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Controlling the Rotor Angle Stability of Single Machine Infinite Bus System in the Presence of Wiener and Alpha-Stable Levy Type Power Fluctuations

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The integration of renewable energy sources into the power systems and the growth of electricity consumption leads to a considerable increase in the power fluctuations. In the first part of this study, the control of the rotor angle stability of single machine infinite bus system in the presence of Wiener type power fluctuations has been achieved by minimizing the corresponding stochastic sensitivity function. In the second part, the power fluctuations have been modeled by alpha-stable Levy processes and since stochastic sensitivity function is not available for alpha-stable Levy processes, then the control of the rotor angle stability has been numerically achieved by minimizing the corresponding rotor angle dispersion for the first time in the literature.

Keywords: Alpha-stable Levy process; Wiener process; power fluctuations; rotor angle stability; single machine infinite bus system.

1. Introduction

The rapidly increasing use of high-variable energy resources and the acceleration in global energy consumption can cause fluctuations which eventually result in loss of synchronism in power systems. In 2006, Continential Europe has experienced a cascading blackout which has been triggered by the high wind generation in Germany [1]. The major blackouts that occurred such as in Italian power system in 2003 [2] and in Turkish power system in 2015 [3] have been caused by the loss of synchronism following a transmission line trip. To maintain the synchronism of the interconnected machines therefore become more important [4]. Rotor angle stability deals with the ability to keep synchronism after a disturbance [5] and the single

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machine infinite bus system (SMIB) is convenient and practical for the rotor angle stability analysis. The stability of SMIB system has been investigated in [6] in which the power fluctuations have been modeled by Gaussian white noise. In [7], the evolution of the probability density function for SMIB has been modeled by Fokker–Kolmogorov–Planck (FKP) equation and the impact of Wiener type load perturbations on the rotor angle stability has been analyzed. In [8], the effects of stochastic excitations in SMIB system have been studied by the p-moment stability of rotor angle. In [9], the wind power and the load uncertainty in SMIB system have been modeled by Wiener processes and an analytical method has been proposed to estimate the probability of transient stability under stochastic disturbances.

Since the analysis of nonlinear stochastic systems by using FKP equation is a difficult task, the quasi-potential function which provides the minimum action related to the steady-state probability distribution have been developed in [10]. The quasi-potential function can be obtained by solving the corresponding Hamilton–Jacobi equation. A quadratic approximation of the quasi-potential have been proposed in [11]. Based on this quadratic approximation, stochastic sensitivity analysis has been presented in [12, 13].

The novelty of this paper lies on two points: In the first part of this paper, the design of controller to achieve the rotor angle stability of SMIB system in the presence of the Wiener type power fluctuations has been built on stochastic sensitivity function. In the second part, the power fluctuations have been modeled by alpha-stable (α-stable) Lévy processes and the control of the rotor angle stability has been numerically achieved in the presence of α-stable Lévy type power fluctuations for the first time in the literature. The motivation for our assumption comes from the modeling of the electricity prices by α-stable Lévy processes in [14] and α-stable periodic autoregressive model for the electricity market in [15]. Since the load has been considered as one of the main factors in determining electricity prices because the sudden demand or supply changes cause sharp spikes in electricity prices then the impulsive and asymmetric load fluctuations have been modeled by α-stable Lévy processes. The nature of observed frequency fluctuations which has been characterized in the form of Lévy-stable distribution in [16] also endorses our α-stable Lévy type power fluctuations modeling.

2. Controlling the Rotor Angle Stability of SMIB System in the Presence of Wiener Type Power Fluctuations

The deterministic swing equations in [5] which govern the rotational nonlinear dynamics of the synchronous machine connected to an infinite bus are given as

$$\begin{aligned} \dot{\delta} &= \omega, \\ M\dot{\omega} &= -D\omega + P_m - P_e, \end{aligned} \tag{1}$$

where δ is the relative rotor angle of synchronous machine, ω is the rotor speed with respect to the synchronous reference, M and D are the moment of inertia and the damping constant, respectively. P_m is the mechanical input power and $P_e = P_{\max}\sin(\delta)$ is the electrical output power in which $P_{\max}$ represents the maximum power output of the synchronous machine. The model SMIB given in (1) is analogous to the Josephson junction and the classical driven pendulum in [17].

The equilibrium points of the SMIB system in (1) satisfy $\bar{\omega} = 0$ and $P_{\max}\sin\bar{\delta} = P_m$. Based on the linearization of the state equations at the equilibrium points, the state $[\bar{\delta}, \bar{\omega}] = [\sin^{-1}(P_m/P_{\max}), 0]$ is the stable equilibrium point (SEP) while the state $[\bar{\delta}_u, \bar{\omega}_u] = [\pi - \sin^{-1}(P_m/P_{\max}), 0]$ corresponds to the saddle point.

In the steady state, there is a balance between the mechanical input power and the electrical output power and the machine runs at a constant speed which leads to the constant relative rotor angle (i.e., at the equilibrium point $\dot{\delta} = 0$).

However, the random load changes, line tripping or loss of generator can cause the imbalance between the mechanical input power and the electrical output power. This imbalance may result in deviation from the rotational speed which leads to the loss of synchronism.

Modeling the imbalance between the mechanical input power and electrical output power as $P_L(t) = \epsilon\xi(t)$ in which ξ is the zero mean Gaussian noise with variance $E\{\xi(t)\xi(\tau)\} = \delta(t-\tau)$ and ϵ is the noise intensity then the dynamics of the stochastic SMIB system can be written as

$$\begin{bmatrix} \dot{\delta} \\ M\dot{\omega} \end{bmatrix} = \begin{bmatrix} w \\ -D\omega + P_m - P_e \end{bmatrix} + \epsilon \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi(t). \tag{2}$$

The trajectories of rotor angle and rotor speed converge either to the SEP or to the stable limit cycle (unique rotating orbit). The rotating orbit can lead to the instability of the sytem which is an undesired situation in power systems. In order to control the rotor angle stability of SMIB system in the presence of Wiener type power fluctuations, we have developed a feedback controller by using the stochastic sensitivity analysis introduced in [12, 13].

It is assumed that the state variables δ and ω are available for measurements and the control input $\mathbf{u}$ is synthesized by the state feedback controller with noisy observations $\mathbf{y}$

$$\begin{aligned} u &= \mathbf{K}(\mathbf{y} - \bar{\boldsymbol{x}}), \\ \mathbf{y}(t) &= \mathbf{x}(t) + \epsilon\mathbf{C}\boldsymbol{\eta}(t), \end{aligned} \tag{3}$$

where $\mathbf{x} = [\delta\ \omega]^T$ and $\bar{\mathbf{x}}$ is the equilibrium state of the deterministic system of (1). The feedback coefficient vector is $\mathbf{K} = [k_1\ k_2]$ with $k_1, k_2 \in \Re$, the intensity matrix of noisy measurement is $\mathbf{C} = \mathrm{diag}(c_1, c_2)$ with $c_1,\ c_2 \in \Re$ and $\boldsymbol{\eta}(t) \in R^2$ is zero mean δ-correlated white Gaussian noise which is uncorrelated with $\xi(t)$ and satisfies $E(\eta(t)\eta^T(\tau)) = \delta(t-\tau)I$ with the identity matrix I.

Considering the control input $\mathbf{u}$ in (3) and assuming the unit inertia constant $M = 1\ p.u.$ in (2), the controlled nonlinear stochastic SMIB system can be written as

$$\dot{\mathbf{x}} = f(\mathbf{x}) + \mathbf{g}u + \epsilon\mathbf{J}\xi(t), \tag{4}$$

where

$$\mathbf{x} = \begin{bmatrix} \delta \\ w \end{bmatrix}, \quad f(\mathbf{x}) = \begin{bmatrix} w \\ -Dw + P_m - P_{\max} \sin \delta \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Applying the procedure given in Appendix, the Hamilton–Jacobi equation for the matrix V can be obtained as

$$(F + GK)^T V + V(F + GK) + V(GK\Phi^T K^T G^T + S)V = 0, \tag{5}$$

where

$$F = \frac{\partial f}{\partial x}(\bar{x}), \quad G = g, \quad S = JJ^T, \quad \Phi = CC^T \tag{6}$$

and (5) has a unique stationary solution $W = V^{-1}$ satisfying the following matrix equation:

$$W(F + GK)^T + (F + GK)W + GK\Phi^T K^T G^T + S = 0, \tag{7}$$

where W is called as stochastic sensitivity matrix in [13] and it quantifies the deviation of the stochastic response of the nonlinear system from the equilibrium state. The unique solution of the (7) can be obtained from the following theorem in [13].

Theorem 1. *For an assigned positive definite stochastic sensitivity matrix W if there exist K such that the eigenvalues of the matrix $(F + GK)$ lie in the left half of the complex plane then an assigned W is attainable and satisfies the following conditions*:

$$P_2(FW + WF^T + S)P_2 = 0, \tag{8}$$

$$W\Phi^{-1}W - FW - WF^T - S \succeq 0, \tag{9}$$

where the projective matrix $P_2 = I - GG^+$, the superscript "+" denotes the pseudoinversion and $\succeq$ refers to the positive semi-definiteness.

The minimization of this deviation can be considered as a control problem which minimizes the stochastic sensitivity function quantified by the cost measure $J(W) = \mathrm{tr}(W^T Q)$ in which $Q \in R^{n\times n}$ is a positive definite symmetrix matrix. Using Theorem 1, the parametric description of the set of pairs (w_{11}, w_{22}) of the attainable stochastic sensitivity matrix W has been obtained as follows:

$$\begin{aligned} &w_{11}^2 w_{22}^2 + 2Dw_{11}^2 w_{22} c_2^2 - w_{11}^2 c_2^2 - P_{\max}^2 w_{11}^2 c_1^2 c_2^2 \\ &\quad - 2P_{\max}\cos(\bar{\delta}) w_{11} w_{22} c_1^2 c_2^2 - w_{22}^2 c_1^2 c_2^2 \geq 0, \end{aligned} \tag{10}$$

where

$$F = \begin{bmatrix} 0 & 1 \\ -P_{max}\cos(\bar{\delta}) & -D \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \Phi = \begin{bmatrix} c_1^2 & 0 \\ 0 & c_2^2 \end{bmatrix}. \tag{11}$$

To construct a feedback controller which minimizes the stochastic sensitivity function, the cost function $J(W) = \mathrm{tr}(W^T Q) = w_{11} + w_{22}$ has been considered in which positive definite symmetrix matrix Q is chosen as identity matrix of order 2.

The minimum of the cost function $J_0 = \min J(W)$ geometrically corresponds to the tangent line to the border of the attainability region and the corresponding pair (w_{11}, w_{22}) of the attainable set provides the minimization of stochastic sensitivity function.

For this minimal attainable value the coefficients of the optimal stabilizing controller can be found using (7) such that

$$k_1 = -\frac{w_{22}}{w_{11}} + P_{\max}\cos(\bar{\delta}) \tag{12}$$

and the coefficient k_2 satisfies the following equation:

$$k_2^2 c_2^2 + 2w_{22}(k_2 - D) + c_1^2\left(\frac{-w_{22}}{w_{11}} + P_{\max}\cos(\bar{\delta})\right)^2 + 1 = 0 \tag{13}$$

which indeed implies that k_2 should be as $k_2 < D$.

The feedback controller designed by Theorem 1 has been based on the linearization of the nonlinear stochastic system around its equilibrium point. These designed coefficients k_1 and k_2 have been verified through the simulations of the nonlinear stochastic SMIB system written in the It ô form

$$d\mathbf{x}(t) = f(\mathbf{x})dt + \mathbf{K}(\mathbf{x} - \bar{\mathbf{x}})dt + \epsilon\boldsymbol{\Upsilon}d\mathbf{B}(t), \tag{14}$$

where the state variable $\mathbf{x} = [\delta\ \omega]^T$, the uncorrelated incremental Wiener process $\mathbf{dB} = [dB^{(1)}\ dB^{(2)}\ dB^{(3)}]^T$ and

$$\boldsymbol{\Upsilon} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & k_1c_1 & k_2c_2 \end{bmatrix}. \tag{15}$$

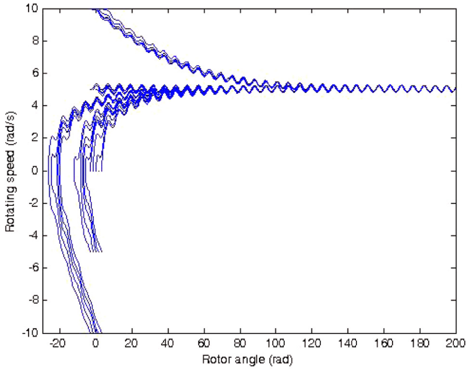
The numerical solution of (14) can be approximated by using the Euler-Maruyama method given in [18] as

$$\mathbf{x}_{t_i} = \mathbf{x}_{t_{i-1}} + f(\mathbf{x}_{t_{i-1}})\tau + \mathbf{K}(\mathbf{x}_{t_{i-1}} - \bar{\mathbf{x}})\tau + \epsilon\boldsymbol{\Upsilon}\triangle\boldsymbol{B}^{\tau}, \tag{16}$$

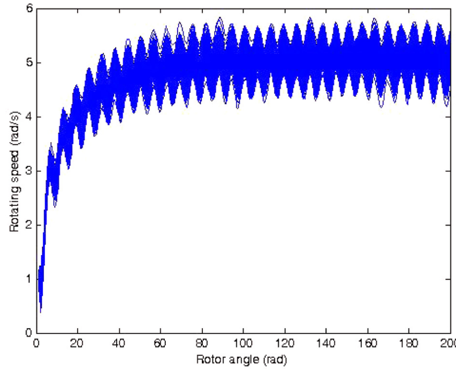
where the increment of the Wiener process defined by $\triangle B_\tau^{(l)} = B(t_i) - B(t_{i-1})$ with $l = 1, 2, 3$ is Gaussian random variable $N(0, \tau)$ where $t_i = i\tau$ for $i = 0, 1, \ldots, n$.

Consider the values of mechanical input power $P_m = 1\ p.u.$, $P_{\max} = 1; p.u.$, the damping coefficient $D = 0.2\ p.u$ and $c_1 = c_2 = c$. In this case the SEP and the saddle point coalesce in a saddle-node bifurcation of fixed point $\bar{\delta} = 1.5708$, $\bar{\omega} = 0$. The initial conditions of $\delta(0)$ and $\omega(0)$ have been selected as 100 evenly spaced points in the interval of $[-\pi, \pi]$ and $[-10, 10]$, respectively.

For the deterministic system in (1) all trajectories converge to a stable limit cycle (unique rotating orbit) as shown in Fig. 1(a). Since for any initial condition the trajectories converge to the unique rotating orbit, the initial point has been chosen as



(a)



(b)

Fig. 1. Phase portrait of rotor angle δ versus rotating speed ω of (a) deterministic ($\epsilon = 0$) for the initial conditions $\delta(0)$, $\omega(0)$ in the interval of $[-\pi, \pi]$ and $[-10, 10]$ (b) uncontrolled stochastic SMIB power system ($\epsilon = 0.001$) for $\delta(0) = 1$, $\omega(0) = 1$.

$\delta(0) = 1$, $\omega(0) = 1$ to visualize the response in the presence of stochastic power fluctuations. The phase portraits of the stochastic responses over 1000 realizations are shown in Fig. 1(b) where the noise intensity $\epsilon = 0.001$. It is seen from Fig. 1(b) that the uncontrolled system in the presence of Wiener type power fluctuations on the load has large-amplitude oscillations and the trajectories of rotor angle and speed converge to the rotating orbit which lead to an unstable operating condition called loss-of-synchronism.

Figure 2 shows the set of pairs (w_{11}, w_{22}) of the attainable stochastic sensitivity matrix W for various intensity values of the noisy measurements "c". As it is seen from Fig. 2 that the increase of c results in the contraction of the attainability region. By using (A.9) it can be calculated that $w_{11} = 2.5$ and $w_{22} = 2.5$ for the uncontrolled stochastic SMIB system of (2).

The optimal stabilizing controller coefficients which minimize the cost function $J(W) = w_{11} + w_{22}$ have been obtained by using the pairs (w_{11}, w_{22}) attainability set. The optimal parameters of attainable stochastic sensitivity matrix and the corresponding optimal controller coefficients are shown in Table 1 for various intensity values of noisy measurements.

Using these optimal controller coefficients the dynamical behavior of the nonlinear stochastic SMIB system in (14) are numerically analyzed over 1000 realizations with the initial condition $\delta(0) = 1$ and $\omega(0) = 1$.

The sample mean and 95% confidence intervals are computed over 1000 realizations. The 95% confidence interval is determined by the 2.5% and 97.5% percentiles of the simulated trajectories which corresponds to the lower and upper bounds of the confidence interval, respectively. The time series of the rotor angle δ with the variations of c are shown in Fig. 3.

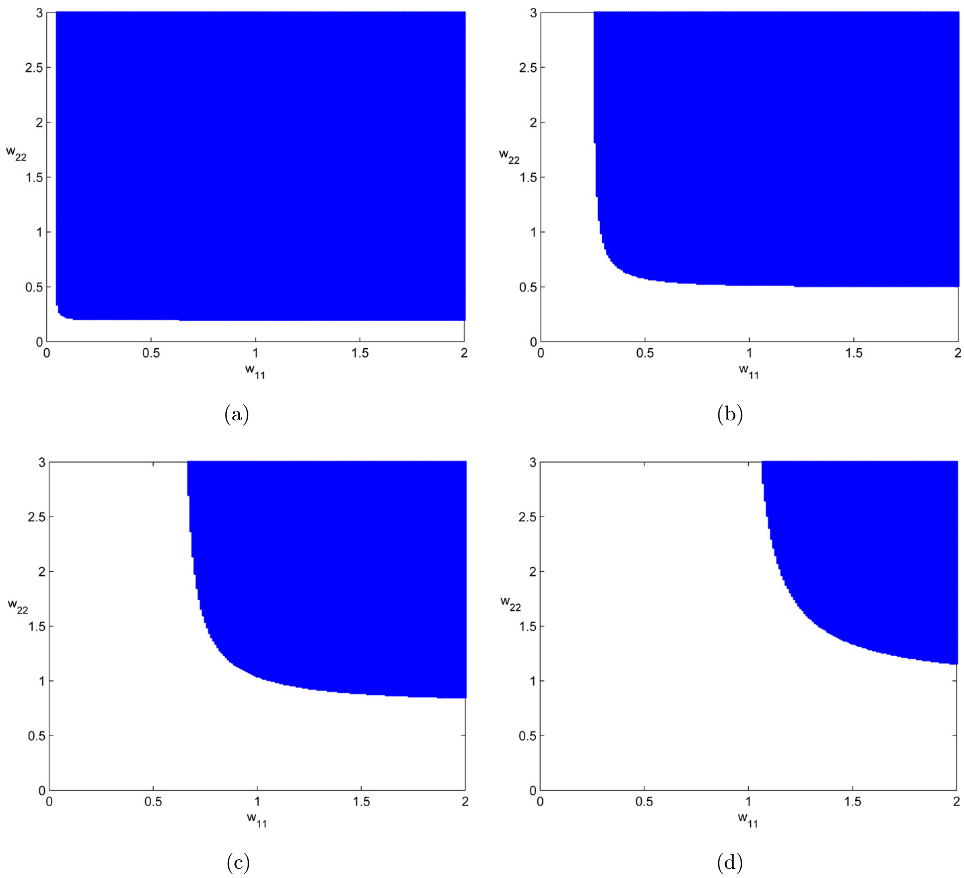


Fig. 2. Attainability region of the stochastic sensitivity matrix W for the mechanical power $P_m = 1$ (a) $c = 0.2$, (b) $c = 0.5$, (c) $c = 0.8$, (d) $c = 1.0$.

The results show that with the optimal controller obtained by stochastic sensitivity analysis, the rotating orbit behavior disappears and the rotor angle trajectories converge to the SEP and the machine becomes stable even if the feedback contains high noisy observations (i.e., for high "c" values). The increase in noisy measurements in feedback controller results in wider confidence intervals.

Table 1. Optimal parameters of attainable stochastic sensitivity matrix and the coefficients of the optimal stabilizing controller.

c	w_{11}	w_{22}	k_1	k_2
0.2	0.07	0.25	−3.5714	−14.8708
0.5	0.4	0.64	−1.60	−6.037
0.8	0.89	1.15	−1.2921	−4.1931
1.0	1.38	1.45	−1.0507	−3.3543

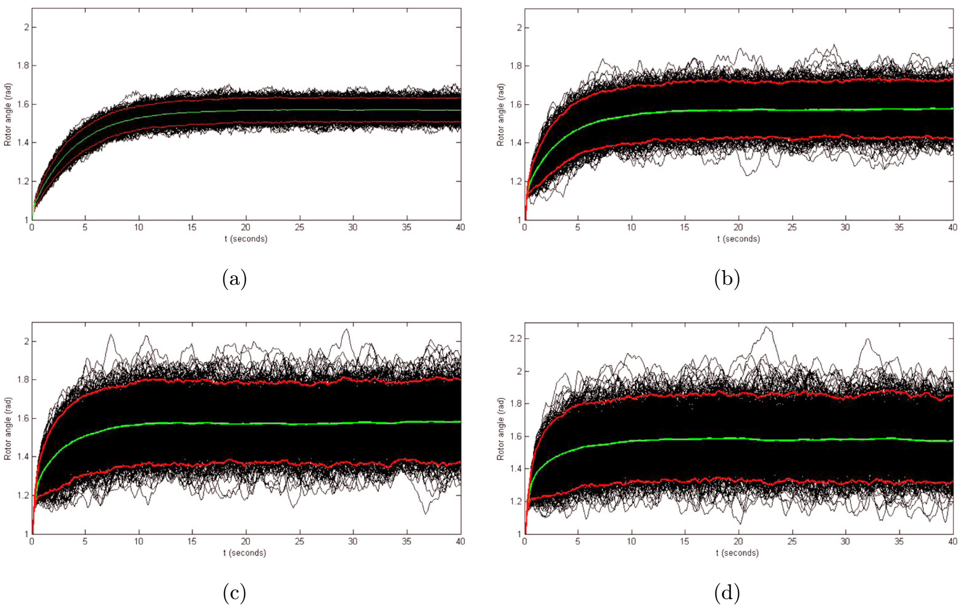


Fig. 3. The stochastic rotor angle responses over 1000 trajectories (black) with optimal regulators, 95% confidence intervals (red), empirical mean (green) for noise intensity (a) $c = 0.2$, (b) $c = 0.5$, (c) $c = 0.8$, (d) $c = 1.0$.

3. Controlling the Rotor Angle Stability in SMIB Power Systems in the Presence of Alpha-Stable Lévy Type Power Fluctuations

As we proposed in [19, 20] the imbalance between the mechanical input power and the electrical output power in SMIB system has been modeled by $P_L(t) = \epsilon L_\alpha(t)$ where ϵ is the noise intensity and $L_\alpha(t)$ is the α-stable Lévy process.

The increments of the Lévy process $dL_\alpha(t) \doteq L_\alpha(t) - L_\alpha(s)$ is α-stable random variable with $S_\alpha((t-s)^{1/\alpha}, \beta, 0)$ for any $0 \leq s < t < \infty$. The α-stable distribution is denoted by $S_\alpha\ (\sigma, \beta, \mu)$ in which the characteristic exponent α $(0 < \alpha \leq 2)$ measures the impulsiveness, σ is scale parameter and σ^α corresponds to the dispersion, μ is the location and β is the skewness parameter. $\beta = 0$ refers to symmetric distribution, $\beta < 0$ to left-skewed distribution and $\beta > 0$ to right-skewed distribution in [21–23]. As the value of characteristic exponent "α" decreases, the impulsiveness increases and hence the tails of the corresponding distributions become heavier.

The increments of the Wiener process $dB(t)$ which has Gaussian distribution with mean μ and variance $2\sigma^2$ is the special case of α-stable Lèvy motion with $S_2(\sigma, 0, \mu)$ where $\alpha = 2$, $\beta = 0$ [21].

An α-stable process with the integral representation [21]

$$\{Y(t), t \in T\} \overset{d}{=} \left\{ \int_S f(t,y) M(dy), t \in T \right\} \tag{17}$$

can be interpreted as a linear combination of independent α-stable variables $M(dy)$ with coefficients $f(t,y)$ satisfying the some mild conditions in [24].

The random measure in (17) have the Lebesque control measure $m(dy) = dy$ then the integral representation of $\{L_\alpha(t), t \geq 0\}$ is as

$$L_\alpha(t) = \int_0^t M(dy) = \int_0^\infty 1_{(0,t)} M(dy), \tag{18}$$

where $1_{(0,t)}$ is the indicator function of set $(0,t)$.

The response $X(t)$ of a linear system to an α-stable input $Y(t)$ is written as [24, 25]

$$X(t) = \int_S g(t,s) dL_\alpha(s) \tag{19}$$

in which

$$g(t,s) = \int_0^t h(t,y) f(y,s) ds, \quad t \geq 0, \tag{20}$$

and $h(t)$ is the unit impulse response function of the linear system.

Then the response $X(t)$ is an α-stable process with the scale [24]

$$\sigma(t) = \left[\int_S |g(t,s)|^\alpha m(ds) \right]^{1/\alpha}. \tag{21}$$

To derive an analytical expression for the rotor angle dispersion of SMIB in the presence of alpha-stable Lévy type power fluctuations, the linearized stochastic SMIB system with the control input $u = k_1(\delta - \bar{\delta}) + k_2(w - \bar{w})$ has been considered as

$$\begin{bmatrix} \dot{\delta} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ k_1 - \cos\bar{\delta} & k_2 - D \end{bmatrix} \begin{bmatrix} \delta \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} P_L, \tag{22}$$

where $P_L = \epsilon L_\alpha(t)$ represents α-stable Lévy type power fluctuations.

The deviation of the rotor angle of the system (22) from the SEP $(\bar{\delta}, \bar{w})$ is defined by $X(t) = \delta(t) - \bar{\delta}$. From (21) the dispersion parameter of the response $X(t)$ have been obtained as

$$\sigma(t)^\alpha = \frac{\epsilon^\alpha}{\lambda^\alpha} \int_0^t |e^{0.5(k_2 - D)z} \sin(\lambda z)|^\alpha dz, \tag{23}$$

in which $f(m,s) = \epsilon\ 1_{(0,m)}(s)$ and the unit impulse response of a linearized SMIB system of (22) is

$$h(t,m) = \frac{1}{\lambda} e^{0.5(k_2 - D)(t-m)} \sin(\lambda(t-m)) \tag{24}$$

Table 2. The coefficients of the optimal stabilizing controller.

α	$t_{\min}$(s)	k_1	k_2
2.0	1.096	−8	−3
1.9	1.6261	−4	−3
1.8	1.8284	−3	−2
1.7	2.3677	−2	−2
1.6	4.0264	−2	−1
1.5	4.1763	−2	−1

with

$$\lambda = \sqrt{\cos\bar{\delta} - k_1 - \frac{(k_2 - D)^2}{4}}. \tag{25}$$

The control problem is to design a state feedback which minimizes the deviation of the response (δ, w) from the SEP $(\bar{\delta}, \bar{w})$ which implies the minimization of the dispersion parameter of the response $X(t)$.

The mechanical power $P_m = 1.0\ p.u.$ and the damping parameter $D = 0.2\ p.u.$ are kept as fixed and the stochastic power fluctuations have been modeled as symmetric α-stable Lévy process. By using (23) the dispersion $\sigma(t)^\alpha$ has been calculated for the range of $k_1, k_2 \in [-10\ \ -1]$ with the noise intensity $\epsilon = 0.001$.

The coefficients of feedback controller k_1 and k_2 which provide the minimum of σ^α(t) have been evaluated as in Table 2 for various impulsiveness parameter α. Table 2 also presents the minimum settling time $t_{\min}$ which is the time elapsing before the state reaches the SEP $(\bar{\delta}, \bar{w})$.

As it is seen from Table 2 the time required to settle down to the SEP increases with the decrease of characteristic exponent (i.e., the increase of impulsiveness).

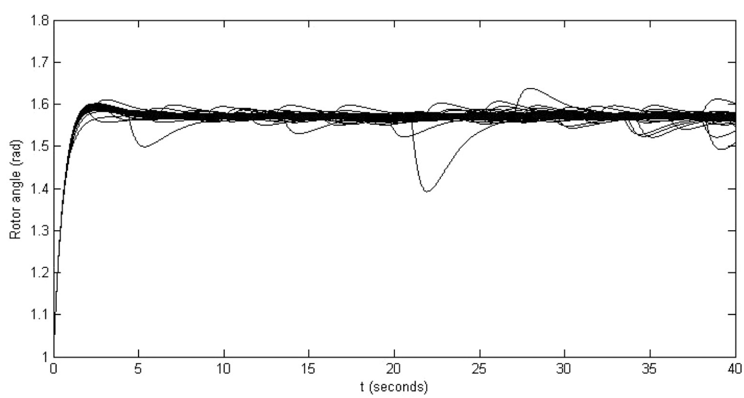


Fig. 4. The stochastic rotor angle responses of nonlinear controlled SMIB system in the presence of α-stable Lévy type power fluctuations with $\alpha = 1.5$, $\beta = 0$ over 1000 realizations.

By using the optimal controller coefficients obtained in Table 2 the numerical responses of rotor angle of nonlinear stochastic SMIB system have been obtained over 1000 realizations as shown in Fig. 4.

The numerical solutions have been approximated by the Euler–Maruyama method given in [18, 22]. The increment of the Lévy process is α-stable random variable $\triangle L^{\tau}_{\alpha,i}$ defined by $\triangle L^{\tau}_{\alpha,i} = L_{\alpha}([t_{i-1}, t_i]) \sim S_{\alpha}(\tau^{1/\alpha}, \beta, \mu)$ with $\tau = t_i - t_{i-1}$ have been generated as in [22].

As it can be seen from Fig. 4 that the responses over 1000 realizations in the presence of α-stable Lévy power fluctuations with $\alpha = 1.5$ have been stabilized and the trajectories of the rotor angle have been converged to the SEP with much narrower confidence interval.

4. Conclusion

The control of the rotor angle stability of SMIB system in the presence of Wiener type power fluctuations has been achieved by minimizing the stochastic sensitivity function. Since there is no analytical derivation for stochastic sensitivity function for α-stable Lévy processes in the literature, we have derived an expression for the rotor angle dispersion of the linearized stochastic SMIB system and by selecting feedback gains, this dispersion has been numerically minimized. The stabilization of the rotor angle for nonlinear stochastic SMIB system has been verified numerically by the designed optimal controllers for both Wiener and α-stable Lévy type power fluctuations.

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Appendix A Stochastic Sensitivity Analysis

Consider a nonlinear stochastic differential equation

$$\dot{x} = f(x) + \epsilon\sigma(x)\xi(t), \tag{A.1}$$

where $x, f(x) \in R^n$, $\sigma(x) \in R^{n\times m}$-matrix function denotes the dependence of disturbances on the states and $\xi(t) \in R^m$ is white Gaussian noise, ϵ is the noise intensity.

The stationary probability density function $\rho(x, \epsilon)$ for the system (A.1) is governed by the following FPK equation [26]:

$$\frac{\epsilon^2}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial^2}{\partial x_j \partial x_k}(a_{ij}\rho) - \sum_{i=1}^{n}\frac{\partial}{\partial x_i}(f_i\rho) = 0, \tag{A.2}$$

where $a_{ij} = [\sigma\sigma^T]_{ij}$. Due to the difficulty of analysis of this equation, the solution of FPK equation have been approximated for the case of low noise intensity based on the quasipotential method in [10] as

$$\Phi(x) = -\lim_{\epsilon\to 0} \epsilon^2 \log \rho(x,\epsilon), \tag{A.3}$$

where the function $\Phi(x)$ is called as quasipotential. Solving (A.3) for $\rho(x,\epsilon)$ gives

$$\rho(x,\epsilon) = \exp(-\Phi(x)/2) \tag{A.4}$$

and by substituting (A.4) into the (A.2) the following Hamilton–Jacobi equation is obtained as

$$f(x)^T \frac{\partial \Phi}{\partial x} + \frac{1}{2}\left(\frac{\partial \Phi}{\partial x}\right)^T S(x) \frac{\partial \Phi}{\partial x} = 0, \tag{A.5}$$

where $S(x) = \sigma(x)\sigma(x)^T$, $\Phi(\bar{x}) = 0$ and $\Phi(x) > 0$ for $x \neq \bar{x}$.

The quadratic approximation of the quasipotential in a small neighborhood of the deterministic equilibrium $\bar{x}$ have been developed in [11] with the positive definite $n \times n$-matrix $V = \frac{\partial^2 \Phi}{\partial x^2}(\bar{x})$ as

$$\Phi(x) \approx \frac{1}{2}(x-\bar{x})^T V (x-\bar{x}). \tag{A.6}$$

By taking derivative of (A.5) twice and substituting V into (A.5) then the following equation for the matrix V is obtained [11]:

$$F^T V + VF + VSV = 0, \tag{A.7}$$

where

$$F = \frac{\partial f}{\partial x}(\bar{x}), \quad S = S(\bar{x}) = \sigma(\bar{x})\sigma^T(\bar{x}) \tag{A.8}$$

and the positive definite matrix $W = V^{-1}$ is the unique solution of the following Lyapunov's equation [11]:

$$FW + WF^T + S = 0, \tag{A.9}$$

where W is called as stochastic sensitivity matrix which characterizes the deviation of the states of stochastic system (A.1) from the equilibrium $\bar{x}$.

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