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A three dimensional dam break flow: Small time behavior

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ABSTRACT

Small time behavior of gravity driven free surface flows resulting from the collapse of a cavity is studied. Initially there is a rigid vertical cylinder of circular cross section starting from the free surface of a liquid and ending at the rigid bottom. The cylinder disappears suddenly and gravity driven flow of the fluid starts. The flow in early stage is described by the potential theory. Attention is paid to the singular behavior of the velocity field at the intersection line between the bottom and the free surface of the cavity. The leading order linear problem is solved by the Fourier series method. The flow velocity is log-singular at the intersection line. In the limiting case where the radius and the center of the cavity approach infinity, the problem is reduced to the classical two dimensional dam break problem where the fluid is initially on one side of a vertical wall (dry bed case). The flow resulting from cavity collapse is a three dimensional dam break flow. It is concluded that the three dimensional effects are important when the radius of the cavity is small compared with its depth and that the local flow near the intersection line of the cavity is governed only by the hydrostatic pressure.

1. Introduction

An intriguing mathematical and fluid mechanical problem is that of the collapsing of a vertical cylindrical cavity under pressure from the surrounding fluid which is of constant depth and of infinite extent horizontally. The cavity covers a region starting from the free surface of the fluid and ending at the rigid bed. Examples of cavity formation are provided by the water entry of projectiles (Truscott et al., 2014), the water exit of underwater vehicles and the underwater explosions.

The flow resulting from the cavity collapse, which will be termed as "cavity collapse flow" for the rest of the paper, can be thought of as the three dimensional dam break flow. When the radius of the cavity is large compared with the fluid depth, the cavity collapse flow resembles the two dimensional dam break flow. This provides a useful check on the validity of the calculations of cavity collapse flow. On the other hand the cavity collapse flow will shed a light on the three dimensional effects of the dam break flow.

A related problem, the dispersion of a column of fluid supported on a rigid horizontal plane under the influence of gravity, was studied by Penney et al. (1952). Short time behavior of the flow is analyzed for a column of semicircular cross section by calculating the initial accelerations at the boundary of the column. Although it is a different problem to the one in this paper, there are mathematical and physical similarities. In both cases the flow at initial stages is treated as potential flow and the solution is sought by Fourier series expansion. Also the conditions of equality of pressure and normal velocity along the interface are the same for both problems.

The small time asymptotic solution can explain the structure of the initial flow, which could be singular and difficult to be described by pure numerical means which work well for smooth, "well behaved" flows. It is mentioned by Lobovský et al. (2014) that in the initial stage of dam break flow there are quantitative differences in the vertical free surface shapes predicted by various numerical methods, theoretical predictions and experimental measurements. Our argument in this paper is that the initial transient behavior near the contact points is better described by asymptotic analysis rather than pure numerical calculations.

In this paper, we investigate the small time behaviour of gravity driven free surface flows of a fluid resulting from the collapse of a cavity which, to the best of our knowledge, has not been studied before. Viscous effects are assumed to be not important for small times and ignored. A justification for that is given at Section 5. It is shown that in the leading order solution the flow velocity is singular at "the intersection line between the bottom and the free surface of the cavity" (which will be termed as "the intersection line of the cavity" for the rest of the paper) and a jet formation similar to the one in the classical dam break problem (Korobkin and Yilmaz, 2009) is expected there. A more difficult

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two dimensional problem involving two different fluids of different depths is studied by Yilmaz et al. (2013) where first order analysis and a singularity investigation at the corner points are carried out. In that problem, known as the wet bed case dam break flow, there are two singular points at the interface: a logarithmic singularity at the bottom point and a power singularity at the top of the interface, where the interface meets the free surface of one of the fluids.

The present analysis is a natural extension of the study of the classical two dimensional dam break problem where there is a fluid only on one side of the wall. In this paper, we consider the three dimensional effects of the dam break flow in the form of a cavity collapse flow. It is found that the larger the radius of the cavity compared with its height, the smaller the three dimensional effects. There is an interesting limiting case of the cylindrical cavity collapse flow: when the radius and the center of the cavity approach infinity, the problem is reduced to the classical two dimensional dam break flow. The other limiting case involves the local flow near the intersection line of the cavity where a singularity of the flow within the mathematical model considered is expected.

Another relevant investigation to the present study is the flow generated when a wall accelerates into a fluid of finite depth with a free surface (King and Needham, 1994). An analytical solution was found for the flow field in which the jet emerges. The jet structure studied in that paper is expected to be similar to the one studied in this paper and in Korobkin and Yilmaz (2009).

In Section 2 the problem is formulated assuming that the fluid is inviscid and incompressible and the leading order solution is derived in Section 3 by a Fourier series expansion. The singularity analysis and the three dimensional effects are discussed in Section 4. Neglect of viscosity is justified a posteriori in Section 5. Finally some conclusions are drawn in Section 6.

2. Formulation of the problem

The unsteady problem of gravity driven free surface flow generated when a vertical cylinder of circular cross section and of depth same as the liquid depth is suddenly removed from the fluid which surrounds it is considered. Eulerian variables are employed to find out about the behaviour of the flow at the initial stages. Initially the liquid is at rest and occupies the region $r' > a, 0 \leq \theta < 2\pi, -H \leq z' \leq 0$, where H is the liquid depth, a is the radius of the cylinder and a prime stands for dimensional variables. The coordinate axes are placed at the free surface with z' axis pointing upward and (r', θ, z') is the cylindrical coordinate system (See Fig. 1). The liquid is assumed inviscid and incompressible. The resulting flow is potential, three dimensional and axisymmetric.

The velocity potential $\phi'(r', z', t')$ satisfies the Laplace's equation in the fluid domain

$$\Delta\phi' = 0 \text{ in } \Omega'(t'), \tag{1}$$

the dynamic free surface condition, that the fluid pressure p' is atmospheric at the free surface,

$$p' = 0 \text{ in } FS'(t') \tag{2}$$

and kinematic free surface conditions, which implies that the fluid particles initially at the free surface remain there,

$$\phi'_{r'} - \zeta'_{z'}\phi'_{z'} - \zeta'_{t'} = 0 \text{ at } FS'_v(t'), \tag{3}$$

$$\phi'_{z'} - \eta'_{r'}\phi'_{r'} - \eta'_{t'} = 0 \text{ at } FS'_h(t'), \tag{4}$$

the slip boundary condition at the rigid bottom,

$$\phi'_{z'} = 0 \text{ at } z' = -H, \tag{5}$$

the radiation condition at infinity,

$$\phi' \to 0 \text{ as } r' \to \infty, \tag{6}$$

and the initial conditions,

$$\phi'(r', z', 0) = 0, \ \eta'(r', 0) = 0, \ \zeta'(z', 0) = 0, \tag{7}$$

$$p'(r', z', 0) = -\rho_0 g z',$$

where $\Omega'(t')$ is the flow region described by

$$\begin{aligned} \Omega'(r', z', t') \quad = \quad & \{(r', z', t') | r' > a + \zeta'(z', t'), \\ & -H \leq z' \leq \eta'(r', t'), t' > 0\}, \end{aligned}$$

$FS'(t')$ is the free surface of the region which is the union of the horizontal free surface $FS'_h(t')$ and the vertical free surface of the cavity $FS'_v(t')$,

$$FS'(t') = FS'_h(t') \cup FS'_v(t'),$$

$$FS'_h(t') = \{(r', z', t') | z' = \eta'(r', t'), r' > a + \zeta'(z', t')\},$$

$$FS'_v(t') = \{(r', z', t') | r' = a + \zeta'(z', t'), -H < z' < \eta'(a, t')\},$$

and the fluid pressure $p'(r', z', t')$ is related to the velocity potential by Bernoulli's equation for unsteady irrotational flow,

$$\frac{p'}{\rho_0} + \frac{\partial\phi'}{\partial t'} + \frac{1}{2}|\nabla\phi'|^2 + g z' = 0,$$

in the fluid domain, g is gravitational acceleration and ρ_0 is the fluid density.

Non dimensional unprimed variables are introduced as follows:

$$r' = Hr, \ z' = Hz, \ t' = Tt, \ \phi' = gHT\phi, \ p' = \rho_0 g H p$$

$$\alpha = a/H, \ \epsilon = gT^2/H,$$

where T is a suitable time scale. The boundary value problem (1)–(7) is rewritten in the non dimensional variables,

$$\Delta\phi = 0 \text{ in } \Omega(t), \tag{8}$$

$$p = 0, \text{ in } FS(t) \tag{9}$$

$$\phi_r - \epsilon\zeta_z\phi_z - \zeta_t = 0 \text{ at } FS_v(t), \tag{10}$$

$$\phi_z - \epsilon\eta_r\phi_r - \eta_t = 0 \text{ at } FS_h(t), \tag{11}$$

$$\phi_z = 0 \text{ at } z = -1, \tag{12}$$

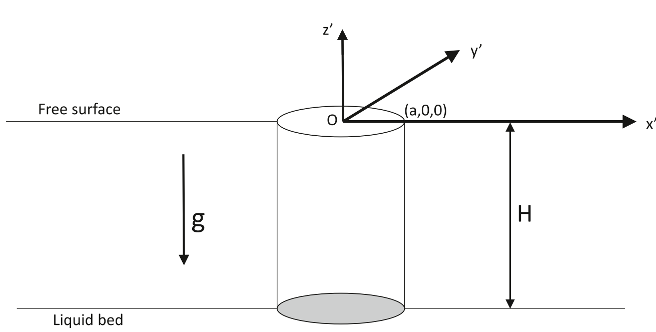

Fig. 1. Flow region of the first problem at initial time $t' = 0$.

$$\phi \to 0 \text{ as } r \to \infty, \tag{13}$$

$$\phi(r,z,0)=0,\ \eta(r,0)=0,\ \zeta(z,0)=0,\ p(r,z,0)=-z, \tag{14}$$

where ϵ is a small number representing the initial stages of the flow,

$$\begin{aligned}
\Omega(r,z,t) &= \{(r,z,t)|r>\alpha+\epsilon\zeta(z,t),\\
&\quad -1\le z\le \epsilon\eta(r,t), t>0\},\\
FS(t) &= FS_h(t)\cup FS_v(t),\\
FS_h(t) &= \{(r,z,t)|z=\epsilon\eta(r,t), r>\alpha+\epsilon\zeta(z,t),\},\\
FS_v(t) &= \{(r,z,t)|r=\alpha+\epsilon\zeta(z,t), -1<z<\epsilon\eta(\alpha,t),\},
\end{aligned}$$

and the unsteady Bernoulli equation in the fluid domain becomes,

$$p+\frac{\partial\phi}{\partial t}+\frac{1}{2}\epsilon|\nabla\phi|^2+z=0.$$

The solution to the problem (8)–(14) is sought in the form, as $\epsilon\to0$,

$$\phi(r,z,t,\epsilon)=\phi_0(r,z,t)+\epsilon\phi_1(r,z,t)+O(\epsilon^2), \tag{15}$$

$$\eta(r,t,\epsilon)=\eta_0(r,t)+\epsilon\eta_1(r,t)+O(\epsilon^2),$$

$$\zeta(z,t,\epsilon)=\zeta_0(z,t)+\epsilon\zeta_1(z,t)+O(\epsilon^2).$$

3. The leading order problem

By substituting the expansions (15) in the boundary value problem (8)–(14), as $\epsilon\to0$, the leading order problem is obtained,

$$\Delta\phi_0=0,\ (r>\alpha, -1<z<0), \tag{16}$$

$$\phi_0=-zt,\ \phi_{0,r}=\zeta_{0,t},\ (r=\alpha, -1<z<0,),$$

$$\phi_0=0,\ \phi_{0,z}=\eta_{0,t},\ (z=0, r>\alpha),$$

$$\phi_{0,z}=0,\ (z=-1, r>\alpha),$$

$$\phi_0\to0,\ (r\to\infty),$$

$$\phi_0(r,z,0)=0,\ \eta_0(r,0)=0,\ \zeta_0(z,0)=0,$$

Notice that along the vertical wall of cavity the vertical velocity is $\frac{\partial\phi_0}{\partial z}=-t$ (see the second equation of (16)) which is true, in particular, approaching the intersection line of the cavity along the lateral free surface, as $z\to-1^+$. However from the fourth equation of (16) we find that the vertical velocity is zero at the rigid bottom, $z=-1$, especially when approaching the intersection line of the cavity: $\partial\phi_0/\partial z\to0$ as $r\to\alpha^+$. That means that the second and fourth boundary conditions of (16) do not match each other at the intersection line of the cavity $r=\alpha, z=-1$, which may cause singularities in the flow field.

The solution to the problem (16) is found by separation of variables,

$$\phi_0(r,z,t)=t\sum_{n=0}^{\infty}A_n\sin(\sigma_n z)K_0(\sigma_n r), \tag{17}$$

where

$$A_n=-2\frac{(-1)^n}{\sigma_n^2}\frac{1}{K_0(\sigma_n\alpha)},\ \sigma_n=\frac{\pi}{2}(2n+1),\ n=0,1,\cdots,$$

and $K_0(x)$ is the modified Bessel function of second kind of order zero.

There are two interesting limiting cases of the problem. In the first one when both the radius and the center of the cavity approach infinity, the two dimensional classical dam break problem is expected to be recovered. As $\alpha\to\infty$ and $r\to\infty$ the modified Bessel function of the second kind of order zero behaves like (see the equation 9.7.2 in Abramowitz and Stegun (1970)),

$$K_0(\sigma_n\alpha)\sim\sqrt{\frac{\pi}{2\sigma_n\alpha}}\exp(-\sigma_n\alpha)\left(1-\frac{1}{8\sigma_n\alpha}+\frac{9}{128}\frac{1}{(\sigma_n\alpha)^2}+O\left((\sigma_n\alpha)^{-3}\right)\right),$$

$$K_0(\sigma_n r)\sim\sqrt{\frac{\pi}{2\sigma_n r}}\exp(-\sigma_n r)\left(1-\frac{1}{8\sigma_n r}+\frac{9}{128}\frac{1}{(\sigma_n r)^2}+O\left((\sigma_n r)^{-3}\right)\right),$$

which implies that the leading order velocity potential has the following limiting behavior

$$\phi_0\sim-2t\sum_{n=0}^{\infty}\frac{(-1)^n}{\sigma_n^2}\sin(\sigma_n z)\exp(-\sigma_n x)\left(1-\frac{1}{2}\frac{x}{\alpha}+\frac{1}{8\sigma_n}\frac{x}{\alpha^2}+O(\alpha^{-3})\right), \tag{18}$$

where $x=r-\alpha$. As $\alpha\to\infty$ Eq. (18) is exactly the same as the Eq. (12) of Korobkin and Yilmaz (2009) which is the leading order outer solution for the classical two dimensional dam break problem.

The behavior of the fluid flow close to the intersection line of the cavity is analyzed in the next section.

4. Singularity analysis and three dimensional effects

The leading order fluid velocity in the radial direction at the cavity wall, $r=\alpha$, is calculated using (17),

$$\frac{\partial\phi_0}{\partial r}(\alpha,z,t)=2t\sum_{n=0}^{\infty}\frac{(-1)^n}{\sigma_n}\frac{K_1(\sigma_n\alpha)}{K_0(\sigma_n\alpha)}\sin(\sigma_n z), \tag{19}$$

where $K_1(x)$ is the modified Bessel function of second kind of order one. To utilize the large argument behavior of the modified Bessel functions the infinite sum in (19), S, is split into two parts,

$$S=S_1+S_2=\sum_{n=0}^{N-1}\frac{(-1)^n}{\sigma_n}\frac{K_1(\sigma_n\alpha)}{K_0(\sigma_n\alpha)}\sin(\sigma_n z)+\sum_{n=N}^{\infty}\frac{(-1)^n}{\sigma_n}\frac{K_1(\sigma_n\alpha)}{K_0(\sigma_n\alpha)}\sin(\sigma_n z), \tag{20}$$

where N is a large number so that the large argument behavior of modified Bessel functions could be used in the second sum in (20),

$$\begin{aligned}
\frac{K_1(\sigma_n\alpha)}{K_0(\sigma_n\alpha)}&\sim\frac{1+\frac{3}{8\sigma_n\alpha}-\frac{15}{128}\frac{1}{(\sigma_n\alpha)^2}+O\left((\sigma_n\alpha)^{-3}\right)}{1-\frac{1}{8\alpha\sigma_n}+\frac{9}{128}\frac{1}{(\sigma_n\alpha)^2}+O\left((\sigma_n\alpha)^{-3}\right)}\\
&\sim1+\frac{1}{2}\frac{1}{\sigma_n\alpha}-\frac{1}{8}\frac{1}{(\sigma_n\alpha)^2}+O\left((\sigma_n\alpha)^{-3}\right).
\end{aligned}$$

Hence the second sum in (20), S_2, becomes,

$$S_2\approx S_3-S_4+\frac{1}{2\alpha}S_5-\frac{1}{2\alpha}S_6-\frac{1}{8\alpha^2}S_7+\frac{1}{8\alpha^2}S_8 \tag{21}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{\sigma_n}\sin(\sigma_n z)-\sum_{n=0}^{N-1}\frac{(-1)^n}{\sigma_n}\sin(\sigma_n z)$$

$$+\frac{1}{2\alpha}\sum_{n=0}^{\infty}\frac{(-1)^n}{\sigma_n^2}\sin(\sigma_n z)-\frac{1}{2\alpha}\sum_{n=0}^{N-1}\frac{(-1)^n}{\sigma_n^2}\sin(\sigma_n z)$$

$$-\frac{1}{8\alpha^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{\sigma_n^3}\sin(\sigma_n z)+\frac{1}{8\alpha^2}\sum_{n=0}^{N-1}\frac{(-1)^n}{\sigma_n^3}\sin(\sigma_n z),$$

where the first and the third sums on the right hand side of (21), S_3 and S_5 respectively, are summed exactly, (see the equations 1.442.3 and 1.444.5 in Gradshteyn and Ryzhik (2007))

$$S_3=\sum_{n=0}^{\infty}\frac{(-1)^n}{\sigma_n}\sin(\sigma_n z)=\frac{1}{\pi}\log\tan\left(\frac{\pi}{4}(1+z)\right), \tag{22}$$

$$S_5 = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sigma_n^2}\sin(\sigma_n z) = \frac{z}{2}. \tag{23}$$

On the other hand there are four finite sums to be considered: S_1, S_4, S_6 and S_8. The sums S_1 and S_4 can be put together as

$$S_1 - S_4 = \sum_{n=0}^{N-1} \frac{(-1)^n}{\sigma_n}\sin(\sigma_n z)\left(\frac{K_1(\sigma_n\alpha)}{K_0(\sigma_n\alpha)} - 1\right). \tag{24}$$

The finite sum S_6 is a function of z only,

$$S_6 = \sum_{n=0}^{N-1} \frac{(-1)^n}{\sigma_n^2}\sin(\sigma_n z). \tag{25}$$

Note that both of the finite sums in (24) and (25) are positive and in the sum S they cancel out each other for $-1 \le z \le 0$; $S_1 - S_4 - S_6/(2\alpha) \sim 0$. This is especially true near the intersection line at $z = -1$. Finally the sum $S_7 - S_8$ is much smaller than the other sums considered and therefore ignored.

Substituting (22) and (23) in (21), then (21) in (20) and then (20) in (19) gives the behavior of the radial velocity along the lateral surface of the cavity ($r = \alpha$),

$$\frac{\partial\phi}{\partial r}(\alpha, z, t) \sim \frac{2t}{\pi}\log\left(\tan\frac{\pi}{4}(1+z)\right) + \frac{zt}{2\alpha} + O(\alpha^{-2}), \tag{26}$$

which implies a logarithmic singularity as $z \to -1$. Note that in the limiting case as $\alpha \to \infty$, the last term on the right hand side of (26) vanishes and the remaining logarithmic term is the same as the one for the horizontal velocity in the classical two dimensional dam break problem, $\partial\phi_0/\partial x$, near the bottom corner point where the vertical free surface meets the rigid bed (see Korobkin and Yilmaz, 2009). Therefore both the two and three dimensional problems have the same leading order local flow near the bottom corner point and the intersection line of the cavity (the contact line) respectively. The form of the radial velocity close to the contact line (26) implies that the local flow is governed only by the hydrostatic pressure.

The shape of the vertical free surface is obtained from (19) and the second equation in (16),

$$\zeta_0(z,t) = t^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{\sigma_n}\frac{K_1(\sigma_n\alpha)}{K_0(\sigma_n\alpha)}\sin(\sigma_n z), \quad -1 < z < 0. \tag{27}$$

The corresponding asymptotic formula for large α is obtained using (26),

$$\zeta_0(z,t) \sim \frac{t^2}{\pi}\log\left(\tan\frac{\pi}{4}(1+z)\right) + \frac{zt^2}{4\alpha} + O(\alpha^{-2}). \tag{28}$$

The vertical free surface shapes, (27) and (28), are plotted in Fig. 2 for various values of α together with the classical two dimensional result (Korobkin and Yilmaz, 2009). Near the intersection line $z = -1$, regardless of the value of α, the free surface shape is governed by the logarithmic term in (28) which is the classical dam break result (Korobkin and Yilmaz, 2009). However away from the intersection line the asymptotic formula (28) models the three dimensional results much better than the two dimensional formula.

The factor α in (27) represents the three dimensional effects of the dam break flow. The vertical free surface (27) is plotted for various values of α in Fig. 3, together with the classical two dimensional dam break flow result. As it was shown in the analysis leading to (18), with increasing values of the radius of the cavity, three dimensional effects become unimportant. A similar observation is made for the initially horizontal free surface (See Fig. 4),

$$\eta_0(r,t) = -t^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{\sigma_n}\frac{K_0(\sigma_n r)}{K_0(\sigma_n\alpha)}, \quad r > \alpha. \tag{28}$$

It is observed from the free surface shapes in Figs. 3 and 4 that with smaller radius of cavity three dimensional effects become more pronounced. The logarithmic singularity at $z = -1$ is clearly visible in Fig. 3

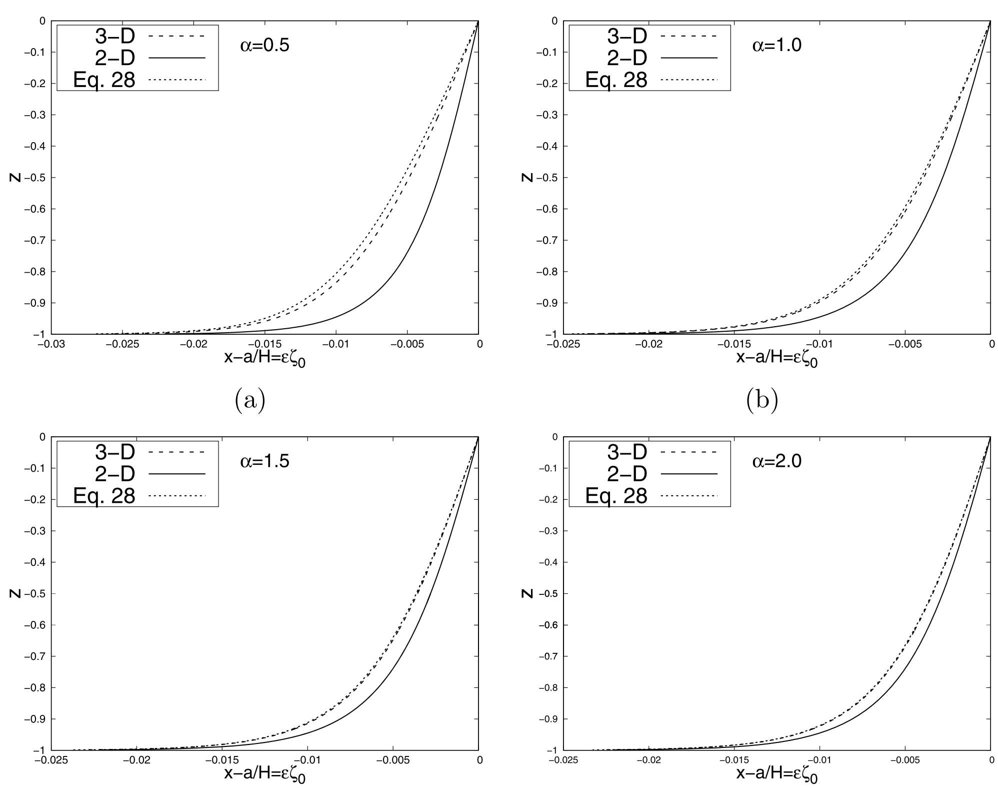

Fig. 2. Shape of the initially vertical free surface for various values of α for small times, $t = 1, \epsilon = 0.01$.

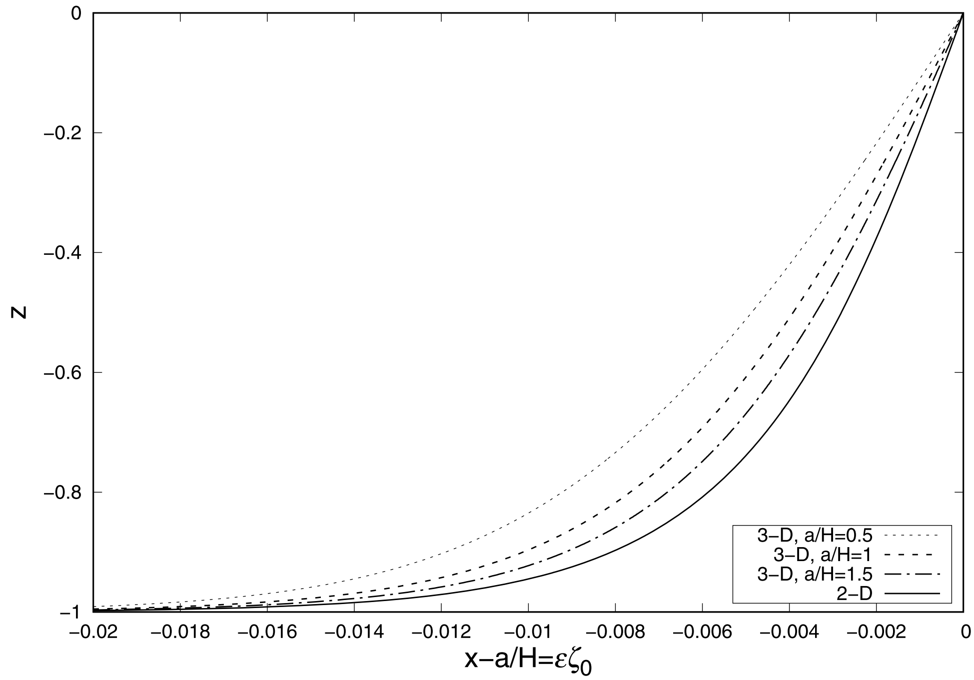

Fig. 3. Shape of the initially vertical free surface for small times, $t = 1, \epsilon = 0.01$.

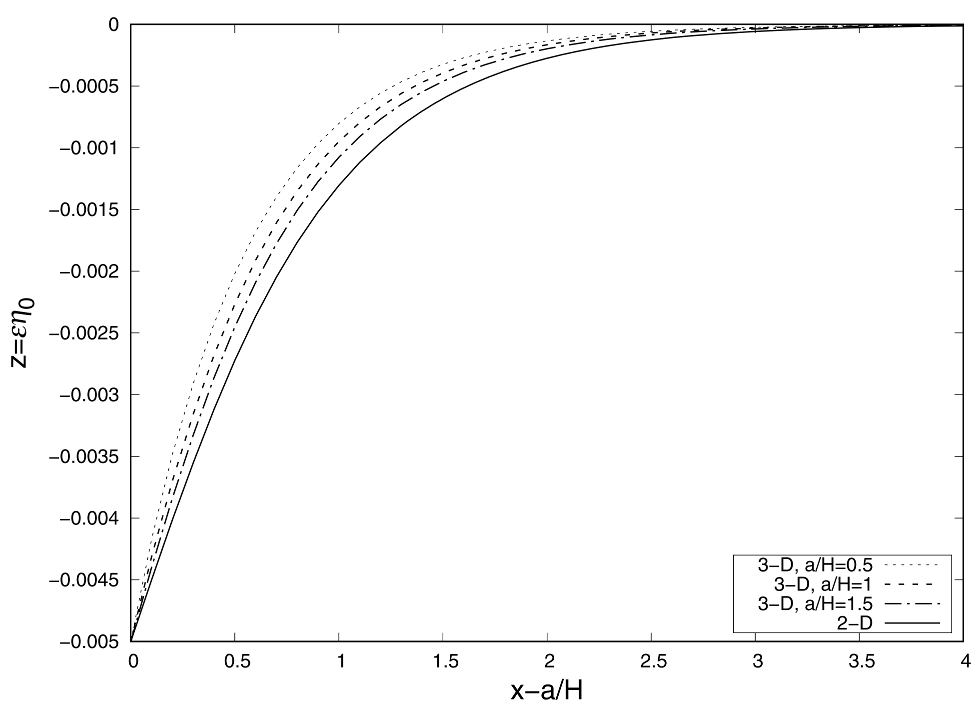

Fig. 4. Shape of the initially horizontal free surface for small times, $t = 1, \epsilon = 0.01$.

and the solution derived in this section should be termed as the leading order "outer" solution since the mathematical model based on the power series in time breaks down near the contact line at the bottom. The outer solution should be corrected by an "inner" solution near the contact line.

5. Nonlinear analysis and viscous effects

First we try to determine the nature of the singularity near the intersection line of the cavity. Due to the axisymmetrical nature of the problem, the singularity analysis could be carried out at each point P on the intersection line of the cavity in a plane passing through the line OP and perpendicular to the unit vector $\mathbf{e}_{y_1}$. Using the polar coordinates r_1 and θ_1 centered at P (see Fig. 5),

$$x_1 = r_1 \cos\theta_1, \; z_1 = r_1 \sin\theta_1,$$

the problem becomes similar to the one considered in Korobkin and Yilmaz (2009). An inner solution near the bottom point P is derived in Korobkin and Yilmaz (2009) and matched with the outer solution using the matching principle of Van Dyke (1964). Following (Korobkin and Yilmaz, 2009), the size of the inner region near the point P at the

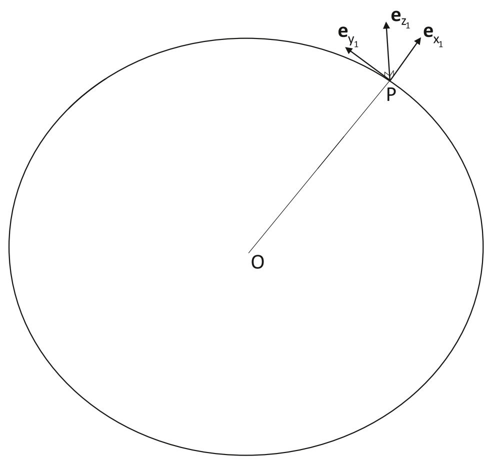

Fig. 5. Coordinates at point P of the intersection line of the cavity.

intersection line of the cavity is given by $-\epsilon\log\epsilon$. We will not pursue inner region calculations in this paper since they would be similar to the ones in Korobkin and Yilmaz (2009).

The neglect of the viscous effects from the present analysis could be justified by comparing the order of magnitude of the acceleration and the viscous terms in the momentum equation. In the main flow region, the order of the acceleration term is $O(g)$ and the order of the viscous term is $O(\nu\sqrt{\epsilon g}/H^{3/2})$. The ratio of the two terms for fresh water is

$$\frac{|\nu\nabla^2\mathbf{u}|}{|\mathbf{u}_t|} = 3.2{\cdot}10^{-7}\frac{\sqrt{\epsilon}}{H^{3/2}}, \tag{29}$$

where for kinematic viscosity $\nu = 1.0034{\cdot}10^{-6}m^2/s$ Procedures (2011) is used. The ratio of the viscous and acceleration terms is quite small and it is safe to ignore the viscous effects in the main flow region. However, close to the intersection line of the cavity the effect of viscosity could be larger than that in the main flow region. Near the intersection line of the cavity $(z = -1, r = a/H)$, the order of the acceleration term is $O(g\ /a)$ and the order of the viscous term is $O(\nu\sqrt{\epsilon g}/(H^{3/2}a^3)$ and the ratio of the two terms for fresh water is

$$\frac{|\nu\nabla^2\mathbf{u}|}{|\mathbf{u}_t|} = 3.2{\cdot}10^{-7}\frac{\epsilon^{-3/2}}{(\log^2\epsilon)H^{3/2}}. \tag{30}$$

Notice that, by comparing the terms in (29) and (30), the viscous effects for the intersection line of the cavity are considerably larger than those in the main flow region. This is especially true when the size of the inner region is too small, that is for very small times. For example, when $\epsilon = 10^{-4}$ then the viscosity is as important as the inertia. Hence, unless we are dealing with very small times (that is when $\epsilon = O(10^{-4})$ or smaller), the viscosity is negligible near the intersection line of the cavity.

6. Conclusions

The collapse of a cavity under pressure from the surrounding fluid with free surface is investigated for small times. The cavity extends all the way from the free surface to the water bed. The problem can be considered as the three dimensional classical dam break problem. In the limiting case when the radius and the center of the cavity approach infinity, the two dimensional dam break solution is recovered. Comparing the free surface shapes with those of the two dimensional classical dam break flow, it is concluded that three dimensional effects are important when the ratio of the radius of the cavity and the water depth is small. At the intersection line of the cavity (the contact line), the flow velocity is log singular which is similar to the corner point singularity of the two dimensional classical dam break problem. It is observed that the local flow near the contact line is governed only by the hydrostatic pressure.

The three dimensional dam break problem considered in this paper is a generalization of the corresponding two dimensional dam break problem. Three dimensional effects become more important when the radius of the cavity becomes smaller compared with the height of the cavity. The neglect of viscosity is justified a posteriori.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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