

DESIGN OF DEMOUNTABLE RECIPROCAL FRAMES WITH NEW GEOMETRIC PROPERTIES

Gülçin ÖZEN¹, Gökhan KIPER² and Koray KORKMAZ³

¹Department of Architecture, Izmir Institute of Technology, Izmir, Turkey

²Department of Mechanical Engineering, Izmir Institute of Technology, Izmir, Turkey

³Department of Architecture, Izmir Institute of Technology, Izmir, Turkey

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ABSTRACT

This study aims to develop missing geometric knowledge for demountable reciprocal frames (RF). While designing a demountable RF, one should know the initial, in-process and final form of the RF. These processes require some specific geometric knowledge. There are some deficiencies about geometric properties in the previous studies about demountable RFs. In this study, the positions and the orientations of the nexors are found by using the Denavit-Hartenberg parameters. This information gives where the joints are placed, how they are oriented and take position according to one another. Besides, the influence of engagement length on the fan height and the base edge are analyzed. Thereby one will be able to find out how much space the RF covers with the known base edge. With the geometric knowledge obtained from this study, demountable RFs having different engagement lengths can be produced with the same nexors.

Keywords: Denavit-Hartenberg Convention, Nexorade, Reciprocal Frame

1. INTRODUCTION

Reciprocal frame (RF) is a three-dimensional beam grillage system arranged to form a closed loop. The name of 'reciprocal frame' was coined by Graham Brown in the UK in 1989 under the title of 'Three Dimensional Structures' to explain the grillage structure in which the beams mutually support each other. The etymology of 'reciprocal' is from the Latin word 'reciprocus' while recus means backward, procus means forwards. Therefore, reciprocity means back and forth, providing an exchange for mutual benefit [1].

Baverel used the term 'nexorade' to explain RFs. The etymology of nexorade comes from the Latin word 'nexor,' meaning link. Nexorade means a family of mutually supported repeating member assemblies. The term creates complications with Graham Brown's RF, but they mean the same thing [2].

Each member of a nexorade is described as a 'nexor.' The simplest form of a nexorade is called a 'fan' (Figure 1 left). In other words, it is the most elementary arrangement of a nexorade. When

nexors are joined, they develop a three-dimensional RF. At the connection of a two nexor cross-section, the minimum perpendicular distance between the centroids of these cross-sections is referred to as 'eccentricity' (e). The position of every nexor depends on the eccentricity. The length of each nexor is denoted by L . The distance between one end of a nexor and an intersection point on a nexor is 'engagement length' (λ) (Figure 1).

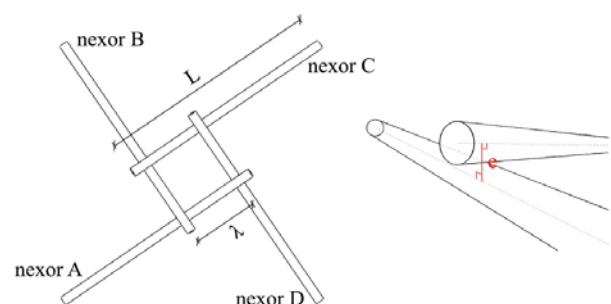


Figure 1: Elements of a nexorade

In RFs, all parameters are dependent on each other. Changing parameters also means changing the form of the RF as explained in Section 2.4. For this

reason, geometric properties should be known in detail to design a demountable RF.

2. GEOMETRIC PROPERTIES OF RECIPROCAL FRAMES

In terms of geometric properties, Chilton and Choo (1992) were the first using, defining, and describing the RF in a scientific paper and specified the principles of RF [3]. Chilton and Choo (1992) explained the main parameters of the static RFs as in Figure 2 [3].

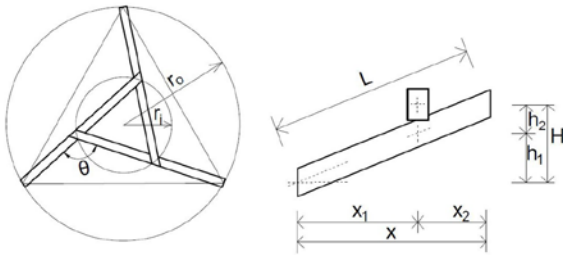


Figure 2: Geometrical parameters that define a fan [4]

Chilton and Choo (1992) have presented the parameters that define the geometry of a static RF and their relation to each other by having rectangular cross-sectioned nexors by using basic trigonometry [3]. Larsen (1996) used the same parameters with the static RFs to find the retractable RF geometry. She analyzed the relationship between ratio h_2/H and number of beams; ratio L/r_0 and number of beams; and horizontal angle (angle between nexors) and vertical angle (slope) (Figure 3) [4].

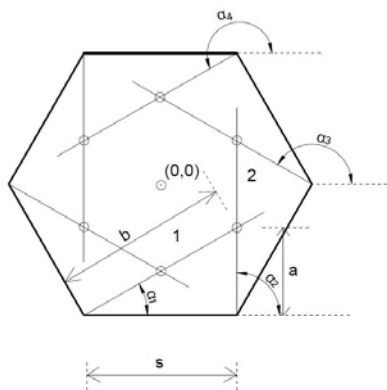


Figure 3: Plan geometry of a retractable RF [4]

Chilton, Choo, and Wilkinson (1998) analysed the relationship between nexor length (L) and angle between nexors (θ); engagement length (λ) and angle θ ; ratio λ/L and angle θ ; inner radius and angle θ for planar RFs and ratio λ/L and height for three-dimensional RFs with the different number of beams [5].

Parigi and Kirkegaard (2014) describes RFs with four parameters: eccentricity, two engagement ratios, and top/down position. They analyze 3-nexor fans with various parameters. Moreover, they observe the dependency of the elements with different positions [6].

The geometric properties of RFs are set in the literature, but there are some deficiencies about the position of nexors and the orientation of the coordinate frames. Moreover, the engagement length, eccentricity, and height relationship have been examined previously, but the influence on engagement length on the height and the base edge has not been reported. These omitted properties, parameters, and relationships are examined in this study.

2.1. Denavit Hartenberg Convention

To understand the geometric properties of a RF fan, the Denavit Hartenberg (DH) convention can be used. DH convention is used for describing the geometry of the links and the relative positions of links with respect to each other. It can be used for linkages, which are mechanisms with joints that have surface contact, such as revolute (R), prismatic (P), helical (H), cylindrical (C), universal (U), and spherical (S) joints. Joints with multi degree-of-freedom (DoF) can be considered as combinations of single DoF joints. A C joint is a combination of an R and a P joint where the R joint axes are in the P joint direction. A U joint is composed of two perpendicularly intersecting R joints, and an S joint can be modeled as a series of three intersecting R joint axes.

Every link connects two or more joints, and every joint connects two and only two links. In DH convention, a Cartesian coordinate system is attached to each link. First, the axes of single DoF joints are numbered (an exception is a C joint where it is not necessary to dissect into R and P joints). The coordinate system attached to each joint axis i is defined according to the following rules:

- z_i -axis is on a joint axis i .
- x_i axis is on the common perpendicular of z_{i-1} and z_i axes.

To describe how a frame (i) relates to the previous frame ($i-1$), four parameters are needed to be known. The four parameters are a , α , d , and θ . While a and α are called link parameters, d and θ

are joint parameters. They are named as link length (a), twist angle (α), link offset (d), and joint angle (θ), respectively. The reason for giving these names is the geometric relationships between two coordinate frames. Link parameters are measured about and along x-axes, and joint parameters are measured about and along z-axes. The right-hand rule is used for finding the direction of a and θ : while the clockwise direction is negative, the counterclockwise direction is positive [7].

In DH convention, some rules are needed to be followed for finding parameters [8]:

- a_{ij} is the distance between the two joint axes, from z_i to z_j along x_j axis, where $j=i+1$.
- α_{ij} is the twist angle between z_i and z_j measured about x_i .
- d_i is the link offset from x_{i-1} to x_i along z_{i-1} axis.
- θ_i is the joint angle between x_{i-1} to x_i measured about z_{i-1} .

A_i^j indicates a homogeneous transformation matrix from frame i to frame $j=i+1$ (Equation 1). The homogeneous transformation matrix is a 4x4 matrix to find the position and orientation from one coordinate frame to another. The homogeneous transformation matrix from frame i to j is as follows:

$$A_i^j = \begin{bmatrix} \cos \theta_i & -\cos \alpha_{ij} \sin \theta_i & \sin \alpha_{ij} \sin \theta_i & a_{ij} \cos \theta_i \\ \sin \theta_i & \cos \alpha_{ij} \cos \theta_i & -\sin \alpha_{ij} \cos \theta_i & a_{ij} \sin \theta_i \\ 0 & \sin \alpha_{ij} & \cos \alpha_{ij} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

A Cartesian coordinate system is defined on the links in compliance with DH convention (Figure 4). Figure 4 is a closed-loop kinematic chain like a reciprocal structure, and the n^{th} coordinate system is identical with the initial (0^{th}) coordinate system [9].

After the coordinates are taken properly, the matrices are created. The changes of the coordinates start with (x_0, y_0, z_0) , turn around the loop, and return to (x_0, y_0, z_0) . After multiplying the matrices (Equation 2) for each coordinate frame, the result is an identity matrix (Equation 3) since it is a closed-loop and the kinematic chain turns back to the starting point at the end [9].

$$I = A_1^n = A_1^2 A_2^3 A_3^4 \dots \dots A_{n-2}^{n-1} A_{n-1}^n \quad (2)$$

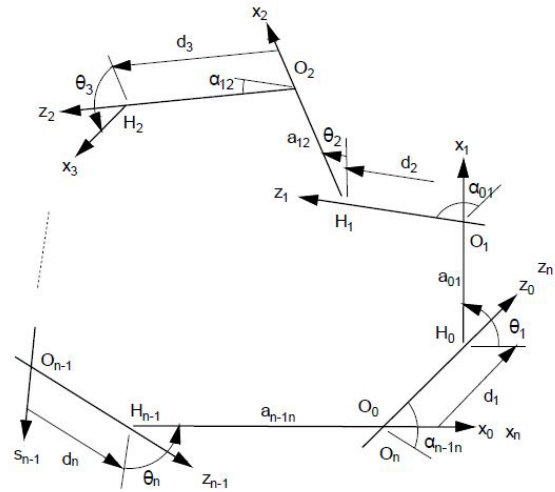


Figure 4: DH parameters on a closed loop kinematic chain [9]

$$I = A_1^n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

2.2. Denavit Hartenberg Convention in Reciprocal Structures

Two nexors can be connected with a series of CRC (cylindrical, revolute, cylindrical) joints in RF. While C joints are set along the link axes, R joints are set along the contact normals. C joint provides rotation around and translation along the defined axis and is a 2-DoF joint. R joint provides rotation between two paired elements and is a 1-DoF joint [9]. A 4-nexor RF having CRC joint series has mobility of 9 (including self-motions of links about their axes).

For a 4-nexor RF having CRC joint series, the coordinate frames are placed on the mechanism, and parameters are defined. " a_{ij} " is the distance between z_i to z_j axes. For RF, all a_{ij} are 0, because all consecutive joint axes intersect. " α_{ij} " is the angle measured from z_i axis to z_j axis about x_j axis. For example, α_{121} in Figure 5 is the angle measured from z_{12} axis to z_1 axis about x_1 axis, which is 270° . " d_i " is the distance between x_{i-1} axis to x_i axis. d_1 can be measured from x_{12} to x_1 axis in Figure 5. The distance between x_{12} to x_1 gives the eccentricity or diameter of the nexor, which is denoted as D in Table 1. Finally, " θ_i " is measured about z_{i-1} axis from x_{i-1} axis to x_i axis. The angle measured from x_{12} to x_1 axis about z_{12} axis gives θ_1 angle (Figure 5). The parameters are listed in Table 1.

Reciprocal structures comprise closed-loop kinematic chains. Therefore, the loop closure equations give the identity matrix as a result (Equation 4).

$$I = A_1^{12} = A_1^2 A_2^3 A_3^4 A_4^5 A_5^6 A_6^7 A_7^8 A_8^9 A_9^{10} A_{10}^{11} A_{11}^{12} A_{12}^1 \quad (4)$$

Equation 5 shows the homogeneous transformation matrix from frame i to j . While the left 3x3 matrix (red) represents rotation, the right 3x1 matrix (blue) represents translation. For example, suppose one wants to know the position or distance from one point to another. In that case, the values in the blue rectangle should be analyzed, or if one wants to find the angle between one coordinate frame to another, the values in the red rectangle should be analyzed.

$$A_i^j = \begin{bmatrix} x_x & y_x & z_x & d_x \\ x_y & y_y & z_y & d_y \\ x_z & y_z & z_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

Table 1: DH parameters for a 4-nexor reciprocal fan for CRC joint series

i	a _{ij}	d _i	α _{ij}	θ _i
1	0	D	270°	θ ₁
2	0	d ₂	0°	θ ₂
3	0	d ₃	90°	180°
4	0	D	270°	θ ₄
5	0	d ₅	0°	θ ₅
6	0	d ₆	90°	180°
7	0	D	270°	θ ₇
8	0	d ₈	0°	θ ₈
9	0	d ₉	90°	180°
10	0	D	270°	θ ₁₀
11	0	d ₁₁	0°	θ ₁₁
12	0	d ₁₂	90°	180°

The distance between two frames (Figure 6) whose coordinates are known can be found with Equation 6:

$$k = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (6)$$

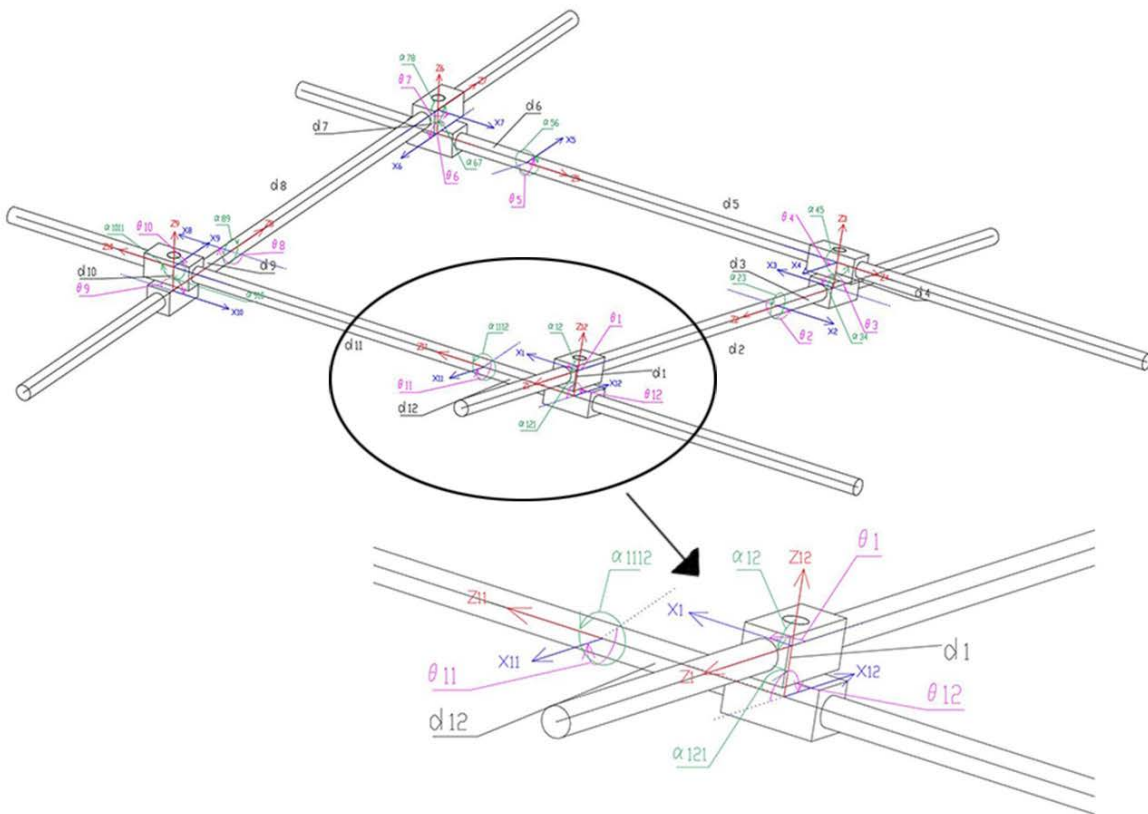


Figure 5: Local coordinate frames for a 4-nexor fan

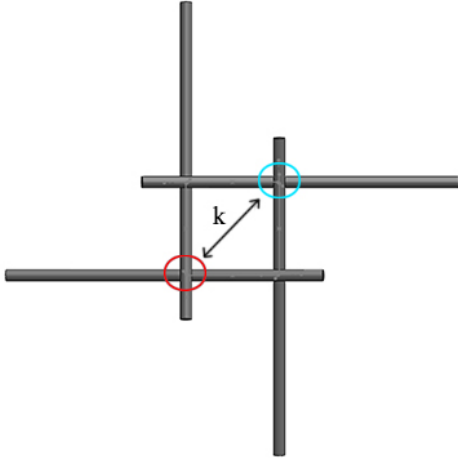


Figure 6: Distance between two coordinate frames

The homogeneous transformation matrices give critical geometric knowledge, which is the rotation and translation data. Constructive details should be considered to create RFs, and DH parameters will give this information. The orientation of the coordinate frames or the position of the joints will be decided according to these parameters. This is illustrated with a case study with changing engagement lengths.

2.2.1. Case Study for a CRC Joint Series RF Fan with DH Convention

DH parameters for $\lambda=40$ cm and $\lambda=50$ cm where $L=140$ cm, $e=4$ cm

DH parameters are found for $\lambda = 40$ cm and $\lambda = 50$ cm where $L = 140$ cm and $e = 4$ cm for CRC joint series RF (Table 2 and Table 3). To find the translation or rotation of the sixth coordinate frame according to first, matrices from first to sixth should be multiplied. These x, y, and z coordinates are used to find the distance between opposite corners. The distance between opposite corners is found and denoted as k_1 for $\lambda = 40$ cm and k_2 for $\lambda = 50$ cm.

$$A_1^6 = \begin{bmatrix} -0,9779 & -0,019546443 & -0,208384 & 40,78186 \\ -0,0195 & -0,982779644 & 0,183748287 & 40,0463 \\ -0,2084 & 0,183744682 & 0,960631394 & 0,582082 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 2: DH parameters for $\lambda = 40$ cm CRC joint series RF fan where $L = 140$ cm, and $e = 4$ cm

i	a_{ij}	d_i	α_{ij}	θ_i
1	0	4	270°	90°
2	0	-30	0°	190,6°
3	0	-10	90°	180°
4	0	4	270°	88,9°
5	0	-30	0°	192,0°
6	0	-10	90°	180°
7	0	4	270°	90°
8	0	-30	0°	190,6°
9	0	-10	90°	180°
10	0	4	270°	88,9°
11	0	-30	0°	192,0°
12	0	-10	90°	180°

The elements of the left 3x3 rotation matrix gives the cosine of the angles between the axes of coordinate frames. The angle between 1st and 6th joint axes are:

$$\begin{aligned} x_x &= 168^\circ & y_x &= 91^\circ & z_x &= 102^\circ \\ x_y &= 91^\circ & y_y &= 169^\circ & z_y &= 79^\circ \\ x_z &= 102^\circ & y_z &= 79^\circ & z_z &= 16^\circ \end{aligned}$$

The right 3x1 matrix gives the coordinates of the origin of the sixth frame. Since the first frame's coordinates are (0,0,0), the distance of the origin of the 6th frame can be found as:

$$k_1 = \sqrt{(40,8)^2 + (40,04)^2 + (0,58)^2} = 57,16 \text{ cm}$$

Table 3: DH parameters for $\lambda = 50$ cm CRC joint series RF fan where $L = 140$ cm, and $e = 4$ cm

i	a_{ij}	d_i	α_{ij}	θ_i
1	0	4	270°	90°
2	0	-40	0°	188,5°
3	0	-10	90°	180°
4	0	4	270°	89,3°
5	0	-40	0°	189,7°
6	0	-10	90°	180°
7	0	4	270°	90°
8	0	-40	0°	188,5°
9	0	-10	90°	180°
10	0	4	270°	89,3°
11	0	-40	0°	189,7°
12	0	-10	90°	180°

$$A_1^6 = \begin{bmatrix} -0,98554 & -0,01256604 & -0,168992126 & 50,628302 \\ -0,01261 & -0,98904071 & 0,147103244 & 50,040511 \\ -0,16899 & 0,147107296 & 0,97457801 & 0,60111105 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The angle between 1st and 6th joint' axes for the $\lambda = 50$ cm case are:

$$\begin{aligned} x_x &= 170^\circ & y_x &= 91^\circ & z_x &= 100^\circ \\ x_y &= 91^\circ & y_y &= 172^\circ & z_y &= 82^\circ \\ x_z &= 100^\circ & y_z &= 82^\circ & z_z &= 13^\circ \end{aligned}$$

2.3. The Impact of Engagement Length on Base Edge and Height

To create a 4-nexor fan, engagement length ($\lambda = 2l$) and eccentricity (e) are given as input and twist

Figure 7). To find these parameters, the necessary formulations are derived.

Figure 7, there are $n = 4$ identical nexors and, $\gamma = 2\pi/n$. In the initial state, the symmetry axes of all nexors (such as OP and OT) are on the same plane. Then all nexors are rotated around their midpoints by an angle Ψ until they are tangent to the adjacent nexors. Since all nexors are rotated with the same angle, the contact points, such as R, are coplanar on the xy- plane.

Let nexor length $L = L_1 + L_2$, $|PQ| = |TS| = l$ and

$$\Psi = \cos^{-1} \left(\sqrt{\frac{(l)^2(1-\cos \gamma)(3-\cos \gamma)+r^2 \sin^2 \gamma \pm \sqrt{[(l)^2(1-\cos \gamma)(3-\cos \gamma)+r^2 \sin^2 \gamma]^2 - 8(l)^4(1-\cos \gamma)^3}}{2(l)^2(1-\cos \gamma)^2}} \right) \quad (8)$$

Also the distance between the first and sixth frames can be found as:

$$k_2 = \sqrt{(50,6)^2 + (50,04)^2 + (0,6)^2} = 71,18 \text{ cm}$$

As can be seen from the results, the orientation and position of the frames change with different engagement lengths. It is essential knowledge while designing a RF. One can decide where the joints should be located with the found position and orientation information.

angle (β), the angle between nexor and the horizontal plane (Ψ), the height of the structure (h), the distance between nexor midpoint and structure's center (b), and base edge (c) are found as an output (

In

the radius of the nexors be $r = |QR| = |RS|$. Then $|QR| + |RS| = 2r = e$ which is eccentricity. Let $|OP| = |OT| = b$ and $|AB| = c$. Ψ , b , h , β , and c are unknown for the configuration. b and Ψ can be found with Equation 7 and 8.

$$b = \frac{(2-(1-\cos \gamma) \cos^2 \Psi) l}{\sin \gamma \cos \Psi} \quad (7)$$

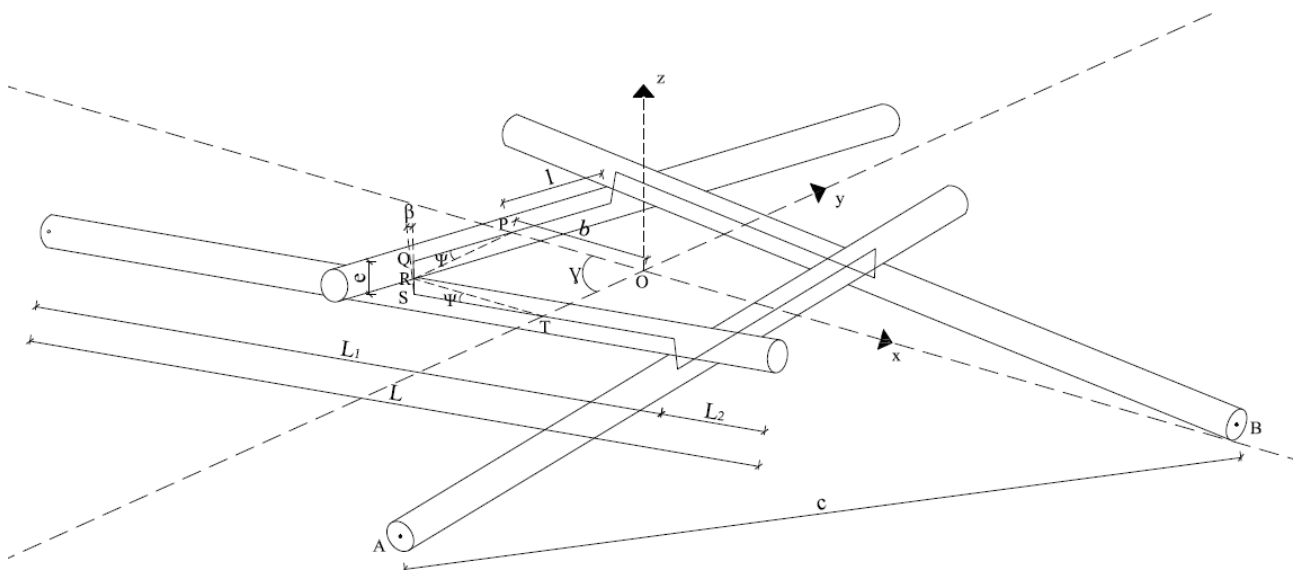


Figure 7: 4-nexor fan parameters

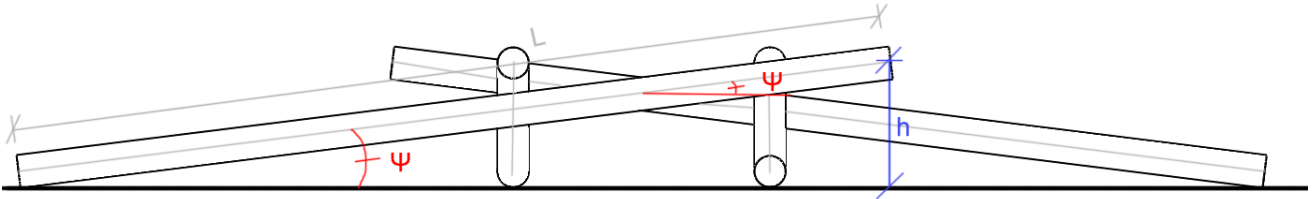


Figure 8: Side view of a 4-nexor fan

If this structure is considered as a roof, Ψ gives the slope, and h is the height of the roof (Figure 8). Taking the total length of one nexor as L , which is a known parameter, h can be found by the following equation (Equation 9):

$$h = \sin \Psi * L \quad (9)$$

After Ψ is found, the twist angle (β) of the nexors can be calculated as two times the angle between SQ and a line perpendicular to TS and parallel to yz - plane (Equation 10). β is α_{ij} in DH parameters.

$$\beta = 2 \cos^{-1} \left(\frac{l \tan \Psi}{r} \right) \quad (10)$$

The area that the RF covers is an important issue from the design point of view. The endpoints of the nexor create a square on the ground for this study. One edge of a square which is named as the base edge (c), can be calculated with formula follows (Equation 11):

$$|AB| = c = \sqrt{2 * [b^2 + [(L_1 - l) * \cos \Psi]^2]} \quad (11)$$

For the inverse solution, to find l and e for given b and Ψ , one needs to use Equations 12 and 13.

$$l = \frac{2b \sin \gamma \cos \Psi}{2 - (1 - \cos \gamma) \cos^2 \Psi} \quad (12)$$

$$e = 2r = 2b \sqrt{\frac{(1 - \cos \gamma) \sin^2 \Psi}{2 - (1 - \cos \gamma) \cos^2 \Psi}} \quad (13)$$

2.3.1 Case Study to Analyze the Impact of Engagement Length on Base Edge and Height

$$\Psi = \cos^{-1} \left(\sqrt{\frac{(25)^2(1 - \cos 90)(3 - \cos 90) + (2)^2 \sin^2 90 \pm \sqrt{[(25)^2(1 - \cos 90)(3 - \cos 90) + (2)^2 \sin^2 90]^2 - 8(25)^4(1 - \cos 90)^3}}{50(1 - \cos 90)^2}} \right)$$

$$= 4,6^\circ$$

$$n = 4, L_1 = 135 \text{ cm}, L_2 = 5 \text{ cm}, L = 140 \text{ cm}, l = 25 \text{ cm}, e = 4 \text{ cm} (r = 2 \text{ cm})$$

In the RF, l is 25 cm, and e is 4 cm. For the given l and e , Ψ , b , h , β , and c is calculated.

Inverse solution for the desired $b=25,2$ cm and $\Psi=4,6^\circ$:

$$l = \frac{25,2 * \sin 90 * \cos(4,6)}{2 - (1 - \cos 90) * \cos^2(4,6)} = 25 \text{ cm}$$

$$e = 2 * 25,2 \sqrt{\frac{(1 - \cos 90) \sin^2(4,6)}{2 - (1 - \cos 90) \cos^2(4,6)}} = 4 \text{ cm}$$

$$b = \frac{(2 - (1 - \cos 90) \cos^2(4,55))}{\sin 90 \cos(4,55)} 25 = 25,2 \text{ cm}$$

$$h = \sin(4,55) * 140 = 11,1 \text{ cm}$$

$$\beta = 2 \cos^{-1} \left(\frac{25 * \tan(4,55)}{2} \right) = 9,1^\circ$$

$$c = \sqrt{2 * [(25,2)^2 + [(135 - 25) * \cos(4,6)]^2]} = 159,1 \text{ cm}$$

Formulas are applied to different engagement lengths from 1 cm to 135 cm to see the change in Ψ , b , h , β , and c . θ value is taken from the DH parameters to see the relation with the other parameters.

Table 4: Parameter changes with the variable engagement length

L ₁	L ₂	L	n	λ	l=λ/2	Ψ (degrees)	b	h	β (degrees)	c	θ (degrees)
135	5	140	4	1	0.5	71.0	2.9	132.4	86.8	62.0	26.6
135	5	140	4	2	1	56.9	3.1	117.3	79.9	103.5	45.5
135	5	140	4	3	1.5	47.1	3.4	102.5	72.4	128.6	57.6
135	5	140	4	6	3	30.7	4.4	71.5	54.1	160.6	74.9
135	5	140	4	7	3.5	27.4	4.8	64.5	49.5	165.1	77.7
135	5	140	4	9	4.5	22.5	5.6	53.7	41.9	170.6	81.6
135	5	140	4	10	5	20.7	6.0	49.4	38.9	172.1	82.8
135	5	140	4	11	5.5	19.1	6.4	45.7	36.2	173.2	83.9
135	5	140	4	12	6	17.7	6.9	42.5	33.8	174.0	84.7
135	5	140	4	15	7.5	14.5	8.2	35.1	28.1	174.9	86.4
135	5	140	4	20	10	11.1	10.6	27.0	21.8	174.0	87.9
135	5	140	4	25	12.5	9.0	13.0	21.9	17.7	172.0	88.6
135	5	140	4	30	15	7.5	15.4	18.3	14.9	169.6	89.0
135	5	140	4	40	20	5.7	20.3	13.9	11.3	164.3	89.4
135	5	140	4	50	25	4.6	25.2	11.1	9.1	159.0	89.6
135	5	140	4	70	35	3.3	35.2	8.0	6.5	149.6	89.8
135	5	140	4	90	45	2.5	45.1	6.2	5.1	142.2	89.9
135	5	140	4	110	55	2.1	55.1	5.1	4.2	137.2	89.9
135	5	140	4	135	67.5	1.7	67.6	4.1	3.4	134.9	89.9

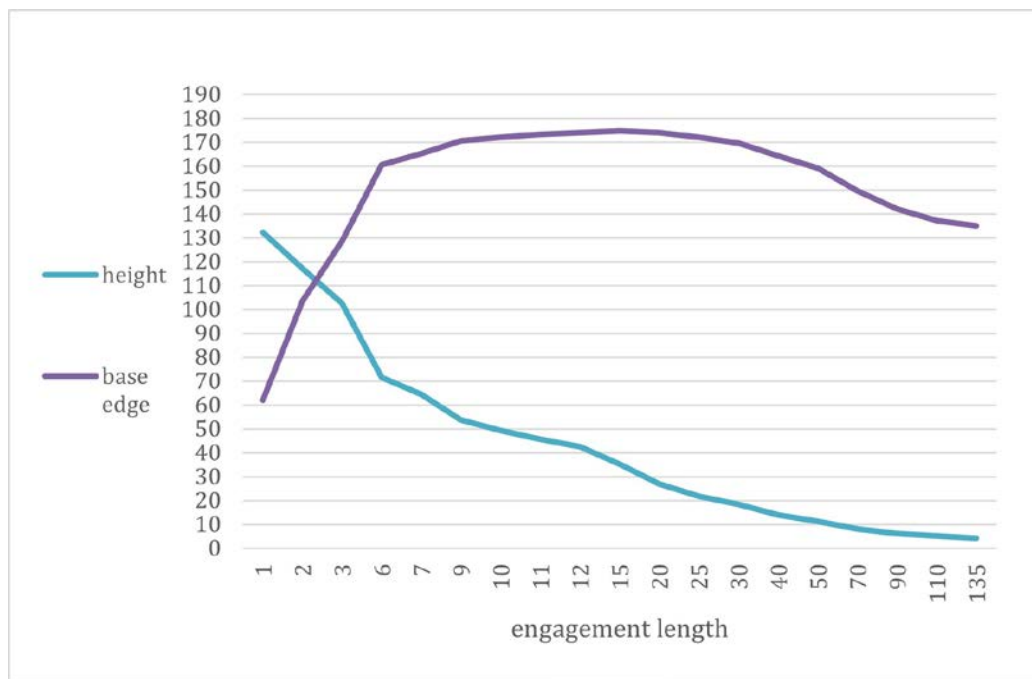


Figure 9: The influence of engagement length on fan height and base edge

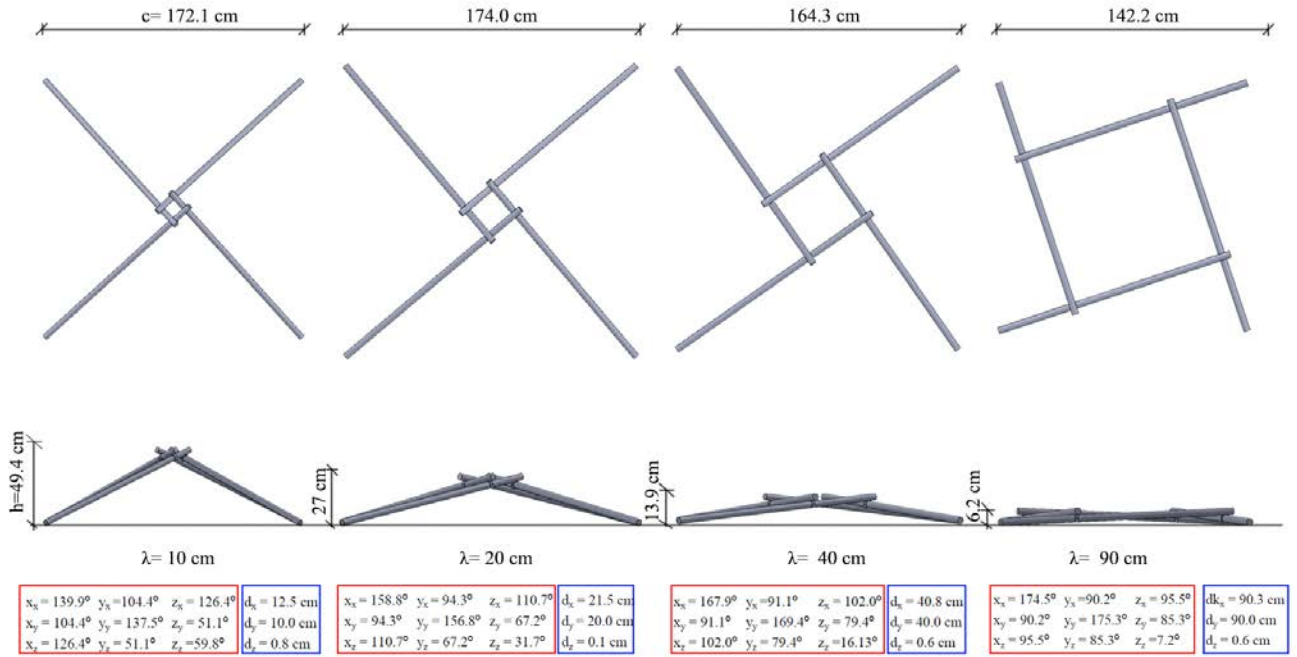


Figure 10: The influence of engagement length on fan height and base edge

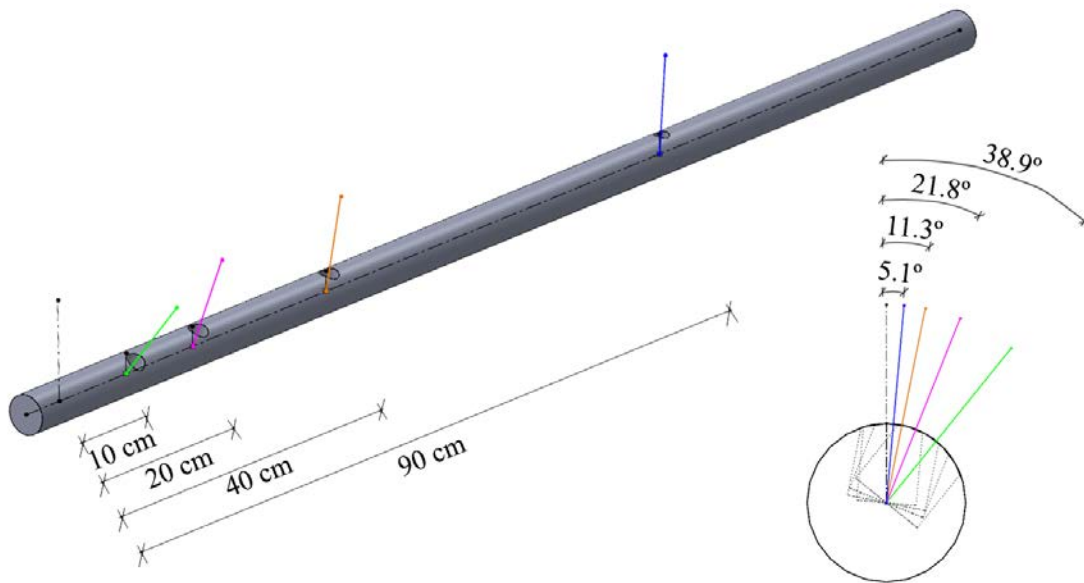


Figure 11: Nexor with several holes for different engagement lengths

Nexor length, eccentricity, and the number of nexors are kept constant while engagement length is increased. It is known from the previous studies that RF gets lower as the engagement length increases. As a result, it can be guessed that Ψ gets smaller as the RF gets closer to the ground. As engagement length increases, the engagement window gets bigger, which means b increases. The twist angle β takes smaller values as the frame gets

closer to the ground. θ takes bigger values, and its value reaches almost 90° . All the parameters monotonically increase or decrease except one. While the base edge is expected to have a maximum value with minimum height, it has the maximum amount when $\lambda = 15$ cm, and then it starts to decrease (Table 4). The graph in Figure 9 shows how the height and base edge change with the increasing engagement length.

With this study, it is possible to obtain different assemblies that can be demountable. When the required nexor length for the desired height or base edge is found, the position and orientation of the joints connecting the nexors is determined (Figure 10). Thus, by drilling holes on the nexor for the joints (Figure 11), RFs with different heights, base edges and engagement lengths that can be disassembled and reassembled as needed.

3. CONCLUSION

In this research, the RF fan is modeled as a kinematic chain with DH parameters for the given eccentricity and engagement length. Homogenous transformation matrices are obtained by using the model and parameters. Thus, it is possible to find the position and orientation of a coordinate frame from these transformation matrices. These position and orientation data are necessary to solve the constructive details when designing demountable RFs. Then, relationships among design parameters are presented to find unknown parameters such as twist angle, the angle between nexors and the horizontal plane, the height of the mechanism, the distance between nexor midpoint and structure's center, and the base edge. Their relations with the increasing engagement length are shown in a table. It is not possible to specify maximum height and maximum base edge independently at the same time with the same four nexors. In order to reach the optimum result, the designer can follow the method suggested in this study or make use of the graph in Figure 9, where the same nexor dimensions are used in the fan.

To conclude, one can find the values of the necessary parameters to obtain different assemblies by demounting the RF and remounting it again using alternative holes on the nexors.

For future work, the goal is to obtain kinetic RFs. For these RFs, the mathematical model proposed in this study is needed. As a preliminary research, "Mobility Analysis of Reciprocal Frames" study was carried out at the International Symposium of Architecture, Technology and Innovation (ATI) [10].

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