On the Sources of CP-violation Contributing to the Electric Dipole Moments

Durmuş A. Demir
Department of Physics, Izmir Institute of Technology, Izmir, TR 35430, Turkey

Yasaman Farzan
Institute for Studies in Theoretical Physics and Mathematics (IPM), PO Box 19395-5531, Tehran, Iran

In the framework of seesaw mechanism embedded in the constrained Minimal Supersymmetric Standard Model (cMSSM), phases of neutrino Yukawa coupling, $\mu$-term and $A$-terms can all contribute to the Electric Dipole Moment (EDM) of the electron. We discuss and classify the situations for which by combined analysis of the upcoming results on $d_e$, $d_H$ and $d_D$ discriminating between these sources will be possible.

I. INTRODUCTION

Recent atmospheric and solar neutrino data [1] as well as the results of KamLAND [2], K2K [3] and MINOS [4] establish nonzero mass for neutrinos. On the other hand, kinematical studies [5] and cosmological observations [6] show that neutrino masses cannot exceed a few eV. Nonzero neutrino masses cannot be accommodated within the Standard Model (SM). Several models have been suggested to attribute tiny yet nonzero masses for neutrinos among which the seesaw mechanism [7] is arguably the most popular one. The seesaw mechanism adds three SM singlet right-handed neutrinos with very heavy Majorana masses to the model. Up to now, we have no clue how large the masses of the right-handed neutrinos are. The scale can lay anywhere between TeV up to $10^{14}$ GeV. If the scale is higher than 1 TeV, the right-handed neutrinos cannot be produced by accelerator technology; thus, we can only learn about the seesaw parameters indirectly through their effects on low energy parameters [8], such as light neutrino mass matrix and slepton mass matrix in the context of supersymmetric seesaw [9]. In this line, any low energy observable which is sensitive to the seesaw parameters deserves special attention.

The $3 \times 3$ neutrino Yukawa matrix introduces six new sources of CP-violation which, in the context of supersymmetric standard model, can induce significantly large contributions to the electric dipole moments of the electron [9, 10] and other particles. So far no finite EDM has been observed [12]:

$$|d_e| < 1.4 \times 10^{-27} \text{ e cm.}$$

However, there are ongoing experiments [13] as well as proposals [14] to improve the present bound by several orders of magnitude. In view of these experiments, it has been suggested to use the EDMs to extract information on the seesaw parameters [15]. However, for deriving any information from $d_e$, we must be aware of other sources of CP-violation that can give a significant contribution to $d_e$.

In the cMSSM, there are two extra sources of CP-violation relevant for the EDMs of leptons: the phases of the $\mu$ parameter and the universal trilinear coupling $a_0$. The phases of $a_0$ and $\mu$ induce $d_e \sim (m_e/m_d)d_d \sim (m_e/m_u)d_u \sim e(m_e/m_d)d_d \sim e(m_e/m_u)d_u$, where $d_u$ and $d_d$, respectively, are the Chromo EDMs (CEDM) of up and down quarks which give rise to EDMs of hadrons and nuclei such as mercury and deuteron. However, the quark EDMs and CEDMs induced by the phases of $Y_{\ell\mu}$ are too small to be detectable in near future. Therefore, if complex $Y_{\ell\mu}$ is the only source of CP-violation, we expect the Deuteron EDM ($d_D$) to be too small to be detectable in the near future ($d_D$ is measured with ionized deuteron which is depleted from electrons). Based on this difference, it has been suggested in [10] to combine the information on $d_D$ and $d_{1H}$ with $d_e$ to disentangle the source of CP-violation. It is also discussed how much the present bounds have to be improved in order to be able to make such a discrimination. In present letter, we review the results obtained in [10].

II. THE MODEL

The seesaw mechanism embedded in the MSSM is described by the superpotential

$$W = Y_{\ell\mu}^i \ell_i H_d L_j - Y_{Y_{\ell\mu}}^j N_i H_u L_j + \frac{1}{2} M_{ij} N_i N_j \mu H_d H_u.$$  

The quark Yukawa couplings, not shown here, are the same as in the MSSM. Here, $i, j$ are generation indices, $L_j$ consist of lepton doublets ($\nu_{Lj}, \ell_{Lj}^c$), and $E_i$ contain left-handed anti-leptons $\ell_{Li}^c$. The superfields $N_i$ contain anti right-handed neutrinos. Without loss of generality, one can rotate and rephase the fields to make Yukawa couplings of charged leptons ($Y_{\ell\mu}$) as well as the mass matrix of the right-handed neutrinos ($M_{ij}$) real diagonal. In what follows, we will use this basis.

In general, the soft supersymmetry breaking terms (the mass-squared matrices and trilinear couplings of the sfermions) can possess flavor-changing entries which facilitate a number of flavor-changing neutral current processes in the hadron and lepton sectors.
The existing experimental data thus put stringent bounds on flavor-changing entries of the soft terms. For instance, flavor-changing entries of the soft terms in the lepton sector can result in sizeable $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. This motivates us to go to the constrained MSSM framework where soft terms of a given type unify at a scale close to the scale of gauge coupling unification. In other words, at the GUT scale, we take

$$L_{soft} = -m_0^2 \tilde{L}_i \tilde{L}_i + \tilde{E}_i \tilde{E}_i + \tilde{N}_i \tilde{N}_i + H_0^u H_0 + H_0^d H_0$$

$$- \frac{1}{2} m_{1/2} (\tilde{B} \tilde{B} + \tilde{W} \tilde{W} + \tilde{g} \tilde{g} + \text{H.c.})$$

$$- (\frac{1}{2} B_H H_d \cdot H_u + \text{H.c.})$$

$$- (\tilde{A}_1^0 \tilde{E}_i H_d \cdot \tilde{L}_j - \tilde{A}_1^0 \tilde{N}_i H_u \cdot \tilde{L}_j + \text{H.c.})$$

$$- (\frac{1}{2} B_\nu M_i \tilde{N}_i \tilde{N}^i + \text{H.c.}).$$

Here $A_\tau = a_0 Y_\tau$ and $A_\nu = a_0 Y_\nu$, where $a_0$ in general can be complex and a source of CP-violation. The last term is the lepton number violating neutrino bilinear soft term which is called the neutrino $B$-term. As has been first shown in [17], the phase of the neutrino $B$-term can induce a contribution to $d_n$. In this letter, for simplicity, we will set $B_\nu = 0$. By rephasing the Higgs fields we can make $B_H$ real; however, the phase of $\mu$ will in general remain nonzero. In this letter, to calculate the effects of the phases of $Y_\nu$, $\mu$ and $a_0$ on the EDMs and CEDMs, we use the results of [11, 16, 18].

III. BOUNDS ON THE EDMs

In this section, we first review the current bounds on $d_D$, $d_{H_0}$ and $d_n$ and the prospects for improving them. We then review how we can write them in terms of $\text{Im}(\mu)$ and $\text{Im}(a_0)$.

- **Neutron EDM, $d_n$:** The present bound on $d_n$ is
  $$d_n < 3.0 \times 10^{-26} \text{ cm at 90\% C.L.}$$

  This bound can be improved considerably by SNS [20] which will be able to probe $d_n$ down to $10^{-28} \text{ cm}$. Here $d_n$ refers to the EDM of the neutron.

- **Mercury EDM, $d_{H_0}$:** The present bound on $d_{H_0}$ is
  $$|d_{H_0}| < 2.1 \times 10^{-28} \text{ cm}$$

  which can be improved by a factor of 2 [21].

- **Deuteron EDM, $d_D$:** The present bound on $d_D$ is too weak to constrain the CP-violating phases; however, there are proposals to probe $d_D$ down to
  $$(1 - 3) \times 10^{-27} \text{ cm}.$$}

Different sources of CP-violation affect the EDMs listed above differently. As a result, in principle, by combining information on these observables we can discriminate between different sources of CP-violation. It is rather straightforward to calculate the EDMs and CEDMs of the elementary particles in terms of the phases of $d_0$ and $\mu$ [11, 16, 18]; however, writing $d_n$, $d_{H_0}$ and $d_D$ in terms of the EDMs and CEDMs of their constituents is more difficult and a subject of debate among the experts. Let us consider them one by one.

- **$d_n(a_q, \hat{d}_q)$:**
  Despite the rich literature on $d_n$ in terms of the quark EDMs and CEDMs, the results are quite model dependent. For example, the SU(3) chiral model [23] and QCD sum rules [24] predict different contributions from $d_n$ and $d_\mu$ to $d_n$. Considering these discrepancies in the literature, we do not use bounds on $d_n$ in our analysis.

- **$d_{H_0}(a_q, \hat{d}_q)$:**
  There is an extensive literature on $d_{H_0}$ [27]. Following Ref. [26], we will interpret the bound on $d_{H_0}$ as
  $$|d_d - \hat{d}_u| < 2 \times 10^{-26} \text{ cm.}$$

- **$d_D(a_q, \hat{d}_q)$:**
  Searches for $d_D$ can serve as an ideal probe for the existence of sources of CP-violation other than complex $Y_\nu$ because i) there is a good prospect of improving the bound on $d_D$ [22]; ii) an ionized deuteron does not contain any electrons and hence we expect only a negligible and undetectable contribution from $Y_\nu$ to $d_D$.

To calculate $d_D$ in terms of quark EDMs and CEDMs, two techniques have been suggested in the literature: i) QCD sum rules [27] and ii) SU(3) chiral theory [25]. Within the error bars, the two models agree on the contribution from $d_d - \hat{d}_u$ which is the dominant one. However, the results of the two models on the sub-dominant contributions are not compatible. Apart from this discrepancy, there is a large uncertainty in the contribution of the dominant term:

$$d_D(a_q, \hat{d}_q) \simeq -e(d_u - \hat{d}_d) \frac{5\ldots 11}{3}.$$ (8)

In this paper we take “the best fit” for our analysis.

IV. NUMERICAL ANALYSIS

In this section, we first describe how we produce the random seesaw parameters compatible with the data. We then describe the results.
what can be inferred from the future data considering different possible situations one by one.

In figures 11, the scatter points marked with "+" represent \( d_e \) resulting from complex \( Y_\nu \). To extract random \( Y_\nu \) and \( M_N \) compatible with data, we have followed the recipe described in [24] and solved the following two equations

\[
Y_\nu^T \frac{1}{M} Y_\nu (v^2 \sin^2 \beta)/2 = U \cdot \Phi \cdot M^\text{diag} \cdot \Phi \cdot U^T
\]

and

\[
h = Y_\nu \text{log} M^\text{GUT} Y_\nu = \begin{bmatrix} a & 0 & d \\ 0 & b & 0 \\ d^* & 0 & c \end{bmatrix},
\]

where \( v = 247 \text{ GeV}, M \) is the mass matrix of the right-handed neutrinos, \( U \) is the mixing matrix of neutrinos with \( s_{13} = 0 \) and \( \Phi \) is \( \text{diag}[1, e^{i\phi_1}, e^{i\phi_2}] \) with random values of \( \phi_1 \) and \( \phi_2 \) in the range \((0, 2\pi)\). Finally, \( M^\text{diag} = \text{diag}[m_1, \sqrt{m_1 + \Delta m_{21}}, \sqrt{m_1 + \Delta m_{31}}] \) where \( m_1 \) picks up random values between 0 and 0.5 eV in a linear scale. The upper limit on \( m_1 \) is what has been found in [31] by taking the dark energy equation of state a free (but constant) parameter.

In order to satisfy the strong bounds on \( \text{Br}(\mu \to e\gamma) \) [12] and \( \text{Br}(\tau \to e\gamma) \) [31], the matrix \( h \), defined in Eq. (10), is taken to have this specific aspect of \( \epsilon \mu \) and \( \mu e \) elements. Actually these branching ratios put bounds on \( (\Delta m^2_L)^\mu e \) and \( (\Delta m^2_L)^{\mu\tau} \) rather than on \( h_{\mu e} \) and \( h_{\mu\tau} \). Notice that only the dominant term of \( \Delta m^2_L \) is proportional to \( h \). There is also a subdominant "finite" contribution to \( \Delta m^2_L \) which is about 10% of the dominant effect and is not proportional to the matrix \( h \) [11]. Nonetheless, for extracting the seesaw parameters, 20% accuracy is enough and we can neglect the subdominant part and take \( \Delta m^2_L \) proportional to the matrix \( h \). In Eq. (10), \( a, b, c \) are real numbers which take random values between 0 and 5. On the other hand, \( |d| \) takes random values between 0 and the upper bound from \( \text{Br}(\tau \to e\gamma) \) [32]. To calculate the upper bound on \( |d| \), we have used the formula derived in Ref. [32]. The phase of \( d \) takes random values between 0 and \( 2\pi \).

To perform this analysis we have taken various values of \( \tan \beta \) and \( a_0 \) and calculated the spectrum of the supersymmetric parameters along the \( m_{1/2} - m_0 \) strips parameterized in Ref. [33]. Notice that Ref. [33] has already removed the parameter range for which color or charge condensation takes place.

In the figures, we have also drawn the present bound on \( d_e \) [12] as well as the limits which can be probed in the future. The present bound is shown by a dashed dark blue line and lies several orders of magnitude above the \( d_e \) from phases of \( Y_\nu \). After five years of data-taking, the Yale group can probe \( d_e \) down to \( 10^{-30} \text{ e cm} \) [13] which is shown with a dot-dashed cyan line in the figures. As demonstrated in the figures, only for large values of \( a_0 \) the effect of complex \( Y_\nu \) on \( d_e \) can be probed by the Yale group and for most of the parameter space the effect remains beyond the reach of this experiment.

There are proposals [14] to use solid state techniques to probe \( d_e \) down to \( 10^{-35} \text{ e cm} \) (shown with dot-dashed yellow line in the figure). In this case, it can be deduced from the figure, we will have a great chance of being sensitive to the effects of the phases of \( Y_\nu \) on \( d_e \). However, unfortunately, the feasibility and time scale of the solid state technique is still uncertain.

In figs. 1 and 4, \( d_e \) resulting from \( \text{Im}[\mu]\) is also depicted. The red solid lines in these figures show \( d_e \) from \( \text{Im}[\mu]\) assuming that the corresponding \( d_{\text{Hg}} \) saturates the present bound [21]. As is well-known, there are uncertainties both in the value of \( m_\mu \) [12] and in the interpretation of \( d_{\text{Hg}} \) in terms of more fundamental parameters \( \tilde{d}_u, \tilde{d}_d \) and \( \tilde{d}_\tau \). To draw this curve we have assumed \( m_\mu = 5 \text{ MeV} \) and \( \tilde{d}_u - \tilde{d}_d < 2 \times 10^{-26} \text{ e cm} \). As shown in the figure, this bound is weaker than even the present direct bound on \( d_e \). The purple dotted lines in figs. 11 and 4, represent \( d_e \) induced by values of \( \text{Im}[\mu]\) that give rise to \( \tilde{d}_u - \tilde{d}_d = 2 \times 10^{-28} \text{ cm} \) (corresponding to \( d_D = 10^{-27} \text{ e cm} \) and \( d_D = 5\epsilon(\tilde{d}_d - \tilde{d}_u) \)). Notice that these curves lie well below the direct bound on \( d_e \) but the Yale group will be able to probe even smaller values of \( d_e \). Similarly in figs. 43, \( d_e \) resulting from \( \text{Im}[a_0]\) is depicted.

In the figures, the bounds from \( d_{\text{Hg}} \) and \( d_D \) appear almost as horizontal lines. This results from the fact that for the \( m_0 - m_{1/2} \) strips that we analyze, \( m_0 \) is almost proportional to \( m_{1/2} \). Using dimensional analysis we can write

\[
\tilde{d}_u - \tilde{d}_d \simeq k_1 \frac{\text{Im}[\mu]}{m_{1/2}^3} \text{ or } \frac{\text{Im}[a_0]}{m_{1/2}^3}, \quad d_e \simeq k_2 \frac{\text{Im}[\mu]}{m_{1/2}^3} \text{ or } \frac{\text{Im}[a_0]}{m_{1/2}^3}
\]

where \( k_1 \) and \( k_2 \) are given by the relevant fermion masses and are independent of \( m_{1/2} \). As a result, for a given value of \( \tilde{d}_u - \tilde{d}_d \), \( \text{Im}[\mu]\) (or \( \text{Im}[a_0]\)) itself is proportional to \( m_{1/2}^3 \), so \( d_e \) will not vary with \( m_{1/2} \).

In the following, we will discuss what can be inferred about the sources of CP-violation from \( d_e \) and \( d_D \) if their values (or the bounds on them) turn out to be in certain ranges. According to Fig. 4 for \( a_0 = 0 \), any signal found by the Yale group implies that there are sources of CP-violation other than the phases of the Yukawa couplings. However, for larger values of \( a_0 \), the effect of \( Y_\nu \) on the EDMs can be observed by the Yale group within 5 years. According to Figs. 21, for \( a_0 \gtrsim 1000 \text{ GeV} \) EDMs originating from complex \( Y_\nu \) can be large enough to be observed by the Yale group. Therefore, if after five years the Yale group reports a null result, we can derive bounds on certain combinations of seesaw parameters and \( a_0 \). At least it will be possible to discriminate between low and high \( a_0 \) values. However, if the Yale group finds that \( \tilde{d}_u - \tilde{d}_d < 10^{-30} \text{ e cm} \), we will not be able to
FIG. 1: Electron EDM for $a_0 = 0$, $\tan \beta = 10$ and $\text{sgn}(\mu) = +$. The scatter points represent $|d_e|$ induced by random complex $Y_\nu$ compatible with the data. The blue dashed line is the present bound on $d_e$. The solid and purple dotted lines show the values of $d_e$ that can be probed in the future.

FIG. 2: Electric dipole moment of the electron for $a_0 = 1000$ GeV, $\tan \beta = 10$ and $\text{sgn}(\mu) = +$. To draw the red solid and purple dotted lines, we have assumed that $\text{Im}[a_0]$ is the only source of CP-violation and have taken $\tilde{d}_d - \tilde{d}_u$ equal to $2 \times 10^{-26}$ cm and $2 \times 10^{-28}$ cm, respectively to derive $\text{Im}[a_0]$. To produce the scatter points, we have assumed that complex $Y_\nu$ is the only source of CP-violation and have randomly produced $Y_\nu$ compatible with the data. The blue dashed lines show the values of $d_e$ that can be probed in the future.

FIG. 3: The same as fig. 2 for $a_0 = 1000$ GeV, $\tan \beta = 20$ and $\text{sgn}(\mu) = +$.

determine whether $d_e$ originates from complex $Y_\nu$ or from more familiar sources such as complex $a_0$ or $\mu$. To be able to make such a distinction, values of $d_D$ down to $10^{-28} - 10^{-29}$ e cm have to be probed which, at the moment, does not seem to be achievable.

If future searches for $d_D$ find $d_D > 10^{-27}$ e cm but the Yale group finds $d_e < 2 \times 10^{-29}$ e cm (this can be tested within only 3 years of data taking by the Yale group), we might conclude that the source of CP-violation is something other than pure $\text{Im}[\mu]$ or $\text{Im}[a_0]$; e.g., the QCD $\theta$-term which can give a significant contribution to $d_D$ but only a negligible contribution to $d_e$. Another possibility is that there is a cancelation between the contributions of $\text{Im}[\mu]$ and $\text{Im}[a_0]$ to $d_e$. The information on $d_u$ would then help us to resolve this ambiguity provided that the theoretical uncertainties in calculation of $d_u$ as well as $d_D$ are sufficiently reduced.

On the other hand, if the Yale group detects $d_e > 2 \times 10^{-29}$ e cm which will be a strong motivation for building a deuteron storage ring and searching for $d_D$. If such a detector finds a null result, within this framework the explanation will be quite non-trivial requiring some fine-tuned cancelation between different contributions.

According to these figures, in the foreseeable future, we will not be able to extract any information on the seesaw parameters from EDMs, because even if we develop techniques to probe $d_e$ as small as $10^{-35}$ e cm, we will not be able to subtract (or dismiss) the effect coming from $\text{Im}[\mu]$ and $\text{Im}[a_0]$ unless we are able to probe $\tilde{d}_u - \tilde{d}_d$ at least 5 orders of magnitude below its present bound which seems impractical. Remember that this is under the optimistic assumptions that the
mass of the lightest neutrino, $m_1$, and $\text{Br}(\tau \rightarrow e\gamma)$ are close to their upper bounds and there is no cancelation between different contributions to the EDMs.

If, in the future, we realize that $m_1$ and $\text{Br}(\tau \rightarrow e\gamma)$ are indeed close to the present upper bounds on them and $a_0 = 0$ ($a_0 = 1000$ GeV) but find $d_e < 10^{-35}$ e cm ($d_e < 10^{-34}$ e cm), we will be able to draw bounds on the phases of $Y_\nu$ which along with the information on the Dirac and Majorana phases of the neutrino mass matrix and the CP-violating phase of the left-handed slepton mass matrix may have some implication for leptogenesis. This is however quite an unlikely situation.

V. SUMMARY

In this work we have studied EDMs of particles in the context of supersymmetric seesaw mechanism. In figs. 14 the values of $d_e$ corresponding to different random complex $Y_\nu$ textures are represented by “+”. For small values of $\tan \beta$ ($\tan \beta < 10$) and $a_0$ ($a_0 < 1000$ GeV), $d_e$ induced by $Y_\nu$ is beyond the reach of the ongoing experiments 13. Such values of $d_e$ can however be probed by the proposed solid state based experiments 14. For larger values of $\tan \beta$ and/or $a_0$, the Yale group may be able to detect the effects of complex $Y_\nu$ on $d_e$. As demonstrated in Figs. 8 and 9 for $\tan \beta = 20$ and $a_0 = 1000 - 2000$ GeV, a large fraction of parameter space yields $d_e$ detectable by the Yale group. However, even in this case we will not be able to extract information on the seesaw parameters from $d_e$ because the source of CP-violation might be $a_0$ and/or $\mu$ rather than $Y_\nu$. If the future searches for $d_D$ 27 find out that $d_D > 10^{-27}$ e cm then we will conclude that there is a source of CP-violation other than complex $Y_\nu$. However, to prove that the dominant contribution to $d_e$ detected by the Yale group comes from complex $Y_\nu$– hence to be able to extract information on the seesaw parameters from it– we should show that $d_D < 10^{-28} - 10^{-29}$ e cm which is beyond the reach of even the current proposals. Notice that for the purpose of discriminating between complex $Y_\nu$ and $a_0/\mu$ as sources of CP-violation, searching for $d_H^2$ is not very helpful because mercury atom contains electron and hence $d_H^2$ obtains a contribution from complex $Y_\nu$. That is while ionized deuteron used for measuring $d_D$ does not contain any electron and the contribution of complex $Y_\nu$ to it is negligible. To obtain information from $d_H$, the theoretical uncertainties first have to be resolved.

Acknowledgments

D.A.D. is grateful to Institute for Studies in Theoretical Physics and Mathematics (IPM) for its generous hospitality while the work on which this talk is based was prepared, and International Centre for Theoretical Physics (ICTP), Turkish Academy of Sciences (through GEBIP grant), and Scientific and Technical Research Council of Turkey (through project 104T503) for their financial supports.