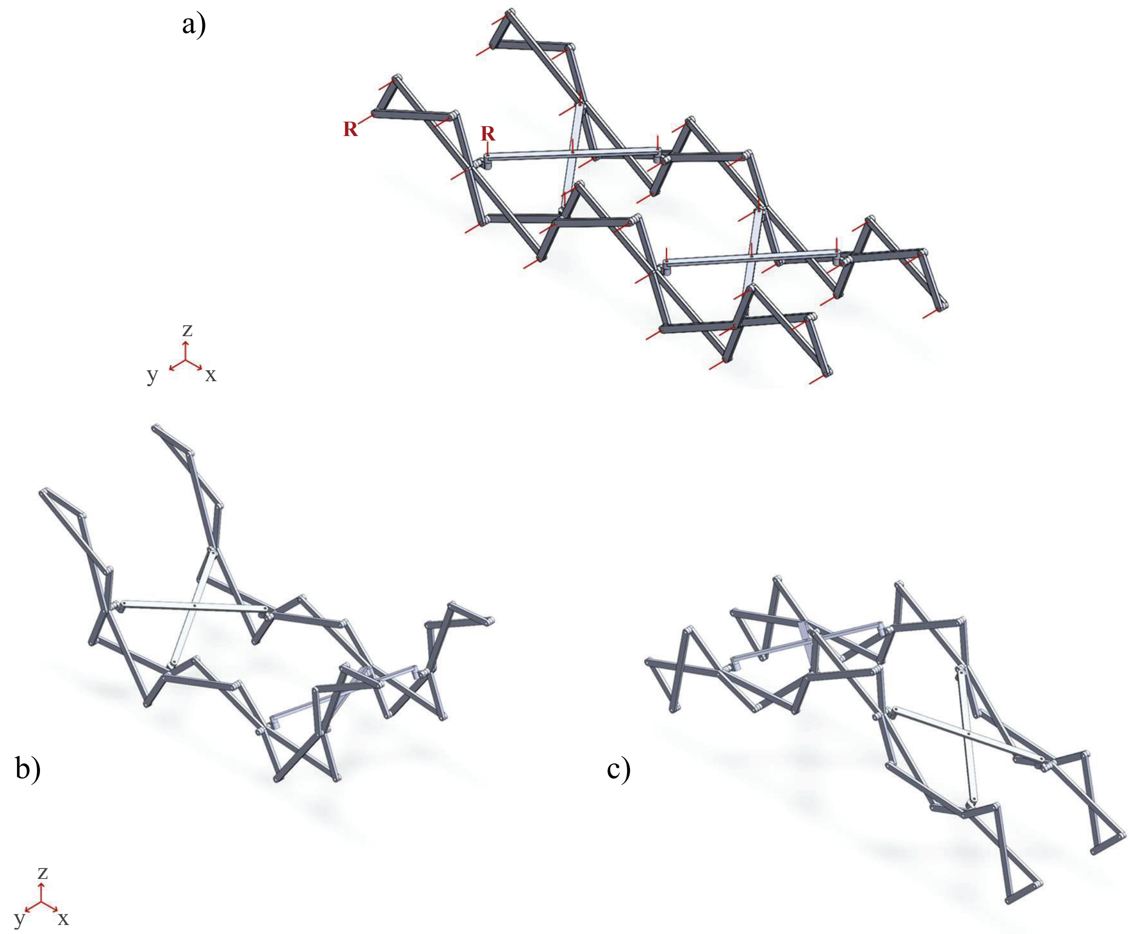


Figure 6.9 A model composed of planar linkages connecting with middle hinged straight bars a) flat position, b) convex vault, c) concave vault.

The second model can be seen in Figure 6.10. Telescopic middle bars are hinged with a joint. They are connecting two parallel scissor linkages with universal joints. Although having universal joints, the model defines a vault when it moves because telescopic bars hinged to each other with R joint.

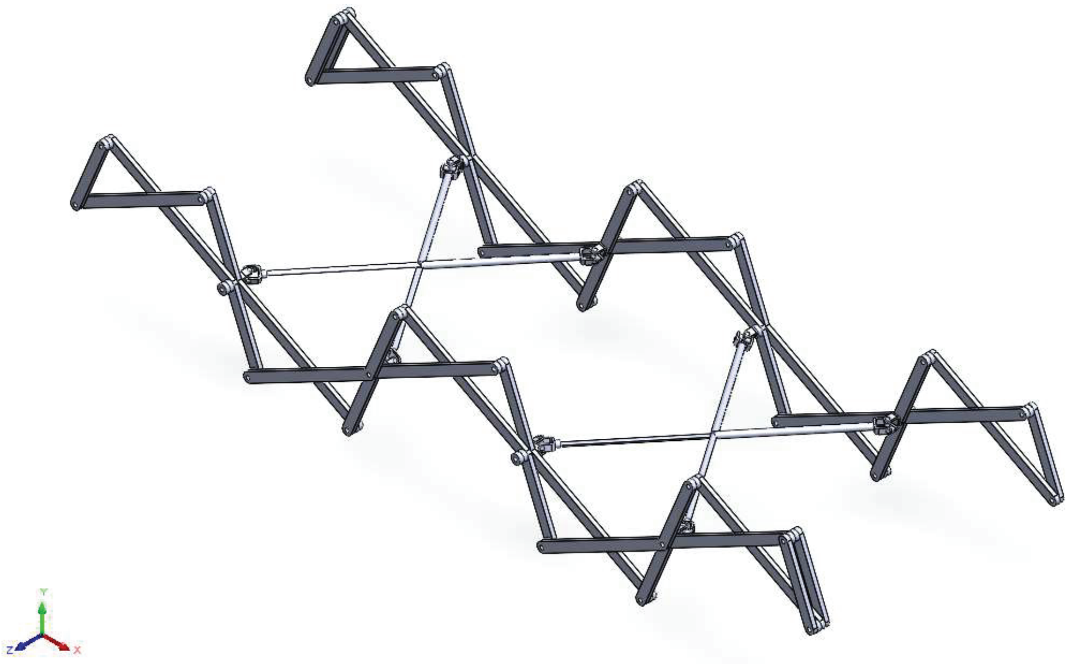


Figure 6.10 A model composed of planar linkages connecting with lazy-tong mechanisms

The third model can be seen in Figure 6.11. In flat position scissor linkages are parallel to each other. In the middle, there are lazy-tong mechanisms. Scissor linkages and lazy tong mechanisms connect with a hinge which has joint axes on both x and z axes. Lazy-tong mechanisms keep connection opposite joints by getting longer and shorter. This model can create both synclastic and anticlastic surfaces. However, it is more than one degree-of-freedom linkage, so it is hard to control and protect the desired form.

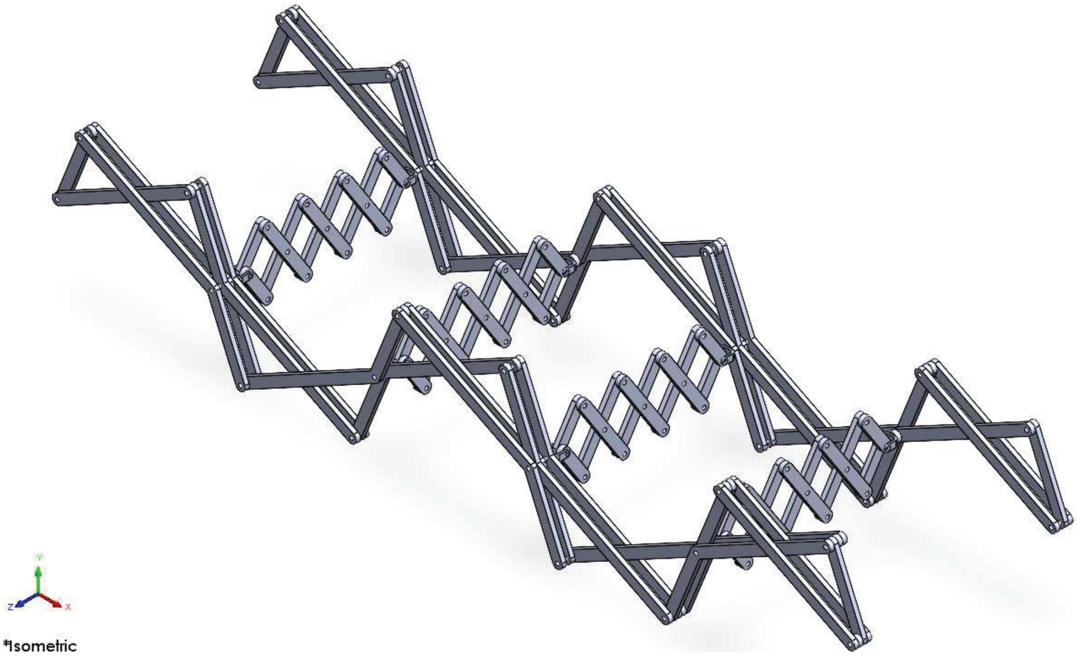


Figure 6.11 A model composed of planar linkages connecting with telescopic middle bars.

The last trial for conoid surface is seen in Figure 6.12. For this model, in the middle, instead of lazy-tong mechanisms prismatic joints are used. Scissor linkages and middle bars are connected with a hinge which has joint axes on both x and z axes. This joint allows middle bars to rotate on yz plane as scissor linkages move. Prismatic joints keep the connection opposite joints by getting longer and shorter. This model can create both synclastic and anticlastic surfaces. However, it is more than one degree-of-freedom linkage, so it is hard to control and protect the desired form.

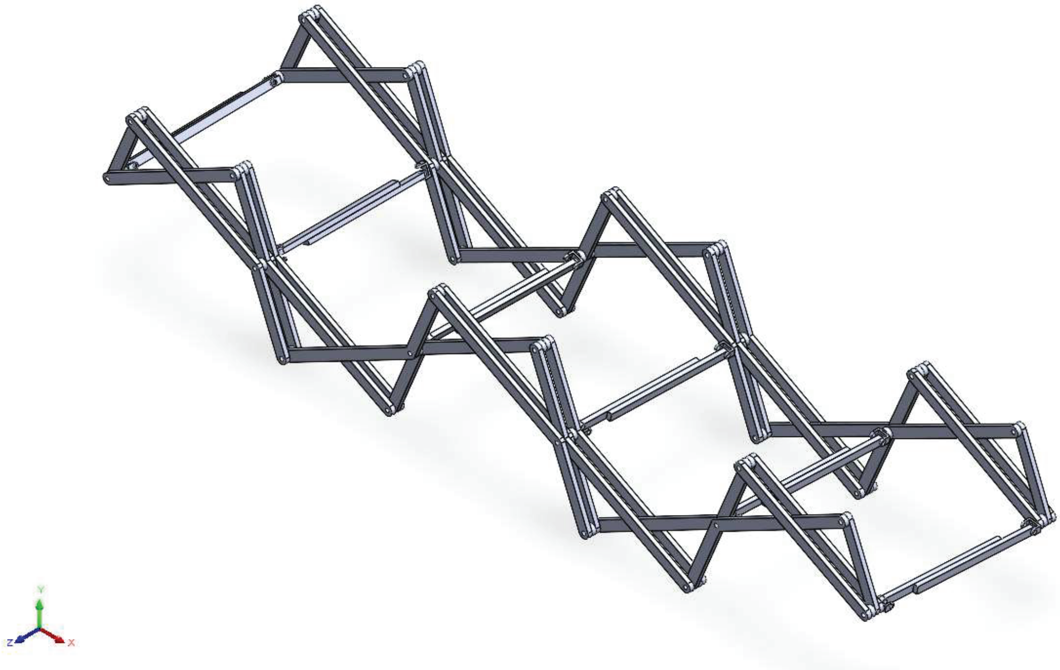


Figure 6.12 A model composed of planar linkages connecting with telescopic middle bars.

6.2.2. Design of Transformable Synclastic Surface with Multiloop Linkage Mechanisms

A spatial scissor linkage can be formed by a network of kinematic linkages of loop assemblies. Different types of scissor units can be obtained as a result of these loop assemblies. There are imaginary lines defining motion characteristics of a scissor unit. These are called normal lines (Kiper et al. 2022). The position of the normal lines can give information about the motion characteristics of a scissor unit. In Figure 6.13, there are two types of scissor units composed of angulated arms. The first one (Figure 6.13a) indicates the motion sequence of a unit that belongs to the deployable system. It can be seen that this scissor unit is obtained as a result of rhombus loop assembly. The second one (Figure 6.13b) shows the motion sequence of a unit that belongs to the transformable system. In Figure 6.13a, it can be observed that the angle between normal lines is fixed, and the scissor unit moves between them. But the other unit, that is obtained as result of antiparallelogram loops, can move from concave to convex. At first normal lines are parallel to each other, but for both concave and convex cases they intersect with each other which means the angle between normal lines change during the motion.

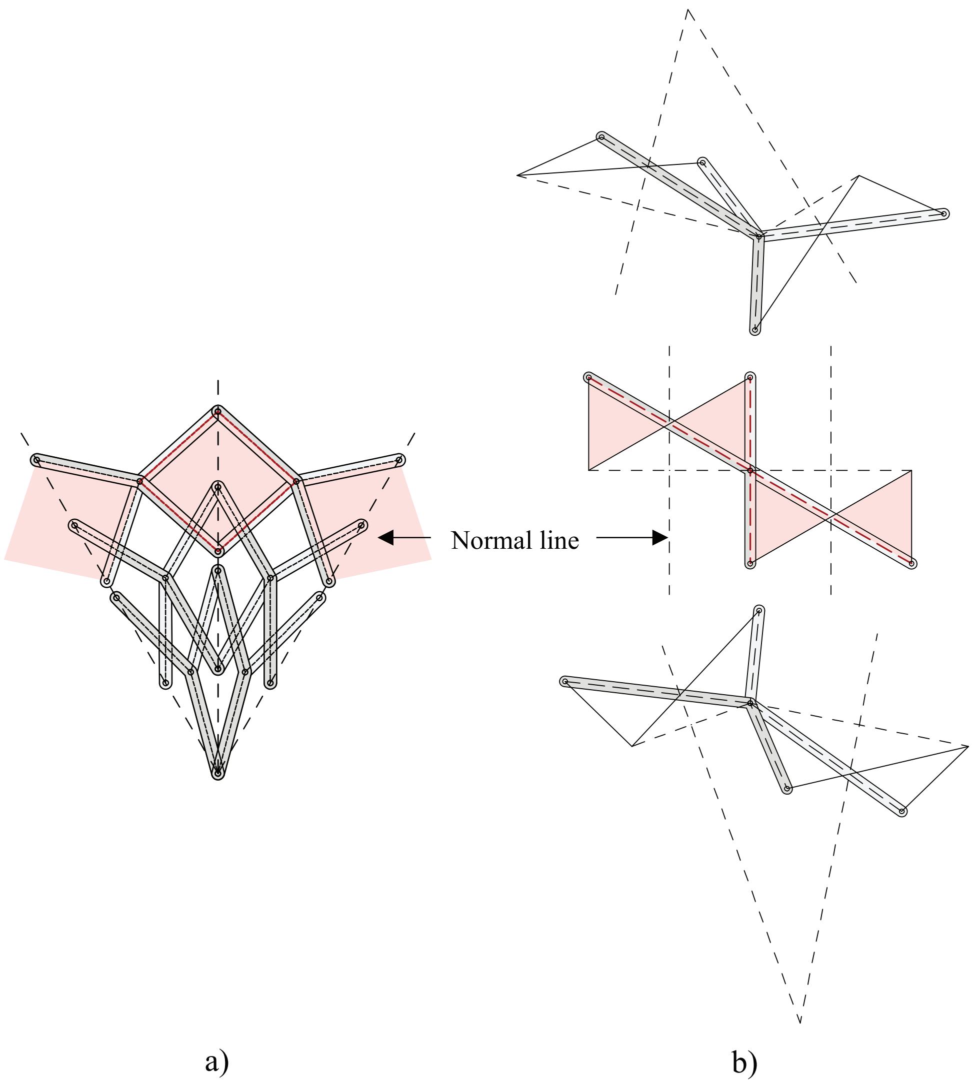


Figure 6.13 Motion sequence comparison of two types of scissor units composed of angulated arms: a) deployable type of scissor unit, b) transformable type of scissor unit.

TG/TR/TCH/TCCH patterned scissor linkage, as mentioned in Table 5.2 Model 5, in chapter 4, is chosen to create one module of a proposed linkage. It is a planar scissor linkage that can transform between concave and convex forms. The 3D-printed version of the model is in Figure 6.14.

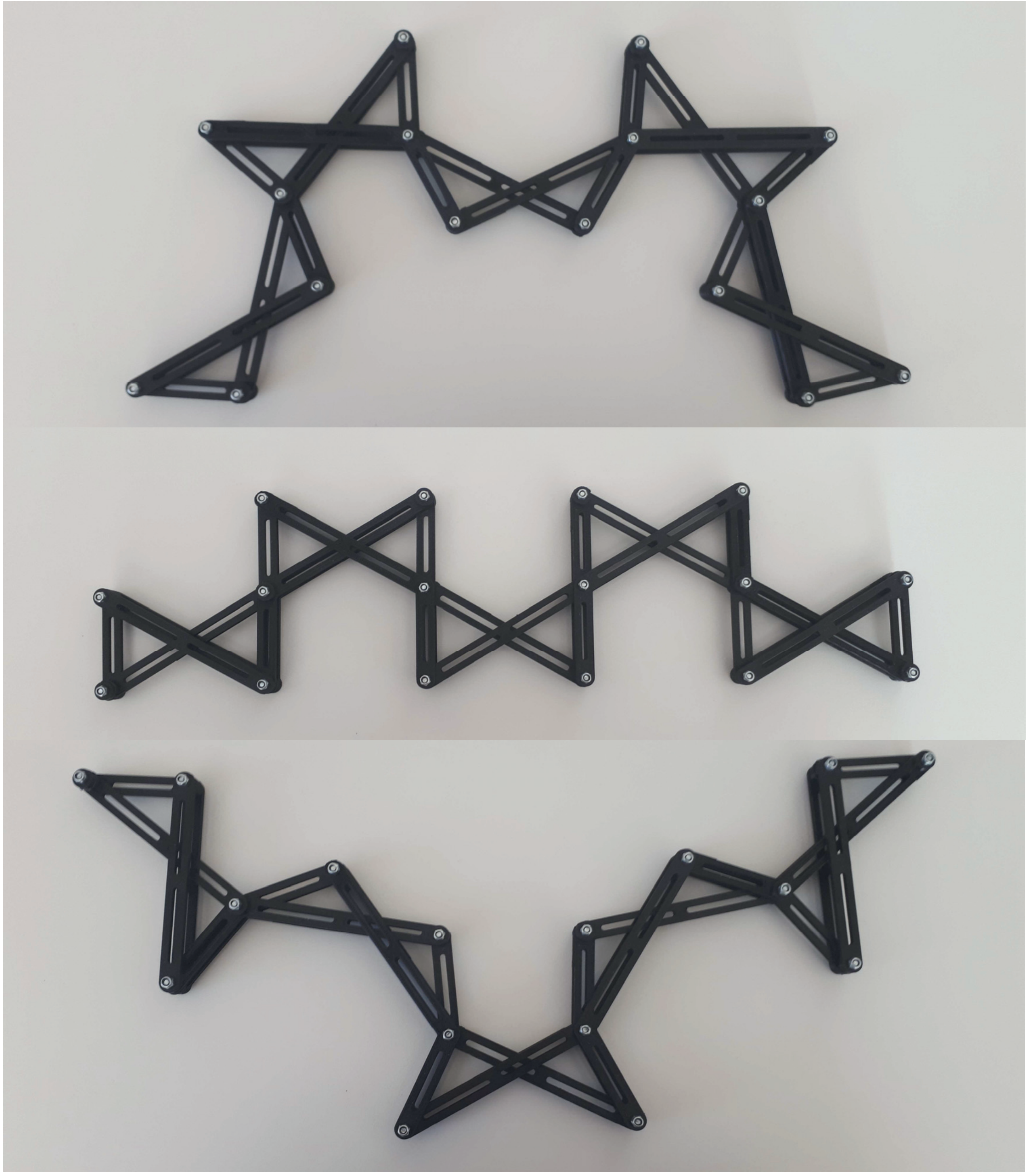


Figure 6.14. 3D Printed model of a scissor linkage with its motion sequence.

As shown in Figure 6.15, by connecting three identical kinematic linkages, one module is constructed with spatial RRR chains. Each linkage that constructs a module is planar. Therefore, the R joint axes in a linkage are parallel to each other. At the connection points where two linkages articulate to each other the axes of the adjacent R joints are perpendicular to each other. As illustrated in the module can transform between concave – flat – convex states.

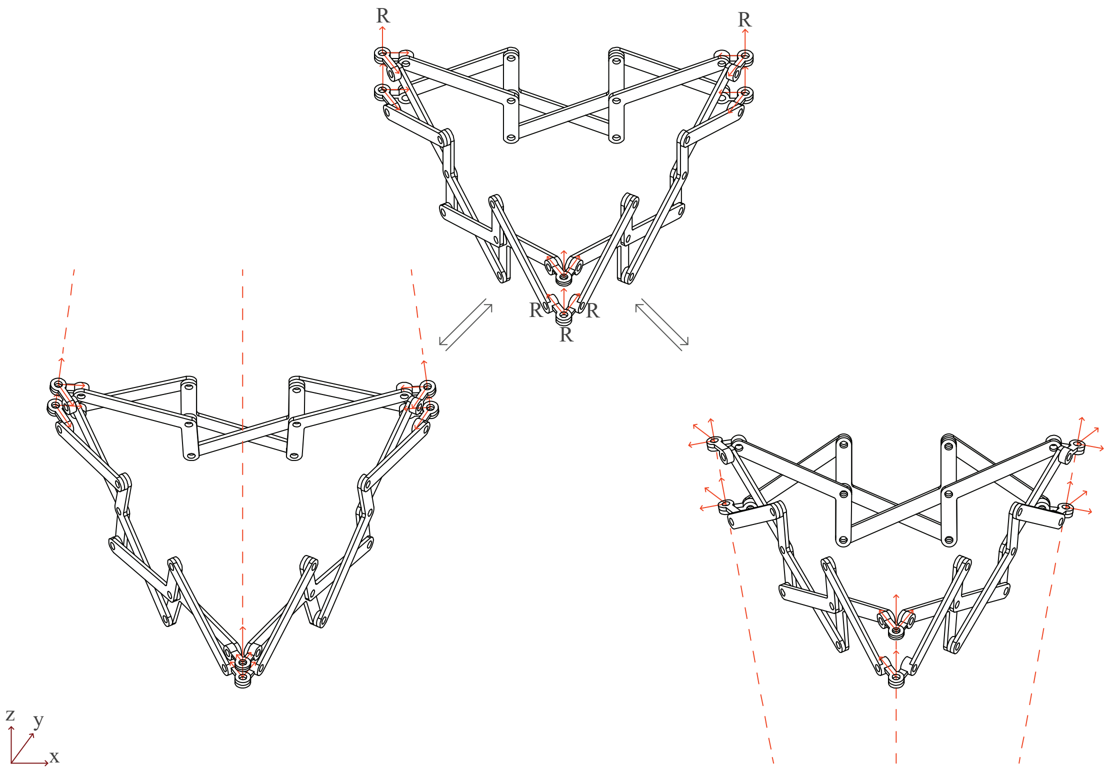


Figure 6.15 A triangular module constructed with antiparallelogram loops can transform concave to convex.

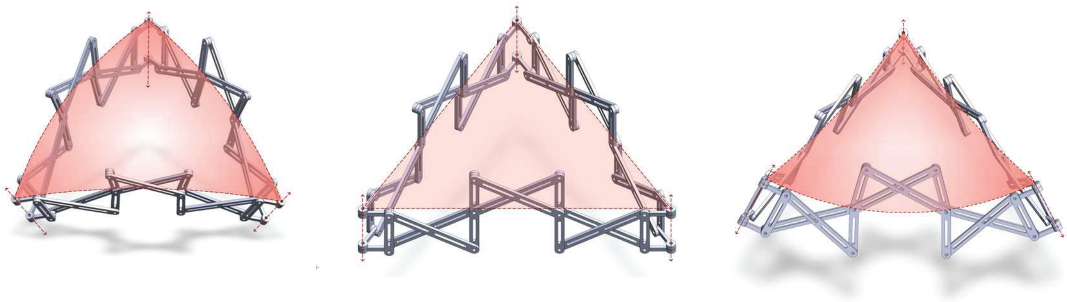


Figure 6.16. The motion sequence of a triangular module transforming concave to convex from left to right.

The 3D-printed version of the module can be seen in Figure 6.17. The model of the module is formed with three 3D-printed planar scissor linkages. The module defines a planar surface in its flat form. It can be observed that the module defines a synclastic surface such that R joint axes of connection points intersect at one point in space for both concave and convex forms.

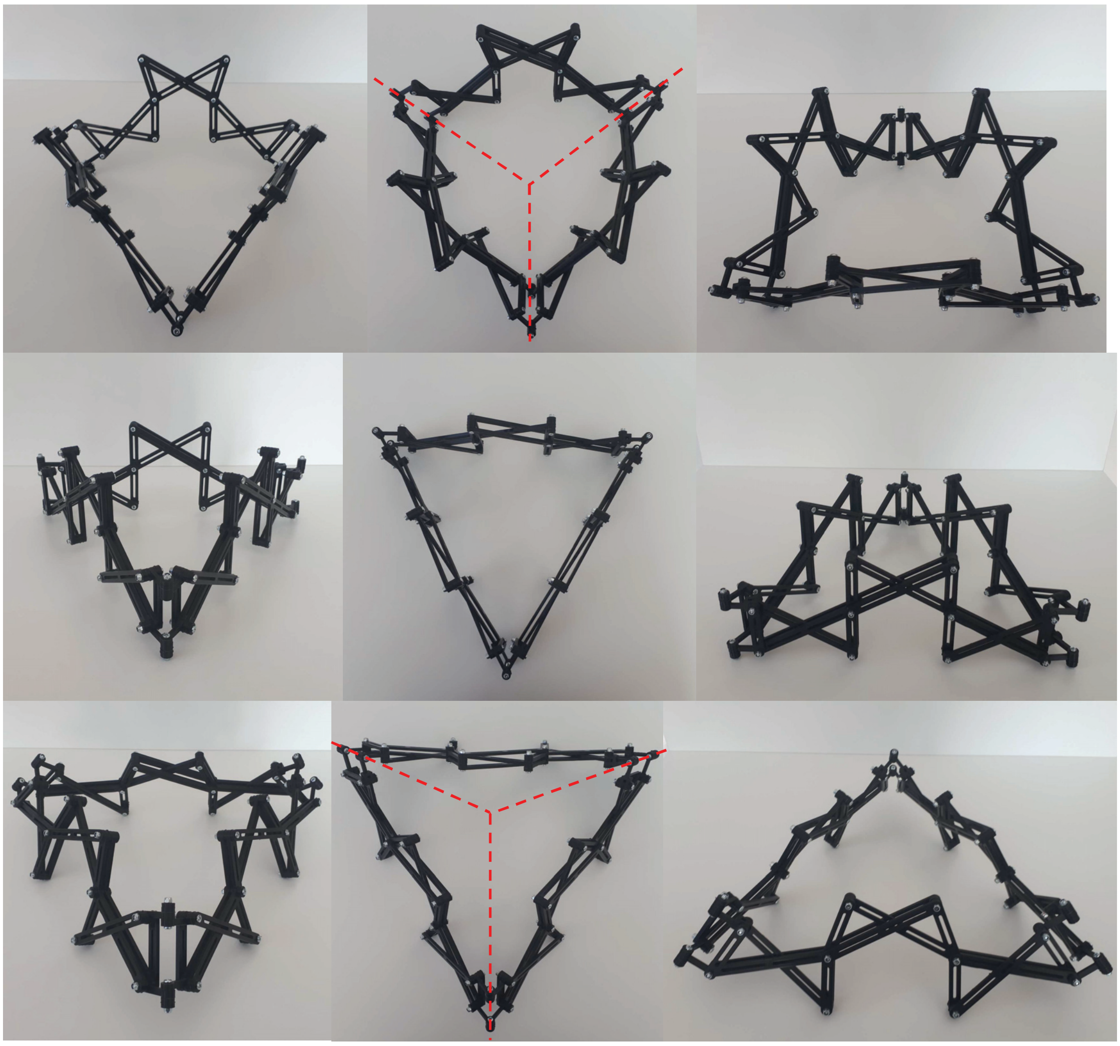


Figure 6.17 3D-printed model of the module with its motion sequence top to bottom concave, flat, and convex.

Spatial scissor linkage is constructed by connecting the planar scissor linkages together in a three-way grid. In order to define a surface in a flat configuration, the structure should be modeled starting from the center to the periphery in a three-way grid order. Planar scissor linkages obtained as a result of loop assemblies have only revolute joints and a single degree of freedom. In order to construct the model, revolute joints are used to assemble the scissor linkages in three directions. Revolute joint axes are simplified in Figure 6.18, where red dots represent the direction of the axis pointing out of the xy plane, and where the dashed lines represent the direction of the axis on xy plane.

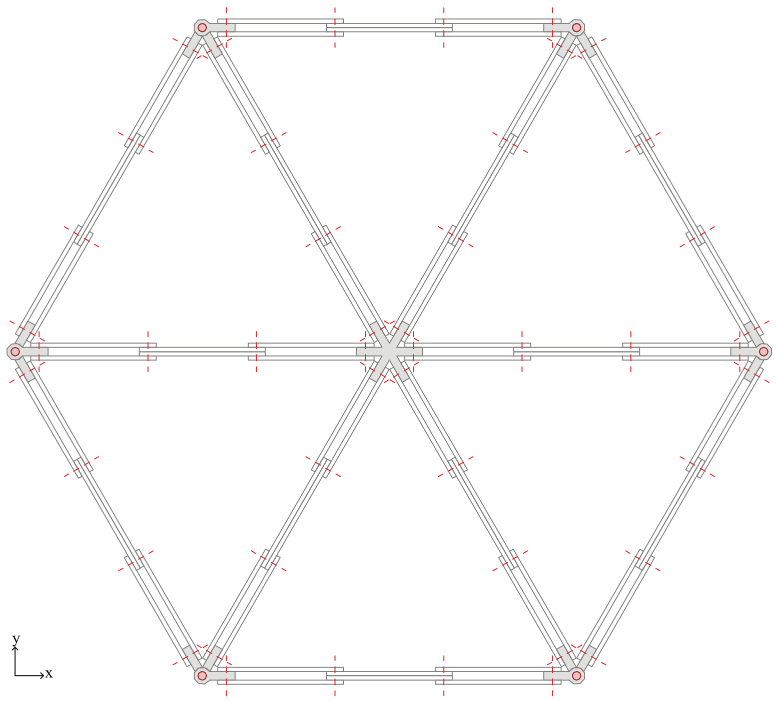
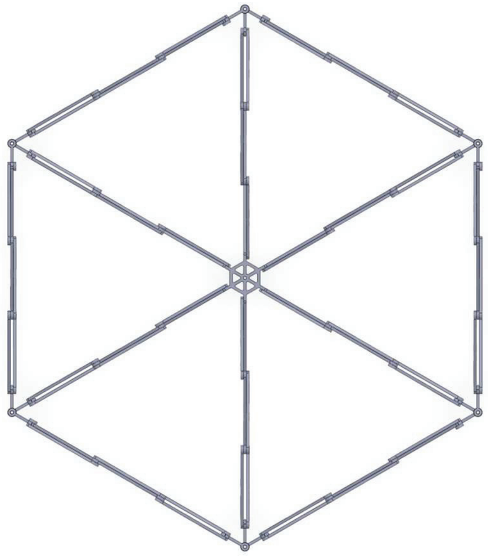
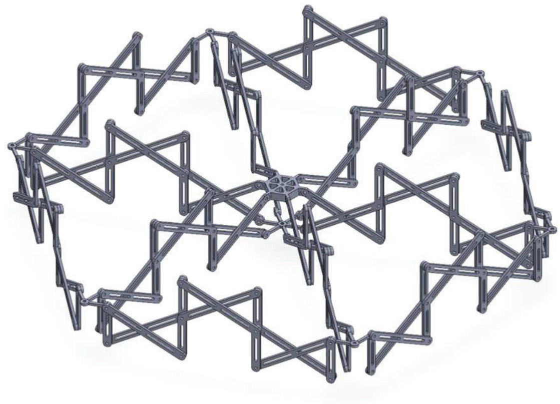


Figure 6.18 Top view of the network constructed scissor linkage.

The linkage modeled in Solidworks is shown in Figure 6.19. In flat form, each module is equilateral triangles. In the center of the linkage, the top hub is fixed, and it connects six modules to each other with the bottom hub. (Figure 6.19b). When the bottom hub is moved towards the fixed joint the whole linkage moves simultaneously; in other words, the linkage has a 1 degree-of-freedom. The linkage can change its form between concave and convex forms.



a)



b)

Figure 6.19 a) Top view of the linkage, b) Flat form of the linkage.

3D printed model of the linkage can be seen in Figure 6.20 to Figure 6.22. The model contains six central and six peripheral linkages. There are two hubs in the center to which the central linkages are connected. The model is animated with the motion of one of the central hubs, as it is seen in Figure 6.21 and Figure 6.22.

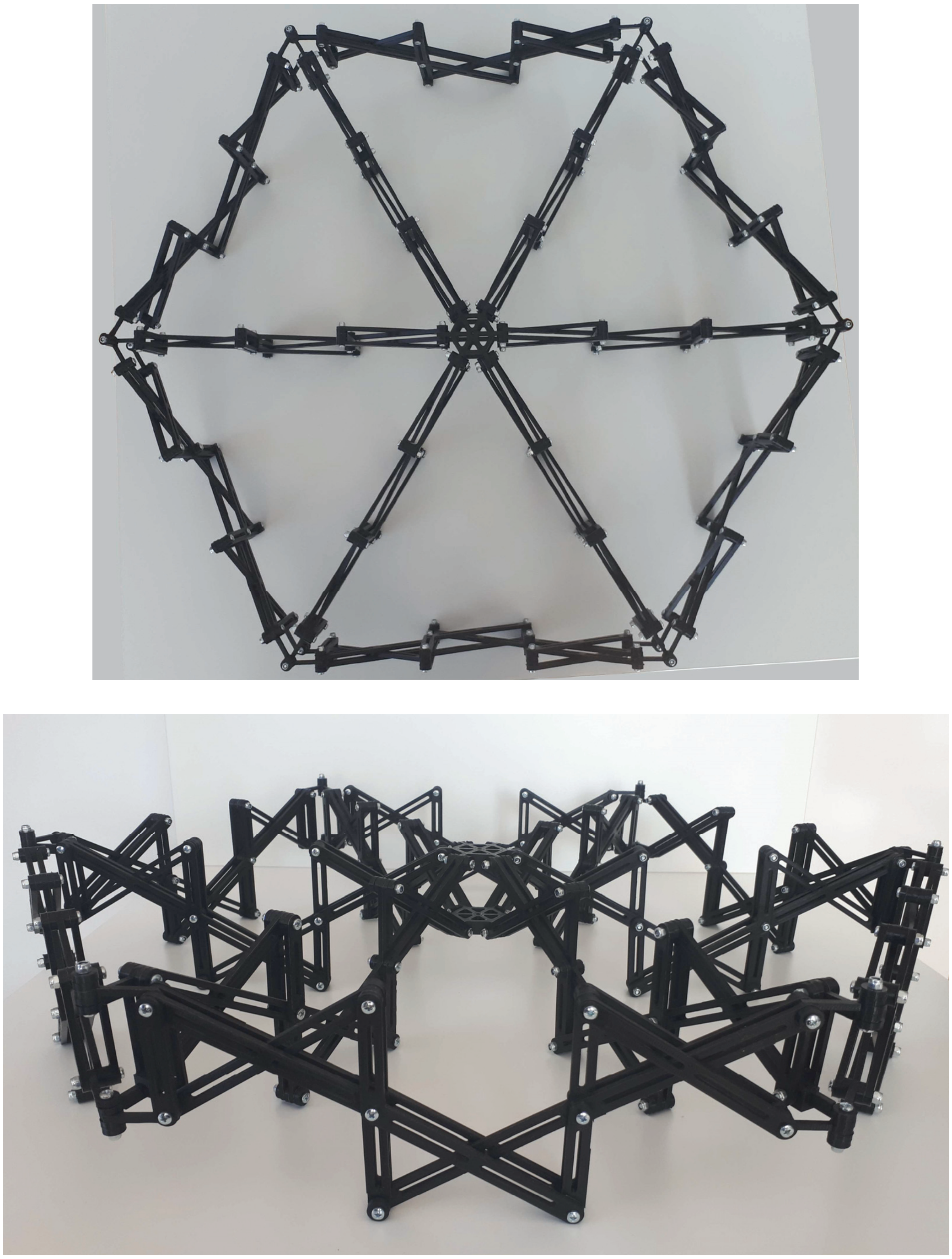


Figure 6.20 3D Printed model of the transformable linkage when it is flat position.

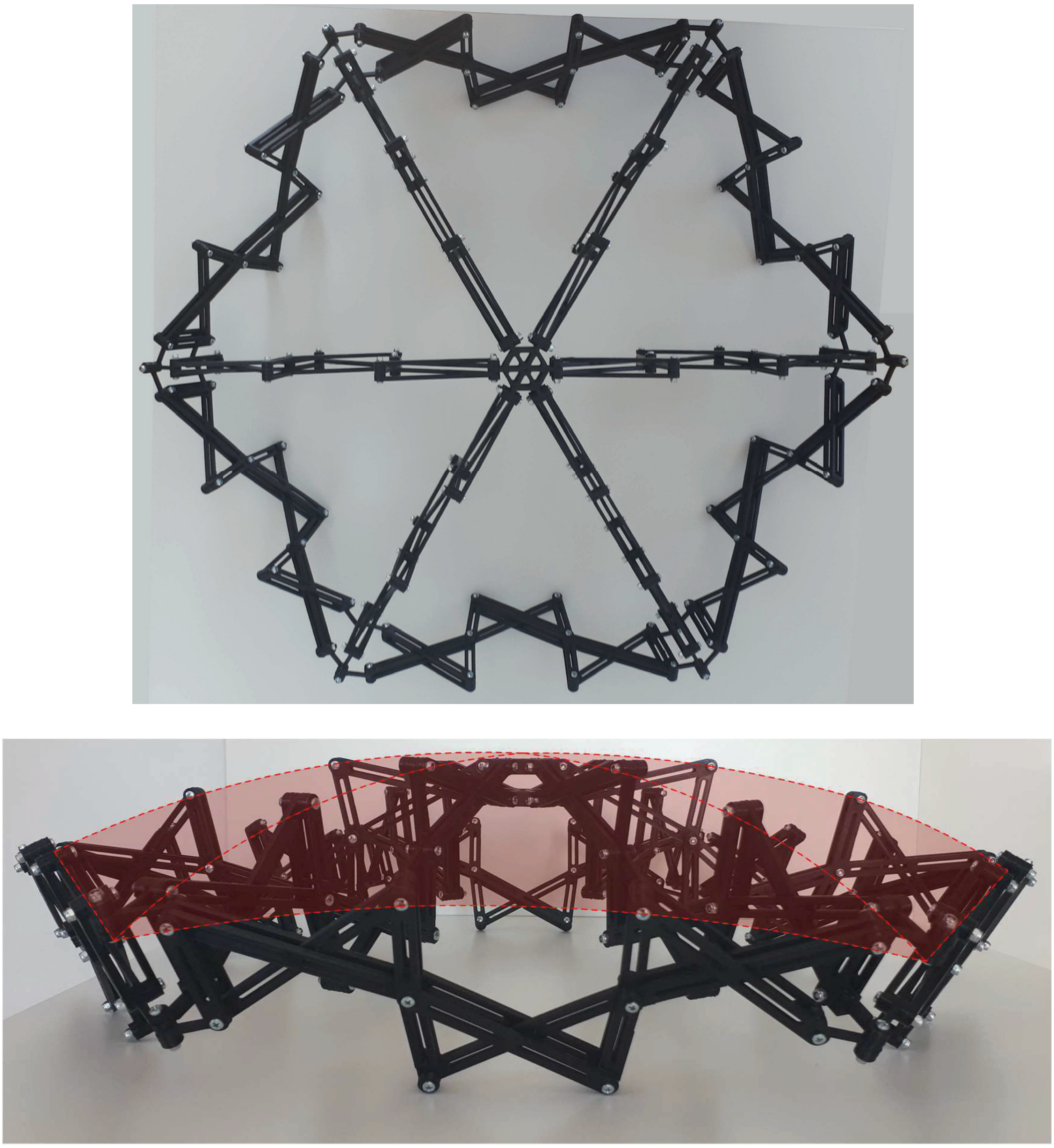


Figure 6.21. 3D-Printed model of the transformable linkage in a concave position

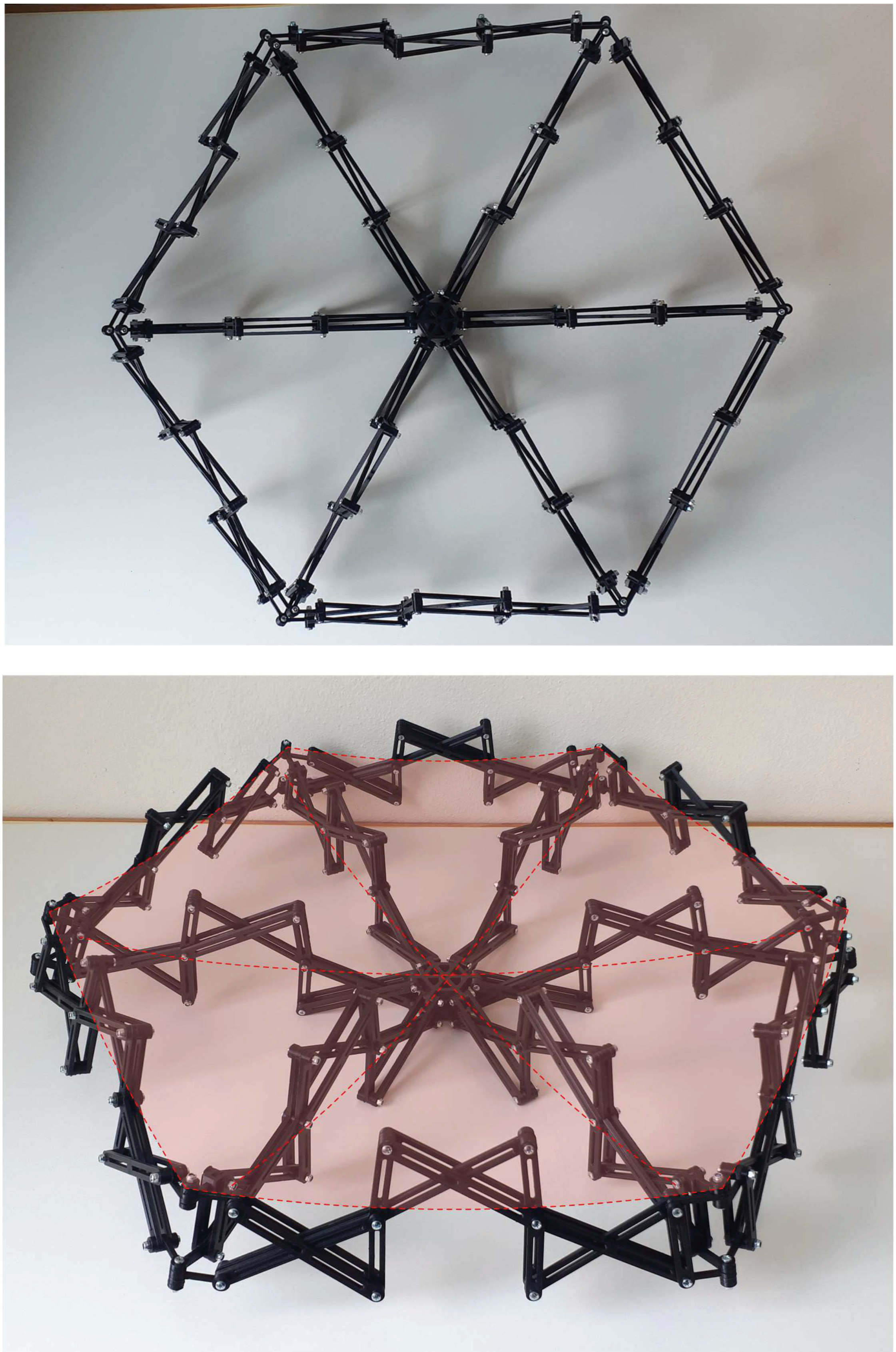


Figure 6.22. 3D Printed model of the transformable linkage when it is convex position.

CHAPTER 7

CONCLUSION

The aim of the study was to design transformable scissor linkages that can change their form between concave and convex forms. The starting point of this dissertation focused on the surface geometry of linkages could have. The main idea was the transformation of the lines and curves into each other as a surface profile.

At first, comprehensive geometric definitions were made, and then the curvature of surfaces was explained in detail. In light of these, surfaces were classified and explained based on their curvature. Then, the literature on scissor structural mechanisms was examined thoroughly based on previous literature survey on geometrical properties. The review of the existing scissor structural mechanisms was investigated in two subcategories: planar scissor linkages and spatial scissor structures.

Examination of planar scissor linkages revealed three types of motion characteristics: scaling deployment, angular deployment, and transformation. Next, the scissor structures in the literature were analyzed according to their surface geometries as monoclastic, synclastic, and anticlastic scissor structures. Then, they were explained based on this classification (Table 2.1).

The basic scissor unit types were also examined as being a component to form scissor linkages. In addition, existing geometrical construction methods of planar scissor linkages were investigated.

According to Table 2.1, in the literature, it was seen that transformable scissor structures defining conoid surface were not studied as the surface that defines the surface of a scissor structure. In this context, one of the aims of the present study was to design transformable conoidal surfaces by transforming the two curves and forming the conoid surface into each other. To do this, scissor linkages obtained as a result of loop assemblies used as a surface profile.

7.1. Achievements and Main Findings of the Study

Planar scissor linkages obtained by the loop assembly method, which duplicates a loop in a certain order, could give three different results in terms of movement types. These are scalable (linear or radial) deployment, angular deployment, and transformation. When the result forms of transformable planar scissor linkages were examined, they always defined a circular curve.

Considering the planar scissor linkage as a directrix of the ruled surface, we can define the forms with conic sections that the planar scissor linkage can take. When a section taken from a conic surface is examined, line, circle, ellipse, parabola, and hyperbola are obtained. It is thought that transformable planar scissor linkages can take ellipse, parabola, or free-form curves as well as circular curves. However, a new approach was needed since the existing loop assembly method was insufficient to obtain scissor linkages that could meet these conditions. This problem could be solved with a new geometric construction method.

In this novel geometric construction method explained in Chapter 4, the desired initial and final forms of the planar scissor linkage could be formed in SolidWorks®. In general, one side of a loop is considered as the radius of a circle. All loops were obtained by mutual matching for initial and final forms. The intended results were achieved both with a kite and its concave form dart and antiparallelogram loops. In addition to the circle-to-circle transformations, other cases, such as parabola-to-parabola and ellipse-to-ellipse, were also tested separately for three loops, and they were successfully applied. The resulting models can transform between concave and convex forms with this novel method.

After that, models were produced to create the conoid surface, which is a ruled surface. Scissor linkages were assumed as directrix of a conoid surface, and between these linkages, some connection details were tried. If a linkage has one degree of freedom, the resulting surface is a vault, which is a monoclastic surface rather than a conoid. When a model was formed to define a conoid surface, linkage was hard to control due to having more than one degree-of-freedom.

So as to overcome the difficulties mentioned above and create a one degree-of-freedom transformable linkage, the linkage was formed with a network of one module, which is composed of three scissor linkages. When one module of the scissor linkage was investigated, it was observed that three linkages could transform between concave and convex forms simultaneously. Creating a network of this module concluded with a synclastic linkage that can successfully change its form between concave and convex forms in accordance with the primary purpose of this study.

7.2. Recommendations for the Future Studies

As a result of this study, the transformable scissor linkage was modeled successfully. The resulting surface of the linkage was a synclastic surface with a circular section. In addition to circles, other synclastic surfaces whose surface profile is an ellipse, or parabola can be studied for further studies. Moreover, linkages defining anticlastic surfaces can be studied.

In this study antiparallelogram loops were taken into consideration only. Along with antiparallelogram loop, kite and dart loops are also used to model transformable scissor linkages. Like the proposed linkage in this dissertation, a module and network possibilities can be studied for kite and dart loops for future studies.

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